Legislative Bargaining and Parties' Patience*

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Abstract

We investigate the relationship between parties' patience and continuation values in legislative bargaining. We consider the legislative bargaining game *without* assuming common discount factor. In the game, it is observed a seemingly paradoxical phenomenon that more patient party obtains less continuation value than less patient party. Also, we show that in some cases, a party's continuation value decreases as she marginally becomes more patient. These seemingly paradoxical results come from the role of patience different from ordinary bargaining games. The role is unique to the majority rule in the legislative bargaining.

Keywords: Parties' patience; Continuation values; Legislative bargaining; Majority rule
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1 Introduction

There have been some studies that analyze the bargaining problem as a non-cooperative game, e.g., Rubinstein (1982), Binmore, Rubinstein and Wolinsky (1986), Gul (1989), and so on. Among the studies, Rubinstein presented a plausible and simple extensive form game and meaningful implications. Rubinsteinesque approach has been applied to many research fields, e.g., political economy, labor economics,[1] and so on.

In political economy, Baron and Ferejohn (1989) modeled the bargaining in the legislature in Rubinsteinesque way. In their legislative bargaining model, there exists more than three legislators. First of all, the nature selects the proposer of policy from among the legislators. The selected legislator proposes a policy and the policy is put on the vote. If a majority of the legislators votes for the proposal, the proposed policy is implemented. Otherwise, the legislative procedure is repeated.

Baron and Ferejohn's approach is one of the most powerful tool in political economy and has been applied to various situations. For example, Baron (1991) analyzed government formation procedure by using the legislative bargaining model, and Diermeier and Fedderson (1998) explained cohesion in legislatures under the confidence vote system in Baron and Ferejohnesque way.

As mentioned above, Baron and Ferejohn's approach has been often used in political economy. However, a very important nature of the model has not been studied. It is how legislators' patience affects the equilibrium expected payoffs. In Baron and Ferejohn and other literatures, common discount factor is assumed. And it has not investigated how the difference and the change of legislators' discount factors have effect on the equilibrium payoffs.

In Rubinstein, discount factors play significantly important role. So, the role of discount factors in Baron and Ferejohn's legislative bargaining model ought to be studied. In the legislative bargaining model, do discount factors play the same role as in Rubinstein's model or different role? If discount factors play different role, how is the role? The purpose of this paper is to answer these questions.

This paper may also be related to two recent papers, Haan and Kooreman (2003) and Piccione and Rubinstein (2003).

Haan and Kooreman present a paradoxical phenomenon in the direct democracy. They formulate the situation that two alternatives are put to the majority vote and assume that there is the cost to vote and the voters can abstain. They prove that, if the cost is sufficiently high, the alternative supported by the minority wins with greater probability than what is supported by the majority.

Piccione and Rubinstein investigate the situation that each player can appropriate wealth of a weaker player than she. They assume that the stronger a player is, the more her initial wealth is and that each player must pay the cost to appropriate a weaker player's wealth. It is proved that, if the cost is sufficiently low, in any equilibrium, there exists a player who is weaker and gets greater payoff than the second strongest player.

In these models, seemingly desirable properties, i.e., the majority support and the strength, are not necessarily bliss. Also in our paper, it may be the case. There may exist a party who is less patient and gets greater payoff than other party. If so, player's patience, which seems to be desirable in bargaining, is not bliss.

The paper is organized as follows: Section 2 describes the political environment as a extensive form game, Section 3 defines the equilibrium concept employed in the paper and proves the existence of equilibria, Section 4 and Section 5 analyze the relationship between parties' patience and continuation values, Section 6 presents two numerical examples, and Section 7 concludes the paper.

[1] For example, Shaked and Sutton (1984) explained involuntary unemployment by using the bargaining model.

2 Political environment

We consider the following political environment, which is the same as Baron and Ferejohn's model except for noncommon discount factors.

There exist three parties in the legislature. Label the parties 1, 2, and 3. Let $N \equiv \{1, 2, 3\}$, i.e., N denote the set of parties. We assume that every member of each party have the same preference, and that no party has a majority of seats and any coalition of two parties has a majority of seats.

The parties make a split-the-pie bargain, i.e., they decide how they distribute political rent by bargaining. We assume that the political rent is a total of 1. We call how to split the rent policy. Let X denote the set of policies: X is the simplex in 3-dimensional Euclidean space, i.e., $X \equiv \left\{\mathbf{x} \in [0,1]^3 \mid \mathbf{1} \cdot \mathbf{x} = 1\right\}$.

The bargaining proceeds in accordance with the following timing. (i) Nature selects a proposer i from the set N of parties in the legislature with probability $\frac{1}{3}$. (ii) The selected proposer i proposes how to split the rent, i.e., a policy $\mathbf{x} \in X$. (iii) Every party $j \in N$ votes on the proposed policy $\mathbf{x}$ simultaneously. Then, if a majority of legislators votes for the proposal, the proposal is implemented and the game ends. Otherwise, the procedure is repeated from (i). Note that by the assumption on legislators' preferences and each party's share of the seats, a majority of parties votes for the proposal if and only if a majority of legislators votes for the proposal.

Each party's payoff is the rent distributed to herself and each party discounts future rent. Thus, party i's payoff is equal to $\delta_i^{t-1} x_i$ when a policy $\mathbf{x} \equiv (x_k)_{k \in N}$ is implemented after the tth vote, where $\delta_i \in (0, 1)$ is i's discount factor.

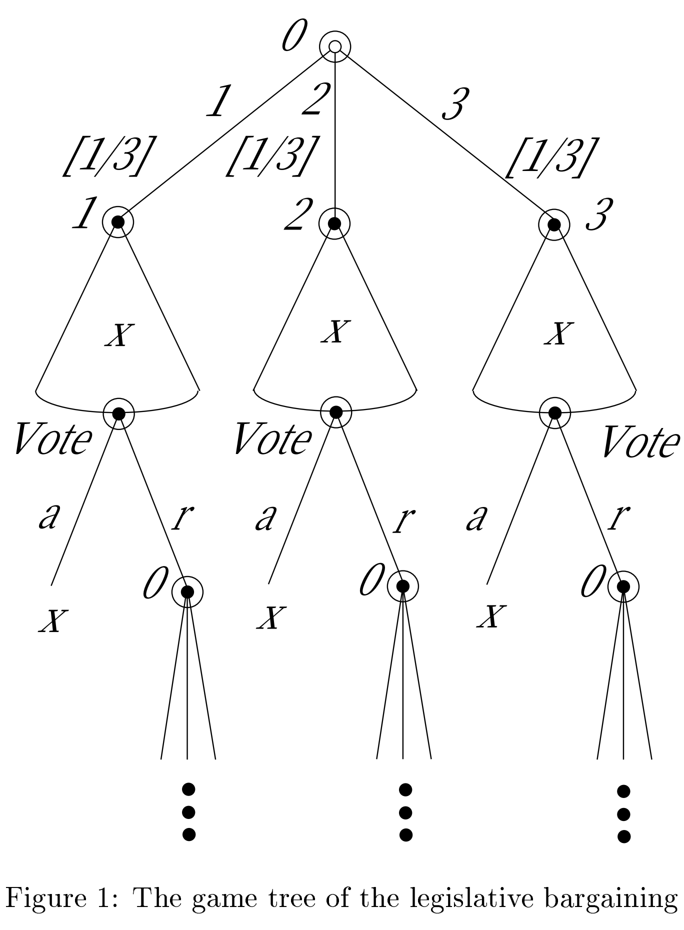

Figure 1: The game tree of the legislative bargaining

According to Osborne and Rubinstein (1994), we can formally describe the game defined above as

$$\eth(\delta) = \langle N, H, P, f, (u_i(\delta_i))_{i\in N}\rangle$$

with $\delta \equiv (\delta_1, \delta_2, \delta_3) \in (0,1)^3$, where each component of $\eth(\delta)$ has the following property. (i) $N \equiv \{1,2,3\}$ is the set of players (parties). (ii) H is the set of histories, which is defined as follows. Let

$$A \equiv \{(a,a,a),(a,a,r),(a,r,a),(r,a,a)\},$$

$$R \equiv \{(r,r,r),(r,r,a),(r,a,r),(a,r,r)\},$$

and

$$X \equiv \{\mathbf{x} \in (0,1)^3 \mid \mathbf{x}\cdot\mathbf{1} = 1\}.$$

We define Z^t as

$$Z^t \equiv \{(p^\tau, \mathbf{x}^\tau, v^\tau)_{\tau=1}^t \mid [\forall\tau \in \{1,\dots,t\} : p^\tau \in N \wedge \mathbf{x}^\tau \in X], [\forall\tau\in\{1,\dots,t-1\} : v^\tau \in R], v^t \in A\}.$$

Let $Z \equiv \bigcup_{t=1}^\infty Z^t$. Z is the set of terminal histories. Finally, we define H as $H \equiv \{h \mid \exists h' : (h,h') \in Z\}$. (iii) $P : H\setminus Z \to \Pi$ is the player function, where $\Pi \equiv \bigcup_{k=1}^4 \{(\pi_l)_{l=1}^k \in (N\cup\{0\})^k \mid \forall i,j \in \{1,\dots,k\} : i\neq j \Rightarrow \pi_i \neq \pi_j\}$ (0 denotes the nature). P is defined as

$$P(h) = \begin{cases} L(h) & \text{if } h = (h', p) \text{ for some } h' \text{ and } p \in N \\ (1,2,3) & \text{if } h = (h',\mathbf{x}) \text{ for some } h' \text{ and } \mathbf{x}\in X \\ 0 & \text{otherwise}\end{cases}$$

for $h \in H\setminus Z$, where $L(h)$ is the last element of h.[2] (iv) For h such that $P(h) = 0$, $f(h)$ is the probability measure on N. Let $f(h)(\{p\}) \equiv \frac{1}{3}$ for $p \in N$. (v) $u_i(\delta_i) : Z \to \mathbb{R}$ is player i's $\in N$ payoff function ($\delta_i \in (0,1)$ reflects player i's time preference). Let

$$u_i(\delta_i)(z^t) \equiv \delta_i^{t-1} x_i^t$$

for $t \in \{1,\dots,\infty\}$ and $z^t \equiv (p^\tau, \mathbf{x}^\tau, v^\tau)_{\tau=1}^t \in Z^t$ with $\mathbf{x}^\tau \equiv (x_i^\tau)_{i\in N}$.

3 Equilibrium

3.1 Equilibrium concept

In this paper, we use the behavior strategy defined below.

Definition 1 Let $i \in N$. σ_i is party i's *behavior strategy* in $\eth(\delta)$ if $\sigma_i(h_i)$ is a probability measure on $A_i(h_i)$ for all $h_i \in H_i$, where H_i and A_i are defined as in the footnote 2.

We denote the set of party i's$\in N$ behavior strategies in $\eth(\delta)$ by Σ_i. Note that Σ_i is not dependent on δ. Let $\Sigma \equiv \prod_{i\in N}\Sigma_i$.

Let $\hat{H}(\eta)$ be defined as $\hat{H}(\eta) \equiv \{h \in H \mid \exists h' \in H : h = (h',\eta)\}$ for $\eta \in N \cup X$. According to Fudenberg and Tirole (1991), $\{\hat{H}(\eta)\}_{\eta\in N\cup X}$ is the payoff-relevant history.[3] We define 'Markovian' as follows.

Definition 2 Let $i\in N$. Party i's behavior strategy $\sigma_i \in \Sigma_i$ in $\eth(\delta)$ is *Markovian* if σ_i satisfies $\forall h_i, h_i' \in \hat{H}(\eta)\cap H_i : \sigma_i(h_i) = \sigma_i(h_i')$ for all $\eta \in N\cup X$.

[2]Take any $i \in N$ and any $h_i \in \{h \in H\setminus Z \mid P(h) = i \vee P(h) = (1,2,3)\} \equiv H_i$. H_i is the set of histories which reach player i's node. Let

$$A_i(h_i) \equiv \begin{cases} X & \text{if } P(h_i) = i \\ \{a, r\} & \text{if } P(h_i) = (1,2,3).\end{cases}$$

Player i chooses her action from $A_i(h_i)$.

[3]$\{\hat{H}(\eta)\}_{\eta\in N\cup X}$ is a partition of the set of histories which reach the node *but the chance*.

'Markovian' means 'stationary.' We denote party i's continuation value induced by a Markovian behavior strategy profile σ in the game $\eth(\delta)$ by $V_i^\delta(\sigma)$

The equilibrium concept employed in the paper is defined as follows.

Definition 3 A behavior strategy profile $\sigma \equiv (\sigma_i)_{i\in N} \in \Sigma$ in $\eth(\delta)$ is *honestly voting Markov perfect equilibrium (honestly voting MPE)* in $\eth(\delta)$ if σ satisfies the following properties.
(i) σ is a subgame perfect equilibrium in $\eth(\delta)$.
(ii) σ_i is Markovian for all $i \in N$.
(iii) For any $i \in N$, any $\mathbf{x} \in X$ and any $h \in \hat{H}(\mathbf{x})$, $\sigma_i(h)(\{a\}) = 1$ if $\mathbf{e}_i \cdot \mathbf{x} \geq \delta_i V_i^\delta(\sigma)$ and $\sigma_i(h)(\{r\}) = 1$ if $\mathbf{e}_i \cdot \mathbf{x} < \delta_i V_i^\delta(\sigma)$, with $(\delta_1, \delta_2, \delta_3) \equiv \delta$.

Properties (i) and (ii) correspond to 'MPE' and property (iii) corresponds to 'honestly voting.' Since any proposal can be passed in equilibrium without 'honestly voting' property, we need 'honestly voting' property. We denote the set of honestly voting MPE in $\eth(\delta)$ by E^δ. For $\delta \equiv (\delta_1, \delta_2, \delta_3) \in (0,1)^3$ and $\{i,j,k\} = N$, let $E^\delta_{i,j,k} : \{=,>\}^2 \to 2^{E^\delta}$ be defined as

$$E^\delta_{i,j,k}(R, R') \equiv \left\{\sigma \in E^\delta \mid \delta_i V_i^\delta(\sigma)\, R\, \delta_j V_j^\delta(\sigma)\, R'\, \delta_k V_k^\delta(\sigma)\right\}$$

for $(R, R') \in \{=,>\}^2$. For instance, $E^\delta_{1,2,3}(=,=)$ represents the set of honestly voting MPE σ in $\eth(\delta)$ such that

$$\delta_1 V_1^\delta(\sigma) = \delta_2 V_2^\delta(\sigma) = \delta_3 V_3^\delta(\sigma),$$

i.e., every party's discounted continuation value is equal.

3.2 A convenient lemma

For $\delta \equiv (\delta_1, \delta_2, \delta_3) \in (0,1)^3$, $h^0 \in \{h \in H \setminus Z \mid P(h) = 0\} \equiv H^0$ and $\sigma \in \Sigma$, we define a game $\eth^S(h^0, \sigma; \delta)$ as

$$\eth^S(h^0, \sigma; \delta) \equiv \langle N, H^S, P^S, f^S, (u_i^S(h^0, \sigma; \delta_i))_{i\in N}\rangle,$$ [4]

where (i)

$$H^S \equiv \{h \mid \exists h' : (h, h') \in Z^S\}$$

with

$$Z^S = \{(p, \mathbf{x}, v) \mid p \in N, \mathbf{x} \in X, v \in \{a, r\}^3\};$$

(ii) $P^S : H^S \setminus Z^S \to \bigcup_{k=1}^4 \{(\pi_l)_{h=l}^k \in (N \cup \{0\})^k \mid \forall i,j \in \{1,\dots,k\} : i \neq j \Rightarrow \pi_i \neq \pi_j\}$ is defined as

$$P^S(h) = \begin{cases} h & \text{if } h \in N \\ (1,2,3) & \text{if } h = (p, \mathbf{x}) \text{ for some } p \in N \text{ and } \mathbf{x} \in X \\ 0 & \text{otherwise} \end{cases}$$

for $h \in H^S \setminus Z^S$;[5] (iii) f^S is a probability measure on N, where $f^S(\{p\}) = \frac{1}{3}$ for all $p \in N$; (iv) $u_i^S(h^0, \sigma; \delta_i) : Z^S \to \mathbb{R}$ for $i \in N$ is defined as

$$u_i^S(h^0, \sigma; \delta_i)(p, \mathbf{x}, v) = \hat{V}_i((h^0, p, \mathbf{x}, v), \sigma; \delta_i)$$

for $(p, \mathbf{x}, v) \in Z^S$, where $\hat{V}_i(h', \sigma; \delta)$ is i's expected payoff in $\eth(\delta)$ calculated by σ and f conditional on h' for $i \in N$, $\delta \in (0,1)^3$, $h' \in H$ and $\sigma \in \Sigma$. We call the game $\eth^S(h^0, \sigma; \delta)$ *segmented game* of $\eth(\delta)$ at h^0 under σ.

