BULLETIN OF THE EARTHQUAKE
RESEARCH INSTITUTE
Vol. 43 (1965), pp. 195-203

14. *Transmission Coefficient of Seismic Waves to Structures.*

By Kiyoshi KANAI,

Earthquake Research Institute.

(Read Feb. 25 and Nov. 24, 1964.—Received Dec. 28, 1964.)

1. Introduction

From the results of the previous investigations we concluded that the vibration of a structure due to earthquake motion should be treated as the multiple reflection phenomena of waves.[1] The results also told us that the most important part of the vibrational damping of a structure is based on that at the time of an earthquake, the vibration energy of a structure dissipating into the ground again as in a wide sense the elastic waves which start from the foundation. Nevertheless, the important problem to engineers which is how much energy of seismic waves gets into a structure and dissipates into the ground again still remains to be investigated.

In order to throw light on the important question of the nature of the transmission coefficient of seismic waves from ground to structure, we carried out an observation of earthquake motions by means of installing horizontal seismographs on the foundation of a building as well as the ground surface a short distance from the building.

2. Theoretical standpoints of the present investigation

(i) Reflection and refraction of seismic waves in a stratified body.

In the first place, the problem of reflection and refraction of seismic waves in a stratified body will be discussed.

Let the axis of z be drawn vertically upwards from the lower boundary of a stratified body of thickness H and let u_2, ρ_2, μ_2; u_1, ρ_1, μ_1 be the displacements, densities and rigidities of the subjacent medium and the stratum respectively.

1) K. KANAI & S. YOSHIZAWA, "Some New Problems of Seismic Vibrations of a Structure. Part 1," *Bull. Earthq. Res. Inst.*, **41** (1963), 825-833.

K. KANAI & S. YOSHIZAWA, "ditto. Part 2," *ditto*, **42** (1964), 237-243.

In the case of distortional waves of purely plane type propagated vertically upwards, the equation of motion of the two media are expressed by $\rho_2(\partial^2 u_2/\partial t^2)=\mu_2(\partial^2 u_2/\partial z^2)$, $\rho_1(\partial^2 u_1/\partial t^2)=\mu_1(\partial^2 u_1/\partial z^2)$.
If the incident waves in the lower medium be of the type: $u_0=A_0 \exp[i(pt-f_2 z)]$, where $p=2\pi/T$, $f_2=V_2 T$, T is the period and V_2 the velocity of the subjacent medium, the reflected and refracted waves at the lower boundary, $z=0$, are expressed by $u_2=A\exp[i(pt+f_2 z)]$, $u_1=B\exp[i(pt-f_1 z)]$, where $f_1=2\pi/V_1 T$ and V_1 is the velocity of the stratum. The boundary conditions at $z=0$ are $u_0+u_2=u_1$, $\mu_2(\partial u_0/\partial z+\partial u_2/\partial z)=\mu_1(\partial u_1/\partial z)$, and substituting the solutions of motions into the boundary conditions, we get

$$\frac{A}{A_0}=\frac{1-\alpha}{1+\alpha}, \quad \frac{B}{A_0}=\frac{2}{1+\alpha}, \tag{1}$$

in which $\alpha=\rho_1 V_1/\rho_2 V_2$. While that at the free surface, $z=H$, is $\partial(u_1+u_1')/\partial z=0$, so that we get

$$B=B', \tag{2}$$

where u_1' and B' are the displacement of the reflected waves at the free surface and the amplitude of it respectively.

If the seismic waves of the type:

$$u_0=F(t) \tag{3}$$

are incident in the subjacent medium, the resultant motions at the free surface, u_s, are expressed by

$$\begin{aligned} u_s=&\frac{4}{1+\alpha}F\left(t-\frac{H}{V_1}\right)+\frac{4}{1+\alpha}\left(\frac{\alpha-1}{\alpha+1}\right)F\left(t-\frac{3H}{V_1}\right) \\ &+\frac{4}{1+\alpha}\left(\frac{\alpha-1}{\alpha+1}\right)^2 F\left(t-\frac{5H}{V_1}\right)+\cdots \\ =&\frac{4}{1+\alpha}\sum_{n=1}^{\infty}\left(\frac{\alpha-1}{\alpha+1}\right)^{n-1}F\left(t-\frac{\overline{2n+1}\,H}{V_1}\right). \end{aligned} \tag{4}$$

