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16. *A Study of Strong Earthquake Motions.*

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1. Introduction

The following empirical formula[1] of the amplitude-period relation of earthquake motions has been obtained from the statistical treatment of the results of spectral analysis of a large number of seismograms which had been recorded deep in the ground where the influence of the vibration characteristics of the ground is very slight. That is

$$A_{m\cdot s}=53T_m^{2.56} \qquad (1)$$

in which $A_{m\cdot s}$ and T_m represent, respectively, the maximum value of displacement spectrum at the spot of 100 km hypocentral distance in micron and the corresponding period in sec. On the other hand, from the previous paper[2] and others, the amplitude-period relation of seismic waves excepting considerably short and long periods can be assumed as follows:

$$\frac{2\pi A}{T}(\equiv\text{velocity})=\text{constant} \qquad (2)$$

in which A and T represent each amplitude and period of seismic waves, respectively. This means that seismic waves of a considerably wide range of period satisfy the nature of energy equipartition. Fig. 1 shows the schematic figures of the above mentioned relation.

In the present investigation, we assume that, concerning an earthquake, A_m in Fig. 1 depends on epicentral distance while T_m in the same figure is independent of epicentral distance. That is to say, the value of A_m at a spot of 100 km epicentral distance corresponds to $A_{m\cdot s}$ in (1).

In this paper, we deal with the relation of the magnitude of an

1) K. KANAI and S. YOSHIZAWA, "The Amplitude and the Period of Earthquake Motions. II", *Bull. Earthq. Res. Inst.*, **36** (1958), 275.

2) *ditto* 1).

earthquake to its period by using the relations expressed by (1) and (2), and we try to find whether or not we can expect these studied results to provide information about waves in destructive earthquakes.

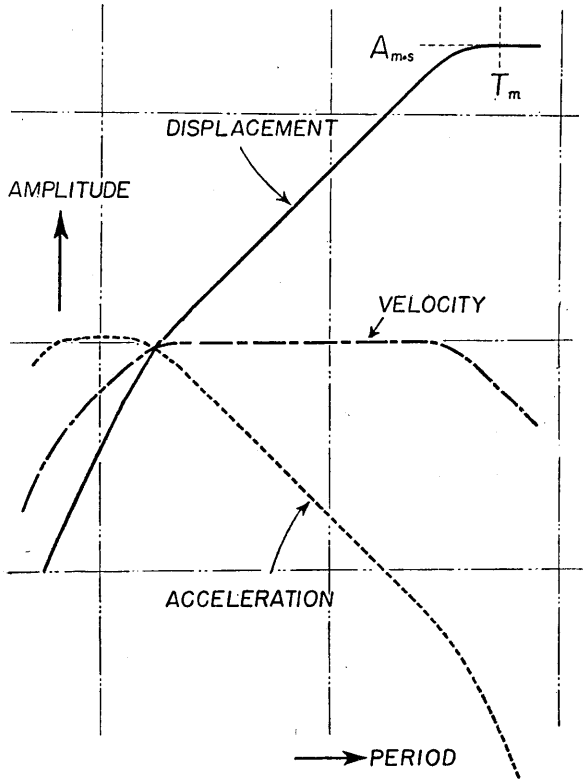

Fig. 1.

2. The case of the Kwanto earthquake of 1923

There are some records of the destructive earthquakes, including that of the Kwanto earthquake of 1923, in Japan, but none of them was successful. They are either incomplete or unreliable.

Now, we try to examine what characteristics of the waves may be expected in an earthquake motion having the same characteristics as the Kwanto earthquake, by using the relations of (1) and (2), and compare the results mentioned above with the analytical results obtained from the incomplete seismogram of the Kwanto earthquake which was recorded at Tokyo.

In the present case, the following formula as the expression of the relation among magnitude M, epicentral distance Δ and maximum amplitude A is adopted. That is

$$\log A_m = M - 1.73 \log \Delta + 0.83^{3)} \quad (3)$$

in which, Δ and A_m are measured in km and micron, respectively. Equating the common logarithm of (1) to the special case of (3) in which $\Delta = 100$ km, we get

