Essays in Oligopoly with Endogenous Sequencing

Toshihiro Matsumura

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Chapter 1

Overview

1.1 Introduction

Cournot, Bertrand, and Stackelberg models have occupied important positions among models of oligopoly. These models have been used in many situations and subjected to intensive discussion.

The familiar Cournot (resp. Bertrand) equilibrium is the Nash equilibrium of the standard Cournot (resp. Bertrand) game where each firm chooses output (resp. price) simultaneously and independently, and the well known Stackelberg equilibrium is the subgame perfect Nash equilibrium of the Stackelberg duopoly game where each firm chooses output or price sequentially. Since both Nash and subgame perfect Nash equilibria are very natural solution concepts, the Cournot, Bertrand, and Stackelberg equilibria are also very natural solutions for each model. This is one of the main reasons why many other stories of oligopoly lacking rigorous game-theoretic foundations, such as the story of conjectural variations in static models, or a symmetric 'leader-leader' equilibrium in Stackelberg-type competition, rapidly lost their influence in the modern theory of industrial organization,[1]

[1] For the story of conjectural variations, see, e.g., Bowley [9] and Frisch [27]. For the story of the 'leader-leader' equilibrium, see Stackelberg [79] and Leontief [42]. For criticism of these stories, see, among others, Fellner [28], Dowrick [24], and Lindh [43]. Regarding conjectural variations, there have been attempts to justify the story through the use of dynamic models. See e.g., Dockner [23] and Cabral [15].

while the Cournot, Bertrand and Stackelberg models are still used as standard models of oligopoly.

Obviously, each of the three models produces a different equilibrium outcome. The equilibrium outcome crucially depends on the sequence of each firm's choice and on strategic variables in the model. This suggests the following question: which model among the three is most appropriate for the analysis of oligopolistic markets? There are two problems. The first problem concerns the strategic valuables (price of output). The second problem concerns the timing (simultaneous-move or sequential-move). This dissertation focuses on the second problem.[2]

Needless to say, the answer depends heavily on the situation under investigation. For example, considering competition between an incumbent firm and a potential new entrant, it is natural to use Stackelberg-type models.[3] The famous model of competition between fishermen (see, e.g., Friedman [30]) is a good example of Cournot-type competition. However, there are many situations where it is not obvious which of the simultaneous-move or sequential-move models is appropriate. In many economic situations, it is more reasonable to assume that firms choose not only what actions to take, but also when to take them. In such situations we must consider whether a simultaneous-move model such as the Cournot model or a sequential-move model such as the Stackelberg model is more suitable for analysis of oligopoly markets.

This problem is not confined to the problem of oligopoly. For example, consider the problem of voluntary provisions of international public goods by each country. This problem was discussed intensively following the Gulf-War in 1991. In most models it is assumed that each country chooses its contributions simultaneously and independently. In fact, each

[2] Many papers discussed the first problem. See e.g., Kreps and Scheinkman [40]. See also Ono [61] and Boyer and Moreaux [10].

[3] See, e.g., Spence [76] and Dixit [20, 21].

country can choose either to announce its contribution as soon as possible, or to wait and observe others' decisions. The same problem arises within a number of situations, such as competition between political parties, or judicial disputes between plaintiffs and defendants in civil affairs.[4] In such situations we should also consider whether a simultaneous-move or a sequential-move model is more appropriate.

A number of economists have investigated this problem of endogenous timing.[5] Most existing models emphasize that Stackelberg outcomes (equilibrium outcomes in a sequential-move game) appear in endogenous timing games but a Cournot or Bertrand outcome (an equilibrium outcome in a simultaneous-move game) does not [See, e.g., Hamilton and Slutsky [37],[6] Robson [66], Anderson and Engers [4], Albæk [2], and Mailath [44]]. These papers suggest that the Stackelberg model is plausible if firms can choose their actions over more than one period, and that the Cournot (or the Bertrand) model is not viable if we consider endogenous timing in duopoly.[7]

Here we take a close look at the model of Hamilton and Slutsky [37], which they call an 'extended game with action commitment'. This model is considered a standard model with endogenous sequencing. The game runs as follows.

In period 1, each duopolist chooses whether to take its action in this period or to wait until the next period. At the beginning of period 2, each firm knows its rival's action. A firm acts in period 2 if it elected to wait in period 1. The payoff for each firm depends on

[4] For judicial dispute, see, among others, Priest [64] and Rubin [69].

[5] von Stackelberg [79] considered this problem in the context of quantity-setting duopoly. He stated that each duopolist wants to be a leader and "cut-throat" competition breaks out where each tries to behave as a leader; thus the Stackelberg equilibrium may be unstable. Leontief [42] formulated the concept of the 'leader-leader' equilibrium by using the above discussion of Stackelberg. Dowrick [24] formally showed that such competition never appears in equilibrium.

[6] We note particularly the extended game with action commitment, which is considered a standard game with endogenous sequencing.

[7] Except for the endogenous timing approach, there are some approaches insisting on the advantage of the Stackelberg over the Cournot. See, among others, Boyer and Moreaux [10], Robson [65] and Basu [5]. Boyer and Moreaux [10] and Robson [65] emphasized that the sequential-move game always has a pure strategy equilibrium while the simultaneous-move game does not.

its and its rival's actions only. (From here on we call this model the H-S model.)

In the H-S model there are four possible outcomes. These are as follows:

(1) firm 1 acts in period 1 and firm 2 acts in period 2 (Stackelberg type);

(2) firm 2 acts in period 1 and firm 1 acts in period 2 (Stackelberg type);

(3) both firms act in period 1 (Cournot or Bertrand type); and

(4) both firms act in period 2 (Cournot or Bertrand type).

They found that three cases ((1)–(3)) are supported as subgame perfect equilibria (Theorem VII). They also found that the outcome of case (3) is weak in the sense that it is supported by weakly dominated strategies, while the outcomes of cases (1) and (2) are not. The Cournot-type (resp. Bertrand-type) outcome is supported only if both firms take Cournot-type (Bertrand-type) actions in period 1, and taking a Cournot-type action in period 1 is weakly dominated by the strategy of waiting until period 2. Thus, the Cournot (Bertrand) outcome is supported by weakly dominated strategies (Theorem VIII).

Since the simultaneous-move outcome is weak, it is not surprising that slight modifications of the H-S model eliminate the Cournot-type or the Bertrand-type equilibrium. Albæk [2] extended the H-S model to an incomplete information game and found that Stackelberg-type outcomes appear in equilibrium but a Cournot-type does not. Robson [66] and Matsumura [48] showed that if small inventory costs are introduced, the equilibrium outcome is of Stackelberg-type only. Anderson and Engers [4] and Mailath [44] discussed models in which only firm 1 has the choice between moving in period 1 or in period 2, and firm 2 is forced to move in period 2.[8] They also found that the Stackelberg-type outcome appears.[9]

[8]Remember that in the H-S model the case (4) never becomes an equilibrium outcome. In their models the case (3) never appears because firm 2 cannot act in period 1.

[9]Here we must emphasize that the models discussed here are not simple extensions of the H-S model. For example, Robson [66] formulated a model with infinite earlier periods, Anderson and Engers [4] and Matsumura [48] formulated n-firm models rather than duopoly models, Mailath [44] formulated an incomplete information game allowing us to consider signalling effects of the Stackelberg leader's action. All we

Existing works have emphasized that the Stackelberg model is more plausible than the Cournot or Bertrand models. The exception is Saloner [71]. The above models permitted firms to choose their actions in one of two or more periods only. On the other hand, Saloner discussed a two-player, two-period model in which duopolists choose their actions over two periods and can take actions in both periods. He modified the Cournot model by allowing for two production periods before the market clears. His model is as follows:

In period 1, each duopolist simultaneously chooses a non-negative output level. At the beginning of period 1 each firm observes its rival's output. In period 2, each firm produces additional (non-negative) output. At the end of period 2, the market opens and the firms sell the total output produced in periods 1 and 2. The payoff for each firm depends on its and its rival's total outputs only.

He showed that many outcomes including both Cournot and Stackelberg outcomes, are found as subgame perfect equilibrium outcomes. His model is different from others discussed above in the sense that he allows a firm taking its action in period 1 to also act in period 2. In his model each firm can increase but not decrease its total output in period 2, while in the H-S model each firm producing in period 1 can neither increase nor decrease its output in period 2. In the H-S model a firm which takes its action in period 1 can completely commit itself to not changing its action. On the other hand, in the Saloner model each firm can commit itself to not decreasing its output, but cannot commit to not increasing its output. In this sense Saloner explicitly considered the incompleteness of commitment.

The incompleteness of commitment is more important in the context of price-setting competition. In the price-setting Stackelberg model as well as the H-S model, the leader can commit to the price which is determined before the follower chooses the price. Consider the following situation: in period 1 firm 1 names its price p_1 and after observing p_1, firm

want to state here is that a special case of each model above becomes an extension of the H-S model.

2 names its price p_2. At the end of period 2 the market opens. This formulation invokes a question: how can firm 1 commit to its price p_1 before the market opens? Is it impossible for firm 1 to change p_1 in period 2? Why is the leader unable to change their actions in period 2 despite the fact that it has enough time to change its price before the market opens? In fact it is an everyday occurrence for firms to change their price. This question makes us doubt the applicability of the Stackelberg model in the context of price-setting competition.

Needless to say, in some situations, the firm may be able to commit itself to its action, so the H-S model and other models discussed above remain important. However, in many economic situations each firm can change its action over periods (e.g., can increase its output). Therefore, the Saloner model is also quite realistic in many situations, and is at least as important as the H-S model.

The Saloner model, however, seems restrictive. First, his model is limited to quantity-setting competition. It is difficult to apply his results to price-setting competition. For example, it is very difficult to rationalize the assumption that each firm can raise its price but cannot discount it. More importantly, he considered the case of strategic substitutes and his result is crucially dependent on this assumption.[10] This assumption is not natural in the context of price-competition.[11]

Second, in his model output produced in periods 1 and 2 is perfectly substitutable for the firm and no inventory costs are incurred. As discussed above, the H-S model as well as the Saloner model has a Cournot-type equilibrium. In the H-S model, however, the

[10] For the concept of strategic substitutes and complements, see Bulow, Geanakoplos and Klemperer [13]. See also Fudenberg and Tirole [31].

[11] Strategic complementality plays a crucial role in many situations. Typical examples are technological switch and industrialization. See, among others, David [19], Katz and Shapiro [39], Farrell and Saloner [25, 26], Murphy, Shleifer, and Vishny [56]. See also Rosenstein-Rodan [67], Okuno-Fujiwara [60], Matsumura and Ueda [53], and Matsumura and Ryser [52]. In particular Farrell and Saloner [25] and Matsumura and Ueda [53] discussed endogenous timing games in the context of technological switch.

equilibrium is weak and easily eliminated if small inventory costs are introduced. Thus, it is important to examine the robustness of the Cournot-type equilibrium in the Saloner model.

Furthermore, both the H-S and the Saloner models seem quite restrictive because both discuss two-period, two-player models only. Here we consider the H-S model. If we consider quantity-setting duopoly, the outcome is either Stackelberg-type or Cournot-type. If we consider a model involving more than two firms, we can analyze more varied situations: a pure simultaneous-move model in which all firms choose their actions at the same time (Cournot model), a pure sequential-move outcome in which all firms choose their actions at different times (generalized Stackelberg outcome), and mixed-type models in which there exists a sequence of periods, in each of which some players may choose their actions simultaneously. A two-period two-player model like the H-S model excludes the possibility of mixed-type outcomes.

1.2 Motivation

This dissertation investigates whether a simultaneous-move model (Cournot or Bertrand) or a sequential-move model (Stackelberg model) is more suitable for analysis of oligopoly markets by introducing endogenous timing into the standard quantity-setting and price-setting oligopoly. As discussed in the previous section, most existing works have emphasized that the Stackelberg model is more plausible than the Cournot or Bertrand models. They suggest that the Cournot and the Bertrand models are adequate only if firms are forced to play simultaneously.

This result is quite surprising because the Cournot and Bertrand models, rather than the Stackelberg model, are standard in oligopoly models. In fact, they are introduced in many models, while the Stackelberg model is usually discussed in relatively limited situations (e.g., in the context of entry-deterrence where the incumbent is determined by historical events). If the result derived in most existing works (i.e., equilibrium outcomes are Stackelberg-type only) is quite robust and does not depend on minor specifications of the models, then we must doubt the applicability of the models based on Cournot-type or Bertrand-type competition.

The main purpose of this dissertation is to investigate under what conditions the Cournot-type or the Bertrand-type outcomes appear in equilibrium.

In Chapters 2 and 3, we take a close look at the Saloner model which explicitly incorporates the incompleteness of the Stackelberg leader's commitment. We will show that the results found in existing works (except for the Saloner model) are crucially dependent on the assumption of complete commitment. We find that under some natural situations, the Cournot or the Bertrand outcome appears in equilibrium.

In Chapter 4, we elaborate the H-S model by formulating a model with more than two firms and more than two periods. We show that Stackelberg-type outcomes never appear in equilibrium if the number of firms is more than two.

In Chapter 5, we investigate endogenous timing in a standard multi-stage commitment game where firms choose two variables over two periods.

1.3 Organization

The remainder of the dissertation is as follows. Chapter 2 analyses the Cournot duopoly model which has more than one production period before the market clears. We introduce small inventory costs in order to check the robustness of Saloner's result, and find that such costs change the result completely. Two Stackelberg outcomes are the only outcomes found in the Cournot model with two production periods when inventory costs are small and positive. In other words, small inventory costs eliminate the Cournot-type equilibrium in a model with two production periods. This result is the same as the results produced by most existing works.

This result, however, depends crucially on the number of production periods. If there are more than two production periods, the Cournot-type outcome appears in equilibrium; therefore, the Cournot-type equilibrium is not so vulnerable in endogenous timing games as it appears at first glance.