Then, in a special case the incident waves, $F(t)$, are sinusoidal type of finite train which synchronize with the natural period of the stratum, the ratio between the maximum amplitude at the free surface, $U_{\max}$, and amplitude of incident waves, A_0, becomes

$$\frac{U_{\max}}{A_0}=\frac{2}{\alpha}\left\{1-\left(\frac{1-\alpha}{1+\alpha}\right)^n\right\}, \tag{5}$$

where n represents the number of successive half waves and $\alpha<1$. Using equation (5), the result of the numerical calculation in the case of $\alpha=1/2$, $1/3$, $1/4$ is shown diagrammatically in Fig. 2.

(ii) The problem of the initial motion of seismic waves.

The positions of the horizontal seismographs, P and Q, and the geological formation as well as N values versus depth are shown in Fig. 3.

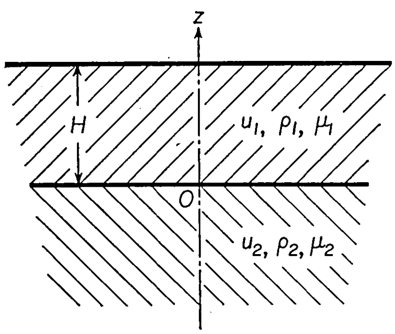

Fig. 1

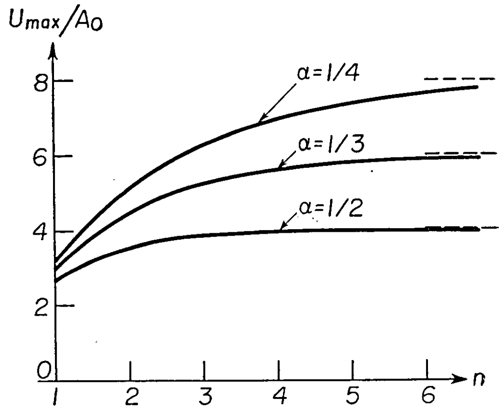

Fig. 2

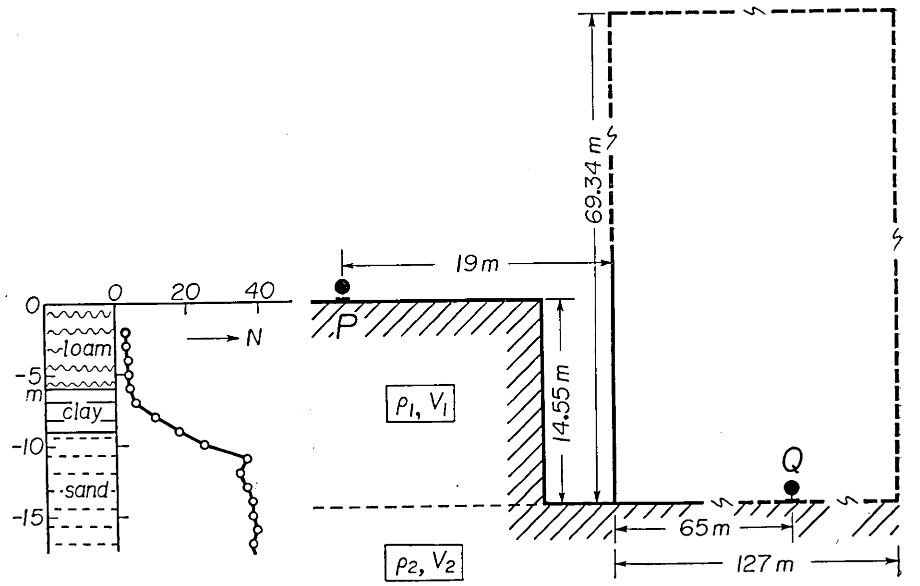

Fig. 3

From equations (1) and (2), the amplitude of the initial motion of seismic waves at P becomes as follows:

$$P_1=\frac{4a_1}{1+\alpha}, \tag{6}$$

in which a_1 is the amplitude of the initial motion of incident waves in the lower medium.