$$\log T_m = 0.39M - 1.70 \,. \quad (4)$$

From (3) and (4), we arrive at the relation,

$$\log \frac{A_m}{T_m} = 0.61M - 1.73 \log \Delta + 2.53 \,. \quad (5)$$

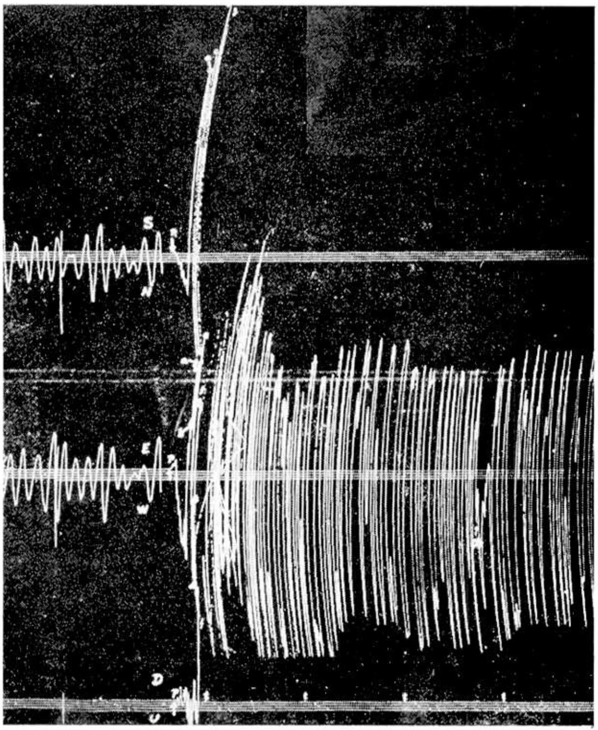

Fig. 2. Seismogram of the Kwanto earthquake.

If equipartition of energy is applicable to the seismic waves, that is to say,

$$\frac{A_m}{T_m}\left(\equiv \frac{A}{T}\right) = \text{constant} \quad (6)$$

amplitude A measured in cm which corresponds to each period T of seismic waves can be expressed by

$$A = 10^{0.61M - 1.73 \log \Delta - 1.47} T \,. \quad (7)$$

And then, acceleration α measured in gal can be written as follows:

$$\alpha = \frac{(2\pi)^2 \cdot 10^{0.61M - 1.73 \log \Delta - 1.47}}{T} \,. \quad (8)$$

3) C. TSUBOI, "Determination of the Gutenberg-Richter Magnitude of Earthquakes occuring in and near Japan", *Journ. Seism. Soc. Japan*, **[ii]**, **7**, No. 3 (1954), 185.

By means of (7) and (8), the values of displacement and acceleration in the cases, $T=0.3$ sec and 1.35 sec, besides the conditions $M=7.9$ and $\Delta=90$, which correspond to the values in Tokyo at the time of the Kwanto earthquake, are calculated. (The periods of 1.35 sec and 0.3 sec have been demonstrated to be the predominant waves of the Kwanto earthquake in Tokyo by Imamura[4] and Ishimoto[5], respectively.) The results of them are as follows:

$$\left.\begin{array}{lll} T=1.35\text{ sec}; & A=1.23\text{ cm}, & \alpha=27\text{ gal}, \\ T=0.3\text{ sec}; & A=0.27\text{ cm}, & \alpha=120\text{ gal}. \end{array}\right\} \quad (9)$$

On the other hand, the values estimated from seismogram by Imamura and Ishimoto are as follows:

$$\left.\begin{array}{lll} T=1.35\text{ sec}; & A=4.43\text{ cm}, & \alpha=100\text{ gal}, \\ T=0.3\text{ sec}; & A=0.6\sim0.8\text{ cm}, & \alpha=250\sim300\text{ gal}. \end{array}\right\} \quad (10)$$

Therefore, (9) and (10) tell us that the values estimated from seismograms are 2.5∼4 times larger than those derived from the empirical formula. Then it may be said that the calculated values are in comparative agreement with the estimated values as taking into consideration the vibration characteristics of the ground[6], because (1) has been obtained from the seismograms at −300 m underground.

3. The case of strong earthquakes in U.S.A.

In this chapter, we compare the values of the strong motion seismograms obtained in U.S.A. as well as the results of their spectral analysis with the values derived from the empirical formulae.

Now, magnitude M, epicentral distance of the observation places Δ and other factors of the earthquake treated here are shown in Table 1. (Among the earthquakes of which the spectral analysis has been carried out[7] all of the earthquakes whose magnitude has been presented[8]

4) A. IMAMURA, "Report of the Investigation of the Kwanto Earthquake", *Rep. Imper. Earthq. Inv. Comm.*, No. 100 A (1925), 27, (in Japanese).

5) M. ISHIMOTO, Kokin-Shoin, Tokyo, (1925), 113, (in Japanese).

6) K. KANAI, "Semi-empirical Formula for the Seismic Characteristics of the Ground", *Bull. Earthq. Res. Inst.*, **35** (1957), 309.

7) J. L. ALFORD, G. W. HOUSNER and R. R. MARTEL, "Spectrum Analysis of Strong-Motion Earthquakes", *1st Tech. Rep. Office Nav. Res., Cotr. N6onr-244, Task Order 25, Project Design.* NR-081-095 (1951).

8) B. GUTENBERG and C. F. RICHTER, "Seismicity of the Earth", *Princeton University Press* (1954).

Table 1.

No.	Date	Epicenter	M	Location	Epicentral distance in km (Δ)
1	X, 3, '41	$40\frac{3}{4}$N, 125W	6.4	Ferndale, Calif.	67
2	II, 9, '41	$40\frac{1}{2}$N, $125\frac{1}{4}$W	6.6	〃	86
3	IX, 12, '38	$40\frac{1}{4}$N, 125W	5.5	〃	65
4	XII, 30, '34	$32\frac{1}{4}$N, $115\frac{1}{2}$W	6.5	El Centro, Calif.	61
5	V, 19, '40	32.7N, $115\frac{1}{2}$W	6.7	〃	22
6	III, 11, '33	33.6N, 118W	6.25	Los Angeles, Subway	55
				Vernon, Calif.	48
7	X, 2, '33	33.8N, 118.1W	5.4	Los Angeles, Subway	37
				Vernon, Calif.	31

are treated here.)