We show the following lemma to calculate equilibria.

[4] N, H^S, P^S, f^S and $u_i^S(h, \sigma; \delta)$ are the set of players, the set of histories, the player function, the probability measure on the set of proposers (parties), and player i's payoff function, respectively.

[5] Take any $i \in N$ and any $h_i \in \{h \in H^S \setminus Z^S \mid P^S(h) = i \vee P^S(h) = (1,2,3)\} \equiv H_i^S$. Let

$$A_i^S(h_i) \equiv \begin{cases} X & \text{if } P^S(h_i) = i \\ \{a, r\} & \text{if } P^S(h_i) = (1,2,3). \end{cases}$$

Player i chooses her action from $A_i^S(h_i)$.

Lemma 1 *Take any $\delta \equiv (\delta_1, \delta_2, \delta_3) \in (0,1)^3$ and $\sigma \in \Sigma$. σ is a subgame perfect equilibrium in $\eth(\delta)$ if and only if for any $h^0 \in H^0$, σ forms a subgame perfect equilibrium in $\eth^S(h^0, \sigma; \delta)$.*

Proof. See Appendix A. **Q.E.D.**

This lemma is a variation of One Deviation Principle.

3.3 A necessary and sufficient condition for $E^{\delta}_{i,j,k}(R,R') \neq \emptyset$

We want to seek a necessary and sufficient condition for $E^{\delta}_{i,j,k}(R,R') \neq \emptyset$ for $\{i,j,k\} = N$ and $(R,R') \in \{=,>\}^2$. For the following arguments, let $\Delta_{i,j,k}(R,R')$ denote the set of δ necessary and sufficient for $E^{\delta}_{i,j,k}(R,R') \neq \emptyset$. Throughout this subsection and Appendix B, let x^l_m denote an amount of rent which proposer l distributes to party m. In the following arguments, we implicitly use Lemma 1.

We want to seek a necessary and sufficient condition for $E^{\delta}_{i,j,k}(=,=) \neq \emptyset$. Suppose that $E^{\delta}_{i,j,k}(=,=) \neq \emptyset$. Take any strategy profile $\sigma \in E^{\delta}_{i,j,k}(=,=)$. Let $(\delta_1,\delta_2,\delta_3) \equiv \delta$. Since $\delta_i V^{\delta}_i(\sigma) = \delta_j V^{\delta}_j(\sigma) = \delta_k V^{\delta}_k(\sigma)$, in σ, every party $h \in \{i,j,k\}$ (i) votes for a policy such that $x^g_h \geq \delta_h V^{\delta}_h(\sigma)$ and against a policy such that $x^g_h < \delta_h V^{\delta}_h(\sigma)$ for $g \in \{i,j,k\}$, and (ii) proposes a policy such that

$$\left(x^h_h, x^h_{h+1}, x^h_{h+2}\right) = \left(1 - \delta_{h+1}V^{\delta}_{h+1}(\sigma), \delta_{h+1}V^{\delta}_{h+1}(\sigma), 0\right)^{6}$$

with probability r_h and a policy such that

$$\left(x^h_h, x^h_{h+1}, x^h_{h+2}\right) = \left(1 - \delta_{h+2}V^{\delta}_{h+2}(\sigma), 0, \delta_{h+2}V^{\delta}_{h+2}(\sigma)\right)$$

with probability $1 - r_h$.[7] Hence, each party h's $\in \{i,j,k\}$ continuation value is as follows:

$$\begin{aligned} V^{\delta}_h(\sigma) = &\frac{1}{3}\left\{r_h\left(1 - \delta_{h+1}V^{\delta}_{h+1}(\sigma)\right) + (1-r_h)\left(1 - \delta_{h+2}V^{\delta}_{h+2}(\sigma)\right)\right\} \\ &+ \frac{1}{3}(1 - r_{h+1})\,\delta_i V^{\delta}_i(\sigma) + \frac{1}{3} r_{h+2}\delta_h V^{\delta}_h(\sigma). \end{aligned}$$

Moreover, utilizing $\delta_i V^{\delta}_i(\sigma) = \delta_j V^{\delta}_j(\sigma) = \delta_k V^{\delta}_k(\sigma)$, we obtain

$$\forall h \in \{i,j,k\} : V^{\delta}_h(\sigma) = \frac{\delta_{h+1}\delta_{h+2}}{\delta_1\delta_2 + \delta_2\delta_3 + \delta_3\delta_1} \tag{1}$$

and

$$(r_i, r_j, r_k) = \left(-\frac{1}{\delta_i} + \frac{2}{\delta_j} - \frac{1}{\delta_k} + \rho, -\frac{2}{\delta_i} + \frac{1}{\delta_j} + \frac{1}{\delta_k} + \rho, \rho\right) \equiv \mathbf{r}(\rho). \tag{2}$$

Thus,

$$\exists \rho \in \mathbb{R} : \mathbf{r}(\rho) \in [0,1]^3$$

must hold. Therefore,

$$\forall h \in \{i,j,k\} : -1 \leq \frac{1}{\delta_h} + \frac{1}{\delta_{h+1}} - \frac{2}{\delta_{h+2}} \leq 1$$

must be satisfied. This expression is a necessary and sufficient condition for $E^{\delta}_{i,j,k}(=,=) \neq \emptyset$.

Hence, we obtain the following lemma.

[6] Throughout the paper, on an index representing a party, for $i \in \{1,2,3\}$, we regard j as identical with i if $i \equiv j$ (mod 3).

[7] For proposer $h \in \{i,j,k\}$, it is not desirable that a proposal does not pass. The reason is as follows. If a proposal does not pass, proposer h's expected payoff is equal to $\delta_h V^{\delta}_h(\sigma)$. And we have

$$\begin{aligned} \left(1 - \delta_{h+1}V^{\delta}_{h+1}(\sigma)\right) - \delta_h V^{\delta}_h(\sigma) &> 1 - \left(\delta_h V^{\delta}_h(\sigma) + \delta_{h+1}V^{\delta}_{h+1}(\sigma) + \delta_{h+2}V^{\delta}_{h+2}(\sigma)\right) \\ &> 1 - \left(V^{\delta}_h(\sigma) + V^{\delta}_{h+1}(\sigma) + V^{\delta}_{h+2}(\sigma)\right) = 0. \end{aligned}$$

It is the case with the following subsections that rejection of a proposal is not desirable for a proposer.

Lemma 2 *Suppose that* $\{i,j,k\}=N$. *Then,*

$$\Delta_{i,j,k}(=,=)=\left\{(\delta_1,\delta_2,\delta_3)\in(0,1)^3 \mid \forall h\in\{i,j,k\}: -1\le\frac{1}{\delta_h}+\frac{1}{\delta_{h+1}}-\frac{2}{\delta_{h+2}}\le 1\right\}. \quad (3)$$

Proof. By the argument above. **Q.E.D.**

This lemma mensions that, when all parties are as patient as one another, there exist equilibria such that all parties have the same discounted continuation value.

Throughout the paper, we use binary relations, $\gg$, $\cong$ and $\simeq$, on $(0,1)$ defined as follows:

$$x\gg y \Leftrightarrow 3\left(\frac{1}{x}-\frac{1}{y}\right)<-1,$$

$$x\cong y \Leftrightarrow -1\le 3\left(\frac{1}{x}-\frac{1}{y}\right)\le 1,$$

and

$$x\simeq y \Leftrightarrow -1\le \frac{1}{x}-\frac{1}{y}\le 1$$

for $x,y\in(0,1)$.

Lemma 2 implies the following corollary.

Corollary 1 *Suppose that* $\{i,j,k\}=N$. *Then,*

$$\forall(\delta_1,\delta_2,\delta_3)\in\Delta_{i,j,k}(=,=):\delta_i\simeq\delta_j\simeq\delta_k\simeq\delta_i.$$

Proof. Almost obvious. **Q.E.D.**

The same logic as above yields the following lemmata.

Lemma 3 *Suppose that* $\{i,j,k\}=N$. *Then,*

$$\Delta_{i,j,k}(=,>)=\left\{(\delta_1,\delta_2,\delta_3)\in(0,1)^3 \mid -1\le 3\left(\frac{1}{\delta_i}-\frac{1}{\delta_j}\right)\le 1\wedge\frac{1}{\delta_i}+\frac{1}{\delta_j}-\frac{2}{\delta_k}<-1\right\}. \quad (4)$$

Proof. By the argument in Subsection B.1. **Q.E.D.**

This lemma mentions that, when two parties are as patient as one another and much more patient than the other party, there exist equilibria such that two more patient parties have the same discounted continuation value greater than the other party's value. Lemma 3 implies the following corollary.

Corollary 2 *Suppose that* $\{i,j,k\}=N$. *Then,*

$$\forall(\delta_1,\delta_2,\delta_3)\in\Delta_{i,j,k}(=,>):\delta_i\cong\delta_j\wedge\delta_i\gg\delta_k\wedge\delta_j\gg\delta_k.$$

Proof. Almost obvious. **Q.E.D.**

Lemma 4 *Suppose that* $\{i,j,k\}=N$. *Then,*

$$\Delta_{i,j,k}(>,=)=\left\{(\delta_1,\delta_2,\delta_3)\in(0,1)^3 \mid -1\le 3\left(\frac{1}{\delta_j}-\frac{1}{\delta_k}\right)\le 1\wedge\frac{1}{\delta_j}+\frac{1}{\delta_k}-\frac{2}{\delta_i}>1\right\}. \quad (5)$$

Proof. By the argument in Subsection B.2. **Q.E.D.**

This lemma mentions that, when two parties are as patient as one another and much less patient than the other party, there exist equilibria such that two less patient parties have the same discounted continuation value smaller than the other party's value. Lemma 4 implies the following corollary.

Corollary 3 *Suppose that* $\{i, j, k\} = N$. *Then,*

$$\forall(\delta_1, \delta_2, \delta_3) \in \Delta_{i,j,k}(>, =) : \delta_i \gg \delta_j \wedge \delta_i \gg \delta_k \wedge \delta_j \cong \delta_k.$$

Proof. Almost obvious. **Q.E.D.**

Lemma 5 *Suppose that* $\{i, j, k\} = N$. *Then,*

$$\Delta_{i,j,k}(>, >) = \left\{(\delta_1, \delta_2, \delta_3) \in (0,1)^3 \mid 3\left(\frac{1}{\delta_i} - \frac{1}{\delta_j}\right) < -1 \wedge 3\left(\frac{1}{\delta_j} - \frac{1}{\delta_k}\right) < -1\right\}. \tag{6}$$

Proof. By the argument in Subsection B.3. **Q.E.D.**

This lemma mentions that, when party i is much more patient than party j and party j is much more patient than party k, there exist equilibria such that party i's discounted continuation value is greater than party j's and party j's is greater than party k's. Lemma 5 implies the following corollary.

Corollary 4 *Suppose that* $\{i, j, k\} = N$. *Then,*

$$\forall(\delta_1, \delta_2, \delta_3) \in \Delta_{i,j,k}(>, >) : \delta_i \gg \delta_j \gg \delta_k.$$

Proof. Almost obvious. **Q.E.D.**

3.4 Some properties of $\Delta_{i,j,k}(R, R')$s and the existence of equilibria

Let

$$\mathfrak{D} \equiv \left\{\Delta_{i,j,k}(R, R') \mid \{i, j, k\} = N \wedge (R, R') \in \{=, >\}^2\right\}.$$

Then, we show some properties of $\Delta_{i,j,k}(R, R')$s as the following lemmata.

Lemma 6 $\forall \Delta \in \mathfrak{D} : \Delta \neq \emptyset$.

Proof. By Lemmata from 2 to 5. **Q.E.D.**

The lemma means the following. There are thirteen cases to be considered: $\delta_1 V_1^\delta(\sigma) = \delta_2 V_2^\delta(\sigma) = \delta_3 V_3^\delta(\sigma)$, $\delta_1 V_1^\delta(\sigma) = \delta_2 V_2^\delta(\sigma) > \delta_3 V_3^\delta(\sigma)$, and so on (♠). Take any case. There exists a honestly voting MPE satisfying the case for some δ.

For a set S, we denote the cardinal number of S by card S.

Lemma 7 card $\mathfrak{D} = 13$ *and* $\forall \{\Delta, \Delta'\} \subset \mathfrak{D} : \Delta \cap \Delta' = \emptyset$.

Proof. By Lemmata from 2 to 5. **Q.E.D.**

According to the lemma, in a game $\eth(\delta)$, if there exists a honestly voting MPE satisfying a case in (♠), there exists no honestly voting MPE satisfying the other cases in (♠).

Lemma 8 $\bigcup_{\Delta \in \mathfrak{D}} \Delta = (0, 1)^3$.

Proof. See Appendix C. **Q.E.D.**

This lemma means that there exists a honestly voting MPE in $\eth(\delta)$ for all $\delta \in (0,1)^3$.

The lemmata above yield the following proposition on the existence of equilibria.

Proposition 1 *Take any $\delta \in (0,1)^3$. There uniquely exist(s) payoff-relevant honestly voting MPE in $\eth(\delta)$, i.e., (i) there exist(s) honestly voting MPE in $\eth(\delta)$, and (ii) for any honestly voting MPE in $\eth(\delta)$, equal continuation value profile is achieved.*

Remark. Take any $\delta \in (0,1)^3$. The set of honestly voting MPE of $\eth(\delta)$ is $E^{\delta}_{i,j,k}(R, R')$, where (i,j,k,R,R') satisfies $\Delta_{i,j,k}(R,R') \ni \delta$. ■

Proof. By Lemmata 7 and 8 and the arguments in Subsection 3.3 and Appendix B. **Q.E.D.**

4 Parties' patience and continuation value (I)

In this section, we investigate whether or not more patient party obtains a larger equilibrium payoff than less patient party or not.

4.1 The result

The result is summarized by Proposition 2. This proposition is one of the two main results in this paper.