Chapter 3 investigates endogenous timing in price-setting duopoly with differentiated goods. First, each firm announces its price; second, it chooses its actual price; and finally the market opens. Once a firm announces a price, it is able to discount it but not raise it. As opposed to other existing endogenous timing models, whether a Bertrand-type or a Stackelberg-type outcome appears depends crucially on the properties of the demand functions. We find three patterns for equilibrium outcomes: one case has the Bertrand-type equilibrium only, another has the Stackelberg-type only, and the third has both types of equilibria. In the most natural case, where the goods are substitutes and strategies are strategic complements, the unique equilibrium outcome is of Bertrand-type.

Chapters 2 and 3 are closely related. Both models discuss the incompleteness of commitment. Although the strategic variables involved in the two models are different, the structures of the models are very similar. Chapter 3 can be seen to extend the Saloner [71] by allowing the case of strategic complements. We discuss this in Chapter 3.

Chapter 4 investigates endogenous sequencing of a quantity-setting oligopoly model. Chapter 4 is the main part of the dissertation and clearly shows the advantage of simultaneous-move models over sequential-move models. We formulate an n-firm, m-period model where each firm chooses both how much to produce and when to produce it. We show that the Stackelberg-type outcome, where each firm produces sequentially, never appears in equilibrium except for two-period duopoly cases.

We investigate two models, with the first model excluding inventory costs and the second allowing for small inventory costs. We find that in the case without inventory costs, at least $n - 1$ firms produce simultaneously in the first period. In the case with small inventory costs, pure strategy equilibria exist if and only if m is 2, and in every equilibrium exactly $n - 1$ firms produce simultaneously in the first period. The former result shows the advantage of the Cournot model rather than the Stackelberg model. The latter suggests that two-period duopoly models are restrictive to investigate games with endogenous sequencing.

In Chapter 5 we introduce endogenous timing into multi-stage duopoly games in which duopolists choose two variables over two periods. We elaborate the two-stage strategic commitment game discussed by Brander and Spencer [12]. Duopolists decide upon their capacity investments and cost-reducing investments. They are allowed to choose which action to take first.

We discuss two types of games; one is a three-stage game in which each duopolist can commit to the order of choices before it chooses its output or cost-reducing investments, and the other is a two-stage game in which it cannot. We find that the outcome discussed by Brander and Spencer [12] never appears in equilibrium. We find that at least one firm chooses its output first. Furthermore, the three-stage game has a unique equilibrium outcome in which both firms choose their outputs first.

The result derived from the former model is quite similar to that of Chapter 4. In fact the model discussed here may be viewed as a variant of the model formulated in Section 2 of Chapter 4. The driving force discussed in Chapter 4 also eliminates the Brander-Spencer type equilibrium. The former model indicates the wide applicability of the result derived in Chapter 4.

Chapter 2

Cournot Duopoly with Three Production Periods

Abstract

This chapter analyses the Cournot duopoly model which has more than one production period before the market clears. Small inventory costs are introduced. When there are more than two periods, the Cournot-type outcome is supported as a subgame perfect equilibrium where production takes place in the first period. However, there is no such an equilibrium with two production periods, and equilibrium outcomes are of Stackelberg-type only. It suggests that investigating only two-stage game is too restrictive for analyzing games with endogenous timing.

This chapter is based on Matsumura [47, 46]. I am grateful to Shinshuke Kambe, Yoshitsugu Kanemoto, Murdoch MacPhee, Hajime Miyazaki, Masahiro Okuno-Fujiwara, Tadashi Sekiguchi and anonymous referees of *the Japanese Economic Review* and Proceeding of APORS. Needless to say, I am responsible for any remaining defects.

2.1 Introduction

There are two famous duopoly models which formulate competition over output quantities: Cournot and Stackelberg models. In particular, Cournot-type competition has been introduced in many models and it is considered a standard type of competition in oligopolistic markets.

In the standard Cournot duopoly model, duopolists are assumed to choose output simultaneously, while in the standard Stackelberg duopoly model, they are assumed to do so sequentially. In both models, the timing is given exogenously. In many economic situations, however, it is more reasonable to assume that firms choose not only what actions to take, but also when to take them. Some economists have investigated this problem. Existing models emphasized that Stackelberg-type outcomes (equilibrium outcomes in a sequential-move game) appear in endogenous timing games but the Cournot-type outcome (an equilibrium outcome in a simultaneous-move game) does not [See, e.g., Hamilton and Slutsky [37], Robson [66], Anderson and Engers [4], Albæk [1, 2], and Mailath [44]]. These papers suggested that the Stackelberg model is more plausible if firms can choose their actions over more than one period.

The exception is the Saloner [71] model. The above models except the Saloner model permitted firms to choose their actions in one of two or more periods only. Saloner discussed a two-player, two-period model in which duopolists choose their actions over two periods and can take actions in both periods. He modified the Cournot model by allowing for two production periods before the market clears. He showed that many outcomes including both Cournot and Stackelberg outcomes, are subgame perfect equilibrium outcomes. In many economic situations, duopolists can choose outcomes over periods; therefore, the Saloner model is more realistic. The Saloner model, however, seems restrictive because output produced in periods 1 and 2 is perfectly substitutable for the firm and no inventory costs are incurred.

In this chapter, we introduce small inventory costs. We find that small inventory costs change the result completely. Equilibrium outcomes other than two Stackelberg outcomes do not exist in the Cournot model with two production periods when inventory costs are small and positive. In other words, small inventory costs eliminate the Cournot-type equilibrium in a model with two production periods. This result is the same as the results produced by other existing works discussed above.

With small inventory costs, a firm chooses to become a Stackelberg leader in order to enjoy the first-mover advantage if the rival produces nothing in the first stage. Therefore, the Cournot-type outcome reaches to an equilibrium only if both firms produce the Cournot outputs in the first period. In the model with two production periods, once the first-period production decisions are made, no strategic moves are left for either firm because no firm chooses its action after observing the second-stage moves. Therefore, the firm which produces the Cournot output in period 1 has no incentive to increase its output in period 2. Given a firm chooses the Cournot output in period 1, the rival chooses to produce nothing in period 1 in order to reduce the inventory cost. Given that the rival produces nothing in period 1, the firm becomes the Stackelberg leader by producing the Stackelberg leader's output in period 1 in order to enjoy the first-mover advantage. Accordingly, the Cournot-type outcome is never attainable as a subgame perfect equilibrium.

This result, however, depends crucially on the number of production periods. If there are more than two production periods, the Cournot-type outcome appears in equilibrium; therefore, the Cournot-type equilibrium is not so vulnerable as it appears at first glance in endogenous timing games. In the model with more than two production periods, once the first-period production decisions are made, two production periods remain. The production decisions made in period 2 have strategic value because each firm chooses its third-stage production after observing the rival's production in period 2. Therefore each firm has an incentive to increase its production in period 2 for the strategic purpose. Accordingly, in order to prevent the firm from making the strategic behavior in period 2, each firm dares to produce the Cournot output in period 1. As a result, the Cournot-type outcome is attainable as a subgame perfect equilibrium outcome in the case of more than two production periods. This result suggests that investigating three-stage games is important for analyzing endogenous timing games.[1]

The remainder of this chapter is organized as follows. In Section 2.2, we formulate a basic model. In Section 2.3, in order to present some benchmarks, we investigate models in which firms are restricted to a specified timing. Section 2.4 discusses the Cournot model with two production periods. We show that small inventory costs eliminate a Cournot outcome from the set of equilibrium outcomes. Section 2.5 investigates the Cournot model with three production periods. We show that the Cournot-type outcome appears in equilibrium. Section 2.6 concludes this chapter.

2.2 The model

In this section, we formulate a three-stage duopoly game. The two firms are denoted as 1 and 2. We often use i or j to refer to a firm, and it is understood that if i denotes 1 in one expression, then j represents 2 and *vice versa.*

The game runs as follows. In the first stage, each firm i ($i = 1,2$) chooses its first-period production $x^i(1) \in [0, \infty)$. At the end of the first stage, each firm knows $x^1(1)$ and $x^2(1)$. In the second stage, each firm i chooses its second-period production $x^i(2) \in [0, \infty)$. At the

[1]Similar principles can apply to Cournot duopoly with more than three production periods. However, we must emphasize that our results are crucially dependent on the assumption that the model has a first period. If we constructs a model with infinite earlier periods, our results do not hold true. We think that a model with infinite earlier periods is quite important, but a model with finite period is also important and well worth discussing. For an important model with infinite earlier periods, see Robson [66].

end of the second stage, each firm knows $x^1(2)$ and $x^2(2)$. In the third stage, each firm i chooses the last-period production $x^i(3) \in [0, \infty)$. At the end of the third stage, the market opens[2] and each firm i sells its total output $x^i \equiv x^i(1) + x^i(2) + x^i(3)$.[3] Firm i's payoff U^i is given by

$$U^i(x^i, x^j, x^i(1), x^i(2)) = V^i(x^i, x^j) - c^i x^i - I^i(x^i(1), x^i(2)) \quad (i = 1, 2)$$

where V^i is firm i's revenue function, c^i is firm i's marginal cost, and I^i is firm i's inventory cost. Here we make the following assumptions about the revenue function of V^i:

Assumption 2.1 (Differentiability). V^i is twice continuously differentiable $(i = 1, 2)$;

Assumption 2.2 (Concavity). $V^i_{11} < 0$ ($i = 1, 2$) ;

Assumption 2.3 (Substitutability). $V^i_2 < 0$ $(i = 1, 2)$;

Assumption 2.4 (Strategic Substitutes). $V^i_{12} < 0$ $(i = 1, 2)$; and

Assumption 2.5 (Stability). $|V^i_{11}| > |V^i_{12}|$ $(i = 1, 2)$

where we use subscripts to denote partial derivatives.

In order to neglect the production-smoothing effect of inventory holding, we assume a constant marginal production cost.[4] We consider a case of fixed inventory costs. Similar cost is discussed in Robson [66], too.[5]

[2] In this model the market opens once. We can easily extend to the model where the market opens at each period. See Matsumura [45].

[3] Some readers may think that each firm can dispose its output at the end of third stage. If we allow it, we must formulate a four-stage game rather than a three-stage game. However, if the production cost c^i is positive, we obtain the similar results in four-stage game, so the extension is not productive. I thank this point for Professor Miyazaki.

[4] For empirical results on the relative importance of the production smoothing effect of inventory holding, see Blanchard [6] and Blinder [7].

[5] It is quite difficult to derive the set of equilibrium outcomes under more general inventory costs functions in the three-period case. But it is relatively easy in the two-period case. We explicitly consider more general costs in two-period in Proposition 2.3.

Assumption 2.6 (Fixed-type Inventory Costs). I^i is given as follows:

$$I^i(x^i(1), x^i(2)) = \begin{cases} \delta(\geq \varepsilon) & \text{if } x^i(1) > 0 \\ \varepsilon(> 0) & \text{if } x^i(1) = 0 \text{ and } x^i(2) > 0 \\ 0 & \text{if } x^i(1) = x^i(2) = 0. \end{cases}$$

Throughout Chapter 2 we use a subgame perfect Nash equilibrium as an equilibrium concept. We restrict our attention to pure strategy equilibria.

2.3 Some benchmarks

In this section we examine the standard Cournot game, which has a single production period. We also examine the standard Stackelberg game.

2.3.1 Cournot game

Each firm i chooses output simultaneously and independently. Given x^j, firm i maximizes its payoff $V^i(x^i, x^j) - c^i x^i$ with respect to x^i. For firm i we define

$$R^i(x^j) = \arg \max_{\{x^i \geq 0\}} (V^i(x^i, x^j) - c^i x^i), \tag{2.1}$$

i.e., $R^i(x^j)$ is firm i's reaction function.

Definition 2.1 (Cournot outcome). A pair (C^1, C^2) of output levels is a Cournot outcome if and only if $C^1 \in R^1(C^2)$ and $C^2 \in R^2(C^1)$.

The stability condition (Assumption 2.5) ensures that $-1 < R^{i'}(x^j) < 0$ and it also ensures the uniqueness of the Cournot equilibrium. Under this condition, (C^1, C^2) is stable for the standard adjustment mechanism: that is,

$$R^i(R^j(x^i)) \begin{cases} > x^i & \text{if } x^i < C^i \\ < x^i & \text{if } x^i > C^i. \end{cases}$$

Assumption 2.7 (Interior solution). $C^i > 0 \;\; (i = 1, 2)$.

2.3.2 Stackelberg game

Firm i chooses x^i and firm j chooses x^j after observing x^i. Given x^i, firm j maximizes its payoff $V^j(x^j, x^i) - c^j x^j$ with respect to x^j. Firm i maximizes its payoff $V^i(x^i, R(x^i)) - c^i x^i$ with respect to x^i. The first order condition of this optimality is $V_1^i + V_2^i R^{j\prime}(x^i) - c^i = 0$.

Definition 2.2 (Stackelberg leader's payoff). Define the Stackelberg leader's payoff $S^i(x^i) \equiv U^i(x^i, R^j(x^i))$.

Assumption 2.8 (Concavity of Stackelberg leader's payoff). $\partial^2 S^i / \partial(x^i)^2 < 0$ $(i = 1, 2)$.

Definition 2.3 (Stackelberg outcome). Let a pair (L^i, F^j) $(i = 1, 2)$ denote a Stackelberg outcome, which is defined by

$$L^i \equiv \arg \max_{\{x^i \geq 0\}} S^i(x^i) \text{ and } F^j \equiv R^j(L^i) \quad (i = 1, 2),$$

where L^i is the Stackelberg leader's output and F^j is the Stackelberg follower's.

As is well known,

$$F^i < C^i < L^i \text{ and } U^i(L^i, F^j) > U^i(C^i, C^j) > U^i(F^i, L^j) \; (j \neq i, \; i = 1, 2) \tag{2.2}$$

where U^i is firm i's payoff.[6]

2.4 Cournot duopoly with two production periods

In this section we examine the subgame which begins at the second stage given that $x^1(1) = x^2(1) = 0$. This is a subgame of the game with three production periods formulated in Section 2.2.