In this case, we adopt the following formula as the expression of the relation among magnitude M, epicentral distance Δ and maximum vibration amplitude A_m. That is

$$\log A_m = M - 3\log\Delta + 2.92 \text{ [9]} \tag{11}$$

where Δ and A_m measured in km and micron, respectively. Equating the common logarithm of (1) to the special case of (11) in which $\Delta =$ 100 km, we get

$$\log T_m = 0.39M - 1.88 \; . \tag{12}$$

From (11) and (12), we arrive at the relation,

$$\log\frac{A_m}{T_m} = 0.61M - 3\log\Delta + 4.80 \; . \tag{13}$$

If equipartition of energy is applicable to the seismic waves, amplitude A measured in cm which corresponds to each period T of seismic waves can be expressed by

$$A = 10^{0.61M - 3\log\Delta + 0.80} \cdot T \; . \tag{14}$$

9) B. GUTENBERG and C. F. RICHTER, "Earthquake Magnitude, Intensity, Energy and Acceleration", *Bull. Seism. Soc. Amer.*, **32** (1942), 163.

And then, acceleration α measured in gal can be written as follows:

$$\alpha=\frac{(2\pi)^2\cdot 10^{0.61M-3\log\Delta+0.80}}{T}. \tag{15}$$

Substituting the values of M and Δ of Table 1 in (15), the accelerations correspond to the periods of Column 3 in Table 2 are calculated. The results of them are shown in Column 5 in Table 2.

Table 2.

	Observation			Calculation	
No.	Maximum acceleration in gal	Predominant period in sec	Ratio of maximum velocity to constant velocity	Acceleration corresponding to τ in gal	α_0/α_c
	α_0	τ	m	α_c	
1	120	0.3	4	22	5
2	70	0.3	4	14	5
3	90	0.2	4	10	9
4	140	0.5	4	20	7
5	300	0.5	4?	300	1
6	50	0.65	2	15	3
6′	150	0.3	2?	41	4
7	50	0.6	2	16	3
7′	110	0.35	3	47	2

It will be seen from Table 2 that the values of the maximum acceleration observed are several times larger than those calculated. The reason of the difference mentioned above can be explained in the same way pointed out in the former chapter, that is, equation (1) has been obtained from the seismograms at -300 m underground and the seismograms treated here are the records of the motions magnified due to the vibration characteristics of ground. If the above explanation is true, the value of α_0/α_c (α_0=observed acceleration, α_c=calculated acceleration) in any earthquake at a place must be definite.

Hereafter, we examine the values of α_0/α_c shown in Table 2 in order to know whether the above explanation is true or not.

The full lines in Figs. 3∼11 show the velocity spectra in the cases of zero damping of the analyzer[10]. It will be seen from Figs. 3∼11

10) *loc. cit.*, 7).

that, in many cases, every velocity takes comparatively the same value throughout every period excepting around period τ (τ is the period correspond to the maximum acceleration.) That is to say, it may be said that the strong earthquake motion at bed rock satisfies the energy equipartition, and at ground surface the amplitude is magnified considerably according to the vibration characteristics of the ground. (Only for reference, the acceleration spectra are shown in Figs. 12~20.) The values of magnification in Column 4 in Table 2 represent the ratios between velocity corresponding to period τ and assumed constant velocity which is represented by broken lines in Figs. 3~11. In each case, the values in Columns 4 and 6 in Table 2 seem somewhat different but the calculated values can be corrected easily by adjusting epicentral distance.

(i) In the cases of Ferndale, California, the values of m in Nos. 1, 2 and 3 and α_0/α_c in Nos. 1 and 2 are nearly equal but α_0/α_c in No. 3 is different from these five. If we adopt 54 km as the value of epicentral distance Δ instead of 65 km in No. 3, the value of α_0/α_c becomes equal to those of Nos. 1 and 2.

(ii) In the cases of El Centro, California, there is a great difference between the values of α_0/α_c as shown in Nos. 4 and 5. As the reason for this, two cases may be considered; one is that the wave forms are complicated as the observation position is near to the point of origin and the other is that the error of calculation of α_c becomes large as the hypocentral distance is small. In No. 4, if we adopt 50 km as the value of Δ instead of 65 km, α_0/α_c becomes nearly equal to m.

(iii) In the case of Los Angeles Subway Terminal and Vernon, California, the values of α_0/α_c as well as m in Nos. 6 and 7 are nearly equal. At any rate, it is a noteworthy result that the amplitude of earthquake motion seems to become maximum at a proper period of each observation station.

4. Conclusions

From the present investigation, we knew that the following empirical formula seems to be applicable to destructive earthquakes. That is

$$A_{m \cdot s} = 53 T_m^{2.56} \,. \qquad (1)$$

At the same time, the present results tell us that, in the case of destructive earthquakes as well, equipartition of energy is recognized in regard to the waves at bed rock, and there is the considerable modification of wave amplitudes according to the vibration characteristics of the ground.