Proposition 2 *Suppose that $\{i^*, i_*, i\} = N$. Take any $\delta \equiv (\delta_1, \delta_2, \delta_3) \in (0,1)^3$ such that $\delta_{i^*} > \delta_{i_*}$. Take any honestly voting MPE σ in $\eth(\delta)$. Then,*

$$\left[\delta \in \Delta_{i^*,i,i_*}(=,>) \wedge \delta_{i_*} < \frac{1}{2}\left(1 + \delta_{i^*} - \frac{\delta_{i^*}}{\delta_i}\right)\right] \tag{7}$$

$$\vee \left[\delta \in \Delta_{i^*,i,i_*}(>,>) \wedge \delta_{i_*} < \frac{\delta_i}{3 - \delta_i}\right] \tag{8}$$

$$\vee \left[\delta \in \Delta_{i,i^*,i_*}(>,>) \wedge \delta_{i_*} < \frac{2}{3}\delta_{i^*}\right] \tag{9}$$

$$\Leftrightarrow V^{\delta}_{i^*}(\sigma) > V^{\delta}_{i_*}(\sigma), \tag{10}$$

$$\left[\delta \in \Delta_{i^*,i,i_*}(=,>) \wedge \delta_{i_*} = \frac{1}{2}\left(1 + \delta_{i^*} - \frac{\delta_{i^*}}{\delta_i}\right)\right] \tag{11}$$

$$\vee \left[\delta \in \Delta_{i^*,i,i_*}(>,>) \wedge \delta_{i_*} = \frac{\delta_i}{3 - \delta_i}\right] \tag{12}$$

$$\vee \left[\delta \in \Delta_{i,i^*,i_*}(>,>) \wedge \delta_{i_*} = \frac{2}{3}\delta_{i^*}\right] \tag{13}$$

$$\Leftrightarrow V^{\delta}_{i^*}(\sigma) = V^{\delta}_{i_*}(\sigma), \tag{14}$$

and

$$\left[\delta \in \Delta_{i^*,i,i_*}(=,>) \wedge \delta_{i_*} > \frac{1}{2}\left(1 + \delta_{i^*} - \frac{\delta_{i^*}}{\delta_i}\right)\right] \tag{15}$$

$$\vee \left[\delta \in \Delta_{i^*,i,i_*}(>,>) \wedge \delta_{i_*} > \frac{\delta_i}{3 - \delta_i}\right] \tag{16}$$

$$\vee \left[\delta \in \Delta_{i,i^*,i_*}(>,>) \wedge \delta_{i_*} > \frac{2}{3}\delta_{i^*}\right] \tag{17}$$

$$\vee\, \delta \in \Delta_{i^*,i_*,i}(=,=) \cup \Delta_{i^*,i_*,i}(=,>) \cup \Delta_{i,i^*,i_*}(>,=) \tag{18}$$

$$\vee\, \delta \in \Delta_{i^*,i_*,i}(>,=) \cup \Delta_{i^*,i_*,i}(>,>) \tag{19}$$

$$\Leftrightarrow V^{\delta}_{i^*}(\sigma) < V^{\delta}_{i_*}(\sigma). \tag{20}$$

Remark. Take any expression from among expressions (7)-(9), (11)-(13) and (15)-(19). Then, there exists δ which satisfies the expression. ■

Proof. By Lemmata from 9 to 12 in Appendix D and Lemma 8. **Q.E.D.**

Thus, more patient party obtains a larger equilibrium payoff if the expressions from (7) to (9) hold, while more patient party obtains a smaller equilibrium payoff if the expressions from (15) to (19) hold. In the following subsections, we shall offer an explanation why this seemingly counter-intuitive result is obtained.

4.2 Three effects of parties' patience on the continuation values

Parties' patience has three effects on their equilibrium continuation values of the game.

Two effects of them are more important than the other. We call one of the two effects *shut-out effect* and another *bargaining-power effect*. Especially the shut-out effect is important and does not appear in ordinary bargaining models.

For the first time, we consider the shut-out effect. In the legislative bargaining, a proposal passes by the majority approval. So, a proposer wants to obtain the approval of only one non-proposer party. Moreover, since a patient party tends to want many rents, the more patient non-proposer party's approval to the proposal is costly for the proposer. Thus, it is natural that the proposer intends to win the approval of the less patient non-proposer party. Hence, a patient party is likely to obtain no rent when she is not a proposer. She is, what is called, shut out by the proposer. So, we call this effect of parties' patience shut-out effect. This effect comes from the majority voting rule[8] and is not familiar in ordinary bargaining models.

On the other hand, the bargaining-power effect can be divided into two effects. We call one of them *recipient's-bargaining-power effect* and another *proposer's-bargaining-power effect*. The recipient's-bargaining-power effect is explained as follows. Consider a non-proposer party which is distributed positive rents to. As long as she is less costly for the proposer than another non-proposer party, the more patient she is, the more the rents that the proposer distributes to her are. In other words, locally, the more patient she is, the stronger her bargaining power against the proposer is, thus she obtains more rents. The proposer's-bargaining-power effect is explained as follows. The proposer distributes positive rents to less costly non-proposer party to win her approval. Naturally, the rents to win less costly party's approval is smaller as she is less patient. Thus, the proposer's rents is greater as the less costly party is less patient.

The other effect is, so to speak, *direct effect*, which comes from discounting each party's rent implemented in the future. But parties' patience actually has no direct effect on the continuation values since the policy is implemented in the first-round bargaining in every honestly voting MPE.

Generally speaking, the shut-out effect is advantageous to less patient party and the bargaining-power effect is to more patient party.

4.3 The intuition of Proposition 2

The intuition of this proposition can be explained by using the shut-out effect and the bargaining-power effect.

Roughly speaking, we can explain the proposition as follows.

In case δ_{i_*} is much smaller than δ_{i^*} and δ_i (the expressions (7)-(9), (11)-(13) and (15)-(17)), i_* is superior to i^* in terms of the shut-out effect and inferior in terms of the bargaining-power effect. If δ_{i_*} is extremely small (the expressions from (7) to (9)), party i_*'s bargainig power is extremely weak, thus party i^*'s advantage in terms of the bargaining-power effect is very significant and so dominates party i_*'s advantage in terms of the shut-out effect. Hence, if δ_{i_*} is extremely small, party i^*'s continuation value is greater than party i_*'s, and otherwise, it is smaller.

[8] We discuss this point in detail in Subsection 4.4.

In case δ_{i^*} and δ_{i_*} have similar value (the expression (18)), party i_* is superior to party i^* in terms of the shut-out effect just in a probability sense. So, we can state that δ_{i^*} is a little greater than δ_{i_*} and i_* is a little superior to party i^* in terms of the shut-out effect. By the interaction between the discount factors and the shut-out effect, in fact, party i^*'s discounted continuation value is equal to party i_*'s, and so party i^* is equivalent to party i_* in terms of the bargaining-power effect. To sum up, we can conclude that party i_*'s continuation value is greater than party i^*'s.

In case δ_{i^*} is much greater than δ_{i_*} and δ_i, and δ_{i_*} is not much smaller than δ_i (the expression (19)), party i^* is shut out by both party i_* and party i. So, party i^* is inferior to party i_* in terms of the shut-out effect. Obviously, the party which party i^* distributes positive rents to is as patient as the party which party i_* does. Thus, party i^* is equivalent to party i_* in terms of the bargaining-power effect. To sum up, we can conclude that party i_*'s continuation value is greater than party i^*'s.

In the following argument, we explain the intuition of the proposition in detail. Here, we focus on the case that $\delta \in \Delta_{i^*,i,i_*}(=,>)$ and explain the intuition in the other cases in Appendix E. For the following arguments, we denote the probability that party i obtains positive rents when she is not the proposer, the rents which party i obtains when she is not the proposer and is distributed positive rents to, and the rents which party i distributes to herself when she is the proposer, by r_h, x_h, and y_h, respectively, in equilibrium for $h \in N$. r_h, x_h, and y_h reflect the shut-out effect, the recipient's-bargaining-power effect, and the proposer's-bargainig-power effect, respectively.

On the expressions (7), (11) and (15) We consider the expressions (7), (11) and (15). Consider the situation that δ satisfies $\delta \in \Delta_{i^*,i,i_*}(=,>)$. The relationship between $(r_{i^*}, x_{i^*}, y_{i^*})$ and $(r_{i_*}, x_{i_*}, y_{i_*})$ is summarized by Table 1, where σ is an arbitrary honestly voting MPE in $\mho(\delta)$.

	i^*		i_*
$r.$	$\frac{1}{2}\left\{\frac{3(\delta_i-\delta_{i^*})}{\delta_{i^*}\delta_i}+\frac{1}{2}\right\}$	$<$	1
$x.$	$\delta_{i^*}V^\delta_{i^*}(\sigma)$	$>$	$\delta_{i_*}V^\delta_{i_*}(\sigma)$
$y.$	$1-\delta_{i_*}V^\delta_{i_*}(\sigma)$	$>$	$1-\delta_{i^*}V^\delta_{i^*}(\sigma)$

Table 1: $r.$, $x.$ and $y.$ in case $\delta \in \Delta_{i^*,i,i_*}(=,>)$

This table is explained as follows.

First, consider r_{i^*} and r_{i_*}. According to the expression (4), when $\delta \in \Delta_{i^*,i,i_*}(=,>)$, δ_{i_*} is much smaller than δ_{i^*} and δ_i. So, when the proposer is party i^* or i, the proposer distributes positive rents to party i_* with probability 1 because party i_*'s approval is less costly for the proposer than another non-proposer party. On the other hand, party i^* is distributed no rent to with probability 1 when the proposer is party i. When the proposer is party i_*, party i^*'s approval is as costly for party i_* as party i's since δ_{i^*} and δ_{i_*} are similar, party i_* randomizes the choice of which party to distribute positive rents to. According to the expression (29), the probability that party i^* obtains positive rent is equal to $\frac{3(\delta_i-\delta_{i^*})}{\delta_{i^*}\delta_i}+\frac{1}{2}$. Thus, we obtain $r_{i^*} < r_{i_*}$. So, in terms of the shut-out effect, party i_* tends to obtain more rents than party i^*.

Next, consider x_{i^*} and x_{i_*}. Since δ_{i^*} is much greater than δ_{i_*}, party i^* has higher threshold to vote for or against the proposal than party i_*. So, x_{i^*} is greater than x_{i_*}, that is, in terms of the recipient's-bargaining-power effect, party i^* tends to obtain more rents than party i_*.

Finally, consider y_{i^*} and y_{i_*}. On the one hand, party i^* wants to win party i_*'s approval when she is the proposer, because δ_{i_*} is much smaller than δ_i and so party i_*'s approval is less costly than party i's. On the other hand, perty i^*'s approval and party i's are indifferent for party i_* as the proposer, because δ_{i^*} and δ_i are similar. Since party i^*'s approver i_* is much less patient than party i_*'s approver i^* or i, party i^* can obtain more rents as the proposer than party i_*, i.e.,

$y_{i^*} > y_{i_*}$. So, in terms of the proposer's-bargaining-power effect, party i^* tends to obtain more rents than party i_*.

These three effects affect the continuation values of parties i^* and i_* in different directions. So, we cannot determine which party's continuation value is greater only from the table. Then, consider the case that δ_{i_*} is extremely small. In the case, party i_* cannot obtain many rents when she is not the proposer, and party i^* need not distribute many rents to i_* when she is the proposer. So, party i^*'s advantage in terms of the bargaining-power effect is very significant. Thus, it is likely that party i^*'s advantage in terms of the bargaining-power effect dominates party i_*'s advantage in terms of the shut-out effect. Hence, if δ_{i_*} is extremely small, party i^*'s continuation value is greater than party i_*'s, and otherwise, party i^*'s continuation value is smaller than party i_*'s. The proposition states that the critical value whether δ_{i_*} is extremely small or not is equal to $\frac{1}{2}\left(1+\delta_{i^*}-\frac{\delta_{i^*}}{\delta_i}\right)$.

4.4 Comparison between the majority rule and the unanimity rule

Proposition 2 states that more patient party's continuation value is smaller than less patient party's under some discount factor profiles. This seemingly paradoxical result comes from the majority rule. To see this, we compare the majority rule with the unanimity rule.

Consider the game which has the same structure as $\eth((\delta_1, \delta_2, \delta_3))$ except for the voting rule. In this game, a proposal passes if and only if all parties vote for the proposal. We denote party h's $\in N$ equilibrium continuation value in the game by V_h^U. For any discount factor profile, V_h^U is calculated as

$$V_h^U = \frac{(1-\delta_{h+1})(1-\delta_{h+2})}{3-2(\delta_1+\delta_2+\delta_3)+\delta_1\delta_2+\delta_2\delta_3+\delta_3\delta_1}. \tag{21}$$

Let $\{i^*, i_*, i\} = N$. Suppose that party i^* is more patient than party i_*, i.e., $\delta_{i^*} > \delta_{i_*}$. Then, we have

$$V_{i^*}^U - V_{i_*}^U = \frac{(\delta_{i^*}-\delta_{i_*})(1-\delta_i)}{3-2(\delta_1+\delta_2+\delta_3)+\delta_1\delta_2+\delta_2\delta_3+\delta_3\delta_1} > 0$$

since $\delta_{i^*} > \delta_{i_*}$ and the denominator is positive:

$$\begin{aligned} 3-2(\delta_1+\delta_2+\delta_3)+\delta_1\delta_2+\delta_2\delta_3+\delta_3\delta_1 &= -(2-\delta_2-\delta_3)\delta_1+3-2(\delta_2+\delta_3)+\delta_2\delta_3 \\ &> -(2-\delta_2-\delta_3)+3-2(\delta_2+\delta_3)+\delta_2\delta_3 \\ &= (1-\delta_2)(1-\delta_3) > 0. \end{aligned}$$

That is, under the unanimity rule, more patient party's continuation value is greater than less patient party's for any discount factor profile.

The reason for this is explained as follows. Under the unanimity rule, a proposal does not pass without every party voting for it. Thus, a proposer has no choice to shut out a party, i.e., cannot help distributing enough positive rents to every non-proposer party. So, the shut-out effect, which is advantageous to less patient party, has no effect under the unanimity rule, and only the bargaining-power effect, which is advantageous to more patient party, remains. Therefore, more patient party obtains greater value.

From the argument above, we can conclude as follows: the majority rule is featured by the shut-out effect, which is advantageous to less patient party, thus less patient party obtains greater continuation value under some discount factor profiles.

5 Parties' patience and continuation value (II)

In this section, we investigate whether or not a party obtains a larger equilibrium payoff as she gets more patient.

5.1 The result

The result is summarized by Proposition 3. This proposition is one of the two main results in this paper.

Proposition 3 *Take any* $\delta \equiv (\delta_1, \delta_2, \delta_3) \in (0,1)^3$*. Take any honestly voting MPE* σ *in* $\eth(\delta)$*. Then, for any* $h \in N$*,*
(i) if and only if

$$\delta \in \left(\bigcup_{R,l,m} \Delta_{l,m,h}(R,>) \right) \cup \left(\bigcup_{l,m} \Delta_{l,h,m}(>,>) \right),$$

then

$$\frac{\partial V_h^\delta(\sigma)}{\partial \delta_h} > 0;$$

(ii) if and only if

$$\delta \in \bigcup_{R,l,m} \Delta_{h,l,m}(>,R),$$

then

$$\frac{\partial V_h^\delta(\sigma)}{\partial \delta_h} = 0;$$

(iii) if and only if

$$\delta \in \bigcup_{(R,R') \neq (>,>),l,m} \Delta_{l,h,m}(R,R'),$$

then

$$\frac{\partial V_h^\delta(\sigma)}{\partial \delta_h} < 0.$$

Proof. By Lemmata from 13 to 16 in Appendix F and Lemma 8. **Q.E.D.**

Thus, a party obtains a larger equilibrium payoff as she gets more patient in case (i), while a party obtains a smaller equilibrium payoff as she gets more patient in case (iii). In the next subsection, we shall offer an explanation why this seemingly counter-intuitive result is obtained.

5.2 The intuition of Proposition 3

The intuition of Proposition 3 can be also explained by the shut-out effect and the bargaining-power effect.

First of all, consider the case that party h's discount factor is far from both the other parties', i.e., the cases (i) and (ii) of the proposition. In these cases, since party h's discount factor is far from both the other parties', party h is not distributed positive rents to *in a probability sense*, that is, every proposer distributes positive rents to party h with certainty or with probability 0. This is why infinitesimal change of δ_h does not affect the shut-out effect. On the other hand, naturally, the greater party h's discount factor δ_h gets, the stronger party h's bargaining power becomes. So, infinitesimal change of δ_h affects the bargaining-power effect[9] in favor of party h. To sum up, infinitesimal change of δ_h is advantageous to party h.

However, in case (ii) of the proposition, party h's discount factor is much greater than the other two parties', so her approval is more costly for a proposer than the other non-proposer party, thus every proposer does not distribute positive rents to party h, Therefore the party h's bargaining power has no effect. Hence, in case (ii) of the proposition, infinitesimal change of δ_h does not affect party h's continuation value.

[9] More precisely, infinitesimal change of δ_h affects the recipient's-bargaining-power effect. It does not *directly* affect the proposer's-bargaining-power effect.