Definition 2.4 (Cumulative production until period 2). Define $q^i(2) \equiv x^i(1) + x^i(2)$.

To compute a subgame perfect Nash equilibrium, the game is solved by backward induction.

[6] See Gal-Or [33] and Dowrick [24].

First we discuss the equilibrium outcome in the third stage given $q^i(2)$ and $q^j(2)$. Let $B^i(x^j : q^i(2))$ denote firm i's reaction function after committing to $q^i(2)$. Since $V^i_{11} < 0$ and $x^i \geq q^i(2)$, $B^i(x^j : q^i(2)) = \max(R^i(x^j), q^i(2))$. The following three lemmas state the equilibrium outcome in the third stage. Lemma 2.1 states that the firm which chooses $q^i(2) \geq C^i$ never increases its output in period 3. Lemma 2.2 states that if both firms' cumulative productions until period 2 are smaller than the Cournot output, a Cournot outcome is realized.

Since firm j determines its output before observing $x^i(3)$, firm i's last-period production $x^i(3)$ has no strategic value. Therefore, firm i has no incentive to produce more than the Cournot output C^i in the last period. Accordingly, firm i does not increase its output provided that $q^i(2) \geq C^i$ (Lemma 2.1). Given that $q^j(2) \geq C^i$, firm j predicts that firm i's total output x^i equals $q^i(2)$. Therefore firm j chooses its total output $x^j = R^j(q^i(2))$ as long as $R^j(q^i(2)) \leq q^j(2)$ (Lemma 2.3). On the other hand, if $(q^i(2), q^j(2)) < (C^i, C^j)$, $x^i \geq q^i(2)$ and $x^j \geq q^j(2)$ are not binding constraints. In this case, the unique equilibrium is a Cournot one (Lemma 2.2).

Lemma 2.1: *If* $q^i(2) \geq C^i$, *then* $x^i(3) = 0$.

Proof: See Appendix A.

Lemma 2.2: *If* $(q^i(2), q^j(2)) \leq (C^i, C^j)$, *then* $(x^i(3), x^j(3)) = (C^i - q^i(2), C^j - q^j(2))$.

Proof: See Appendix A.

Lemma 2.3: *If* $q^i(2) < C^i$, *and* $q^j(2) > C^j$, *then* $x^i(3) = \max(R^i(q^j(2)) - q^i(2), 0)$.

Proof: See Appendix A.

From Lemmas 2.1–2.3, we know the equilibrium outcome in the last stage given $q^i(2)$ and $q^j(2)$. Next, we investigate the second-stage production given that $x^1(1) = x^2(1) = 0$. We assume that the inventory cost ε is positive but sufficiently small, namely;

Assumption 2.9 (Small Inventory costs). Inventory cost ε is small such that $0 < \varepsilon < V^i(L^i, F^j) - V^i(C^i, C^j) - c^i(L^i - C^i)$ $(j \neq i,\ i = 1, 2)$.

Proposition 2.1 states that at least one firm produces nothing in period 2 and chooses to wait until the last period.

Proposition 2.1: *Suppose that Assumptions 2.1 — 2.9 are satisfied. Suppose that $x^1(1) = x^2(1) = 0$. Then there is no equilibrium in which both $x^1(2)$ and $x^2(2)$ are positive.*

Proof: See Appendix A.

A positive inventory cost is essential. If other things are equal, firm i strictly prefers waiting until period 3 to producing $x^i(2) \leq C^i$, while if the inventory cost is zero, whether production is made in period 2 or in period 3 is a matter of indifference to the firm.

Next we consider the equilibria in the two-stage game. Proposition 2.2, which is derived straightforwardly from Proposition 2.1, states that the equilibrium outcomes are of Stackelberg-type only. Proposition 2.1 states that one firm chooses to wait until period 3. Given this, the rival chooses to become a Stackelberg leader because the Stackelberg leader's profit is larger than the Cournot's as long as inventory costs are small. The strict proof is presented in the Appendix A.

Proposition 2.2: *Suppose that Assumptions 2.1–2.9 are satisfied. Suppose that $x^1(1) = x^2(1) = 0$. Then (x^{1*}, x^{2*}) is a subgame perfect equilibrium outcome if and only if $(x^{1*}, x^{2*}) \in \{(L^1, F^2), (F^1, L^2)\}$.*

Proof: See Appendix A.

The existence of small inventory costs eliminates the Cournot-type equilibrium. This result is quite different from Saloner [71], in which the inventory cost is zero. Why do small inventory costs eliminate the Cournot-type equilibrium? Given that firm j chooses a smaller output than the Cournot output C^j, firm i produces more than C^i in the first period and enjoys the first-mover advantage. Therefore, the Cournot-type equilibrium is realized only if both firms choose the Cournot output in the first period. Suppose that there is no inventory cost. As Lemma 2.1 states, firm i never produces any additional output in the second period if it chooses C^i in the first period. Firm j weakly prefers waiting until the last period over producing the Cournot output C^j in the first period because waiting enables firm j to respond optimally when firm i chooses $x^i(1) > C^i$. In other words, $x^j(1) = C^j$ is weakly dominated by $x^j(1) = 0$. If we introduce positive inventory costs, $x^j(1) = C^j$ is strictly dominated by $x^j(1) = 0$; therefore, small inventory costs eliminate the Cournot-type equilibrium.

Some readers may think that the inventory cost discussed here is too restrictive. the Cournot-type outcome fails to become equilibrium one under more general inventory cost functions. Proposition 2.3 states that the Cournot-type outcome does not appear in equilibrium under more general situations.

Assumption 2.10 (Positive inventory costs). $I^i(0,0) = 0$ and $I^i(0, x^i(2)) > 0 \ \forall x^i(2) > 0$. $(i = 1, 2)$.

Assumption 2.11 (Small inventory costs). Suppose that firm i is a Stackelberg leader producing in period 2 only and firm j is a follower producing in period 3 only. Then the inventory costs is small such that firm i's payoff is larger than its Cournot counterpart $U^i(C^i, C^j, 0, 0)$ $(i \neq j, i = 1, 2)$.

Proposition 2.3: *Suppose that* $x^1(1) = x^2(1) = 0$. *Suppose that Assumptions 2.1–2.5, 2.7–2.8, and 2.10–2.11 are satisfied. Then (i) in equilibrium one firm produces in period 1 only and the other produces in period 2 only, and (ii) there is no equilibrium where* $(x^1, x^2) = (C^1, C^2)$.

Proof: See Appendix A.

This result is quite similar to those produced by other endogenous timing models (see, e.g., Hamilton and Slutsky [17], Robson [illegible], Anderson and Engers [illegible], and Albaek [illegible]). This result makes us doubt whether the Cournot model, which permits simultaneous moves only, is adequate for analyzing behaviors of oligopolistic firms in the real world.[1]

2.5 Cournot duopoly with three production periods

In this section we show that the Cournot outcome is one of equilibrium outcomes if there are more than two production periods. We assume that the first-stage inventory cost δ is small enough that the following two inequalities are satisfied.

Assumption 2.12 (Small Inventory Cost). The inventory cost δ is positive and small such that,

$$\delta < \bar{\delta} \equiv [illegible], \text{ and} \tag{2.8}$$

$$0 < \delta < [illegible]. \tag{2.9}$$

Definition 2.5 We define [illegible]

Definition 2.6 We denote the better solutions of R^1 and R^2 by $\bar{S}$.

Here we describe the set of equilibrium outcomes in the three-period case.

Proposition 2.6 *Suppose that Assumptions 2.1-2.4 and 2.11 are satisfied. Then (x^{1*}, x^{2*}) is a subgame perfect equilibrium outcome in the game with three production periods if and only if $(x^{1*}, x^{2*}) \in E \cup \{(L^1, F^2), (F^1, L^2)\}$ where $E = \{(x^1, x^2) \mid (x^1, x^2) \in R, x^1 \leq$ [illegible] x^1, and $x^2 \leq$ [illegible]$\}$.*

Proof. See Appendix 2.

Corollary *Suppose that Assumptions 2.1–2.8 and 2.11 are satisfied. Then there is a subgame perfect equilibrium where $(x^1, x^2) = (C^1, C^2)$.*

[1] There are many models in which firms are assumed to be faced with Cournot-type competition. See, among others, Brander and Spencer [12], Fershtman and Judd [illegible], and [illegible] [illegible].

This result is quite similar to those produced by other endogenous timing models [See, e.g., Hamilton and Slutsky [37], Robson [66], Anderson and Engers [4], and Albæk [2]]. This result makes us doubt whether the Cournot model, which permits simultaneous-moves only, is adequate for analyzing behaviors of oligopolistic firms in the real world.[7]

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In this section we show that the Cournot outcome is one of equilibrium outcomes if there are more than two production periods. We assume that the first-stage inventory cost δ is small enough that the following two inequalities are satisfied:

Assumption 2.12 (Small Inventory Costs). The inventory cost δ is positive and small such that

$$0 < \delta < V^i(L^i, F^j) - c^i L^i - V^i(C^i, C^j) - c^i C^i, \text{ and} \tag{2.3}$$

$$0 < \delta < V^i(C^i, C^j) - c^i C^i - V^i(F^i, L^j) - c^i F^i. \tag{2.4}$$

Definition 2.5 We define $\bar{x}^i$ by $V^j(R^j(\bar{x}^i), \bar{x}^i) - c^i R^j(\bar{x}^i) - \delta = V^j(F^j, L^i) - c^j F^j$.

Definition 2.6 We denote the outer envelope of R^1 and R^2 by R.

Here we describe the set of equilibrium outcomes in the three-period case.

Proposition 2.4: *Suppose that Assumptions 2.1–2.8 and 2.11 are satisfied. Then* (x^{1*}, x^{2*}) *is a subgame perfect equilibrium outcome in the game with three production periods if and only if* $(x^{1*}, x^{2*}) \in E \cup \{(L^1, F^2), (F^1, L^2)\}$ *where* $E = \{(x^1, x^2) \mid (x^1, x^2) \in R,\ x^1 \leq \bar{x}^1,$ *and* $x^2 \leq \bar{x}^2\}$.

Proof: See Appendix A.

Corollary: *Suppose that Assumptions 2.1–2.8 and 2.11 are satisfied. Then there is a subgame perfect equilibrium where* $(x^1, x^2) = (C^1, C^2)$.

[7] There are many models in which firms are assumed to be faced with Cournot-type competition. See, among others, Brander and Spencer [12], Brander and Lewis [11], and Fershtman and Judd [29].

Proof: See Appendix A.

The above results state that many outcomes including Cournot and Stackelberg outcomes appear in equilibrium if we allow three production periods rather than two periods. Proposition 2.4 also states that if we choose $\delta \to 0$, the set of equilibrium outcome converges to that of Saloner [71]. Note that $\bar{x}^i \to L^i$ if we choose $\delta \to 0$.

Here we make a rough sketch why the Cournot-type outcome appears in equilibrium if we allow three production periods. As discussed in the previous section, the Cournot-type outcome is realized only if both firms choose the Cournot output in the first period. In the model with three production periods, once the first-period production decisions are made, two production periods remain. Firm i's second-period production $x^i(2)$ has strategic value because firm j chooses its last-period production $x^j(3)$ after observing firm i's second-stage production $x^i(2)$. Suppose that firm i chooses the Cournot output C^i in the first period but firm j chooses to produces nothing in order to reduce the inventory cost. If firm i infers that firm j waits until period 3, it increases its total output x^i by choosing $x^i(2) = L^i - C^i$ in order to reduce the last-period production of firm j. To avoid the above strategic behavior in the second period by firm i, firm j also chooses the Cournot output C^j in the first period.[8] Accordingly, the Cournot-type outcome is supported in the models with three or more production periods.

Firm j dares to produce its Cournot output C^j in order to prevent the rival (firm i) from being a Stackelberg leader. If two periods remain after the first period, the Cournot-type outcome appears in equilibrium. Therefore, in a model with more than three production periods, the Cournot outcome becomes an equilibrium one.

If we consider other kinds of inventory cost function, the set of equilibrium outcomes

[8] Strategic value of inventory-holding discussed in many situations. See, e.g., Saloner [70], Rotemberg and Saloner [68], and Matsumura [45].

changes. However, the Cournot-type outcome appears in equilibrium if the marginal inventory cost is small enough. Reducing first-period production from the Cournot output by firm j increases firm i's output. If the marginal inventory cost is sufficiently small, the inventory cost reduced by a small first-period production is negligible, while expansion of the second-stage output by firm i reduces firm j's payoff significantly. Therefore, firm j chooses its Cournot output in the first production period in order to prevent the rival from adding the output strategically.

2.6 Concluding remarks

As Saloner [71] showed, if there are two production periods before the market clears, many equilibria, including Cournot and Stackelberg equilibria, are realized. Equilibria other than two Stackelberg equilibria are vulnerable, however, in the sense that these are supported by weakly dominated strategy. This is why small inventory costs eliminate these equilibria as is shown in Section 2.4. The result that equilibrium outcomes are of Stackelberg-type is the same as the results produced by other endogenous timing models.

This chapter showed that the Cournot-type equilibrium is not so vulnerable as it appears at first glance because this equilibrium exists and is supported by undominated strategies in a more than two production periods counterpart. Three production periods are important because they allow firms to take a strategic action in the subgame after the first period. If there are more than two production periods, each firm produces the Cournot output in the first period in order to prevent the rival from making the strategic moves. Therefore, the Cournot-type outcome fails to be equilibrium outcome in a model with two production periods only.

Cournot model is quite natural, especially when firms are faced with perfectly symmetric situations. The Cournot-type outcome is the unique symmetric equilibrium outcome among the pure strategy equilibrium outcomes in the Cournot duopoly with three production periods. Thus, it may be natural that the Cournot-type outcome becomes the focal point in endogenous timing games. [See, e.g., Myerson [57] pp. 111-112. See also Schelling [72].]