If we assume (i) magnitude of earthquake, (ii) epicentral distance from engineering judgement according to the results of statistical studies of seismicity for (iii) space as well as (iv) time, we can obtain the spectrum of seismic waves at bed rock. And if, (v) vibration characteristics of the ground are added to them, the seismic factor of the ground for earthquake engineering can be estimated resonably.

At any rate, the results of the present investigations show promise of giving information about the waves to be expected in destructive earthquakes, by means of the general application of the many results of investigations concerning common earthquakes.

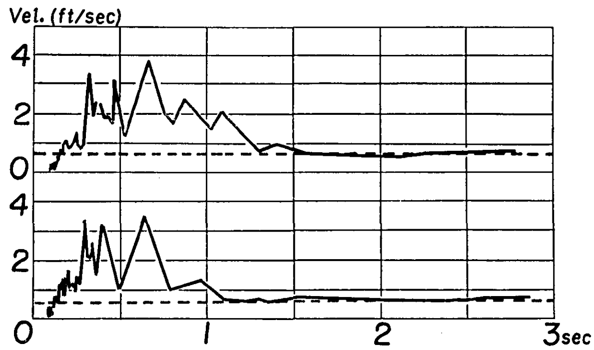

Fig. 3. Velocity spectra for Ferndale, California; earthquake of Oct. 3, 1941. Components: N 45 E (lower), S 45 E (upper).

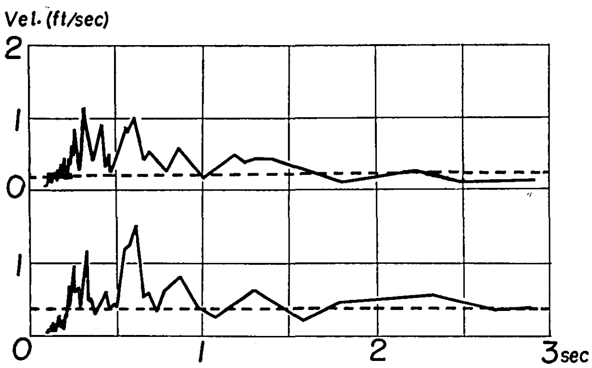

Fig. 4. Velocity spectra for Ferndale, California; earthquake of Feb. 9, 1941. Components: N 45 E (lower), S 45 E (upper).

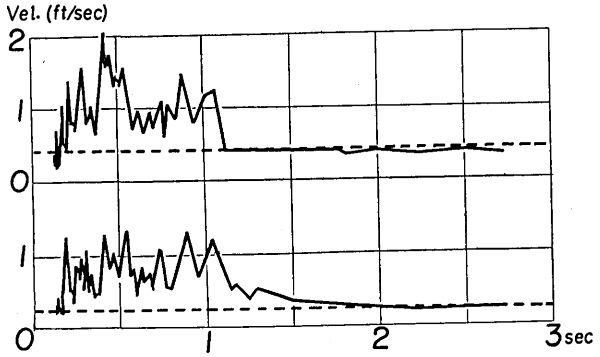

Fig. 5. Velocity spectra for Ferndale, California; earthquake of Sept. 11, 1938. Components: N 45 E (lower), S 45 E (upper).

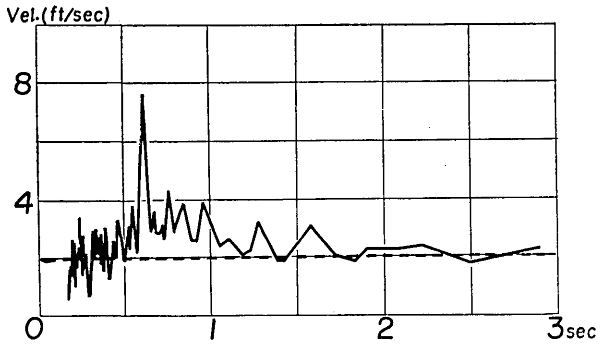

Fig. 6a. Velocity spectrum for El Centro, California; earthquake of Dec. 30, 1934. Component N–S.

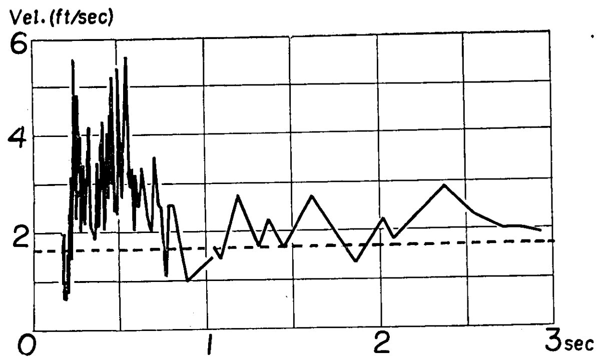

Fig. 6b. Velocity spectrum for El Centro, California; earthquake of Dec. 30, 1934. Component E–W.