In case (iii) of the proposition, change of δ_h has a seemingly paradoxical effect on party h's continuation value. The case of (iii) is the situation that there exists a party which as patient as party h. Label such party h' and the othe party $h''$for the convinience sake. When party h'' is the proposer, she distributes positive rents to party h with some probability r and to party h' with probability $1 - r$, because party h's approval is as costly as party h''s. If party h marginally gets more patient, the proposer h'' comes to hate party h and so decreases r. Hence, the shut-out effect gets disadvantageous to party h as she becomes more patient. On the other hand, party h's bargaining power is stronger as she is more patient. However, party h sharply competes with party h' for political rents which the proposer h'' distributes because party h is as patient as party h'. So, party h cannot claim very many rents even if party h's becomes more patient. This is why it is to a small extent that the bargaining-power effect becomes more advantageous to party h as she is more patient. Thus, the shut-out effect dominates the bargaining-power effect. Hence, the more patient party h becomes, the smaller her continuation value gets.

In this section, it is also meaningful to compare the majority rule with the unanimity rule. Party h's equilibrium continuation value under the unanimity rule is given by the expression (21). From the expression, we obtain

$$\frac{\partial V_h^U}{\partial \delta_h} = \frac{(1-\delta_{h+1})(1-\delta_{h+2})(2-\delta_{h+1}-\delta_{h+2})}{\{3-2(\delta_1+\delta_2+\delta_3)+\delta_1\delta_2+\delta_2\delta_3+\delta_3\delta_1\}^2} > 0.$$

As explained in the previous section, the shut-out effect does not affect the continuation values under the unanimity rule. Hence, increase of δ_h implies increase of party h's continuation value. From this consideration, we can conclude that the seemingly paradoxical phenomenon in case (iii) of the proposition comes from the characteristic of the majority rule, i.e., the shut-out effect.

6 Numerical examples

The following two examples help us understand Propositions 2 and 3. Let δ_i denote party i's discount factor for $i \in N$ in this section.

Example 1 We consider the case that $\delta_2 = \delta_3 = \frac{1}{2}$. Then, we have

$$\left(\delta_1, \frac{1}{2}, \frac{1}{2}\right) \in \begin{cases} \Delta_{2,3,1}(=,>) & \text{iff } \delta_1 < \frac{2}{5} \\ \Delta_{1,2,3}(=,=) & \text{iff } \frac{2}{5} \leq \delta_1 \leq \frac{2}{3} \\ \Delta_{1,2,3}(>,=) & \text{iff } \delta_1 > \frac{2}{3}, \end{cases}$$

$$V_1^\delta(\sigma) = \begin{cases} \frac{3}{11-8\delta_1} & \text{iff } \delta_1 < \frac{2}{5} \\ \frac{1}{1+4\delta_1} & \text{iff } \frac{2}{5} \leq \delta_1 \leq \frac{2}{3} \\ \frac{3}{11} & \text{iff } \delta_1 > \frac{2}{3}, \end{cases}$$

and

$$V_h^\delta(\sigma) = \begin{cases} \frac{4(1-\delta_1)}{11-8\delta_1} & \text{iff } \delta_1 < \frac{2}{5} \\ \frac{2\delta_1}{1+4\delta_1} & \text{iff } \frac{2}{5} \leq \delta_1 \leq \frac{2}{3} \\ \frac{4}{11} & \text{iff } \delta_1 > \frac{2}{3} \end{cases}$$

for $h = 2, 3$. Figure 2 describes δ_1-$V_i^\delta(\sigma)$ relationship.

Example 2 Now we consider the case that $\delta_2 = \frac{1}{3}$ and $\delta_3 = \frac{2}{3}$. Then, we have

$$\left(\delta_1, \frac{1}{3}, \frac{2}{3}\right) \in \begin{cases} \Delta_{3,2,1}(>,>) & \text{iff } \delta_1 < \frac{3}{10} \\ \Delta_{3,2,1}(>,=) & \text{iff } \frac{3}{10} \leq \delta_1 \leq \frac{3}{8} \\ \Delta_{3,1,2}(>,>) & \text{iff } \frac{3}{8} < \delta_1 < \frac{6}{11} \\ \Delta_{3,1,2}(=,>) & \text{iff } \frac{6}{11} \leq \delta_1 \leq \frac{6}{7} \\ \Delta_{1,3,2}(>,>) & \text{iff } \delta_1 > \frac{6}{7}, \end{cases}$$

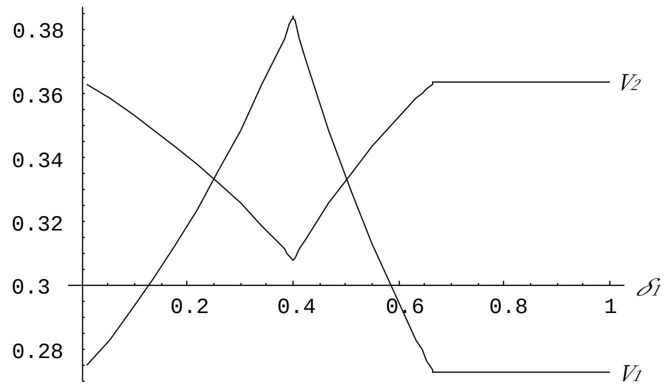

Figure 2: δ_1-$V_i^\delta(\sigma)$ relationship in case $\delta_2 = \delta_3 = \frac{1}{2}$

$$V_1^\delta(\sigma) = \begin{cases} \frac{7}{24-17\delta_1} & \text{iff } \delta_1 < \frac{3}{10} \\ \frac{2}{3+8\delta_1} & \text{iff } \frac{3}{10} \leq \delta_1 \leq \frac{3}{8} \\ \frac{6}{21-8\delta_1} & \text{iff } \frac{3}{8} < \delta_1 < \frac{6}{11} \\ \frac{8}{14+15\delta_1} & \text{iff } \frac{6}{11} \leq \delta_1 \leq \frac{6}{7} \\ \frac{14}{47} & \text{iff } \delta_1 > \frac{6}{7}, \end{cases}$$

$$V_2^\delta(\sigma) = \begin{cases} \frac{9(1-\delta_1)}{24-17\delta_1} & \text{iff } \delta_1 < \frac{3}{10} \\ \frac{6\delta_1}{3+8\delta_1} & \text{iff } \frac{3}{10} \leq \delta_1 \leq \frac{3}{8} \\ \frac{3(3-2\delta_1)}{21-8\delta_1} & \text{iff } \frac{3}{8} < \delta_1 < \frac{6}{11} \\ \frac{3(2+\delta_1)}{14+15\delta_1} & \text{iff } \frac{6}{11} \leq \delta_1 \leq \frac{6}{7} \\ \frac{15}{47} & \text{iff } \delta_1 > \frac{6}{7}, \end{cases}$$

and

$$V_3^\delta(\sigma) = \begin{cases} \frac{8(1-\delta_1)}{24-17\delta_1} & \text{iff } \delta_1 < \frac{3}{10} \\ \frac{1+2\delta_1}{3+8\delta_1} & \text{iff } \frac{3}{10} \leq \delta_1 \leq \frac{3}{8} \\ \frac{2(3-\delta_1)}{21-8\delta_1} & \text{iff } \frac{3}{8} < \delta_1 < \frac{6}{11} \\ \frac{12\delta_1}{14+15\delta_1} & \text{iff } \frac{6}{11} \leq \delta_1 \leq \frac{6}{7} \\ \frac{18}{47} & \text{iff } \delta_1 > \frac{6}{7}. \end{cases}$$

Figure 3 describes δ_1-$V_i^\delta(\sigma)$ relationship.

7 Conclusion

We observed a seemingly paradoxical phenomenon that, in the legislative bargaining, more patient party's continuation value is smaller than less patient party's under some discount factor profiles (Section 4). Moreover, it was shown that under some discount factor profiles, when a party marginally becomes more patient, her continuation value decreases (Section 5). These seemingly paradoxical phenomena do not appear in ordinary bargaining models.

The cause of these phenomena occuring is explained as follows. In the legislative bargaining, the decision is made according to the majority rule. A proposer can make her proposal pass by winning only one non-proposer party's approval. Patient party's approval is costly for a proposer. Thus, a proposer wants the less patient non-proposer party's approval. So, the more patient non-proposer party is distributed no rent to. The more patient party is, so to speak, shut out by a proposer. This effect of patience is unique to the legislative bargaining, more exactly, to the majority rule. This paper summarized above.

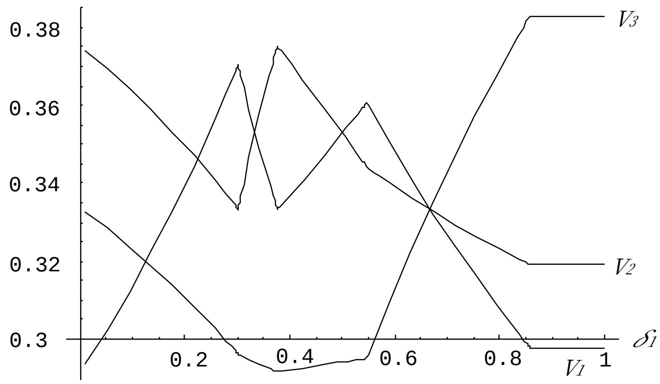

Figure 3: δ_1-$V_i^\delta(\sigma)$ relationship in case $\delta_2 = \frac{1}{3}$ and $\delta_3 = \frac{2}{3}$

By the way, the model can be extended in some ways.

Endogenizing the discount factors First of all, we consider endogenizing the discount factors. Since common discount factor is not assumed in $\eth(\delta)$, we can endogenize discount factors as follows. Add to the game above the 0th stage where each party simultaneously decides her own discount factor. Formally, we construct the game as follows. Take $\epsilon \in (0,1)$. In the 0th stage, party $i \in N$ decides her own discount factor from $\Delta_i \equiv \{k\epsilon\}_{k=1}^{n_\epsilon}$ with $n_\epsilon \equiv -\left[-\frac{1-\epsilon}{\epsilon}\right]$. Note that $n_\epsilon \geq 1$ for any $\epsilon \in (0,1)$. After $\delta \in \prod_{i \in N} \Delta_i$ is selected, the $\eth(\delta)$ is played. Let $\hat{\eth}(\epsilon)$ denote this extended game.

For any $\epsilon \in (0,1)$, the game $\hat{\eth}(\epsilon)$ has an equilibrium in which every party selects ϵ. The reason is as follows. For $\epsilon \in (0,1)$ such that $n_\epsilon = 1$, the claim above is trivially obvious. We consider the case that $n_\epsilon \geq 2$. Without loss of generality, it is sufficient to party 1 has no incentive to deviate from ϵ. Assume that party 1 deviates from ϵ to $k\epsilon$ with $k \in \{2, \ldots, n_\epsilon\}$. $(k\epsilon, \epsilon, \epsilon) \in \Delta_{1,2,3}(>,=)$ because

$$-1 \leq 3\left(\frac{1}{\epsilon} - \frac{1}{\epsilon}\right) \leq 1$$

and

$$\frac{1}{\epsilon} + \frac{1}{\epsilon} - \frac{2}{k\epsilon} \geq \frac{1}{\epsilon} + \frac{1}{\epsilon} - \frac{2}{2\epsilon} = \frac{1}{\epsilon} > 1.$$

Thus, from the expression (30), party 1 obtains the expected payoff of

$$\frac{2-\epsilon}{6-\epsilon} \tag{22}$$

under $(k\epsilon, \epsilon, \epsilon)$. On the other hand, since

$$-1 \leq \frac{1}{\epsilon} + \frac{1}{\epsilon} - \frac{2}{\epsilon} \leq 1,$$

we have $(\epsilon, \epsilon, \epsilon) \in \Delta_{1,2,3}(=,=)$. Thus, from the expression (1), party 1 obtains the expected payoff of

$$\frac{1}{3} \tag{23}$$

under $(\epsilon, \epsilon, \epsilon)$. Comparing the expression (22) with the expression (23), we obtain

$$\frac{1}{3} - \frac{2-\epsilon}{6-\epsilon} = \frac{2\epsilon}{3(6-\epsilon)} > 0.$$

So, party 1 has no incentive to deviate from ϵ in $\hat{\eth}(\epsilon)$ with $n_\epsilon \geq 2$. To sum up, for any $\epsilon \in (0,1)$, the game $\hat{\eth}(\epsilon)$ has an equilibrium in which every party selects ϵ. That is, in this equilibrium, every party selects as small discount factor as possible. Particularly, every party's equilibrium discount factor converges to 0 as the grid of selectable discount factor profiles gets as fine as possible.

Interpretation of this extended model and the extreme and eccentric result is to be considered in the future.

Generalizing the number of parties We can extend the model above as there are many parties. Let n be an odd integer and $n \geq 1$. Let $N \equiv \{1, \ldots, n\}$ instead of $N \equiv \{1, 2, 3\}$. Assume that every coalition of $\frac{n+1}{2}$ parties has the majority of the seats in the legislature and that every coalition of $\frac{n-1}{2}$ parties does not have the majority of the seats in the legislature.

Let δ_i and V_i denote party i's $\in N$ discount factor and equilibrium continuation value, respectively. Consider equilibria in which $\delta_i V_i = \delta_j V_j$ for all $i, j \in N$. Let

$$\mathfrak{C}_i \equiv \left\{ C \in 2^{N \setminus \{i\}} \mid \text{card } C = \frac{n-1}{2} \right\}$$

for $i \in N$. For $i \in N$ and $C \in \mathfrak{C}_i$, define $\mathbf{x}_i(C) \equiv (x_j)_{j=1}^n$ with

$$x_j \equiv \begin{cases} \delta_j V_j & \text{if } j \in C \\ 0 & \text{if } j \in N \setminus \{i\} \setminus C \\ 1 - \sum_{h \in C} \delta_h V_h & \text{if } j = i \end{cases}$$

for $j \in N$. In equilibrium, every party $i \in N$ seems to propose a policy $\mathbf{x}_i(C)$ with probability $r_i(C) \in [0, 1]$ for all $C \in \mathfrak{C}_i$ and a policy $\mathbf{x} \in X \setminus \{\mathbf{x}_i(C) \mid C \in \mathfrak{C}_i\}$ with probability 0. V_i and $r_i(C)$ must satisfy

$$V_i = \frac{\sum_{C \in \mathfrak{C}_i} r_i(C) \left(1 - \sum_{j \in C} \delta_j V_j\right) + \sum_{j \neq i} \sum_{C \in \mathfrak{C}_j \wedge C \ni i} r_j(C) \delta_i V_i}{n}$$

and

$$\sum_{C \in \mathfrak{C}_i} r_i(C) = 1$$

for all $i \in N$, and

$$\delta_1 V_1 = \cdots = \delta_n V_n.$$

Let R denote the set of $(r_i(C))_{i \in N, C \in \mathfrak{C}_i}$ satisfying the equations above. Note that

$$\dim R = \sum_{i \in N} \text{card } \mathfrak{C}_i = \frac{n!}{\left(\frac{n-1}{2}!\right)^2} \equiv L.$$

There are equilibria such that $\delta_i V_i = \delta_j V_j$ for all $i, j \in N$ if and only if there are discount factor profiles such that $R \cap [0, 1]^L \neq \emptyset$.

In the n-party case, it is difficult to solve the model. However, n-party model seems to obtain the result similar to 3-party model.

Asymmetric information It is natural that a party does not know how patient the other parties are. So, treating a party's discount factor as her private information is more realistic approach.