However, we cannot deny the importance of the Stackelberg model. We should consider the refinement of these equilibria. The question remains for future research.

Chapter 3

A Two-Stage Price-Setting Duopoly: Bertrand or Stackelberg

Abstract

This chapter investigates endogenous timing in price-setting duopoly with differentiated goods. First, each firm announces its price; second, it chooses its actual price; and finally the market opens. Once a firm announces a price, it is able to discount it but not raise it. As opposed to existing works emphasizing the advantage of the Stackelberg model over the Bertrand, in our model whether the Bertrand-type or the Stackelberg-type outcomes appear in equilibrium depends crucially on the properties of the demand functions. We find three patterns of equilibrium outcomes; one case has the Bertrand-type equilibrium only, another has the Stackelberg-type only, and the other has both types of equilibria. In the most natural case where the goods are substitutes and strategies are strategic complements, the unique equilibrium outcome is of Bertrand-type.

This chapter is based on Matsumura [49]. I am grateful to Haruo Imai, Shingo Ishiguro, Hideshi Itoh, Yashushi Iwamoto, Tatsuaki Kuroda, Murdoch MacPhee, Akira Okada, Yoshiyasu Ono, Tetsuya Shinkai and participants of the seminar at Institute of Economic Research of Kyoto University and Annual Meeting of Western Department of Japan Association of Economics and Economics at Fukuoka University for their helpful comments and suggestions. Needless to say, I am responsible for any remaining defects.

3.1 Introduction

Without any doubt, the Bertrand model is one of the most important models of oligopoly. In the standard Bertrand duopoly model, each firm simultaneously and independently chooses its price. The Stackelberg model has also received much attention in the context of price-competition.[1] In the price-setting Stackelberg duopoly model, one firm (the Stackelberg leader) chooses its price before the other firm (the Stackelberg follower) chooses its price. The Bertrand model formulated a simultaneous-move game and the Stackelberg model formulated a sequential-move game. Both models have been intensively discussed in many economic situations. In particular, Bertrand-type competition has been introduced in many models and is considered one of the representative types of competition in oligopolistic markets.

In both Bertrand and Stackelberg models, the timing is given exogenously. In many economic situations, however, it is more reasonable to assume that firms choose not only what actions to take, but also when to take them. Some economists have investigated this problem. Existing models emphasized that Stackelberg type outcomes (equilibrium outcomes in a sequential-move game) appear in endogenous timing games but a Bertrand type outcome (an equilibrium outcome in a simultaneous-move game) does not [See Hamilton and Slutsky [37], Robson[66], Anderson and Engers [4], Albæk[2], Mailath [44], and Matsumura [47]].[2] These papers suggested that a sequential-move game like the Stackelberg model is more plausible than a simultaneous-move game like the Bertrand model if firms are able to choose when to take their actions. In particular, Robson [66] explicitly for-

[1] Usually Stackelberg model is discussed in the context of quantity-setting competition rather than price-setting competition. However, price-setting Stackelberg model has also been investigated in various situations. See, among others, Ono [61, 62], Ito and Ono [38], and Robson [66].

[2] There are some exceptions. See Saloner [71] and Matsumura [46], which emphasized that both simultaneous-move and sequential-move outcomes appear in equilibria. However these models analyze quantity-setting competition rather than price-setting, and a straightforward adjustment to price competition is not possible.

mulated a model discussing endogenous timing in Bertrand competition and finds that a Bertrand-type outcome never appears in equilibrium.

In Hamilton and Slutsky [37], which is one of the most important works in this field, each firm chooses whether or not to take its action in period 1. The firm which took its action in period 1 can choose no action in period 2. In order to take some actions in period 2, the firm must not take its action in period 1. In other words, once a firm chose its action in period 1, it never changes its choice in period 2. This assumption is also made in Robson [66], Anderson and Engers [4], and Albæk [2].

In the price-setting Stackelberg model, as well as Hamilton and Slutsky [37] and Robson [66], the leader can commit to the price determined before the follower chooses the price. Consider the following situation: in period 1 firm 1 names its price p_1 and after observing p_1, firm 2 names its price p_2. At the end of period 2 the market opens. This formulation invokes a question: how can firm 1 commit to its price p_1 before the market opens? Is it impossible for firm 1 to change p_1 in period 2? Why the leader cannot change their actions in period 2 even if it has enough time to change its price before the market opens? This question makes us doubt the applicability of the Stackelberg model in the context of price-setting competition.

Firm 1 may be able to commit to its price by advertising or contracting. Firm 1 can announce its price by advertising. For example, in Japan if a firm raises its announced price just before the market opens without any rational reason, the firm may be prosecuted because such devious strategy is against anti-monopoly regulation. A firm can make contracts with potential clients, through which they are granted the option of purchasing firm 1's goods at the price p_1. Firm 1 can offer the following contract to all potential consumers; "I will sell you my products at price p_1 if you accept this contract until the end of period 2." The clients who receive this contract can refuse to buy at any price larger than p_1.

Thus, firm 1 cannot choose a price larger than p_1.

However, in the examples above, the firm can easily cut the price named in the previous period.[3] Usually consumers prefer a lower price, so there is no reason for them to reject the discount. The rival (firm 2) may not welcome the price-cutting of firm 1, but it is difficult to prevent firm 1 from cutting its price because such a action is against anti-monopoly regulations.

Under these conditions, it is natural to assume that the price commitment is incomplete: each firm can commit to an upper-bound of its price but not to a lower-bound. In other words, each firm can discount its stated price but cannot raise it.[4] In this chapter we incorporate the above incompleteness of price commitment into a two-period price-setting duopoly game.[5] We formulate the following model: in the first stage each firm announces its price, in the second stage each firm chooses its price, and finally the market opens. Neither firm raises its announced price in the second stage.

We find that the equilibrium outcomes depend on the properties of each firm's demand function. In some cases the equilibrium outcome is of Bertrand-type only, in some cases the equilibrium outcomes are of Stackelberg-type only, and in other cases both Stackelberg and Bertrand outcomes are found in equilibrium. In the most natural cases, where goods of firms are substitutes and strategies are strategic complements, the unique equilibrium outcome is of Bertrand-type. As opposed to other existing endogenous timing games like Robson [66], this result suggests that the Bertrand model is never less plausible than the

[3] For example, Matsushita Electric Industrial discounted the announced price of a computer game, whose name is 'real', before the market opened.

[4] Note that our discussion is quite different from those of price-rigidity. The theories of price-rigidity emphasized that firms seldom cut the prices. However, these theories discuss the firms' price policies **after** the market opens, not the commitment **before** the market opens. Furthermore, they never say that a firm **cannot** cut its price. See, among others, Carlton[14], Nishimura [59], and Blinder [8].

[5] Similar incompleteness of commitment in the context of quantity-setting duopoly was discussed in Saloner [71], Robson [65], Pal [63], and Matsumura [47]. All of them considered strategic substitutes case only. See also Spence [77] and Fudenberg and Tirole [32].

Stackelberg model in realistic situations.

The remainder of this chapter is organized as follows. In Section 3.2, we formulate the model. In Section 3.3, in order to present some benchmarks, we investigate models in which firms are restricted to a specified timing. Section 3.4 discusses the equilibrium of the model formulated in Section 3.2. Section 3.5 concludes this chapter. All proofs of propositions are presented in the Appendix B.

3.2 The model

In this section, we formulate a two-stage duopoly game.[6] The two firms are denoted as 1 and 2. We often use i or j to refer to a firm, and it is understood that if i denotes 1 in an expression, then j represents 2 and *vice versa.*

The game runs as follows. In the first stage, each firm i (i = 1,2) independently announces its price $p_i(1) \in [0, \infty]$. If firm i chooses $p_i(1) = \infty$, it implies that firm i does not announce a price in the first stage.

At the end of the first stage, each firm knows $p_1(1)$ and $p_2(1)$. In the second stage, each firm i independently chooses its actual price $p_i \in [0, p_i(1)]$. At the end of the second stage, the market opens and each firm i sells its output at the price p_i. In our model, p_i is never larger than $p_i(1)$. This formulation reflects the assumption that each firm cannot raise its announced price, but can discount it.[7] For simplicity, we neglect production costs.[8] Firm

[6] We can easily extend it to a model with more than two periods like Robson [66] and similar principles can apply to this model.

[7] Some readers may think that each firm can easily raise its announced price. Instead of considering the announcement of the price, we can formulate a model where each firm commits by contracts, as is discussed in the Introduction.

[8] If we consider production costs explicitly, all propositions hold true under the following condition: the marginal costs are constant, or firm i's payoff is given by the following instead of (3.1):

$$U_i(p_i, p_j) \equiv \max_{\{p_i \geq 0,\ x_i \leq x_i(p_i, p_j)\}} (p_i x_i - c_i(x_i))$$

where c_i is firm i's cost function and $c_i'' \geq 0$.

i's payoff U_i is given by

$$U_i(p_i, p_j) \equiv p_i x_i(p_i, p_j) \qquad (i = 1, 2) \tag{3.1}$$

where $x_i : R_+^2 \to R_+$ is firm i's demand function. We make the following assumptions about the demand functions:

Assumption 3.1 (Differentiability). x_i is twice continuously differentiable $\quad (i = 1, 2)$;

Assumption 3.2 (Downward-sloping demand). $\partial x_i / \partial p_i < 0 \quad (i = 1, 2)$; and

Assumption 3.3 (Concavity of payoff functions). $\partial^2 U_i / \partial p_i^2 < 0 \quad (i = 1, 2)$.

Assumption 3.1 ensures a smooth demand function.[9] Assumption 3.2 excludes Giffen's paradox.

3.3 Some benchmarks

In this section we investigate models in which firms are restricted to a specified timing by way of establishing benchmarks. We examine the Bertrand and Stackelberg models.

3.3.1 Bertrand game

Each firm i $(i = 1, 2)$ chooses its price simultaneously and independently in period 2 only. Given p_j, firm i maximizes its payoff $U_i(p_i, p_j)$ with respect to p_i. The first order condition of this optimality is

$$x_i + p_i \frac{\partial x_i}{\partial p_i} = 0 \qquad (i = 1, 2). \tag{3.2}$$

Definition 3.1 (Reaction function). Let $R_i(p_j)$ $(i = 1, 2)$ denote firm i's reaction function, which is defined by $R_i(p_j) \equiv \arg\max_{\{p_i \geq 0\}} (p_i x_i(p_i, p_j))$.

Definition 3.2 (Bertrand equilibrium). Let a pair (B_1, B_2) of price levels denote a Bertrand equilibrium (Bertrand outcome), where $B_1 \in R_1(B_2)$ and $B_2 \in R_2(B_1)$.

[9] Assumption 3.1 seems innocuous, but it is restrictive. This assumption excludes a case of homogeneous goods. We discuss a case of homogeneous goods in Example 2 of Section 3.4.

From (3.2), we have

$$R_i'(p_j) = -\frac{\partial x_i/\partial p_j + p_i(\partial^2 x_i/\partial p_i \partial p_j)}{2(\partial x_i/\partial p_i) + p_i(\partial^2 x_i/\partial p_i^2)} \qquad (i = 1, 2) \tag{3.3}$$

From (3.3) and Assumption 3.3, we have that $\text{sign}(R_i') = \text{sign}(\partial x_i/\partial p_j + p_i(\partial^2 x_i/\partial p_i \partial p_j))$. If $\partial R_i/\partial p_j$ is positive (negative), firm i's reaction curve has upward-sloping (downward-sloping); thus strategies are strategic complements (substitutes). Here we make the following assumption:

Assumption 3.4 (Stability condition). If $(R_i(p_j), p_j) \in R^2_{++}$, then $0 <| R_i'(p_j) |< 1$.

Assumption 3.4 is a well known stability condition. Under this condition, it is known that (B_1, B_2) is stable for the standard adjustment mechanism: that is,

$$R_i(R_j(p_i)) \begin{cases} > p_i & \text{if } p_i < B_i \\ = p_i & \text{if } p_i = B_i \\ < p_i & \text{if } p_i > B_i. \end{cases} \qquad (i = 1, 2) \tag{3.4}$$

This stability condition guarantees the uniqueness of the Bertrand equilibrium.

3.3.2 Stackelberg game

Firm i chooses p_i in period 1 and firm j chooses p_j after observing p_i in period 2. Given p_i, firm j maximizes its payoff with respect to p_j. Firm i maximizes its payoff $U_i(p_i, R_j(p_i))$ with respect to p_i. The first order condition of this optimality is

$$x_i + p_i(\frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial p_j} R_j') = 0. \tag{3.5}$$

Definition 3.3 (Stackelberg leader's payoff). Define the Stackelberg leader's payoff $U_i^L(p_i) \equiv U_i(p_i, R_j(p_i))$.

Assumption 3.5 (Concavity of Stackelberg leader's payoff). $\partial^2 U_i^L/\partial p_i^2 < 0 \quad (i = 1, 2)$

Definition 3.4 (Stackelberg equilibrium). Let a pair (L_i, F_j) $(i = 1, 2)$ denote a Stackelberg equilibrium (Stackelberg outcome), which is defined by

$$L_i \equiv \arg \max_{\{p_i \geq 0\}} U_i^L(p_i) \text{ and } F_j \equiv R_j(L_i) \quad (i = 1, 2),$$

where L_i is the Stackelberg leader's price and F_j is the Stackelberg follower's.

Here we note a result concerning L_i and B_i.

Result 3.1: $B_i > L_i$ *if and only if* $(\partial x_i / \partial p_j)\ R_j' < 0 \quad (i = 1, 2)$

This is straightforwardly derived from (3.5), Assumption 3.5, and the definition of B_i.

3.4 Equilibrium outcomes

In this section we discuss equilibrium outcomes of the endogenous timing game formulated in Section 3.2. At the end of the section we present three examples. We use a subgame perfect Nash equilibrium as our equilibrium concept. We restrict our attention to pure strategy equilibria.