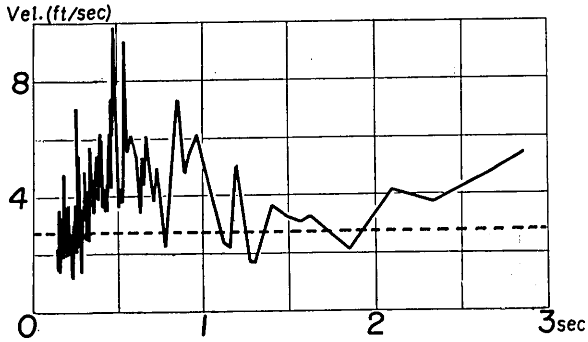

Fig. 7a. Velocity spectrum for El Centro, California; earthquake of May 18, 1940. Component N-S.

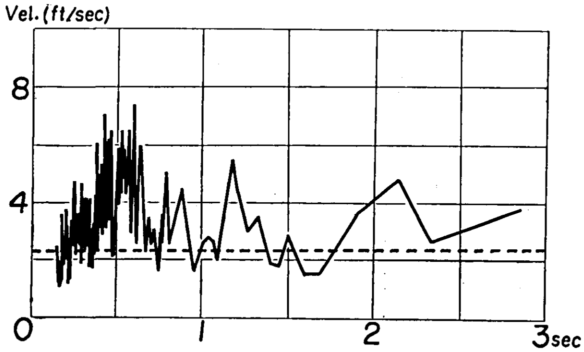

Fig. 7b. Velocity spectrum for El Centro, California; earthquake of May 18, 1940. Component E-W.

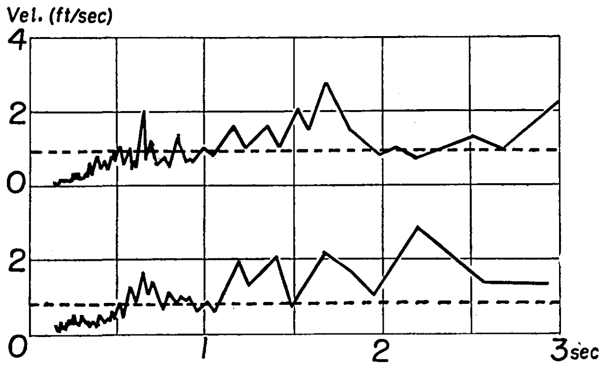

Fig. 8. Velocity spectra for Los Angeles Subway Terminal; earthquake of March 10, 1933. Components: N 39 E (lower), N 51 W (upper).

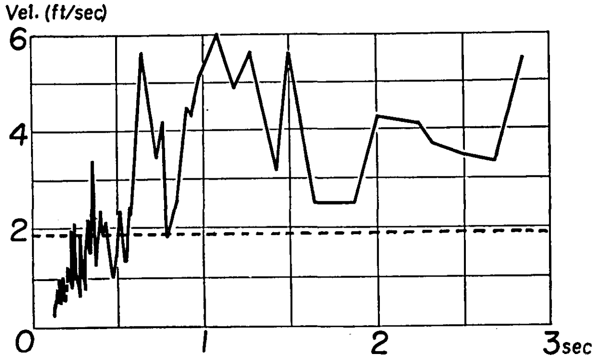

Fig. 9a. Velocity spectrum for Vernon, California; earthquake of March 10, 1933. Component N 08 E.

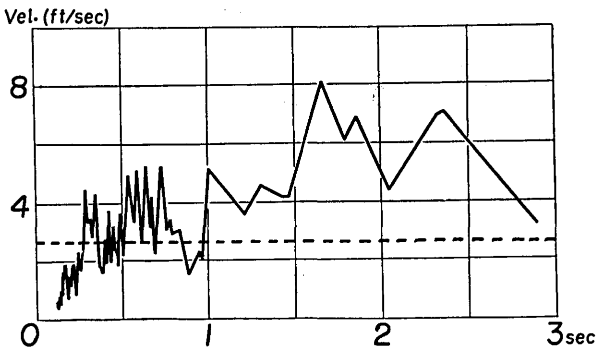

Fig. 9b. Velocity spectrum for Vernon, California; earthquake of March 10, 1933. Component S 82 E.

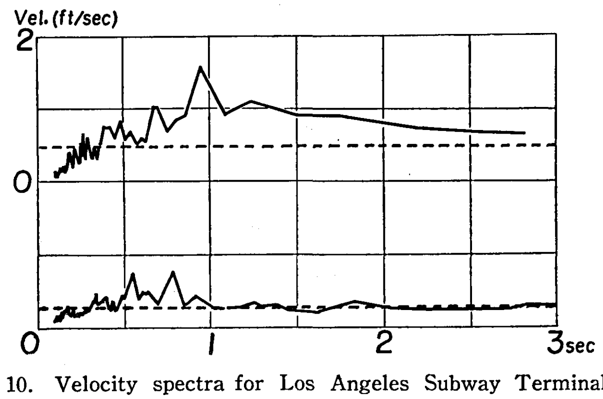

Fig. 10. Velocity spectra for Los Angeles Subway Terminal; earthquake of Oct. 2, 1933. Components: N 39 E (lower), N 51 W (upper).