Appendix

A Proof of Lemma 1

Proof. The necessity is obvious. Thus, the proof is complete if we verify the sufficiency. Fix an arbitrary $\sigma \equiv (\sigma_i)_{i\in N} \in \Sigma$ such that for any $h^0 \in H^0$, σ forms a subgame perfect equilibrium in $\eth^S(h^0, \sigma, \delta)$. Take any $i \in N$ and any $h \in H$ such that $P(h) = i \vee P(h) = (1, 2, 3)$. And, take any $\sigma_i' \in \Sigma_i$ such that σ_i' is equivalent to σ_i except that $\sigma_i'(h) \neq \sigma_i(h)$. For the first time, consider the case that $P(h) = i$. For h, we define h' as $(h', i) = h$. σ forms a subgame perfect equilibrium in $\eth^S(h', \sigma, \delta)$. Thus, we can symbolically write

$$\int_{\mathbf{x}\in[0,1]^3} \left(\int_{v\in\{a,r\}^3} u_i^S(h', \sigma; \delta)(i, \mathbf{x}, v) d\sigma(h', i, \mathbf{x})(\{v\}) \right) d\sigma_i(h', i)(\{\mathbf{x}\})$$
$$\geq \int_{\mathbf{x}\in[0,1]^3} \left(\int_{v\in\{a,r\}^3} u_i^S(h', \sigma; \delta)(i, \mathbf{x}, v) d\sigma(h', i, \mathbf{x})(\{v\}) \right) d\sigma_i'(h', i)(\{\mathbf{x}\}).$$

Using $\hat{V}$, we can rewrite this expression as

$$\hat{V}_i((h', i), \sigma; \delta) \geq \hat{V}_i((h', i), (\sigma_i', \sigma_{-i}); \delta). \tag{24}$$

Secondly, consider the case that $P(h) = (1, 2, 3)$. Let $(h', p, \mathbf{x}) = h$. σ forms a subgame perfect equilibrium in $\eth^S(h', \sigma, \delta)$. Thus, we can symbolically write

$$\int_{v\in\{a,r\}^3} u_i^S(h', \sigma; \delta)(p, \mathbf{x}, v) d\sigma(h', p, \mathbf{x})(\{v\})$$
$$\geq \int_{v_i\in\{a,r\}} \left(\int_{v_{-i}\in\{a,r\}^2} u_i^S(h', \sigma; \delta)(p, \mathbf{x}, v) d\sigma_{-i}(h', p, \mathbf{x})(\{v_{-i}\}) \right) d\sigma_i'(h', p, \mathbf{x})(\{v_i\}).$$

Using $\hat{V}$, we can rewrite this expression as

$$\hat{V}_i((h', p, \mathbf{x}), \sigma; \delta) \geq \hat{V}_i((h', p, \mathbf{x}), (\sigma_i', \sigma_{-i}); \delta) \tag{25}$$

Hence, from the inequalites (24) and (25), the following inequality holds in both cases:

$$\hat{V}_i(h, \sigma; \delta) \geq \hat{V}_i(h, (\sigma_i', \sigma_{-i}); \delta).$$

By this inequality and One Deviation Principle, it is verified that σ is a subgame perfect equilibrium. **Q.E.D.**

B A necessary and sufficient condition for $E_{i,j,k}^{\delta}(R, R') \neq \emptyset$ in the other cases

B.1 A necessary and sufficient condition for $E_{i,j,k}^{\delta}(=, >) \neq \emptyset$

We want to seek a necessary and sufficient condition for $E_{i,j,k}^{\delta}(=, >) \neq \emptyset$.

Suppose that $E_{i,j,k}^{\delta}(=, >) \neq \emptyset$. Take any strategy profile $\sigma \in E_{i,j,k}^{\delta}(=, >)$. Let $(\delta_1, \delta_2, \delta_3) \equiv \delta$. Since $\delta_i V_i^{\delta}(\sigma) = \delta_j V_j^{\delta}(\sigma) > \delta_k V_k^{\delta}(\sigma)$, in σ, (i) every party $h \in \{i, j, k\}$ votes for a policy such that $x_h^g \geq \delta_h V_h^{\delta}(\sigma)$ and against a policy such that $x_h^g < \delta_h V_h^{\delta}(\sigma)$ for $g \in \{i, j, k\}$, (ii) party i proposes a policy such that

$$\left(x_i^i, x_j^i, x_k^i\right) = \left(1 - \delta_k V_k^{\delta}(\sigma), 0, \delta_k V_k^{\delta}(\sigma)\right),$$

(iii) party j proposes a policy such that

$$\left(x_i^j, x_j^j, x_k^j\right) = \left(0, 1 - \delta_k V_k^\delta(\sigma), \delta_k V_k^\delta(\sigma)\right),$$

and (vi) party k proposes a policy such that

$$\left(x_i^k, x_j^k, x_k^k\right) = \left(\delta_i V_i^\delta(\sigma), 0, 1 - \delta_i V_i^\delta(\sigma)\right)$$

with probability r and a policy such that

$$\left(x_i^k, x_j^k, x_k^k\right) = \left(0, \delta_j V_j^\delta(\sigma), 1 - \delta_j V_j^\delta(\sigma)\right)$$

with probability $1-r$. Hence, each party's continuation value is as follows:

$$V_i^\delta(\sigma) = \frac{1}{3}\left(1 - \delta_k V_k^\delta(\sigma)\right) + \frac{1}{3} \cdot 0 + \frac{1}{3} r \delta_i V_i^\delta(\sigma),$$

$$V_j^\delta(\sigma) = \frac{1}{3}\left(1 - \delta_k V_k^\delta(\sigma)\right) + \frac{1}{3}(1-r)\delta_j V_j^\delta(\sigma) + \frac{1}{3} \cdot 0,$$

and

$$V_k^\delta(\sigma) = \frac{1}{3}\left\{r \delta_i V_i^\delta(\sigma) + (1-r)\delta_j V_j^\delta(\sigma)\right\} + \frac{1}{3}\delta_k V_k^\delta(\sigma) + \frac{1}{3}\delta_k V_k^\delta(\sigma).$$

Moreover, utilizing $\delta_i V_i^\delta(\sigma) = \delta_j V_j^\delta(\sigma)$, we obtain

$$V_i^\delta(\sigma) = \frac{2\delta_j(1-\delta_k)}{(3-2\delta_k)(\delta_i+\delta_j) - \delta_i\delta_j}, \tag{26}$$

$$V_j^\delta(\sigma) = \frac{2\delta_i(1-\delta_k)}{(3-2\delta_k)(\delta_i+\delta_j) - \delta_i\delta_j}, \tag{27}$$

$$V_k^\delta(\sigma) = \frac{\delta_i+\delta_j-\delta_i\delta_j}{(3-2\delta_k)(\delta_i+\delta_j) - \delta_i\delta_j}, \tag{28}$$

and

$$r = \frac{3(\delta_j-\delta_i)}{2\delta_i\delta_j} + \frac{1}{2}. \tag{29}$$

These must satisfy $\delta_i V_i^\delta(\sigma) > \delta_k V_k^\delta(\sigma)$ and $0 \le r \le 1$. Hence,

$$\delta_i \frac{2\delta_j(1-\delta_k)}{(3-2\delta_k)(\delta_i+\delta_j) - \delta_i\delta_j} > \delta_k \frac{\delta_i+\delta_j-\delta_i\delta_j}{(3-2\delta_k)(\delta_i+\delta_j) - \delta_i\delta_j}$$

and

$$0 \le \frac{3(\delta_j-\delta_i)}{2\delta_i\delta_j} + \frac{1}{2} \le 1$$

must hold. Therefore,

$$-1 \le 3\left(\frac{1}{\delta_i} - \frac{1}{\delta_j}\right) \le 1$$

and

$$\frac{1}{\delta_i} + \frac{1}{\delta_j} - \frac{2}{\delta_k} < -1$$

must be satisfied. These two expressions are a necessary and sufficient condition for $E_{i,j,k}^\delta(=,>) \neq \emptyset$.

B.2 A necessary and sufficient condition for $E^{\delta}_{i,j,k}(>,=) \neq \emptyset$

We want to seek a necessary and sufficient condition for $E^{\delta}_{i,j,k}(>,=) \neq \emptyset$.

Suppose that $E^{\delta}_{i,j,k}(>,=) \neq \emptyset$. Take any strategy profile $\sigma \in E^{\delta}_{i,j,k}(>,=)$. Let $(\delta_1,\delta_2,\delta_3) \equiv \delta$. Since $\delta_i V^{\delta}_i(\sigma) > \delta_j V^{\delta}_j(\sigma) = \delta_k V^{\delta}_k(\sigma)$, in σ, (i) every party $h \in \{i,j,k\}$ votes for a policy such that $x^g_h \geq \delta_h V^{\delta}_h(\sigma)$ and against a policy such that $x^g_h < \delta_h V^{\delta}_h(\sigma)$ for $g \in \{i,j,k\}$, (ii) party i proposes a policy such that

$$\left(x^i_i, x^i_j, x^i_k\right) = \left(1-\delta_j V^{\delta}_j(\sigma), \delta_j V^{\delta}_j(\sigma), 0\right)$$

with probability r and a policy such that

$$\left(x^i_i, x^i_j, x^i_k\right) = \left(1-\delta_k V^{\delta}_k(\sigma), 0, \delta_k V^{\delta}_k(\sigma)\right)$$

with probability $1-r$, (iii) party j proposes a policy such that

$$\left(x^j_i, x^j_j, x^j_k\right) = \left(0, 1-\delta_k V^{\delta}_k(\sigma), \delta_k V^{\delta}_k(\sigma)\right),$$

and (vi) party k proposes a policy such that

$$\left(x^k_i, x^k_j, x^k_k\right) = \left(0, \delta_j V^{\delta}_j(\sigma), 1-\delta_j V^{\delta}_j(\sigma)\right).$$

Hence, each party's continuation value is as follows:

$$V^{\delta}_i(\sigma) = \frac{1}{3}\left\{r\left(1-\delta_j V^{\delta}_j(\sigma)\right) + (1-r)\left(1-\delta_k V^{\delta}_k(\sigma)\right)\right\} + \frac{1}{3}\cdot 0 + \frac{1}{3}\cdot 0,$$

$$V^{\delta}_j(\sigma) = \frac{1}{3}\left(1-\delta_k V^{\delta}_k(\sigma)\right) + \frac{1}{3}\delta_j V^{\delta}_j(\sigma) + \frac{1}{3} r \delta_j V^{\delta}_j(\sigma),$$

and

$$V^{\delta}_k(\sigma) = \frac{1}{3}\left(1-\delta_j V^{\delta}_j(\sigma)\right) + \frac{1}{3}(1-r)\delta_k V^{\delta}_k(\sigma) + \frac{1}{3}\delta_k V^{\delta}_k(\sigma).$$

Moreover, utilizing $\delta_j V^{\delta}_j(\sigma) = \delta_k V^{\delta}_k(\sigma)$, we obtain

$$V^{\delta}_i(\sigma) = \frac{\delta_j + \delta_k - \delta_j\delta_k}{3(\delta_j+\delta_k) - \delta_j\delta_k}, \tag{30}$$

$$V^{\delta}_j(\sigma) = \frac{2\delta_k}{3(\delta_j+\delta_k) - \delta_j\delta_k}, \tag{31}$$

$$V^{\delta}_k(\sigma) = \frac{2\delta_j}{3(\delta_j+\delta_k) - \delta_j\delta_k}, \tag{32}$$

and

$$r = \frac{3(\delta_k - \delta_j)}{2\delta_j\delta_k} + \frac{1}{2}. \tag{33}$$

These must satisfy $\delta_i V^{\delta}_i(\sigma) > \delta_j V^{\delta}_j(\sigma)$ and $0 \leq r \leq 1$. Hence,

$$\delta_i \frac{\delta_j + \delta_k - \delta_j\delta_k}{3(\delta_j+\delta_k) - \delta_j\delta_k} > \delta_j \frac{2\delta_k}{3(\delta_j+\delta_k) - \delta_j\delta_k}$$

and

$$0 \leq \frac{3(\delta_k - \delta_j)}{2\delta_j\delta_k} + \frac{1}{2} \leq 1$$

must hold. Therefore,

$$-1 \leq 3\left(\frac{1}{\delta_j} - \frac{1}{\delta_k}\right) \leq 1$$

and

$$\frac{1}{\delta_j} + \frac{1}{\delta_k} - \frac{2}{\delta_i} > 1$$

must be satisfied. These two expressions are a necessary and sufficient condition for $E^{\delta}_{i,j,k}(>,=) \neq \emptyset$.

B.3 A necessary and sufficient condition for $E^{\delta}_{i,j,k}(>,>) \neq \emptyset$

We want to seek a necessary and sufficient condition for $E^{\delta}_{i,j,k}(>,>) \neq \emptyset$.

Suppose that $E^{\delta}_{i,j,k}(>,>) \neq \emptyset$. Take any strategy profile $\sigma \in E^{\delta}_{i,j,k}(>,>)$. Let $(\delta_1, \delta_2, \delta_3) \equiv \delta$. Since $\delta_i V^{\delta}_i(\sigma) > \delta_j V^{\delta}_j(\sigma) > \delta_k V^{\delta}_k(\sigma)$, in σ, (i) every party $h \in \{i,j,k\}$ votes for a policy such that $x^g_h \geq \delta_h V^{\delta}_h(\sigma)$ and against a policy such that $x^g_h < \delta_h V^{\delta}_h(\sigma)$ for $g \in \{i,j,k\}$, (ii) party i proposes a policy such that

$$\left(x^i_i, x^i_j, x^i_k\right) = \left(1 - \delta_k V^{\delta}_k(\sigma), 0, \delta_k V^{\delta}_k(\sigma)\right),$$

(iii) party j proposes a policy such that

$$\left(x^j_i, x^j_j, x^j_k\right) = \left(0, 1 - \delta_k V^{\delta}_k(\sigma), \delta_k V^{\delta}_k(\sigma)\right),$$

and (vi) party k proposes a policy such that

$$\left(x^k_i, x^k_j, x^k_k\right) = \left(0, \delta_j V^{\delta}_j(\sigma), 1 - \delta_j V^{\delta}_j(\sigma)\right).$$

Hence, each party's continuation value is as follows:

$$V^{\delta}_i(\sigma) = \frac{1}{3}\left(1 - \delta_k V^{\delta}_k(\sigma)\right) + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot 0,$$

$$V^{\delta}_j(\sigma) = \frac{1}{3}\left(1 - \delta_k V^{\delta}_k(\sigma)\right) + \frac{1}{3}\delta_j V^{\delta}_j(\sigma) + \frac{1}{3} \cdot 0,$$

and

$$V^{\delta}_k(\sigma) = \frac{1}{3}\left(1 - \delta_j V^{\delta}_j(\sigma)\right) + \frac{1}{3}\delta_k V^{\delta}_k(\sigma) + \frac{1}{3}\delta_k V^{\delta}_k(\sigma).$$

Moreover, parametering the values, we obtain

$$V^{\delta}_i(\sigma) = \frac{(3 - \delta_j)(1 - \delta_k)}{9 - 3\delta_j - 6\delta_k + \delta_j \delta_k}, \tag{34}$$

$$V^{\delta}_j(\sigma) = \frac{3(1 - \delta_k)}{9 - 3\delta_j - 6\delta_k + \delta_j \delta_k}, \tag{35}$$

and

$$V^{\delta}_k(\sigma) = \frac{3 - 2\delta_j}{9 - 3\delta_j - 6\delta_k + \delta_j \delta_k}. \tag{36}$$

These must satisfy $\delta_i V^{\delta}_i(\sigma) > \delta_j V^{\delta}_j(\sigma) > \delta_k V^{\delta}_k(\sigma)$. Hence,

$$\delta_i \frac{(3 - \delta_j)(1 - \delta_k)}{9 - 3\delta_j - 6\delta_k + \delta_j \delta_k} > \delta_j \frac{3(1 - \delta_k)}{9 - 3\delta_i - 6\delta_k + \delta_j \delta_k} > \delta_k \frac{3 - 2\delta_j}{9 - 3\delta_j - 6\delta_k + \delta_j \delta_k}$$

must hold. Therefore,

$$3\left(\frac{1}{\delta_i} - \frac{1}{\delta_j}\right) < -1$$

and

$$3\left(\frac{1}{\delta_j} - \frac{1}{\delta_k}\right) < -1$$

must be satisfied. These two expressions are a necessary and sufficient condition for $E^{\delta}_{i,j,k}(>,>) \neq \emptyset$.

C Proof of Lemma 8

We prove Lemma 8 in two ways. One of them (Proof 1) is a abstract way and another (Proof 2) is in accordance with the arguments in Section 3.3. In Proof 1, we use Kakutani's Fixed Point Theorem not in ordinary way.