Definition 3.5 (The set of equilibrium outcomes). Let E denote the set of equilibrium outcomes. A pair $(p_1, p_2) \in E$ if and only if there is a subgame perfect equilibrium in which firm 1 chooses p_1 and firm 2 chooses p_2.

Here we classify demand functions into the following four cases:

$$\text{case (1): } \partial x_i / \partial p_j > 0 \text{ and } \partial R_i / \partial p_j > 0 \quad (i = 1, 2)$$

$$\text{case (2): } \partial x_i / \partial p_j > 0 \text{ and } \partial R_i / \partial p_j < 0 \quad (i = 1, 2)$$

$$\text{case (3): } \partial x_i / \partial p_j < 0 \text{ and } \partial R_i / \partial p_j > 0 \quad (i = 1, 2)$$

$$\text{case (4): } \partial x_i / \partial p_j < 0 \text{ and } \partial R_i / \partial p_j < 0 \quad (i = 1, 2)$$

Case (1) is the most natural case. In cases (1) and (2) the goods of firms are substitutes. In cases (1) and (3) each firm has an upward-sloping reaction curve and in cases (2) and (4) each firm has a downward-sloping reaction curve. Case (1) is considered a standard case of price-setting competition.

In cases (3) and (4) the firms' goods are complements. There are some important examples of complementary goods in price-setting competition. For example, consider the following situation: firm 1 and firm 2 are retailers located at the same district. If firm 1 raises its price, the number of consumers visiting the district decreases, resulting in a reduction in firm 2's demand [See, e.g., Krugman [41], and Matsuyama [54]].

3.4.1 Cases (1) and (4)

In this subsection, we discuss the case of strategic complements in which goods are substitutes (case (1)) and the case of strategic substitutes in which goods are complements (case (4)). We find that the equilibrium outcome is of Bertrand-type only.

Proposition 3.1: *Suppose that Assumptions 3.1–3.5 are satisfied. Suppose that* $sign(\partial x_i/\partial p_j) = sign(\partial R_i/\partial p_j)$ $(i = 1, 2)$. *Then the set of equilibrium outcomes* E *is* $\{(B_1, B_2)\}$.

Proof: See Appendix B.

The intuition behind Proposition 3.1 is quite clear. In case (1), since the demand of each firm is increasing in the rival's price, the Stackelberg leader has an incentive to raise the rival's price. Since the rival's reaction curve has a positive slope, the leader wants to commit a higher price in order to induce a higher price from the rival. Therefore, if firm i commit to a price in the first stage, it chooses $L_i > B_i$. However, the rival (firm j) knows that $L_i > R_i(F_j)$ and that firm i can cut its price in the second stage. Thus the price announced by firm i is never credible and the price announcement never serves as a commitment device. As a result, the unique equilibrium outcome is of Bertrand-type.

In case (4), since the demand of each firm is decreasing in the rival's price, the Stackelberg leader has incentive to cut the rival's price. Since the rival's reaction curve has a negative slope, the leader wants to commit to a higher price in order to induce a lower price from the rival, but as above this commitment is not credible.

In case (1), there is a second-mover advantage and in case (4), there is a first-mover advantage.[10] Proposition 3.1 states that there are cases in which the Bertrand equilibrium appears whether or not the first-mover has advantage.

3.4.2 Case (2)

In this subsection, we discuss the case of strategic substitutes in which the goods are substitutes (case (2)). We find that there are many equilibrium outcomes including Bertrand and Stackelberg outcomes.

Proposition 3.2: *Suppose that Assumptions 3.1–3.5 are satisfied. Suppose that* $\partial x_i/\partial p_j > 0$ *and* $\partial R_i/\partial p_j < 0$ $(i = 1, 2)$. *Then the set of equilibrium outcomes* $E = \{(p_1, R_2(p_1)) \mid p_1 \in [L_1, B_1]\} \cup \{(R_1(p_2), p_2) \mid p_2 \in [L_2, B_2]\}$

Proof: See Appendix B.

The reason why the Stackelberg outcome (L_1, F_2) appears in equilibrium is as follows: in case (2), since firm 1's demand is increasing in p_2, firm 1 has an incentive to increase p_2. Since firm 2 has a reaction curve with a negative slope, firm 1 wants to commit to a lower price in order to induce a higher price from the rival; thus $L_1 < B_1$. In our model firm 1 is able to commit to a lower price. Therefore, the announcement of the price by firm 1 is credible, and thus the Stackelberg outcome appears.

However, Bertrand equilibrium outcome (B_1, B_2) is also one of the equilibrium outcomes. Suppose that firm 2 chooses $p_2(1) = B_2$. Then firm 1 cannot induce a lower price than B_2 from firm 2, so firm 1 loses the incentive to become a leader by announcing a low price like L_1. A rigorous proof of Proposition 3.2 is given in the Appendix B.

[10] See Gal-Or [33] and Dowrick [24].

3.4.3 Case (3)

In this subsection, we discuss the case of strategic complements in which the goods are complements (case (3)). We find that the equilibrium outcome is of Stackelberg-type only.

Proposition 3.3: *Suppose that Assumptions 3.1–3.5 are satisfied. Suppose that* $\partial x_i/\partial p_j < 0$ *and* $\partial R_i/\partial p_j > 0$ $(i = 1, 2)$. *Then the set of equilibrium outcomes* $E = \{(L_1, F_2), (F_1, L_2)\}$.
Proof: See Appendix B.

The reason why the Stackelberg outcome (L_1, F_2) becomes an equilibrium one is almost the same as in case (2). As opposed to other cases, the Bertrand outcome (B_1, B_2) is not found in equilibrium. Since the goods are complements, firm 1 has an incentive to decrease p_2. Since firm 2 has a reaction curve with a positive slope, firm 1 wants to commit a lower price in order to induce a lower price from the rival; thus $L_1 < B_1$. As we will prove in Lemma 3.3 in the Appendix B, the Bertrand outcome can appear only when $(p_1(1), p_2(1)) \geq (B_1, B_2)$. Given $p_2(1) \geq B_2$, firm 1 chooses $p_1(1) = L_1 < B_1$ in order to induce the price-cutting of firm 2; thus, by contradiction the Bertrand outcome is never found in equilibrium. A rigorous proof is given in the Appendix B.

3.4.4 Examples

Finally, we present some examples, which are discussed in many works.

Example 1 (additively separable demand function)
Suppose that the demand is additively separable, i.e., x_i can be denoted as $x_i = y_i(p_i) + z_i(p_j)$. This example was discussed by Robson [65]. In this case, we have that $\mathrm{sign}(R_i') = \mathrm{sign}(\partial x_i/\partial p_j)$. From Proposition 3.1, the equilibrium outcome is of Bertrand-type only.

Example 2 (homogeneous goods)
Consider a Bertrand duopoly in which firm 1's (firm 2's) unit cost is c_1 (c_2), respectively.

Without loss of generality we assume that $c_1 \geq c_2 \geq 0$. The duopolists produce perfectly substitutable commodities for which the market demand function is given by $X(p)$ (quantity as a function of price). The demand of firm i is given as follows:

$$x_i = \begin{cases} X(p_i) & \text{if } p_i < p_j \\ (1/2)X(p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

where the market demand X is decreasing in the equilibrium price $p \equiv \min(p_1, p_2)$.

In this example, the goods are substitutes, and the strategies are strategic complements. This example does not satisfy Assumption 3.1, so we cannot directly apply the result of Proposition 3.1 to this case. We find that any price lies in the interval $[\min(c_1, p_2^M), c_2]$ becomes one of the equilibrium prices where p_2^M is the monopoly price of firm 2. Furthermore, if we eliminate the equilibria which are supported by weakly dominated strategies, the equilibrium price becomes unique and is equal to $\min(c_1, p_2^M)$. This is exactly the same as the Bertrand equilibrium price.[11]

Example 3 (entry-deterrence)

Consider a model with fixed entry costs. Suppose that the goods are substitutes. In the first stage, each firm simultaneously and independently decides whether or not to enter the market. When firm i decides to enter, it chooses $p_i(1) \in [0, \infty)$. At the end of the first stage, each firm i knows whether or not firm j entered the market, and observe $p_j(1)$. In the second stage, if firm i has already entered the market, then it chooses the actual price $p_i \in [0, p_i(1)]$. If firm i has not entered the market yet, it decides whether or not to enter the market and chooses the actual price $p_i \in [0, \infty)$ after entering the market.

It is assumed that due to the entry costs firm i's profit (payoff) never becomes positive if $p_j \leq \bar{p}_j$. In other words, firm j is able to deter the entry of firm i if firm j names a price

[11] If we use the contingent demand function defined by Shubik [75] instead the demand function discussed above, we can derive the Bertrand equilibrium as the unique equilibrium price without using equilibrium refinement discussed above. See also Kreps and Scheinkman [40] and Boyer and Moreaux [10].

which is smaller than or equal to $\bar{p_j}$ before firm i enters the market. We assume that

$$0 < \Pi_i(B_i, B_j) < \Pi_i(\bar{p_i})$$

where (B_i, B_j) is a Bertrand equilibrium, and $\Pi_i(\bar{p_i})$ is firm i's payoff when firm i names the limit price $\bar{p_i}$ and deters the entry of firm j. We also assume that $\bar{p_i} \leq p_i^M$ where p_i^M is firm i's monopoly price.

If firm i chooses an aggressive price policy like limit pricing, the rival becomes less aggressive (giving up the entry.). So this example is closely related to the case (2). Needless to say, we cannot directly apply the result of Proposition 3.2 to this example. However, we can easily derive the result similar to Proposition 3.2 from this example. The following outcomes are supported as subgame perfect equilibria: (1) the Bertrand-type outcome where both firms enter the market in the first stage; and (2) the Stackelberg-type outcome where firm i deters the entry of firm j by choosing the limit price $\bar{p_i}$.

Example 3 above can be used in the context of entry-deterrence. In most entry-deterrence models there is an incumbent before the game. Our example 3 states that even if there is no incumbent and some potential new entrants are faced with competition, one firm succeeds in deterring the rival's entry.

3.5 Concluding remarks

One of the main messages of existing works on endogenous timing games is that we should pay more attention to sequential-move models (Stackelberg models) rather than simultaneous-move models (Cournot or Bertrand models), because only the Stackelberg-type outcomes are found in equilibrium in endogenous timing games [See Hamilton and Slutsky [37], Robson [66], Anderson and Engers [4], and Albæk [2]].

However, existing works pay little attention to the following problem: how the leader commit to its action before the market opens. In reality, there is a considerable possibility that the leader deviate from its stated price before the market opens. In this chapter we introduce incompleteness of price-commitment. We find that the Bertrand-type outcome is the unique equilibrium outcome in the most natural case where the strategies are strategic complements and goods are substitutes. From this result, as opposed to other works, we should emphasize that the Bertrand model is more plausible than the Stackelberg one in the context of price-setting competition.

In this chapter we examine a one-shot game. In reality, firms are faced with long-run competition. If we consider a repeated game with endogenous timing, each firm may strongly attach to the price it announced because a discount of the price may cause price war in the near future.[12] We should consider the effectiveness of price-commitment by a leader under long-run competition. This remains for future research.

[12] See, e.g., Green and Porter [35].

Chapter 4

Quantity-Setting Oligopoly with Endogenous Sequencing

Abstract

This chapter investigates endogenous sequencing of a quantity-setting oligopoly model. We formulate an n-firm, m-period model where each firm chooses both how much to produce and when to produce it. We investigate two models, the first does not include any inventory costs and the second includes small inventory costs. We find that in the cases without inventory costs, at least n - 1 firms simultaneously produce in the first period. In the cases with small inventory costs, pure strategy equilibria exist only if m is 2, and in every equilibrium exactly n - 1 firms simultaneously produce in the first period. As opposed to existing works emphasizing the advantage of the Stackelberg over the Cournot, these results show that the generalized Stackelberg equilibrium where each firm produces sequentially never appears in equilibrium except for duopoly cases.

This chapter is based on Matsumura [48, 50]. I am grateful to Murdoch MacPhee, Masuyuki Nishijima, and Makoto Okamura
for their helpful discussions. Needless to say, I am responsible for any remaining defects.

4.1 Introduction

Cournot and Stackelberg models have occupied important positions among the models of oligopoly. These models have been introduced in many models and subjected to intensive discussion. The Cournot model formulated a simultaneous-move game, while the Stackelberg model formulated a sequential-move game. If we consider a simple quantity-setting duopoly game, the game is either a sequential-move game (Stackelberg game) or a simultaneous-move game (Cournot game).

If we consider a model involving more than two firms, we can analyze more varied situations: a pure simultaneous-move model in which all firms choose their actions at the same time (Cournot model), a pure sequential-move model in which all firms choose their actions at different times (generalized Stackelberg model),[1] and mixed-type models in which there is a sequence of periods in each of which some players choose their actions simultaneously.

Each of the models above has been intensively discussed in various contexts. Obviously, each model produces a different equilibrium outcome, i.e., the equilibrium outcome crucially depends on the sequence of each firm's choice in the model. This suggests the following question: which of the models is natural for analysis of oligopolistic markets? The aim of this chapter is to find an answer to the problem by introducing endogenous sequencing into a standard quantity-setting oligopoly model.

In many economic situations, it is more reasonable to assume that firms choose not only what actions to take, but also when to take them. A number of economists have investigated this problem. Hamilton and Slutsky, Robson [66], Albæk [1, 2], Mailath [44] and Matsumura [47] discussed duopoly games with endogenous sequencing and emphasized that Stackelberg-type outcomes appear in endogenous timing games but a Cournot-type outcome

[1] For the concept of generalized Stackelberg model, see Robson [65] and Anderson and Engers [4].

does not. Although in some duopoly models, both Stackelberg-type and Cournot-type outcomes appear in equilibria,[2] it is known that the existence of Cournot-type equilibrium is heavily dependent on minor model specifications while Stackelberg-type equilibria are robust.[3] However, these are restrictive in the sense that they investigated duopoly rather than n-firm oligopoly.[4]

In this chapter, we formulate n-firm, m-period, quantity-setting oligopoly models in which each firm can choose when to take its action as well as what action to take. First, we formulate a basic model where each firm's payoff depends only on its own output level and that of other firms. We find that there are two types of equilibria: either all firms simultaneously produce, or n - 1 firms simultaneously produce first and thereafter one firm produces, and that no other equilibrium exists. In other words, most firms (all or all but one) produce simultaneously in every equilibrium. We think that this result shows the advantage of the simultaneous-move model (the Cournot model) rather than the sequential-move model (the generalized Stackelberg model).