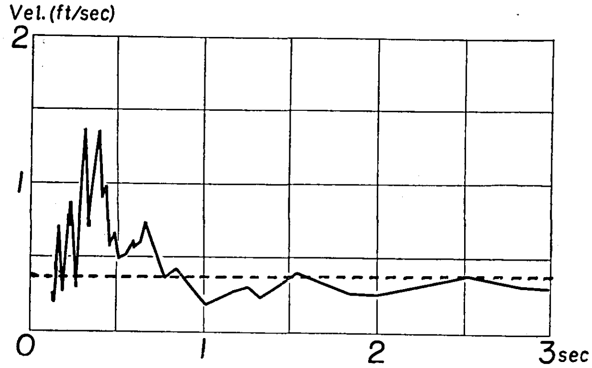

Fig. 11a. Velocity spectrum for Vernon, California; earthquake of Oct. 2, 1933. Component N 08 E.

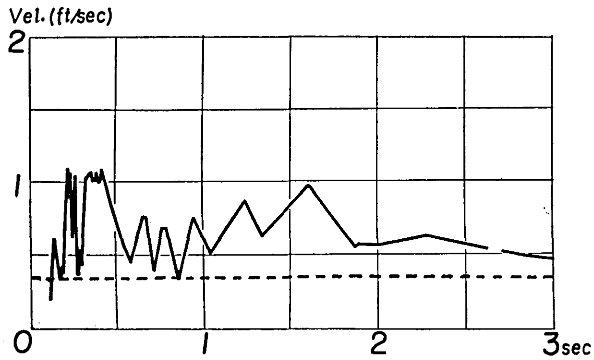

Fig. 11b. Velocity spectrum for Vernon California; earthquake of Oct. 2, 1933. Component S 82 E.

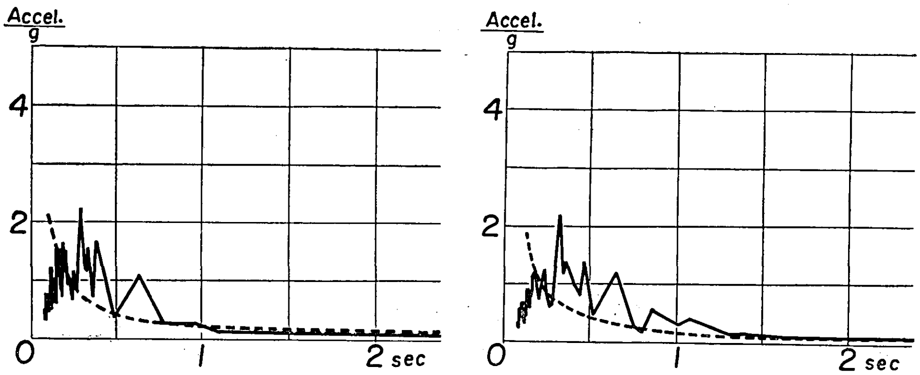

Fig. 12. Acceleration spectra for Ferndale, California; earthquake of Oct. 3, 1941. Components: N 45 E (left), S 45 E (right).

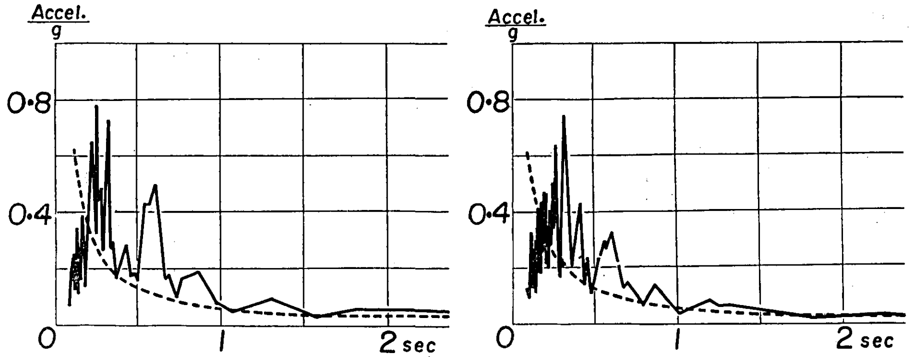

Fig. 13. Acceleration spectra for Ferndale, California; earthquake of Feb. 9, 1941. Components: N 45 E (left), S 45 E (right).

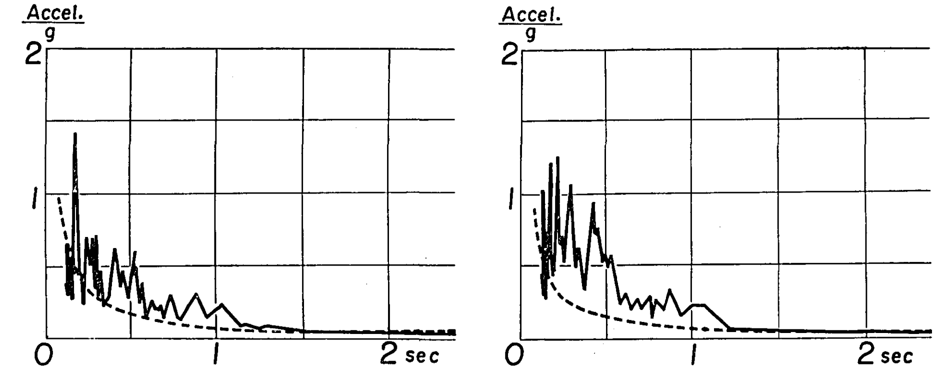

Fig. 14. Acceleration spectra for Ferndale, California; earthquake of Sept. 11, 1938. Components: N 45 E (left), S 45 E (right).