Proof 1. Take an arbitrary $\delta \equiv (\delta_j)_{j\in N} \in (0,1)^3$. Define $F^\delta : [0,1]^9 \rightrightarrows [0,1]^9$ as follows: for $\mathbf{z} \equiv (r^j, x^j_{j+1}, x^j_{j+2})_{j=1}^3 \in [0,1]^9$,

$$F^\delta(\mathbf{z}) = \prod_{j=1}^{3} \left(R^\delta_j(\mathbf{z}) \times X^\delta_j(\mathbf{z}) \right),$$

where

$$R^\delta_i(\mathbf{z}) = \begin{cases} \{1\} & \text{if } v^\delta_{i+1}(\mathbf{z}) < v^\delta_{i+2}(\mathbf{z}) \\ [0,1] & \text{if } v^\delta_{i+1}(\mathbf{z}) = v^\delta_{i+2}(\mathbf{z}) \\ \{0\} & \text{if } v^\delta_{i+1}(\mathbf{z}) > v^\delta_{i+2}(\mathbf{z}) \end{cases}$$

and

$$X^\delta_i(\mathbf{z}) = \left\{ \left(v^\delta_{i+1}(\mathbf{z}), v^\delta_{i+2}(\mathbf{z}) \right) \right\},$$

for $i \in N$ with

$$v^\delta_i(\mathbf{z}) = \delta_i \cdot \frac{1}{3} \left[\left\{ 1 - r^i \delta_{i+1} x^i_{i+1} - \left(1 - r^i\right) \delta_{i+2} x^i_{i+2} \right\} + \left(1 - r^{i+1}\right) \delta_i x^{i+1}_i + r^{i+2} \delta_i x^{i+2}_i \right].$$

Obviously, $[0,1]^9$ is compact and convex, and F^δ is upper-semicontinuous and has nonempty and convex values. Hence, Kakutani's Fixed Point Theorem yields $\exists \mathbf{z}^* \in [0,1]^9 : \mathbf{z}^* \in F^\delta(\mathbf{z}^*)$. Pick $\mathbf{z}^* \in [0,1]^9$ such that $\mathbf{z}^* \in F^\delta(\mathbf{z}^*)$. Let $(r^{j*}, x^{j*}_{j+1}, x^{j*}_{j+2})_{j=1}^3 \equiv \mathbf{z}^*$. Consider a strategy profile s such that every party $i \in N$ votes for a proposal $(x_j)_{j\in N}$ if and only if $x_i \geq v^\delta_i(\mathbf{z}^*)$ and proposes $(1 - x^{i*}_{i+1}, x^{i*}_{i+1}, 0)$ with probability r^{i*} and $(1 - x^{i*}_{i+2}, 0, x^{i*}_{i+2})$ with probability $1 - r^{i*}$. Taking $\mathbf{z}^* \in F^\delta(\mathbf{z}^*)$ into account, we can show that s is a honestly voting MPE. Hence, it is shown that $\eth(\delta)$ has honestly voting MPE for all $\delta \in (0,1)$. Therefore, $\forall \delta \in (0,1)^3 \exists \Delta \in \mathfrak{D} : \delta \in \Delta$. **Q.E.D.**

Proof 2. The proof is complete by showing $(\delta_1, \delta_2, \delta_3) \in \bigcup_{\Delta\in\mathfrak{D}} \Delta$ for any $(\delta_1, \delta_2, \delta_3) \in (0,1)^3$. Invoking the symmetry, it is sufficient to verify $(\delta_1, \delta_2, \delta_3) \in \bigcup_{\Delta\in\mathfrak{D}} \Delta$ for any $(\delta_1, \delta_2, \delta_3) \in (0,1)^3$ such that $\delta_1 \geq \delta_2 \geq \delta_3$. Take any $(\delta_1, \delta_2, \delta_3) \in (0,1)^3$ such that $\delta_1 \geq \delta_2 \geq \delta_3$. We consider the following four cases: (I) $3\left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right) \geq -1 \wedge 3\left(\frac{1}{\delta_2} - \frac{1}{\delta_3}\right) \geq -1$, (II) $3\left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right) \geq -1 \wedge 3\left(\frac{1}{\delta_2} - \frac{1}{\delta_3}\right) < -1$, (III) $3\left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right) < -1 \wedge 3\left(\frac{1}{\delta_2} - \frac{1}{\delta_3}\right) \geq -1$, and (IV) $3\left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right) < -1 \wedge 3\left(\frac{1}{\delta_2} - \frac{1}{\delta_3}\right) < -1$.

(I) $3\left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right) \geq -1 \wedge 3\left(\frac{1}{\delta_2} - \frac{1}{\delta_3}\right) \geq -1$ *(Ineq.I).* We consider the three cases: (I-i) $\frac{1}{\delta_2} + \frac{1}{\delta_3} - \frac{2}{\delta_1} > 1$, (I-ii) $\frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{\delta_3} < -1$, and (I-iii) $\frac{1}{\delta_2} + \frac{1}{\delta_3} - \frac{2}{\delta_1} \leq 1 \wedge \frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{\delta_3} \geq -1$.

(I-i) $\frac{1}{\delta_2} + \frac{1}{\delta_3} - \frac{2}{\delta_1} > 1$ *(Ineq.I-i).* $\delta_2 \geq \delta_3$ and the second inequality of the expression (Ineq.I) yields $-1 \leq 3\left(\frac{1}{\delta_2} - \frac{1}{\delta_3}\right) \leq 0$. This inequality and the expression (Ineq.I-i) imply $(\delta_1, \delta_2, \delta_3) \in \Delta_{1,2,3}(>, =)$.

(I-ii) $\frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{\delta_3} < -1$ *(Ineq.I-ii).* $\delta_1 \geq \delta_2$ and the first inequality of the expression (Ineq.I) yields $-1 \leq 3\left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right) \leq 0$. This inequality and (Ineq.I-ii) imply $(\delta_1, \delta_2, \delta_3) \in \Delta_{1,2,3}(=, >)$.

(I-iii) $\frac{1}{\delta_2} + \frac{1}{\delta_3} - \frac{2}{\delta_1} \leq 1 \wedge \frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{\delta_3} \geq -1$ *(Ineq.I-iii).* $\delta_1 \geq \delta_2 \geq \delta_3$ and the expression (Ineq.I-iii) yield

$$0 \leq \frac{1}{\delta_2} + \frac{1}{\delta_3} - \frac{2}{\delta_1} \leq 1 \tag{37}$$

and

$$-1 \leq \frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{\delta_3} \leq 0. \tag{38}$$

Furthermore, these inequalities imply

$$-1 \leq \frac{1}{\delta_3} + \frac{1}{\delta_1} - \frac{2}{\delta_2} \leq 1. \tag{39}$$

From inequalities (37) to (39), we obtain $(\delta_1, \delta_2, \delta_3) \in \Delta_{1,2,3}(=,=)$.

(II) $3\left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right) \geq -1 \wedge 3\left(\frac{1}{\delta_2} - \frac{1}{\delta_3}\right) < -1$ *(Ineq.II)*. We consider the two cases: (II-i) $\frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{\delta_3} \geq -1$ and (II-ii) $\frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{\delta_3} < -1$.

(II-i) $\frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{\delta_3} \geq -1$ *(Ineq.II-i)*. $\delta_1 \geq \delta_2 \geq \delta_3$ and the inequality (Ineq.II-i) yield

$$-1 \leq \frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{\delta_3} \leq 0, \tag{40}$$

This inequality and the first inequality of the expression (Ineq.II) yield

$$\frac{1}{\delta_2} + \frac{1}{\delta_3} - \frac{2}{\delta_1} \leq 1.$$

Moreover, by $\delta_1 \geq \delta_2 \geq \delta_3$,

$$0 \leq \frac{1}{\delta_2} + \frac{1}{\delta_3} - \frac{2}{\delta_1} \leq 1. \tag{41}$$

Furthermore, the inequalities (40) and (41) yield

$$-1 \leq \frac{1}{\delta_3} + \frac{1}{\delta_1} - \frac{2}{\delta_2} \leq 1. \tag{42}$$

Hence, from the inequalities (40) to (42), we obtain $(\delta_1, \delta_2, \delta_3) \in \Delta_{1,2,3}(=,=)$.

(II-ii) $\frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{\delta_3} < -1$ *(Ineq.II-ii)*. $\delta_1 \geq \delta_2$ and the first inequality of the expression (II) yield $0 \leq 3\left(\frac{1}{\delta_2} - \frac{1}{\delta_1}\right) \leq 1$. From this inequality and the inequality (Ineq.II-ii), we obtain $(\delta_1, \delta_2, \delta_3) \in \Delta_{1,2,3}(=,>)$.

(III) $3\left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right) < -1 \wedge 3\left(\frac{1}{\delta_2} - \frac{1}{\delta_3}\right) \geq -1$ *(Ineq.III)*. We consider the two cases: (III-i) $\frac{1}{\delta_2} + \frac{1}{\delta_3} - \frac{2}{\delta_1} \leq 1$ and (III-ii) $\frac{1}{\delta_2} + \frac{1}{\delta_3} - \frac{2}{\delta_1} > 1$.

(III-i) $\frac{1}{\delta_2} + \frac{1}{\delta_3} - \frac{2}{\delta_1} \leq 1$ *(Ineq.III-i)*. $\delta_1 \geq \delta_2 \geq \delta_3$ and the inequality (Ineq.III-i) yield

$$0 \leq \frac{1}{\delta_2} + \frac{1}{\delta_3} - \frac{2}{\delta_1} \leq 1. \tag{43}$$

This inequality and the second inequality of the expression (III) yield $\frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{\delta_3} \geq -1$. Moreover, by $\delta_1 \geq \delta_2 \geq \delta_3$,

$$-1 \leq \frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{\delta_3} \leq 0. \tag{44}$$

Furthermore, the inequalities (43) and (44) yield

$$-1 \leq \frac{1}{\delta_3} + \frac{1}{\delta_1} - \frac{2}{\delta_2} \leq 1. \tag{45}$$

Hence, from the inequalities (43) to (45), we obtain $(\delta_1, \delta_2, \delta_3) \in \Delta_{1,2,3}(=,=)$.

(III-ii) $\frac{1}{\delta_2} + \frac{1}{\delta_3} - \frac{2}{\delta_1} > 1$ *(Ineq.III-ii)*. $\delta_2 \geq \delta_3$ and the second inequality of the expression (III) yield $0 \leq 3\left(\frac{1}{\delta_3} - \frac{1}{\delta_2}\right) \leq 1$. From this inequality and the inequality (Ineq.III-ii), we obtain $(\delta_1, \delta_2, \delta_3) \in \Delta_{1,2,3}(>,=)$.

(IV) $3\left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right) < -1 \wedge 3\left(\frac{1}{\delta_2} - \frac{1}{\delta_3}\right) < -1$ *(Ineq.IV)*. Obviously, $(\delta_1, \delta_2, \delta_3) \in \Delta_{1,2,3}(>,>)$.
Q.E.D.

D Lemmata to prove Proposition 2

Lemma 9 *Suppose that* $\{i, j, k\} = N$. *Take any* $\delta \equiv (\delta_1, \delta_2, \delta_3) \in \Delta_{i,j,k}(=,=)$. *Take any honestly voting MPE* σ *in* $\partial(\delta)$. *Then,*

$$\forall \{l, m\} \subset N : \delta_l \lesseqgtr \delta_m \Rightarrow V_l^\delta(\sigma) \gtreqless V_m^\delta(\sigma).$$

Proof. From Proposition 2, $\sigma \in E_{i,j,k}^\delta(=,=)$. Hence, the definition of $E_{i,j,k}^\delta(=,=)$ yields

$$\delta_i V_i^\delta(\sigma) = \delta_j V_j^\delta(\sigma) = \delta_k V_k^\delta(\sigma).$$

Therefore, the proof is complete. **Q.E.D.**

Lemma 10 *Suppose that* $\{i, j, k\} = N$. *Take any* $\delta \equiv (\delta_1, \delta_2, \delta_3) \in \Delta_{i,j,k}(=,>)$. *Take any honestly voting MPE* σ *in* $\partial(\delta)$. *Then,*

$$\delta_i \lesseqgtr \delta_j \Rightarrow V_i^\delta(\sigma) \gtreqless V_j^\delta(\sigma), \tag{46}$$

and

$$\delta_k \lesseqgtr \frac{1}{2}\left(1 + \delta_l - \frac{\delta_l}{\delta_m}\right) \Rightarrow V_l^\delta(\sigma) \gtreqless V_k^\delta(\sigma) \tag{47}$$

for $\{l, m\} = \{i, j\}$.

Proof. *On the expression (46).* From Proposition 2, $\sigma \in E_{i,j,k}^\delta(=,>)$. The definition of $E_{i,j,k}^\delta(=,>)$ yields $\delta_i V_i^\delta(\sigma) = \delta_j V_j^\delta(\sigma)$. Therefore, the proof is complete for the expression (46).

On the expression (47). From Proposition 2, $\sigma \in E_{i,j,k}^\delta(=,>)$. The definition of $E_{i,j,k}^\delta(=,>)$ yields

$$V_i^\delta(\sigma) = \frac{2\delta_j(1-\delta_k)}{(3-2\delta_k)(\delta_i+\delta_j) - \delta_i\delta_j} \tag{48}$$

and

$$V_k^\delta(\sigma) = \frac{\delta_i + \delta_j - \delta_i\delta_j}{(3-2\delta_k)(\delta_i+\delta_j) - \delta_i\delta_j}. \tag{49}$$

So,

$$V_j^\delta(\sigma) - V_i^\delta(\sigma) = \frac{-\delta_i + \delta_i\delta_j + \delta_j - 2\delta_j\delta_k}{(3-2\delta_k)(\delta_i+\delta_j) - \delta_i\delta_j},$$

of which the denominator is obviously positive. Hence,

$$\operatorname{sgn}(V_i^\delta(\sigma) - V_k^\delta(\sigma)) = \operatorname{sgn}(-\delta_i + \delta_i\delta_j + \delta_j - 2\delta_j\delta_k) = \operatorname{sgn}\left(\frac{1}{2}\left(1 + \delta_i - \frac{\delta_i}{\delta_j}\right) - \delta_k\right).$$

Thus, we obtain

$$\delta_k \lesseqgtr \frac{1}{2}\left(1 + \delta_i - \frac{\delta_i}{\delta_j}\right) \Rightarrow V_i^\delta(\sigma) \gtreqless V_k^\delta(\sigma). \tag{50}$$

Similarly,

$$\delta_k \lesseqgtr \frac{1}{2}\left(1 + \delta_j - \frac{\delta_j}{\delta_i}\right) \Rightarrow V_j^\delta(\sigma) \gtreqless V_k^\delta(\sigma). \tag{51}$$

Therefore, the proof is complete for the expression (47). **Q.E.D.**

Lemma 11 *Suppose that* $\{i, j, k\} = N$. *Take any* $\delta \equiv (\delta_1, \delta_2, \delta_3) \in \Delta_{i,j,k}(>,=)$. *Take any honestly voting MPE* σ *in* $\partial(\delta)$. *Then,*

$$\forall l \in \{j, k\} : V_i^\delta(\sigma) < V_l^\delta(\sigma) \tag{52}$$

and

$$\delta_j \lesseqgtr \delta_k \Rightarrow V_j^\delta(\sigma) \gtreqless V_k^\delta(\sigma). \tag{53}$$

Proof. *On the expression (52).* From Proposition 2, $\sigma \in E^{\delta}_{i,j,k}(>,=)$. Hence, we obtain

$$V_i^{\delta}(\sigma) = \frac{\delta_j + \delta_k - \delta_j \delta_k}{3(\delta_j + \delta_k) - \delta_j \delta_k}$$

and

$$V_j^{\delta}(\sigma) = \frac{2\delta_k}{3(\delta_j + \delta_k) - \delta_j \delta_k}.$$

So,

$$V_j^{\delta}(\sigma) - V_i^{\delta}(\sigma) = \frac{\delta_k - \delta_j + \delta_j \delta_k}{3(\delta_j + \delta_k) - \delta_j \delta_k}. \tag{54}$$

By the way, the definition of $\Delta_{i,j,k}(>,=)$ yields

$$-1 \leq 3\left(\frac{1}{\delta_j} - \frac{1}{\delta_k}\right) \leq 1,$$

which implies

$$\delta_k - \delta_j + \delta_j \delta_k \geq \frac{2}{3}\delta_j \delta_k > 0.$$

And the denominator of the right hand side of expression (54) is obviously positive. Therefore,

$$V_j^{\delta}(\sigma) > V_i^{\delta}(\sigma).$$

Similarly,

$$V_k^{\delta}(\sigma) > V_i^{\delta}(\sigma).$$

Hence, the proof is complete for the expression (52).