Next, we allow each firm's payoff to depend on when it produces as well as how much it and others produce. In reality, each firm's profit depends on the timing of production. For example, production in an earlier stage may increase inventory or interest costs (see, e.g., Robson [66] and Matsumura [47]), or the firm produces first may lose the opportunity to obtain a better production technology or useful information about the demand or rivals' cost conditions (see, e.g., Gal-Or [34], Albæk [2], Chamley and Gale [16], and Matsumura and Ueda [53]). We introduce small inventory costs, as typical costs of producing in earlier

[2]See, e.g., Saloner [71].

[3]In the game with action commitment formulated by Hamilton and Slutsky [37], the Cournot-type equilibrium exists but is supported by weakly dominated strategies. Thus the Cournot-type equilibrium is unstable with resect to even small perturbations for the model, See also Albæk [2], and Matsumura [47].

[4]Anderson and Engers [4] formulated an n-player game with endogenous timing and found that the equilibrium outcome is of generalized Stackelberg-type (i.e., perfectly sequential-move outcome). Their model differs from our model, however, in that not all firms can choose their timing.

periods, into the basic model. We find that pure strategy equilibria exist only if m is 2, and in every equilibrium exactly n - 1 firms simultaneously produce in the first period. In case of duopoly, the equilibrium outcomes is of Stackelberg-type only. Thus, in duopoly this result is quite similar to that of existing works emphasizing the advantage of the Stackelberg over the Cournot. However, as opposed to existing works, this result states that generalized Stackelberg equilibrium never appears in equilibrium except for duopoly.[5]

The remainder of this chapter is organized as follows. In Section 4.2, we formulate a basic model. In Section 4.3, we discuss two-stage oligopoly models with exogenous sequencing as a benchmark. Section 4.4 discusses the equilibria of the basic model. Section 4.5 modifies the basic model by introducing inventory costs. Section 4.6 concludes the chapter. All proofs are presented in the Appendix C.

4.2 The basic model

In this section, we formulate a (m+1)-stage, n-firm oligopoly game. Both m and n are larger than one. The set of firms is denoted by N where $N = \{1, 2, ..., n\}$. Throughout Chapter 4 we use a perfect Bayesian equilibrium as our equilibrium concept.

In the first stage (period 0), each firm $i \in N$ chooses the timing $e_i \in \{1, 2, ..., m\}$ where $e_i = t$ implies that firm i chooses period t. The set of firms choosing period t is denoted by N^t. At this time, each firm $j \neq i$ does not observe e_i.[6] Each firm $i \in N^1$ independently

[5]We must emphasize that our results are crucially dependent on the assumption that each of our models has a first period. If we constructs a model with infinite earlier periods, most of our propositions do not hold true. We think that a model with infinite earlier periods is quite important, but a model with finite period is also important and well worth discussing. For an important model with infinite earlier periods, see Robson [66].

[6]If we assume that all firms observe it, the game becomes an extension of the extended game with observable delay formulated by Hamilton and Slutsky [37]. On the other hand, if we assume that it is unobservable, the game becomes an extension of the extended game with action commitment formulated in the same paper. As we state in the Introduction of Chapter 1, we note particularly the result derived from the latter game of Hamilton and Slutsky. This is why we assume that it is unobservable. If we assume that it is observable to all firms, as well as in Hamilton and Slutsky the Cournot-type outcome is supported by a subgame perfect equilibrium under the assumptions made in Sections 4.2 and 4.3.

chooses the output quantity $x_i \in [0, \bar{x}]$ in period 1.

At the beginning of period $t(\geq 2)$, each firm observes actions taken in period $t-1$; as a result, each firm knows which firms belong to N^{t-1}. In period t, each firm $j \in N^t$ independently chooses the output quantity $x_j \in [0, \bar{x}]$.

At the end of period m, the market opens and each firm sells its own output. Firm i's payoff $U_i : R^2_+ \to R$ is given by $U_i(x_i, X_{-\{i\}})$ where $X_{-\{i\}} \equiv \sum_{j \in N \setminus i} x_j$.[7] We now make the following assumptions about the payoff function:

Assumption 4.1 (Symmetric players).[8] $U_i(a,b) = U_j(a,b)\ \forall i,j \in N$, and $(a,b) \in R^2_+$;

Assumption 4.2 (Differentiability). U_i is twice continuously differentiable ;

Assumption 4.3 (Substitutability). $\partial U_i / \partial X_{-\{i\}} < 0$;

Assumption 4.4 (Strategic substitutability) $\partial^2 U_i / \partial x_i \partial X_{-\{i\}} < 0$;

Assumption 4.5 (Concavity). $\partial^2 U_i / \partial x_i^2 < 0$; and

Assumption 4.6 (Interior solution).[9] $\partial U_i(0,.)/\partial x_i > 0$, and $\partial U_i(\bar{x},.)/\partial x_i < 0$.

Definition 4.1 (leader, follower, intermediate, and the last intermediate). We call firms producing in period 1 leaders, call firms followers if they produce in period $t' > 1$ and no firm produces in period $t > t'$, and call other firms intermediates. We call firms the last intermediates if they are intermediates and no intermediate produces after them.

Before investigating the equilibrium outcomes of the game, we discuss the standard Cournot game where all firms simultaneously produce.

In the Cournot game, given the outputs of other firms, each firm $i \in N$ chooses its output quantity. Each firm maximize its payoff $U_i(x_i, X_{-\{i\}})$ with respect to x_i. For firm i

[7] We can easily extend the model to differentiated goods model.

[8] This assumption is not essential but greatly economizes the space.

[9] Some readers may think that this assumption is too strong to derive general results. However, we can derive results similar to our propositions without this assumption. For example, instead of Proposition 4.3 discussed in Section 4.3, we can derive the following result without Assumption 4.6: "In equilibrium the number of firms producing positive output in period $t > 1$ is at most one."

we define

$$R_i(X_{-\{i\}}) \equiv \arg \max_{\{x_i \geq 0\}} U_i(x_i, X_{-\{i\}}), \tag{4.1}$$

i.e., $R_i(X_{-\{i\}})$ is firm i's reaction function. Here we make an assumption about R_i.

Assumption 4.7 (Stability condition). $|\partial R_i / \partial X_{-\{i\}}| \leq 1/2$).

Assumption 4.7 is more restrictive than the standard stability condition ($|\partial R_i / \partial X_{-\{i\}}| < 1$). As shown the following Lemma 4.1, this assumption ensures the disadvantage of a follower.

4.3 Two-period games with exogenous sequencing

Before investigating the equilibrium outcomes of the game with endogenous sequencing, we discuss s-firm, 2-stage games with exogenous sequencing as a benchmark. Discussion here is an extension of Dowrick [24] duopoly models to oligopoly models.[10] We will show that leaders have advantage in cases of strategic substitutes. The following results are used for the proof of our main results discussed in the following two sections. From here on, we restrict our attention to pure strategy equilibria.

We consider the following two-stage games with exogenous timing. The set of players is $S \subseteq N$. Each firm $h \notin S$ has already produced before the game. Let $\tilde{X}$ denote the total output of firms which do not belong to S, and $\tilde{X}$ is given exogenously.

In the first stage, after observing $\tilde{X}$ each firm $i \in S^* \subseteq S$ simultaneously chooses its output. In the second stage, after observing $\tilde{X}$ and the outputs of firms which belong to S^*, each firm $j \in S \backslash S^*$ simultaneously chooses its output.

(1): the Cournot game

Suppose that $|S^*| \geq 1$ and $S^* = S$, i.e., all firms produce in the first stage.

[10] Strictly speaking, Dowrick formulated duopoly models under more general situations. For example, he discussed strategic complements cases as well as strategic substitutes cases, while we restrict our attention to strategic substitutes cases.

Definition 4.2 (Cournot output). Let $C(S, \tilde{X})$ denote the equilibrium output of the Cournot game above, where $\tilde{X} \equiv \sum_{\{i \in N \setminus S\}} x_i$. It is given by $C = R\Big((|S|-1)C + \tilde{X}\Big)$, where R is the reaction function defined in Section 4.2.

The existence and the uniqueness of the equilibrium is guaranteed by the stability condition (Assumption 4.7).

(2): the game with followers

Suppose that $S^* \subset S$ and $S^* \neq \emptyset$.

Definition 4.3 (the outputs of leaders and followers). Let $L(S, S^*, \tilde{X})$ denote each firm i's ($i \in S^*$) equilibrium output and $F(S, S^*, \tilde{X})$ denote each firm j's ($j \in S \setminus S^*$) equilibrium output. In cases of one follower (i.e., $|S \setminus S^*| = 1$), they are given by

$$F = R(|S^*|L + \tilde{X}), \text{ and } L \in \arg\max_x U\Big(x, (|S^*|-1)L + R((|S^*|-1)L + \tilde{X} + x) + \tilde{X}\Big).$$

In cases of more than one follower (i.e., $|S \setminus S^*| > 1$), they are given by

$$F = C(S \setminus S^*, \tilde{X} + |S^*|L), \text{ and } L \in \arg\max_x U\Big(x, \tilde{X} + (|S^*|-1)L + |S \setminus S^*|\bar{C}\Big)$$

where $\bar{C} \equiv C(S \setminus S^*, \tilde{X} + (|S^*|-1)L + x)$.

Assumption 4.8 (The existence and the uniqueness of the equilibrium).[11] For any $S^* \subseteq S \subseteq N$, and $\tilde{X} \in R_+$, there exists a pure strategy equilibrium and the equilibrium is unique in each two-stage game with exogenous sequencing formulated above.

Here we show that each follower strictly prefers the Cournot outcome to the follower's outcome if the number of followers is one.[12]

[11] The assumption of uniqueness is for simplicity, but the assumption of existence is essential. Obviously, if no pure strategy equilibrium exists in the games with exogenous sequencing above, then no pure strategy equilibrium exists in the game with endogenous sequencing.

[12] In cases of duopoly, this result, as well as Lemma 4.2, is shown by Gal-Or [33] and Dowrick [24].

Lemma 4.1: *Suppose that Assumptions 4.1–4.8 are satisfied. Suppose that* $|S| \geq 2$. *Then*

$$U_i\Big(C(S,\tilde{X}),(|S|-1)C(S,\tilde{X})+\tilde{X}\Big) > U_i\Big(F(S,S\backslash\{i\},\tilde{X}),(|S|-1)L(S,S\backslash\{i\},\tilde{X})+\tilde{X}\Big)$$

$\forall\ i \in N, \tilde{X} \in R_+$, *and* $S \subseteq N$.

Proof: See Appendix C.

Next we show that if the number of leaders is one, the leader strictly prefers the leader's outcome to the Cournot outcome.

Lemma 4.2: *Suppose that Assumptions 4.1–4.8 are satisfied. Suppose that* $|S| \geq 2$. *Then* $U_i\Big(C(S,\tilde{X}),(|S|-1)C(S,\tilde{X})+\tilde{X}\Big) < U_i\Big(L(S,\{i\},\tilde{X}),(|S|-1)F(S,\{i\},\tilde{X})+\tilde{X}\Big)\ \forall\ i \in N$, $\tilde{X} \in R_+$, *and* $S \subseteq N$.

Proof: See Appendix C.

4.4 Equilibrium outcome

In this section we discuss the equilibrium outcomes of the basic model formulated in Section 4.2. We restrict our attention to pure strategy equilibria.

Proposition 4.1: *Suppose that Assumptions 4.1–4.8 are satisfied. Then the number of followers is at most one in every equilibrium.*

Proof: See Appendix C.

Proposition 4.2: *Suppose that Assumptions 4.1–4.8 are satisfied. Then no intermediate exists in equilibrium.*

Proof: See Appendix C.

From Proposition 4.2, we have that each firm is either a leader or a follower in equilibrium; thus pure sequential-move outcomes (generalized Stackelberg outcomes) where no firms simultaneously produces in the same period never appear in equilibrium. The following Proposition 4.3 states that the number of leaders is n or n - 1 in every equilibrium, and that such equilibria exist.[13] Proposition 4.3 states that the number of leaders rather than followers increases when the number of players increases.

Proposition 4.3: *Suppose that Assumptions 4.1–4.8 are satisfied. Then, (i) there exist an equilibrium where all firms become leaders, (ii) there exist equilibria where all but one firm become leaders, and (iii) no other pure strategy equilibrium exists.*

Proof: See Appendix C.

4.5 A model with inventory costs

In our basic model, as well as in Hamilton and Slutsky [37], each firm's payoff depends on its own production level and that of others but does not directly depend on the timing of production. In reality, each firm's profit depends on the timing of production as well as on its own and others' level of production. For example, production in an earlier stage may increase the inventory costs or interest costs, or the firm producing first may miss the opportunity to obtain a better production technology or useful information about demand or rivals' cost conditions.

In this section, we allow each firm's payoff to depend on when it produces as well as the level of production by introducing small inventory costs into the basic model.[14]

Here we define the new payoff function of each firm i, V_i, as follows:

$$V_i(x_i, X_{-\{i\}}, e_i) \equiv U_i(x_i, X_{-\{i\}}) - \varepsilon I_i(e_i, x_i)$$

[13] Some readers may think that a refinement of the equilibria discussed by Hamilton and Slutsky [37] (elimination of weakly dominated strategies) can be applied to this model. If the number of firms is two, we can eliminate the Cournot-type equilibrium by this refinement. In cases of more than two firms, however, we cannot eliminate it , i.e., the Cournot-type equilibrium is supported by undominated strategies.