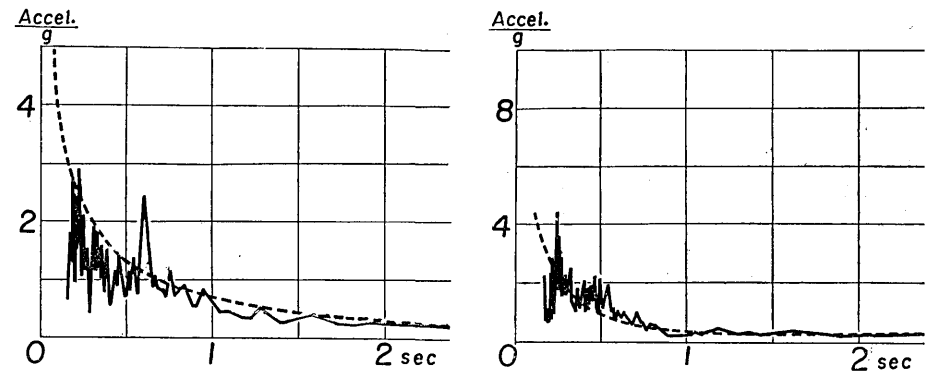

Fig. 15. Acceleration spectra for El Centro, California; earthquake of Dec. 30, 1934. Components: N-S (left), E-W (right).

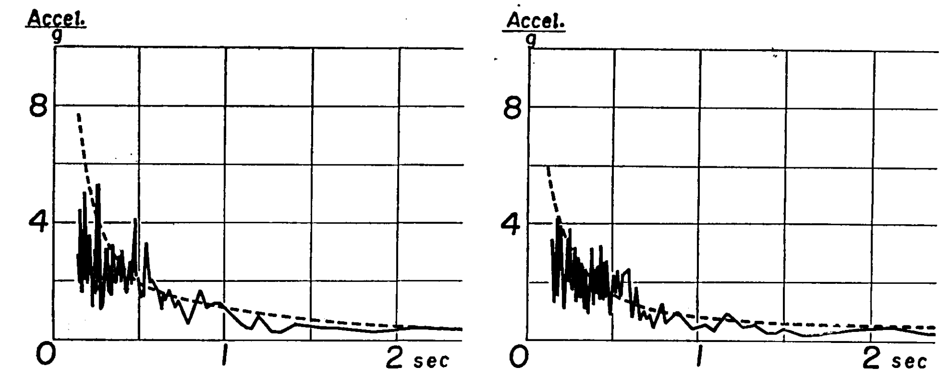

Fig. 16. Acceleration spectra for El Centro, California; earthquake of May 18, 1940. Components: N–S (left), E–W (right).

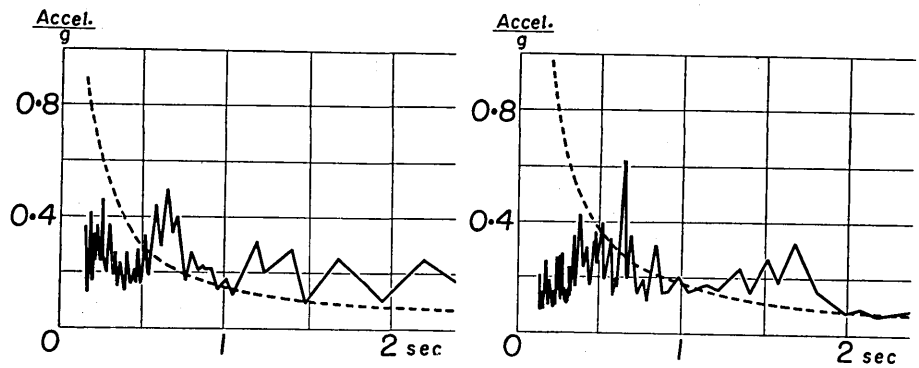

Fig. 17. Acceleration spectra for Los Angeles Subway Terminal; earthquake of March 10, 1933. Components: N 39 E (left), N 51 W (right).

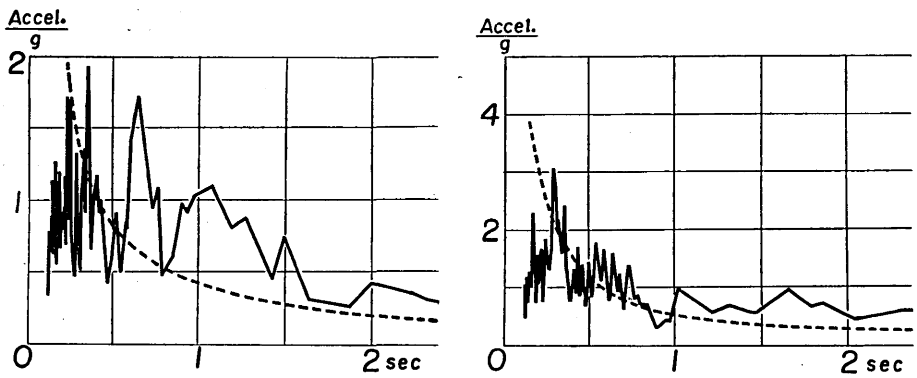

Fig. 18. Acceleration spectra for Vernon, California; earthquake of March 10, 1933 Components: N 08 E (left), S 82 E (right).