On the other hand, the amplitude of the initial motion of seismic waves at Q can be written as follows:

$$Q_1=a_1\gamma, \tag{7}$$

in which γ is the transmission coefficient of seismic waves from ground to structure. From equations (6) and (7), we obtain the following relation, that is,

$$\frac{Q_1}{P_1}=\frac{\gamma(1+\alpha)}{4}. \tag{8}$$

(iii) The problem of the maximum amplitude of seismic waves.

The problem concerning the maximum amplitude of seismic waves is not simple. As a first approximation, the maximum amplitude of seismic waves at the free surface occurs in the condition corresponding to equation (5) and takes the value as follows:

$$P_2=\frac{2a_2}{\alpha}, \tag{9}$$

in which, a_2 is the amplitude of the initial motion of incident waves in the lower medium which synchronizes with the natural period of the stratum. Because, as seen in Fig. 2, when a successive number of half waves is larger than 3, the magnification of amplitude in the stratum becomes nearly equal to the asympototic value and actual seismic waves may be considered as to hold the above condition.[2)]

On the other hand, when the height of a building is larger than the wave length of seismic waves, the maximum amplitude of seismic waves at Q may be considered as follows:

$$Q_2=a_3\gamma, \tag{10}$$

2) K. KANAI & M. SUZUKI, "Analytical Results of the Acceleration Seismograms obtained at Tokyo and Yokohama," *Bull. Earthq. Res. Inst.*, **32** (1954), 189-197.

in which a_3 is the maximum amplitude of incident waves in the lower medium. As it is not difficult to adopt practically the case in which a_2 is nearly equal to a_3, from equations (9) and (10) we get the following relation, that is,

$$\frac{Q_2}{P_2} \doteqdot \frac{\gamma\alpha}{2}. \tag{11}$$

Using equations (8) and (11), we can estimate the values of α as well as γ as follows:

$$\alpha = \frac{1}{2\left(\frac{Q_1}{P_1}\right)\left(\frac{P_2}{Q_2}\right)-1}, \tag{12}$$

$$\gamma = \frac{2}{\alpha}\left(\frac{Q_2}{P_2}\right). \tag{13}$$

3. An example of estimating the value of transmission coefficient

In order to estimate the value of transmission coefficient of seismic waves from ground to structure, we made simultaneous observations of earthquake motions at the basement of a seventeen stories and two basements steel framed reinforced concrete building and the ground surface near it. The rough sketch of the observation site is shown in Fig. 3 and the constants of the horizontal seismographs used here are presented in Table 1. (The periods of the building and of the ground of 14 m thickness are respectively 0.89 sec and 0.3 sec.)

Table 1. Constants of the horizontal seismographs

Position		Period in sec	Damping	Magnification
P	EW	1.0	8:1	120
	NS	1.0	13:1	110
Q	EW	1.0	8:1	135
	NS	1.0	14:1	120

The typical records of the simultaneous observations of earthquake motions are shown in Figs. 4 and 5.

The data of the amplitude of the initial motion of S waves and those of the maximum amplitude of earthquake motion observed in the present investigation are tabulated in Tables 2 and 3, respectively.

Substituting the mean values of two kinds of amplitude ratio presented in Tables 2 and 3 in equations (12) and (13), we obtain the values of α and γ as follows:

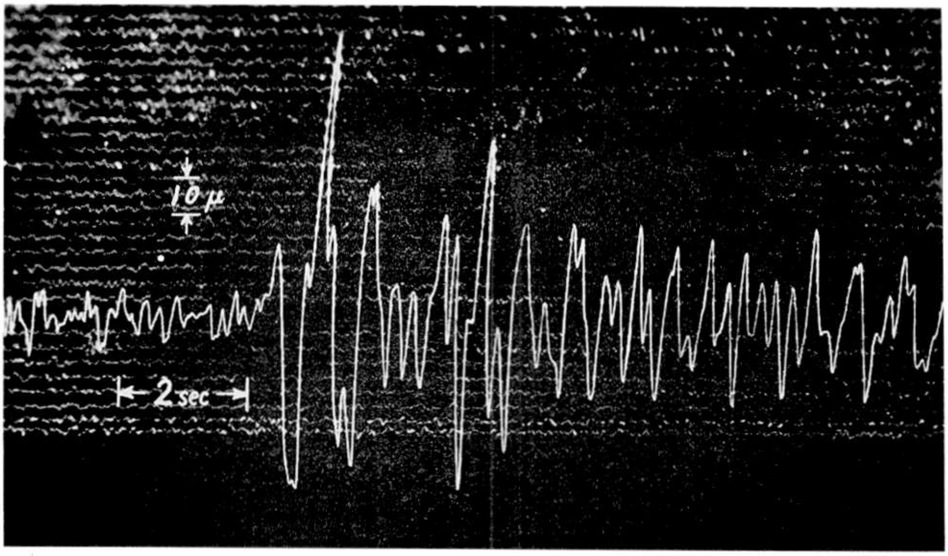

Fig. 4a. Earthquake No. 78. Seismogram observed at the ground surface

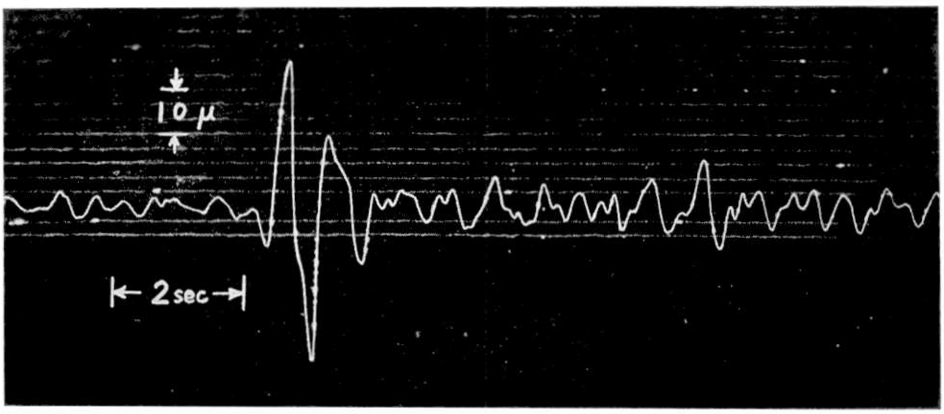

Fig. 4b. Earthquake No. 78. Seismogram observed at the basement of a building

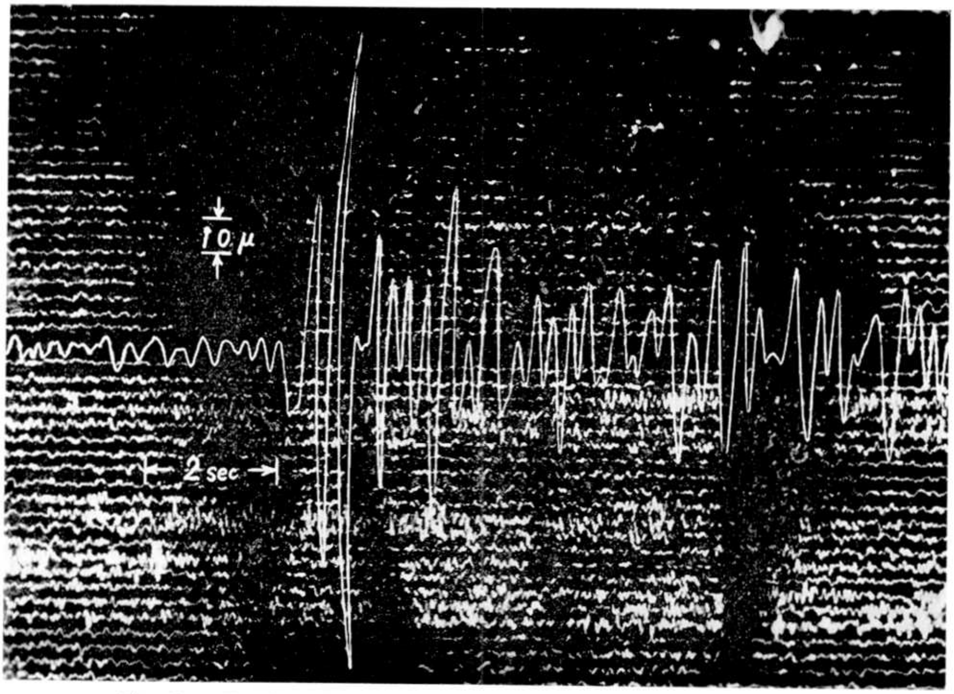

Fig. 5a. Earthquake No. 67. Seismogram observed at the ground surface

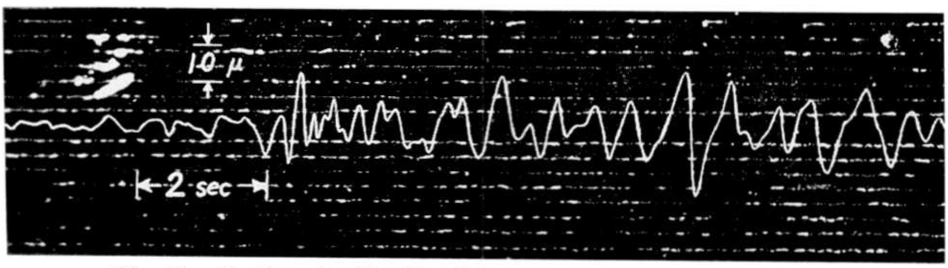

Fig. 5b. Earthquake No. 67. Seismogram observed at the basement of a building

$$\alpha=0.38, \tag{14}$$

$$\gamma=1.28. \tag{15}$$

Equation (15) means that the amplitude of the earthquake motions on the basement is 1.28 times that of the incident waves registered there, in other words, the basement amplitude is 0.64 times the earthquake motions on the outcrop of the lower medium on which the building stands.

The result of the present investigation also tells us that the effective impedance (effective rigidity × effective density) of a tall building is

Table 2. Amplitude of the initial motion of S waves

Earthq. No.	Date	Initial motion in micron				Basement/Ground	
		Basement		Ground			
		EW	NS	EW	NS	EW	NS
44	'63 Oct. 7		0.8		1.8		0.46
67	〃 〃 19		8.3		19		0.44
73	〃 〃 22	3.0		7.5		0.40	
78	〃 〃 26	8.9		21		0.43	
Mean						0.42	0.45