[14] If the inventory costs are sufficiently large, no firm wants to become a Stackelberg leader. Hence the unique equilibrium outcome is of Cournot-type, where firms produce in the last period only. In order to eliminate such an obvious case, we consider the case of small costs.

where ε is a constant and $I_i : \{1, 2, ..m\} \times [0, \bar{x}] \to R_+$.[15] We call εI_i inventory costs of firm i. Here we make the following assumptions:

Assumption 4.9. Assumptions 4.1–4.8 hold true when we replace U_i with V_i ; and

Assumption 4.10. I_i is bounded and strictly decreasing in e_i.

The following Propositions 4.4 and 4.5 state that, if ε is positive and small, the equilibrium outcomes in two-period duopoly are of Stackelberg type only, while no such an equilibrium exists in oligopoly involving more than two firms.

Proposition 4.4: *Suppose that Assumptions 4.9 and 4.10 are satisfied. Suppose that $m = 2$. Then there exists $\bar{\varepsilon}$ such that for every $0 < \varepsilon < \bar{\varepsilon}$, there exist pure strategy equilibria and the number of leaders is n - 1 in every equilibrium.*

Proof: See Appendix C.

Proposition 4.5: *Suppose that Assumptions 4.9 and 4.10 are satisfied. Suppose that $m > 2$. Then there exists $\bar{\varepsilon}$ such that for every $0 < \varepsilon < \bar{\varepsilon}$, no pure strategy equilibrium exists.*

Proof: See Appendix C.

We should not emphasize the non-existence of pure strategy equilibrium. In our model we allow each firm to produce in one of m periods only. If we allow firms produces over m periods (like Saloner [71]), Proposition 4.5 does not hold true. For example if we allow each firm to produce additional output in every period and restrict our attention to fixed-type inventory costs discussed in Chapter 2, we can show that in cases where m are larger than 2, :

(1) Cournot-type equilibrium where all firms produce in period 1 only exists;

(2) semi-Cournot-type equilibria where only one firm produces in the last period and the

[15] Similar costs are discussed by Robson [65, 66] and Matsumura [47].

others produce in period 1 only exist; and

(3) the generalized Stackelberg-type equilibria never appears in equilibrium except for duopoly.

Anyway, we cannot rationalize the generalized Stackelberg model.

Finally, we note the case of negative inventory costs. Although this may be unrealistic, we present a result as a benchmark.[16] Proposition 4.6 states that the equilibrium outcome is of Cournot-type only in cases of negative ε.[17]

Proposition 4.6: *Suppose that Assumptions 4.9 and 4.10 are satisfied. Suppose that ε is negative. Then a unique equilibrium exists, in which all firms become leaders.*

Proof: See Appendix C.

4.6 Concluding remarks

In many economic situations, it is more reasonable to assume that firms choose not only what actions to take, but also when to take them. Some economists have investigated this problem. Hamilton and Slutsky [37], Robson [66], Albæk [2], Mailath [44] and Matsumura [47] discussed duopoly games with endogenous sequencing and emphasized that Stackelberg-type outcomes appear but a Cournot-type outcome does not. These papers suggested that the Stackelberg model is more plausible than the Cournot model if firms can choose when to take their actions. It is also well known that in duopoly cases the Stackelberg equilibrium outcomes are quite robust for small perturbations such as the introduction of small inventory costs, or informational advantage of waiting.

In this chapter we stress that the above result is misleading. We find that, without

[16] The assumption of negative ε implies that an earlier production economizes on the costs. It is possible if production costs increase over time. See Pal [63].

[17] In the duopoly case, Pal [63] derives the results similar to our Propositions 4.4 and 4.6 by extending the Saloner [71] model.

inventory costs, most firms (all or all but one) take their actions in the first period and the number of followers is at most one. If the number of firms is large, the number of leaders acting simultaneously also becomes large; thus the equilibrium outcome becomes very close to that of Cournot-type. This result shows the advantage of the simultaneous-move model (the Cournot model) rather than the sequential-move model (the generalized Stackelberg model).

We also find that, with small and positive inventory costs, the result is crucially dependent on n (the number of players) and m (the number of periods). In the cases with small inventory costs, pure strategy equilibria exist only if m is 2, and in every equilibrium exactly n - 1 firms simultaneously produce in the first period. This result shows that the generalized Stackelberg-type equilibrium where each firm produces sequentially never appears in equilibrium except for duopoly. This also implies that two-period duopoly models are restrictive to investigate games with endogenous sequencing.

Independently, Nishijima [58] formulated a different oligopoly model with endogenous sequencing and derives a result similar to our Proposition 4.3. In addition to our assumptions (except for the stability condition (Assumption 4.7)), he requires three conditions which imply that the first-mover enjoys an advantage in various situations. In order to derive our Proposition 4.3, we use two of Nishijima's three conditions, but we derive them from other standard assumptions rather than assuming. More importantly, we do not need the first condition of him, which is the most restrictive condition among the three in the sense that it is not satisfied in a broad class of standard quantity-setting oligopoly models.[18]

Our model can easily be extend to a quantity-setting oligopoly with differentiated goods, but we find it difficult to apply our results to a price-setting oligopoly. It is natural to consider strategic complements cases in the context of price-setting competition, but our

[18] For example, it is not always satisfied even in linear-demand, linear-cost cases.

results are heavily dependent on the assumption of strategic substitutes (Assumption 4.4).[19] In the context of price-setting oligopoly there is a famous model investigating the sequences of firms' choices, a price-leadership model.[20] We should extend our oligopoly model in order to analyze more general situations such as these covered by the Hamilton and Slutsky [37] duopoly model, and this remains for future research.[21]

[19] For example, Proposition 4.3 (i) and (ii), hold true for strategic complements but (iii) does not. In some cases of strategic complements, pure sequential-move outcomes are found in equilibrium.

[20] See, e.g., Ono [61, 62] and Ito and Ono [38].

[21] If we restrict our attention to two-period cases, we obtain more general results. See Matsumura [48].

Chapter 5

Endogenous Timing in a Two-Stage Strategic Commitment Game

Abstract

This chapter presents an investigation of the endogenous timing in multi-stage duopoly games in which duopolists choose two variables over two periods. We elaborate the two-stage strategic commitment game discussed by Brander and Spencer [12]. Duopolists decide their capacity investments and cost-reducing investments and they are allowed to choose which action to take first. We discuss two types of games; one is a three-stage game in which each duopolist can commit to the order of choices before it chooses its capacity investment or cost-reducing investments, and the other is a two-stage game in which it cannot. We find that at least one firm chooses its capacity investment first. Furthermore, the three-stage game has the unique equilibrium outcome in which both firms choose their outputs first.

This chapter is based on Matsumura [51]. I am grateful to Lim Chin, Yoshitsugu Kanemoto, Kazuharu Kiyono, and Yuval Shilony for their helpful comments on the early draft. I am also indebted to an anonymous referee for detailed and precious suggestions. Needless to say, I am responsible for any remaining defects.

5.1 Introduction

Many economists have investigated models in which firms compete in terms of more than one variable over multi-period.[1] Strategic commitments in the upstream stage, which distort their actions in the downstream stage, have been intensively discussed. Brander and Spencer [12] developed a two-stage duopoly game in which duopolists chose cost-reducing investment in the first stage and chose output in the second stage. They found that the first-stage investment serves as a commitment device. The strategic value of cost-reducing investment induces firms to make very aggressive investment if the strategies in the second stage are strategic substitutes. Such aggressive investment by both firms, however, accelerates competition in the second stage resulting in reducing their profits.

Many devices other than cost-reducing investment are known to serve as commitment devices, and these have the same kind of strategic values. The strategic values of these devices are, however, crucially dependent on the time structure of the game. Brander and Spencer also showed that the strategic value of cost-reducing investment disappears if duopolists choose both cost-reducing and output at the same time; therefore the equilibrium investment is smaller than that in the two-stage counterpart.[2] If we restrict our attention to the two-stage competition, the equilibrium cost-reducing investment level may drastically change if cost-reducing investment are determined after output.

If we regard cost-reducing investment as R & D investment, it is natural to assume, as they did, that the cost-reducing investment is chosen before output. In many actual situations, however, duopolists can choose which action to take first between more than one action rather than choosing actions by an exogenously fixed order. For example, consider the following situation: a firm chooses capacity investment for production and cost-saving

[1] See, e.g., Spence [78], Shaked and Sutton [74], Brander and Lewis [11], Fershtman and Judd [29], and Chu and Nishimura [17]

[2] See also Dasgupta and Stiglitz [18].

investment for distribution. The firm's output capacity depends on its investment for production and per-output distribution cost depends on its investment for distribution. In this case, it is natural to assume that the firm can control which investment to make first.

In this chapter, we investigate two-player, multi-stage games in which duopolists choose two kinds of actions (cost-reducing investment and output) over two periods. If they choose cost-reducing investment first, they choose output in the next stage and *vice versa*.[3] We discuss two games; one is a three-stage game in which firms can commit to the order of their choices before they choose its output or investments, and the other is a two-stage game in which they cannot commit. We find that in both games there is no equilibrium in which both firms choose cost-reducing investment first, while there is an equilibrium in which both firms choose output first. In the two-stage game, there are asymmetric equilibria in which one firm chooses output first and the other chooses cost-reducing investment first. On the other hand the three-stage game has a unique equilibrium, in which each firm commits to choosing output first. By making a larger cost-reducing investment, each firm reduces its marginal cost and commit to a larger output. However, this commitment is indirect. A firm can make a direct commitment by choosing its output first. If its rival chooses cost-reducing investment first, the firm can reduces its rival's output more effectively by producing output first. This is why there is no equilibrium in which both firms choose cost-reducing first in both games.

In the two-stage game, given that the firm chooses output first, its output is decided before the rival's action; therefore, there is no room for the rival to make a strategic behavior. Hence, given that the firm chooses output in the first stage, it is a matter of indifference for the rival whether or not it chooses output in the first stage. Therefore, both symmetric

[3] We implicitly assume that production level of each firm is equal to its capacity. Strictly speaking, it is possible that each firm chooses output which is strictly smaller than its capacity. Even if we allow this possibility, we obtain the same results derived in this chapter. I thank Professor Shilony for this point.

and asymmetric equilibria exist.

On the other hand, in the three-stage game each firm can choose which action to take first before choosing its output quantity. By committing to choosing output before cost-reducing investment, each firm shows that it never becomes a Stackelberg follower; as a result, it can reduce the output of its rival and increase its profit. This is why both firms choose output first in the three-stage game.

This chapter is organized as follows. In Section 5.2, we formulate the model of the two-stage game in which duopolists cannot commit to the order of the choice. In Section 5.3, in order to present some benchmarks, we discuss the games in which the timing is exogenously given. Section 5.4 analyzes the equilibrium outcomes in the endogenous timing game. Section 5.5 considers the three-stage game in which duopolists can commit to the order of decision. Section 5.6 concludes this chapter.

5.2 The model

In this section we formulate a two-stage game. Throughout this chapter, we consider two symmetric firms which choose two variables over two periods. The two firms are denoted as 1 and 2. We often use i or j to refer to a firm, and it is understood that if i denotes 1 in one expression, then j represents 2 and *vice versa*. Firm i ($i = 1, 2$) chooses the two variables a^i and b^i over two periods. For concreteness, we take a^i and b^i to be firm i's output and cost-saving distribution investment, respectively. Firm i's payoff U^i is given by

$$U^i(a^i, a^j, b^i) \equiv V^i(a^i, a^j) - c^i(a^i, b^i)$$

where $V^i(a^i, a^j)$ is firm i's gross profit, which does not include the distribution cost. Firm i's distribution cost is $c^i(a^i, b^i)$, which includes distribution investment cost. Duopolists have identical gross profit functions and distribution cost functions. We assume that

$$V^i_2 < 0, V^i_{12} < 0, c^i_1 > 0, \text{ and } c^i_{12} < 0 \ (i = 1, 2)$$

where we use subscripts to denote partial derivatives. The inequality $c^i_{12} < 0$ implies that the increase of distribution investment reduces both average and marginal distribution costs: that is, b^i is cost-reducing investment. The inequalities $V^i_2 < 0$ and $V^i_{12} < 0$ imply that a^i and a^j are substitutes in the sense that increasing the output of good j decreases the total and marginal profit of firm i.[4] We assume the second order conditions and the stability condition; namely, we assume the following inequalities:

$$V^i_{11} - c^i_{11} < 0, c^i_{22} < 0, c^i_{22}(V^i_{11} - c^i_{11}) - (c^i_{12})^2 > 0, \mid V^i_{11} - c^i_{11} \mid > \mid V^i_{12} \mid \ (i = 1, 2).$$

These assumptions are standard in the field of industrial organization.

The game runs as follows. In the first stage, each firm i simultaneously and independently chooses its first stage action $e^i(1) \in S = R_+ \times \{a, b\}$. $e^i(1) = (x, a)$ implies that firm i chooses a^i in the first period and sets $a^i = x$. After this first stage, each firm knows $e^1(1)$ and $e^2(1)$. In the second stage, each firm i simultaneously and independently chooses its second stage action $e^i(2) \in R_+$. $e^i(2) = y$ implies that firm i chooses

$$\begin{array}{ll} a^i = y & \text{if } e^i(1) = (\ .b) \\ b^i = y & \text{if } e^i(1) = (\ .a) \end{array}.$$

Throughout Chapter 5, we use a subgame perfect Nash equilibrium as an equilibrium concept. We restrict our attention to pure strategy equilibria.

5.3 Some benchmarks

Before discussing the endogenous timing game, we examine equilibrium outcomes in the games in which the timing is exogenously given. For simplicity, we assume that there is a unique equilibrium and it is an interior solution in the following three games.

[4] Some readers may think that the assumption of strategic substitutes ($V^i_{12} < 0$) is too restrictive, but Proposition 5.1, which is one of our main results, hold true in the case of strategic complements ($V^i_{12} > 0$).