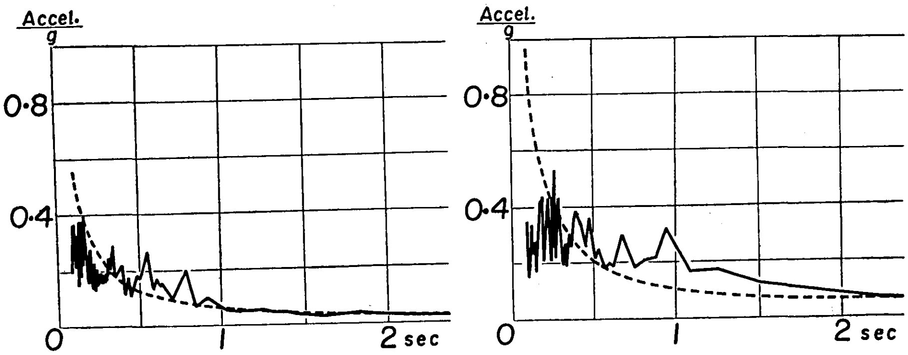

Fig. 19. Acceleration spectra for Los Angeles Subway Terminal; earthquake of Oct. 2, 1933. Components: N 39 E (left), N 51 W (right).

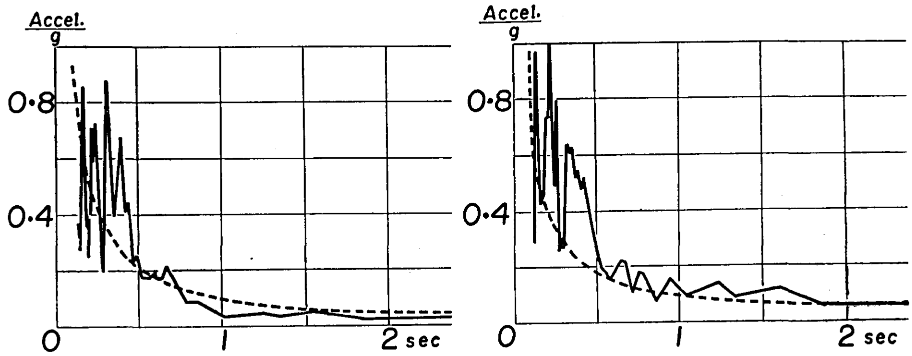

Fig. 20. Acceleration spectra for Vernon, California; earthquake of Oct. 2, 1933. Components: N 08 E (left), S 82 E (right).

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16. 強震動に関する 1 つの考察

地震研究所 金 井 清

前論文において，地震動の周期と振幅の関係について，次の関係式を得た．即ち，

$$A_{m \cdot s}=53T_m^{2.56}\,, \qquad (1)$$

$$\frac{2\pi A}{T}(\equiv V)=一定 \qquad (2)$$

ここに，A は振幅，Tは周期，V は速度，$A_{m \cdot s}$ は震央距離 100 km における変位スペクトルの最大値(ミクロン)，T_m はその周期である．

本研究では，普通の大きさの地震で得られた，これらの実験式が，はたして構造物に破壊をもたらすような地震動にもあてはまるかどうかをしらべたものである．

研究の方法は，先づ，震央距離，変位振幅とマグニチュードとの関係式として，坪井並びにグーテンベルグのものをとり，これらの式と (1), (2) の式を結び付けて，新しい実験式を導き出した．即ち，坪井式と (1), (2) から，変位振幅 A (cm)，加速度振幅 α (gal) は，

$$A=10^{0.61M-1.73\log\varDelta-1.47}T, \qquad (7)$$

$$\alpha=\frac{(2\pi)^2 10^{0.61M-1.73\log\varDelta-1.47}}{T} \qquad (8)$$

となり，グーテンベルグ式と (1), (2) からは

$$A=10^{0.61M-3\log\varDelta+0.80}T, \qquad (14)$$

$$\alpha=\frac{(2\pi)^2 10^{0.61M-3\log\varDelta+0.80}}{T} \qquad (15)$$

となつた．

次に，関東地震の東京に於ける地震記象から推定された値及びアメリカ合衆国で得られた強震記録並びにその解析結果と (7), (8) 及び (14), (15) を使つた計算結果とを比較した．その結果は，地盤による震動の増幅を考慮に入れると，計算値と実測値が相当によく合うことを示した．

要するに，本研究によつて，普通地震に関する数多い研究結果から強震動の性質を推定することが，無意味でないということがわかつたわけである．