						0.44	

Table 3. Maximum amplitude. ( ); period in sec

Earthq. No.	Date	Maximum amplitude in micron				Basement/Ground	
		Basement		Ground			
		EW	NS	EW	NS	EW	NS
60	'63 Oct. 16	5.9 (0.3)		33 (0.3)		0.18	
72	〃 〃 21	5.9 (0.2)	9.2 (0.2)	29 (0.3)	31 (0.3)	0.20	0.30
73	〃 〃 22	16 (0.3)	20 (0.3)	96 (0.3)	70 (0.2)	0.17	0.29
Mean						0.18	0.30
						0.24	

approximately half of that of the soil on which the building stands.

In conclusion the author wishes to express his hearty thanks to the members of the Design Division of the Taisei Construction Co., Ltd. for their contribution to this investigation and also to Dr. T. Tanaka and Messrs. T. Suzuki, T. Morishita and K. Osada who took part in the observation and to Miss S. Yoshizawa who assisted him in preparing this paper.

14. 構造物への地震波の伝達係数

地震研究所 金 井 清

前回の研究結果で，地震動による構造物振動が，構造物内での波動の重複反射の現象にほかならないことと，構造物の振動減衰性は，その内部では無視できるぐらい小さくて，ほとんどの部分が，基礎と土地との境界面で起ることが立証された．

それにしても，構造物内に生ずる歪量は，構造物下に到達した地震波のエネルギーがどれだけ構造物内に伝達されるかということに支配されるものであり，また，他方において，一度構造物内に入つたエネルギーがどれだけ基礎部分から地中に逸散するかということにも深い関係をもつものである．

本研究では，構造物へ入る地震波のエネルギーを推定する方法を数理的にしらべ，実際の地震観測の結果に適用してみた．その結果，この建物については，構造物基礎部分の振動は，入射波のそれの 1.28 倍であり，言い換えると，構造物基礎部分の振動は，その基礎附近の土層が露頭になつている時の振動の 0.64 倍であることがわかつた．