On the expression (53). From Proposition 2, $\sigma \in E^{\delta}_{i,j,k}(>,=)$. The definition of $E^{\delta}_{i,j,k}(>,=)$ yields $\delta_j V_j^{\delta}(\sigma) = \delta_k V_k^{\delta}(\sigma)$. Therefore, the proof is complete for the expression (53). **Q.E.D.**

Lemma 12 *Suppose that $\{i,j,k\} = N$. Take any $\delta \equiv (\delta_1, \delta_2, \delta_3) \in \Delta_{i,j,k}(>,>)$. Take any honestly voting MPE σ in $\eth(\delta)$. Then,*

$$V_i^{\delta}(\sigma) < V_j^{\delta}(\sigma), \tag{55}$$

$$\delta_k \lesseqgtr \frac{2}{3}\delta_j \Rightarrow V_j^{\delta}(\sigma) \gtreqless V_k^{\delta}(\sigma) \tag{56}$$

and

$$\delta_k \lesseqgtr \frac{\delta_j}{3 - \delta_j} \Rightarrow V_i^{\delta}(\sigma) \gtreqless V_k^{\delta}(\sigma). \tag{57}$$

Proof. *On the expression (55).* From Proposition 2, $\sigma \in E^{\delta}_{i,j,k}(>,>)$. The definition of $E^{\delta}_{i,j,k}(>,>)$ yields

$$V_i^{\delta}(\sigma) = \frac{(3 - \delta_j)(1 - \delta_k)}{9 - 3\delta_j - 6\delta_k + \delta_j \delta_k},$$

and

$$V_j^{\delta}(\sigma) = \frac{3(1 - \delta_k)}{9 - 3\delta_j - 6\delta_k + \delta_j \delta_k}.$$

So,

$$V_j^{\delta}(\sigma) - V_i^{\delta}(\sigma) = \frac{\delta_j (1 - \delta_k)}{9 - 3\delta_j - 6\delta_k + \delta_j \delta_k} > 0.$$

On the expression (56). From Proposition 2, $\sigma \in E^{\delta}_{i,j,k}(>,>)$. The definition of $E^{\delta}_{i,j,k}(>,>)$ yields

$$V_j^{\delta}(\sigma) = \frac{3(1 - \delta_k)}{9 - 3\delta_j - 6\delta_k + \delta_j \delta_k}$$

and

$$V_k^\delta(\sigma) = \frac{3 - 2\delta_j}{9 - 3\delta_j - 6\delta_k + \delta_j\delta_k}.$$

So,

$$V_j^\delta(\sigma) - V_k^\delta(\sigma) = \frac{2\delta_j - 3\delta_k}{9 - 3\delta_j - 6\delta_k + \delta_j\delta_k}.$$

Hence,

$$\operatorname{sgn}\left(V_j^\delta(\sigma) - V_k^\delta(\sigma)\right) = \operatorname{sgn}(2\delta_j - 3\delta_k).$$

Thus, we obtain

$$\delta_k \lesseqgtr \frac{2}{3}\delta_j \Rightarrow V_j^\delta(\sigma) \gtreqless V_k^\delta(\sigma).$$

On the expression (57). From Proposition 2, $\sigma \in E_{i,j,k}^\delta(>,>)$. The definition of $E_{i,j,k}^\delta(>,>)$ yields

$$V_i^\delta(\sigma) = \frac{(3 - \delta_j)(1 - \delta_k)}{9 - 3\delta_j - 6\delta_k + \delta_j\delta_k}$$

and

$$V_k^\delta(\sigma) = \frac{3 - 2\delta_j}{9 - 3\delta_j - 6\delta_k + \delta_j\delta_k}.$$

So,

$$V_i^\delta(\sigma) - V_k^\delta(\sigma) = \frac{\delta_j - \delta_k(3 - \delta_j)}{9 - 3\delta_j - 6\delta_k + \delta_j\delta_k}.$$

Hence,

$$\operatorname{sgn}\left(V_i^\delta(\sigma) - V_k^\delta(\sigma)\right) = \operatorname{sgn}(\delta_j - \delta_k(3 - \delta_j)).$$

Thus, we obtain

$$\delta_k \lesseqgtr \frac{\delta_j}{3 - \delta_j} \Rightarrow V_i^\delta(\sigma) \gtreqless V_k^\delta(\sigma).$$

Q.E.D.

E Intuition of Proposition 2: In detail

On the expressions (8), (12) and (16) For the expressions (8), (12) and (16), the intuition is also explained by the similar logic as the expressions (7), (11) and (15).

Consider the situation that δ satisfies $\delta \in \Delta_{i^*,i,i_*}(>,>)$. The relationship between $(r_{i^*}, x_{i^*}, y_{i^*})$ and $(r_{i_*}, x_{i_*}, y_{i_*})$ is summarized by Table 2, where σ is an arbitrary honestly voting MPE in $\eth(\delta)$.

	i^*		i_*
$r.$	0	$<$	1
$x.$			$\delta_{i_*}V_{i_*}^\delta(\sigma)$
$y.$	$1 - \delta_{i_*}V_{i_*}^\delta(\sigma)$	$>$	$1 - \delta_i V_i^\delta(\sigma)$

Table 2: $r.$, $x.$ and $y.$ in case $\delta \in \Delta_{i^*,i,i_*}(>,>)$

This table is explained as follows.

According to the expression (6), when $\delta \in \Delta_{i^*,i,i_*}(>,>)$, δ_{i_*} is much smaller than δ_i, and δ_i is much smaller than δ_{i^*}. So, when the proposer is party i^* or i, the proposer distributes positive rents of $\delta_{i_*}V_{i_*}^\delta(\sigma)$ to party i_* with probability 1 because party i_*'s approval is less costly for the proposer than another non-proposer party. On the other hand, party i^* is distributed no rent to with probability 1 when the proposer is party i_* or i. So, in terms of the shut-out effect and the recipient's-bargaining-power effect, party i_* tends to obtain more rents than party i^*.

Next, consider the proposer's-bargaining-power effect. On the one hand, party i^* wants to win party i_*'s approval when she is the proposer, because δ_{i_*} is much smaller than δ_i and so party i_*'s approval is less costly than party i's one. On the other hand, party i_* wants to win party i's approval when she is the proposer, because δ_i is much smaller than δ_{i^*} and so party i's approval is less costly than party i^*'s one. Since party i^*'s approver i_* is much less patient than party i_*'s approver i, party i^* can obtain more rents as the proposer than party i_δ, i.e., $y_{i^*} > y_{i_*}$. So, in terms of the proposer's-bargaining-power effect, party i^* tends to obtain more rents than party i_*.

As mensioned above, the shut-out effect and the recipient's-bargaining-power effect, and the proposer's-bargaining-power effect affect the continuation values of parties i^* and i_* in different directions. So, we cannot determine which party's continuation value is greater only from the table. Then, consider the case that δ_{i_*} is extremely small. In the case, party i_* cannot obtain many rents when she is not the proposer, and party i^* need not distribute many rents to i_* when she is the proposer. So, party i_*'s advantage in terms of the shut-out effect and the recipient's-bargaining-power effect is very small, and party i^*'s advantage in terms of the proposer's-bargaining-power effect is very significant. Thus, it is likely that party i^*'s advantage in terms of the proposer's bargaining-power effect dominates party i_*'s advantage in terms of the other effects. Hence, if δ_{i_*} is extremely small, party i^*'s continuation value is greater than party i_*'s, and otherwise, party i^*'s continuation value is smaller than party i_*'s. The proposition states that the critical value of δ_{i_*} is equal to $\frac{\delta_i}{3-\delta_i}$.

On the expressions (9), (13) and (17) For the expressions (9), (13) and (17), the intuition is also explained by the similar logic as the expressions (7), (11) and (15). The relationship between $(r_{i^*}, x_{i^*}, y_{i^*})$ and $(r_{i_*}, x_{i_*}, y_{i_*})$ is summarized by Table 3, where σ is an arbitrary honestly voting MPE in $\eth(\delta)$.

	i^*		i_*
$r.$	$\frac{1}{2}$	$<$	1
$x.$	$\delta_{i^*} V^{\delta}_{i^*}(\sigma)$	$>$	$\delta_{i_*} V^{\delta}_{i_*}(\sigma)$
$y.$	$1-\delta_{i_*} V^{\delta}_{i_*}(\sigma)$	$>$	$1-\delta_{i^*} V^{\delta}_{i^*}(\sigma)$

Table 3: $r.$, $x.$ and $y.$ in case $\delta \in \Delta_{i,i^*,i_*}(>,>)$

This table is explained as follows.

First, consider r_{i^*} and r_{i_*}. According to the expression (6), when $\delta \in \Delta_{i,i^*,i_*}(>,>)$, δ_{i_*} is much smaller than δ_{i^*}, and δ_{i^*} is much smaller than δ_i. So, when the proposer is party i^* or i, the proposer distributes positive rents to party i_* with probability 1 because party i_*'s approval is less costly for the proposer than another non-proposer party. On the other hand, by the same logic, party i^* is distributed no rent to with probability 1 when the proposer is party i, and positive rents with probability 1 when the proposer is party i_*. Thus, we obtain $r_{i^*} < r_{i_*}$. So, in terms of the shut-out effect, party i_* tends to obtain more rents than party i^*.

Next, consider x_{i^*} and x_{i_*}. Since δ_{i^*} is much greater than δ_{i_*}, party i^* has higher threshold to vote for or against the proposal than party i_*. So, x_{i^*} is greater than x_{i_*}, that is, in terms of the recipient's-bargaining-power effect, party i^* tends to obtain more rents than party i_*.

Finally, consider y_{i^*} and y_{i_*}. On the one hand, party i^* wants to win party i_*'s approval when she is the proposer, because δ_{i_*} is much smaller than δ_i and so party i_*'s approval is less costly than party i's one. On the other hand, party i_* wants to win party i^*'s approval when she is the proposer, because δ_{i^*} is much smaller than δ_i and so party i^*'s approval is less costly than party i's one. Since party i^*'s approver i_* is much less patient than party i_*'s approver i^*, party i^* can obtain more rents as the proposer than party i_δ, i.e., $y_{i^*} > y_{i_*}$. So, in terms of the proposer's-bargaining-power effect, party i^* tends to obtain more rents than party i_*.

These three effects affect the continuation values of parties i^* and i_* in different directions. So, we cannot determine which party's continuation value is greater only from the table. Then, consider the case that δ_{i_*} is extremely small. In the case, party i_* cannot obtain many rents when she is not the proposer, and party i^* need not distribute many rents to i_* when she is the proposer. So, party i^*'s advantage in terms of the bargaining-power effect is very significant. Thus, it is likely that party i^*'s advantage in terms of the bargaining-power effect dominates party i_*'s advantage in terms of the shut-out effect. Hence, if δ_{i_*} is extremely small, party i^*'s continuation value is greater than party i_*'s, and otherwise, party i^*'s continuation value is smaller than party i_*'s. The proposition states that the critical value of δ_{i_*} is equal to $\frac{2}{3}\delta_{i^*}$.

On $\delta \in \Delta_{i^*,i_*,i}(=,=)$ of the expression (18) Next, we consider $\delta \in \Delta_{i^*,i_*,i}(=,=)$ of the expression (18). The relationship between $(r_{i^*}, x_{i^*}, y_{i^*})$ and $(r_{i_*}, x_{i_*}, y_{i_*})$ is summarized by Table 4, where σ is an arbitrary honestly voting MPE in $\eth(\delta)$.

	i^*		i_*
$r.$	$\frac{1}{2}\left(1+\frac{2}{\delta_{i^*}}-\frac{1}{\delta_{i_*}}-\frac{1}{\delta_i}\right)$	$<$	$\frac{1}{2}\left(1-\frac{1}{\delta_{i^*}}+\frac{2}{\delta_{i_*}}-\frac{1}{\delta_i}\right)$
$x.$	$\delta_{i^*}V_{i^*}^{\delta}(\sigma)$	$=$	$\delta_{i_*}V_{i_*}^{\delta}(\sigma)$
$y.$	$1-\delta_i V_i^{\delta}(\sigma)$	$=$	$1-\delta_i V_i^{\delta}(\sigma)$

Table 4: $r.$, $x.$ and $y.$ in case $\delta \in \Delta_{i^*,i_*,i}(=,=)$

This table is explained as follows.

According to the expression (3), when $\delta \in \Delta_{i^*,i_*,i}(=,=)$, δ_{i^*}, δ_{i_*} and δ_i are very similar. That is, each party is as patient as other party. Thus, all parties claim the same amount of rents. So, we obtain $x_{i^*} = x_{i_*}$ and $y_{i^*} = y_{i_*}$.

Since all parties claim the same amount of rents, each party randomizes the choice of which party to distribute positive rents to. From the expression (2), when she is not the proposer, party i^* is distributed positive rents to with probability

$$r_{i^*} = \frac{1}{2}\left[\rho + \left\{1 - \left(-\frac{2}{\delta_{i^*}} + \frac{1}{\delta_{i_*}} + \frac{1}{\delta_i} + \rho\right)\right\}\right] = \frac{1}{2}\left(1 + \frac{2}{\delta_{i^*}} - \frac{1}{\delta_{i_*}} - \frac{1}{\delta_i}\right),$$

and party i_* is distributed positive rents to with probability

$$r_{i_*} = \frac{1}{2}\left\{\left(-\frac{1}{\delta_{i^*}} + \frac{2}{\delta_{i_*}} - \frac{1}{\delta_i} + \rho\right) + (1-\rho)\right\} = \frac{1}{2}\left(1 - \frac{1}{\delta_{i^*}} + \frac{2}{\delta_{i_*}} - \frac{1}{\delta_i}\right).$$

Therefore, we obtain

$$r_{i_*} - r_{i^*} = \frac{3}{2}\frac{\delta_{i^*} - \delta_{i_*}}{\delta_{i^*}\delta_{i_*}} > 0$$

because $\delta_{i^*} > \delta_{i_*}$. In short, party i^* tends to be avoided since she is more patient.

To sum up, party i_*'s continuation value is greater than party i^*'s.

On $\delta \in \Delta_{i^*,i_*,i}(=,>)$ of the expression (18) Next, we consider $\delta \in \Delta_{i^*,i_*,i}(=,>)$ of the expression (18). The relationship between $(r_{i^*}, x_{i^*}, y_{i^*})$ and $(r_{i_*}, x_{i_*}, y_{i_*})$ is summarized by Table 5, where σ is an arbitrary honestly voting MPE in $\eth(\delta)$.

This table is explained as follows.

According to the expression (4), when $\delta \in \Delta_{i^*,i_*,i}(=,>)$, δ_{i^*} and δ_{i_*} are very similar. That is, party i^* is as patient as party i_*. Thus, party i^* and party i_* claim the same amount of rents. So, we obtain $x_{i^*} = x_{i_*}$. Furthermore, according to the expression (4), when $\delta \in \Delta_{i^*,i_*,i}(=,>)$, δ_i is much smaller than δ_{i^*} and δ_{i_*}. That is, party i is much less patient than party i^* and party i_*. So, when the proposer is party i^* or i_*, she wants party i's approval, which is less costly than

	i^*		i_*
$r.$	$\frac{1}{2}\left\{\frac{3(\delta_{i_*}-\delta_{i^*})}{2\delta_{i^*}\delta_{i_*}}+\frac{1}{2}\right\}$	$<$	$\frac{1}{2}\left\{\frac{3(\delta_{i^*}-\delta_{i_*})}{2\delta_{i^*}\delta_{i_*}}+\frac{1}{2}\right\}$
$x.$	$\delta_{i^*}V_{i^*}^{\delta}(\sigma)$	$=$	$\delta_{i_*}V_{i_*}^{\delta}(\sigma)$
$y.$	$1-\delta_i V_i^{\delta}(\sigma)$	$=$	$1-\delta_i V_i^{\delta}(\sigma)$

Table 5: $r.$, $x.$ and $y.$ in case $\delta \in \Delta_{i^*,i_*,i}(=,>)$

another non-proposer party's. Thus, $y_{i^*} = y_{i_*} = 1 - \delta_i V_i(\sigma)$. Therefore, party i^* is equivalent to party i_* in terms of the bargaining-power effect.

Next, consider the shut-out effect. Since party i is much less patient than party i^* and party i_*, party i^* does not distribute positive rents to party i_* when she is the proposer, and *vice versa*. Party i randomizes the choice of which party to distribute positive rents to when she is the proposer, because party i^* is as patient as party i_* and so party i^*'s approval is as costly as party i_*'s. From the expression (29), party i distributes positive rents to party i^* with probability $\frac{3(\delta_{i_*}-\delta_{i^*})}{2\delta_{i^*}\delta_{i_*}}+\frac{1}{2}$ and to party i_* with probability $\frac{3(\delta_{i^*}-\delta_{i_*})}{2\delta_{i^*}\delta_{i_*}}+\frac{1}{2}$. Thus, we obtain

$$r_{i^*} = \frac{1}{2}\left\{\frac{3\left(\delta_{i_*}-\delta_{i^*}\right)}{2\delta_{i^*}\delta_{i_*}}+\frac{1}{2}\right\}$$

and

$$r_{i_*} = \frac{1}{2}\left\{\frac{3\left(\delta_{i^*}-\delta_{i_*}\right)}{2\delta_{i^*}\delta_{i_*}}+\frac{1}{2}\right\}.$$

Therefore,

$$r_{i_*} - r_{i^*} = \frac{3\left(\delta_{i^*}-\delta_{i_*}\right)}{2\delta_{i^*}\delta_{i_*}} > 0$$

because $\delta_{i^*} > \delta_{i_*}$. In short, party i^* tends to be avoided since she is more patient.

To sum up, party i_*'s continuation value is greater than party i^*'s.

On $\delta \in \Delta_{i,i^*,i_*}(>,=)$ of the expression (18) Next, we consider $\delta \in \Delta_{i,i^*,i_*}(>,=)$ of the expression (18). The relationship between $(r_{i^*}, x_{i^*}, y_{i^*})$ and $(r_{i_*}, x_{i_*}, y_{i_*})$ is summarized by Table 6, where σ is an arbitrary honestly voting MPE in $\eth(\delta)$.

	i^*		i_*
$r.$	$\frac{1}{2}\left[1+\left\{\frac{3(\delta_{i_*}-\delta_{i^*})}{2\delta_{i^*}\delta_{i_*}}+\frac{1}{2}\right\}\right]$	$<$	$\frac{1}{2}\left[1+\left\{\frac{3(\delta_{i^*}-\delta_{i_*})}{2\delta_{i^*}\delta_{i_*}}+\frac{1}{2}\right\}\right]$
$x.$	$\delta_{i^*}V_{i^*}^{\delta}(\sigma)$	$=$	$\delta_{i_*}V_{i_*}^{\delta}(\sigma)$
$y.$	$1-\delta_{i_*}V_{i_*}^{\delta}(\sigma)$	$=$	$1-\delta_{i^*}V_{i^*}^{\delta}(\sigma)$

Table 6: $r.$, $x.$ and $y.$ in case $\delta \in \Delta_{i,i^*,i_*}(>,=)$

This table is explained as follows. According to the expression (5), when $\delta \in \Delta_{i,i^*,i_*}(>,=)$, δ_{i^*} and δ_{i_*} are very similar. That is, party i^* is as patient as party i_*. Thus, party i^* and party i_* claim the same amount of rents. So, we obtain $x_{i^*} = x_{i_*}$. Furthermore, according to the expression (5), when $\delta \in \Delta_{i,i^*,i_*}(>,=)$, δ_i is much greater than δ_{i^*} and δ_{i_*}. That is, party i is much more patient than party i^* and party i_*. So, party i^* wants party i_*'s approval, which is less costly than party i's, when she is the proposer, and *vice versa*. Taking account of the fact that party i^* and party i_* claim the same amount of rents, we obtain $y_{i^*} = y_{i_*}$. Therefore, party i^* is equivalent to party i_* in terms of the bargaining-power effect.

Next, consider the shut-out effect. Since party i^* and party i_* is much less patient than party i, party i^* distribute positive rents to party i_* with certainty when she is the proposer, and *vice* *versa*. Party i randomizes the choice of which party to distribute positive rents to when she is the proposer, because party i^* is as patient as party i_* and so party i^*'s approval is as costly as party i_*'s. From the expression (**??**), party i distributes positive rents to party i^* with probability $\frac{3(\delta_{i_*}-\delta_{i^*})}{2\delta_{i^*}\delta_{i_*}}+\frac{1}{2}$ and to party i_* with probability $\frac{3(\delta_{i^*}-\delta_{i_*})}{2\delta_{i^*}\delta_{i_*}}+\frac{1}{2}$. Thus, we obtain

$$r_{i^*}=\frac{1}{2}\left[1+\left\{\frac{3\left(\delta_{i_*}-\delta_{i^*}\right)}{2\delta_{i^*}\delta_{i_*}}+\frac{1}{2}\right\}\right]$$

and

$$r_{i_*}=\frac{1}{2}\left[1+\left\{\frac{3\left(\delta_{i^*}-\delta_{i_*}\right)}{2\delta_{i^*}\delta_{i_*}}+\frac{1}{2}\right\}\right].$$

Therefore,

$$r_{i_*}-r_{i^*}=\frac{3\left(\delta_{i^*}-\delta_{i_*}\right)}{2\delta_{i^*}\delta_{i_*}}>0$$

because $\delta_{i^*}>\delta_{i_*}$. In short, party i^* tends to be avoided since she is more patient.

To sum up, party i_*'s continuation value is greater than party i^*'s.

On $\delta\in\Delta_{i^*,i_*,i}(>,=)$ of the expression (19) Next, we consider $\delta\in\Delta_{i^*,i_*,i}(>,=)$ of the expression (19). The relationship between $(r_{i^*},x_{i^*},y_{i^*})$ and $(r_{i_*},x_{i_*},y_{i_*})$ is summarized by Table 7, where σ is an arbitrary honestly voting MPE in $\eth(\delta)$.

	i^*		i_*
$r.$	0	$<$	$\frac{1}{2}\left[1+\left\{\frac{3(\delta_i-\delta_{i_*})}{2\delta_{i_*}\delta_i}+\frac{1}{2}\right\}\right]$
$x.$			$\delta_{i_*}V_{i_*}^{\delta}(\sigma)$
$y.$	$1-\delta_iV_i^{\delta}(\sigma)$	$=$	$1-\delta_iV_i^{\delta}(\sigma)$

Table 7: $r.$, $x.$ and $y.$ in case $\delta\in\Delta_{i^*,i_*,i}(>,=)$

This table is explained as follows. According to the expression (5), when $\delta\in\Delta_{i^*,i_*,i}(>,=)$, δ_{i_*} and δ_i are much smaller than δ_{i^*}. That is, party i^* is much more patient than party i_* and party i. So, party i^*'s appropval is more costly for the proposer than party i_* and party i. Thus, when the proposer is party i_* or party i, she does not distribute positive rents to party i^* and does to the non-proposer party but party i^* with certainty. Hence, $r_{i_*}>r_{i^*}=0$. This implies that party i_* tends to obtain more rents than party i^* in terms of the shut-out effect and the recipient's-bargaining-power effect.

Next, consider the proposer's-bargaining-power effect. According to the expression (5), when $\delta\in\Delta_{i^*,i_*,i}(>,=)$, δ_{i_*} and δ_i are very similar. That is, party i_* is as patient as party i. So, party i_*'s approval is as costly for the proposer as party i. Thus, when party i^* is the proposer, she randomizes the choice of which party of i_ast and i to distribute positive rents to. On the other hand, as mentioned above, party i_* distributes positive rents to party i with certainty when she is proposer. Taking account of the fact that party i_* is as patient as party i and so party i_* and party i claim the same amount of rents, we can conclude that $y_{i_*}=y_{i^*}$. That is, party i^* is equivalent to party i_* in terms of the proposer's-bargaining-power effect.

To sum up, party i_*'s continuation value is greater than party i^*'s.

On $\delta\in\Delta_{i^*,i_*,i}(>,>)$ of the expression (19) Finally, we consider $\delta\in\Delta_{i^*,i_*,i}(>,>)$ of the expression (19). The relationship between $(r_{i^*},x_{i^*},y_{i^*})$ and $(r_{i_*},x_{i_*},y_{i_*})$ is summarized by Table 8, where σ is an arbitrary honestly voting MPE in $\eth(\delta)$.

This table is explained as follows.

According to the expression (6), when $\delta\in\Delta_{i^*,i_*,i}(>,=)$, δ_{i_*} and δ_i are much smaller than δ_{i^*}. That is, party i^* is much more patient than party i_* and party i. So, party i^*'s approval is more

	i^*		i_*
$r.$	0	$<$	$\frac{1}{2}$
$x.$			$\delta_{i_*} V_{i_*}^{\delta}(\sigma)$
$y.$	$1 - \delta_i V_i^{\delta}(\sigma)$	$=$	$1 - \delta_i V_i^{\delta}(\sigma)$

Table 8: $r.$, $x.$ and $y.$ in case $\delta \in \Delta_{i^*, i_*, i}(>,>)$

costly for the proposer than party i_* and party i. Thus, when the proposer is party i_* or party i, she does not distribute positive rents to party i^* and does to the non-proposer party but party i^* with certainty. Hence, $r_{i_*} > r_{i^*} = 0$. This implies that party i_* tends to obtain more rents than party i^* in terms of the shut-out effect and the recipient's-bargaining-power effect.

Next, consider the proposer's-bargaining-power effect. According to the expression (6), when $\delta \in \Delta_{i^*, i_*, i}(>,>)$, δ_i is much smaller than δ^{i_*} and δ_{i_*}. That is, party i is much less patient than party i^* and party i_*. So, party i's approval is less costly for the proposer than party i^* and party i_*. Thus, when the proposer is party i^* or party i_*, she distributes positive rents to party i with certainty. So, we can conclude that $y_{i_*} = y_{i^*}$. That is, party i^* is equivalent to party i_* in terms of the proposer's-bargaining-power effect.

To sum up, party i_*'s continuation value is greater than party i^*'s.

F Lemmata to prove Proposition 3

Lemma 13 *Suppose that $\{i,j,k\} = N$. Take any $\delta \equiv (\delta_1, \delta_2, \delta_3) \in \Delta_{i,j,k}(=,=)$. Take any honestly voting MPE σ in $\eth(\delta)$. Then,*

$$\forall h \in \{i,j,k\} : \frac{\partial V_h^{\delta}(\sigma)}{\partial \delta_h} < 0.$$

Proof. Party h's$\in \{i,j,k\}$ continuation value in σ is given by the expression (1). Hence, we obtain

$$\forall h \in \{i,j,k\} : \frac{\partial V_h^{\delta}(\sigma)}{\partial \delta_h} = -\frac{\delta_{h+1}\delta_{h+2}(\delta_{h+1} + \delta_{h+2})}{(\delta_1\delta_2 + \delta_2\delta_3 + \delta_3\delta_1)^2} < 0.$$

Q.E.D.

Lemma 14 *Suppose that $\{i,j,k\} = N$. Take any $\delta \equiv (\delta_1, \delta_2, \delta_3) \in \Delta_{i,j,k}(=,>)$. Take any honestly voting MPE σ in $\eth(\delta)$. Then,*

$$\frac{\partial V_i^{\delta}(\sigma)}{\partial \delta_i} < 0,$$

$$\frac{\partial V_j^{\delta}(\sigma)}{\partial \delta_j} < 0,$$

and

$$\frac{\partial V_k^{\delta}(\sigma)}{\partial \delta_k} > 0.$$

Proof. Party i's continuation value in σ, j's and k's are given by the expression (26), (27) and (28), respectively. Hence, we obtain

$$\frac{\partial V_i^{\delta}(\sigma)}{\partial \delta_i} = -\frac{2\delta_j (1-\delta_k)(3-\delta_j-2\delta_k)}{\{(3-2\delta_k)(\delta_i+\delta_j) - \delta_i\delta_j\}^2} < 0,$$

$$\frac{\partial V_j^\delta(\sigma)}{\partial \delta_j} = -\frac{2\delta_i (1-\delta_k)(3-\delta_i-2\delta_k)}{\{(3-2\delta_k)(\delta_i+\delta_j)-\delta_i\delta_j\}^2} < 0,$$

and

$$\frac{\partial V_k^\delta(\sigma)}{\partial \delta_k} = \frac{2(\delta_i+\delta_j-\delta_i\delta_j)(\delta_i+\delta_j)}{\{(3-2\delta_k)(\delta_i+\delta_j)-\delta_i\delta_j\}^2} > 0.$$

Q.E.D.

Lemma 15 *Suppose that* $\{i,j,k\} = N$. *Take any* $\delta \equiv (\delta_1, \delta_2, \delta_3) \in \Delta_{i,j,k}(>,=)$. *Take any honestly voting MPE* σ *in* $\eth(\delta)$. *Then,*

$$\frac{\partial V_i^\delta(\sigma)}{\partial \delta_i} = 0,$$

$$\frac{\partial V_j^\delta(\sigma)}{\partial \delta_j} < 0,$$

and

$$\frac{\partial V_k^\delta(\sigma)}{\partial \delta_k} < 0.$$

Proof. Party i's continuation value in σ, j's and k's are given by the expression (30), (31) and (32), respectively. Hence, we obtain

$$\frac{\partial V_i^\delta(\sigma)}{\partial \delta_i} = 0,$$

$$\frac{\partial V_j^\delta(\sigma)}{\partial \delta_j} = -\frac{2\delta_k(3-\delta_k)}{\{3(\delta_j+\delta_k)-\delta_j\delta_k\}^2} < 0,$$

and

$$\frac{\partial V_k^\delta(\sigma)}{\partial \delta_k} = -\frac{2\delta_j(3-\delta_j)}{\{3(\delta_j+\delta_k)-\delta_j\delta_k\}^2} < 0.$$

Q.E.D.

Lemma 16 *Suppose that* $\{i,j,k\} = N$. *Take any* $\delta \equiv (\delta_1, \delta_2, \delta_3) \in \Delta_{i,j,k}(>,>)$. *Take any honestly voting MPE* σ *in* $\eth(\delta)$. *Then,*

$$\frac{\partial V_i^\delta(\sigma)}{\partial \delta_i} = 0,$$

$$\frac{\partial V_j^\delta(\sigma)}{\partial \delta_j} > 0,$$

and

$$\frac{\partial V_k^\delta(\sigma)}{\partial \delta_k} > 0.$$

Proof. Party i's continuation value in σ, j's and k's are given by the expression (34), (35) and (36), respectively. Hence, we obtain

$$\frac{\partial V_i^\delta(\sigma)}{\partial \delta_i} = 0,$$

$$\frac{\partial V_j^\delta(\sigma)}{\partial \delta_j} = \frac{3(1-\delta_k)(3-\delta_k)}{(9-3\delta_j-6\delta_k+\delta_j\delta_k)^2} > 0,$$

and

$$\frac{\partial V_k^\delta(\sigma)}{\partial \delta_k} = \frac{(3-2\delta_j)(6-\delta_j)}{(9-3\delta_j-6\delta_k+\delta_j\delta_k)^2} > 0.$$

Q.E.D.

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