5.3.1 Output first game

We define (a^C, b^C) as the equilibrium outcome in the game where output is determined in the first stage and cost-reducing investment is determined in the second stage. The first order condition for firm i is given by the following equation:

$$c_2^i = 0 \ (i = 1, 2). \tag{5.1}$$

We define $B^i(a^i)$ as firm i's optimal cost-reducing investment b^i given its output a^i. $B^i(a^i)$ is given by $c_2^i(a^i, B^i(a^i)) = 0$. In the first stage, firm i maximizes U^i with respect to a^i. The first order condition for firm i in the first stage is given by

$$V_1^i - c_1^i = 0 \ (i = 1, 2) \tag{5.2}$$

where we use (5.1). We assume that the second order conditions are satisfied.

Note that (a^C, b^C) coincides with the Nash equilibrium outcome of the single-stage game in which each firm i chooses a^i and b^i at the same time.

5.3.2 Cost-reducing investment first game

We consider the game in which cost-reducing investment is determined in the first stage and output is determined in the second stage. We define (a^B, b^B) as the equilibrium outcome in this game. In the second stage, firm i maximizes U^i with respect to a^i given b^i and b^j. The first order condition for firm i is given by the following equation:

$$V_1^i - c_1^i = 0 \ (i = 1, 2). \tag{5.3}$$

We define $R^i(a^j : b^i) = \arg\max_{\{a^i\}}(V^i(a^i, a^j) - c^i(a^i, b^i))$, i.e., R^i is firm i's reaction function and given by $V_1^i(R^i(a^j : b^i), a^j) - c_1^i(R^i(a^j : b^i), b^i) = 0$. We define $(a^{i*}(b^i, b^j), a^{j*}(b^j, b^i))$ as the equilibrium outcome in the subgame given b^i and b^j. By definition, $a^{i*}(b^i, b^j) = R^i(a^{j*}(b^j, b^i) : b^i)$ and $a^{j*}(b^j, b^i) = R^j(a^{i*}(b^i, b^j) : b^j)$.

In the first stage, firm i maximizes U^i with respect to b^i. The first order condition for firm i is given by

$$V_2^i \frac{\mathrm{d}a^j}{\mathrm{d}b^i} - c_2^i = -V_2^i \frac{c_{12}^i V_{12}^j}{(V_{11}^i - c_{11}^i)(V_{11}^j - c_{11}^j) - V_{12}^i V_{12}^j} - c_2^i = 0 \ (i = 1, 2) \tag{5.4}$$

where we use (5.3). Here we compare b^C with b^B.

Result 5.1: $b^C < b^B$ *and* $U(a^C, a^C, b^C) > U(a^B, a^B, b^B)$.

Proof: See Propositions 2 and 3 in Brander and Spencer [12].

5.3.3 Asymmetric game

Consider the game in which firm i chooses output a^i and firm j chooses cost-reducing investment b^j in the first stage. We define $(a^i, a^j, b^i, b^j) = (a^L, a^F, b^L, b^F)$ as the equilibrium outcome in this game. In the second stage, the first order conditions for firm i and firm j are given by the following equations, respectively:

$$c_2^i = 0 \text{ and } V_1^j - c_1^j = 0 \ (i = 1, 2). \tag{5.5}$$

In the first stage, firm i's and firm j's first order conditions are given by the following equations, respectively:

$$(V_1^i - c_1^i) - V_2^i \frac{V_{12}^j}{V_{11}^j - c_{11}^j} = 0 \text{ and } c_2^j = 0 \ (i = 1, 2). \tag{5.6}$$

a^L, a^F, b^L and b^F satisfy the following four equations:

$$a^L = \arg\max_{\{a^i\}} V^i(a^i, R^j(a^i : b^F)) - c^i(a^i : b^L) \tag{5.7}$$

$$a^F = R^j(a^L : b^F) \tag{5.8}$$

$$b^L = B^i(a^L) \tag{5.9}$$

$$b^F = B^j(a^F). \tag{5.10}$$

In this game firm i is a Stackelberg leader and firm j is a Stackelberg follower for output.

5.4 Endogenous timing game

In this section we investigate the endogenous timing game formulated in Section 5.2. There are four possible pure strategy equilibria; two are symmetric equilibria in which both firms choose the same variables in the first stage, and two are asymmetric equilibria in which one firm chooses a first and the other chooses b first. In this section we show that there is no equilibrium in which both firms choose b first.

Proposition 5.1: *(i) There is an equilibrium in which $e^1(1) = e^2(1) = (a^C, a)$, (ii) there is an equilibrium in which $(a^i, b^i, a^j, b^j) = (a^L, b^L, a^F, b^F)$ $(i = 1, 2)$, and (iii) no other pure strategy equilibrium outcome exists.*

Proof: (i) Suppose that firm i's strategy as follows:

$$e^i(1) = (a^C, a), e^i(2) = \begin{cases} B^i(a^i) & \text{if } e^i(1) = (a^i, a) \\ R^i(a^j : b^i) & \text{if } e^i(1) = (b^i, b) \text{ and } e^j(1) = (a^j, a) \\ a^{i*}(b^i, b^j) & \text{if } e^i(1) = (b^i, b) \text{ and } e^j(1) = (b^j, b). \end{cases}$$

We show that firm j's best response is to do the same as firm i. To compute a subgame perfect Nash equilibrium, the game is solved by backward induction. First, consider the second stage action. By definitions of $B^j(a^j), R^j(a^i : b^i)$ and $a^{j*}(b^j, b^i)$, firm j's best response in the second stages is

$$e^j(2) = \begin{cases} B^j(a^j) & \text{if } e^j(1) = (a^j, a) \\ R^j(a^i : b^j) & \text{if } e^j(1) = (b^j, b) \text{ and } e^i(1) = (a^i, a) \\ a^{j*}(b^j, b^i) & \text{if } e^j(1) = (b^j, b) \text{ and } e^i(1) = (b^i, b). \end{cases}$$

Next, consider the first stage action. Given that $e^i = (a^C, a)$, firm i chooses $b^i = B^i(a^C) = b^C$ in the next stage and b^i is not affected by $e^j(1)$. Thus, firm j maximizes U^j with respect to a^j and b^j given that $a^i = a^C$ and $b^i = b^C$. Optimal a^j and b^j are given by (5.1) and (5.2). Since $a^j = a^C$ and $b^j = b^C$ satisfy (5.1) and (5.2), we have that $e^j(1) = (a^C, a)$ is one of firm j's best responses.

(ii) We construct a particular profile of strategies which yields a subgame perfect equilibrium. We show that following firm j's strategy is its best response to the following firm i's strategy in all subgames and *vice versa*. Suppose that firm i's and j's strategies are as follows:

$$e^i(1) = (a^L, a), e^i(2) = \begin{cases} B^i(a^i) & \text{if } e^i(1) = (a^i, a) \\ R^i(a^j : b^i) & \text{if } e^i(1) = (b^i, b) \text{ and } e^j(1) = (a^j, a) \\ a^{i*}(b^i, b^j) & \text{if } e^i(1) = (b^i, b) \text{ and } e^j(1) = (b^j, b) \end{cases}$$

$$e^j(1) = (b^F, b), e^j(2) = \begin{cases} B^j(a^j) & \text{if } e^j(1) = (a^j, a) \\ R^j(a^i : b^j) & \text{if } e^j(1) = (b^j, b) \text{ and } e^i(1) = (a^i, a) \\ a^{j*}(b^j, b^i) & \text{if } e^j(1) = (b^j, b) \text{ and } e^i(1) = (b^i, b). \end{cases}$$

In the proof of Proposition 5.1 (i), we have already shown that the above strategies in the second stage are best responses for both firms. Thus we consider the first stage actions. First, we show that the above strategy of firm j is its best response. Given that $e^i(1) = (a^L, a)$, $e^i(2) = B^i(a^L)$. Thus $e^j(1)$ and $e^j(2)$ do not affect firm i's actions. Given that $a^i = a^L$ and $b^i = b^L$, (5.8) and (5.10) ensure that the optimal action of firm j is $(a^j, b^j) = (a^F, b^F)$. Since firm i's actions do not depend on firm j's actions, both $e^j(1) = (a^F, a)$ and $e^j(1) = (b^F, b)$ are firm j's best responses in the first stage. Therefore, firm j's strategy above is one of its best responses.

Next, we show that firm i's strategy above is its best response. Suppose that firm i chooses a first. (5.7) and (5.9) ensure that $a^i = a^L$ is the optimal level of a^i given that $e^j(1) = (b^F, b)$. Thus, all we have to show is that firm i cannot increase its payoff by choosing b first. We prove it by contradiction.

Suppose that given $e^j(1) = (b^F, b)$, $e^i(1) = (b^i, b)$ is firm i's best response, while $e^i(1) = (a^L, a)$ is not. If $e^i(1) = (b^i, b)$ and $e^j(1) = (b^F, b)$, then $e^i(2) = a^{i*}(b^i, b^F)$ and $e^j(2) = a^{j*}(b^F, b^i)$. Given firm j's strategy above, firm i takes the following strategy: $e^i(1) = (a^{i*}(b^i, b^F), a)$ and $e^i(2) = B^i(a^i)$. In the second stage, firm j chooses $a^j = R^j(a^{i*}(b^i, b^F) : b^F) = a^{j*}(b^F, b^i)$. Accordingly, this deviation by firm i does not affect a^i and a^j; thus this deviation does not decrease firm i's payoff. Since $U^i(a^L, a^F, B^i(a^L)) \geq$

$U^i(a^{i*}(b^i,b^F),a^{j*}(b^F,b^i),B^i(a^{i*}(b^i,b^F))\geq U^i(a^{i*}(b^i,b^F),a^{j*}(b^F,b^i),b^i)$, we have that $e^i(1)=(a^L,a)$ is one of firm i's best responses, a contradiction.

(iii) Suppose that both firms choose a first. In this case the unique equilibrium outcome is the one described in Proposition 5.1 (i) because the output first game has a unique equilibrium. Suppose that firm i chooses a first and firm j chooses b first. In this case the unique equilibrium is the one described in Proposition 5.1 (ii). Under these conditions, all we have to show is that there is no equilibrium in which both firms choose b first. Suppose that $e^1(1)=e^2(1)=(b^B,b)$ and $e^1(2)=e^2(2)=a^B$ on the equilibrium path. Given firm j's strategy, firm i takes the following strategy: $e^i(1)=(a^B,a)$, and $e^i(2)=B^i(a^B)$. In the second stage, firm j chooses $a^j=R^j(a^B:b^B)=a^B$. Accordingly, this deviation from the equilibrium strategy by firm i does not affect a^j. From (5.4), we have that $b^B\neq B^i(a^B)$. Therefore, $U^i(a^B,a^B,B^i(a^B))>U^j(a^B,a^B,b^B)$. Under these conditions, the above deviation from the equilibrium strategy by firm i increases firm i's payoff, a contradiction. Q.E.D.

In the strategic substitutes cases, the profits in the Brander and Spencer model (b is decided before a) are smaller than in the Cournot counterpart (a is decided before b). Thus, firms' profits in the endogenous timing game are larger than in the Brander and Spencer counterpart if we restrict our attention to symmetric pure strategy equilibrium. If strategies are strategic complements, however, the profits in the Brander and Spencer model are larger than in the Cournot counterpart, but $(a^1,b^1,a^2,b^2)=(a^B,b^B,a^B,b^B)$ never arises in the endogenous timing game; i.e., there is no equilibrium in which both firms choose cost-reducing investment first.[5] Thus, we should note that a Cournot outcome arises in the endogenous timing game not because the Cournot outcome is more favorable for duopolists than the Brander and Spencer outcome, but because direct commitment by

[5] Note that we do not use the condition that $V_{12}<0$ in the proof of Proposition 5.1.

output is more effective than indirect commitment by cost-reducing investment.

As with other endogenous timing games,[6] however, this game also has asymmetric equilibria. Suppose that firm i chooses a first. Since firm j cannot affect the output of firm i, firm j loses an incentive to make strategic commitment; thus choosing a first or not is a matter of indifference for firm j. Therefore, given that firm i chooses a first, choosing b first is one of best responses for firm j. Accordingly, asymmetric equilibria exist.

5.5 Preplay commitment game

In the previous section, we showed that there are three pure strategy equilibria (one symmetric and two asymmetric equilibria). We can compare the payoffs of firms in the three equilibrium outcomes. As Gal-Or [33] and Dowrick [24] showed, if strategies are strategic substitutes (i.e., $V^i_{12} < 0$), the Stackelberg leader has the first mover advantage and the following Result 5.2 is satisfied:

Result 5.2: $U^i(a^L, a^F, b^L) > U^i(a^C, a^C, b^C) > U^i(a^F, a^L, b^F)$.

In this section we discuss a three-stage game in which firms can commit to the order of choices before they choose the amounts of outputs and investments. The game runs as follows. In the first stage, each firm i simultaneously and independently chooses the timing $e^i \in \{a, b\}$. $e^i = a$ implies that firm i chooses output a^i before cost-reducing investment b^i and $e^i = b$ implies *vice versa*.[7] After this first stage, both firms know e^1 and e^2. In the second stage, firm i chooses a^i if $e^i = a$ and b^i otherwise. In the third stage, each firm chooses the one which is not chosen in the previous stage.

Consider the subgame which follows from the end of the first stage. The equilibrium

[6] See, e.g., Robson [66], Anderson and Engers [4]. See also Hamilton and Slutsky [37], and Albæk [1, 2].

[7] Some readers may think that this formation is too restrictive. We can modify the alternatives in the first stage as follows: $e^i \in \{a, b, ab, e, n\}$ in which $e^i = ab$ implies that firm i choose both a^i and b^i in the second stage, $e^i = e$ implies that firm i choose both a^i and b^i in the third stage, and $e^i = n$ implies that firm i does not commit to the order of decision. Proposition 5.2 holds true in this case.

outcome in the subgame depends on e^1 and e^2. Discussions in Section 5.3 stated that firm i's equilibrium payoff in the subgames after the first stage commitments is as follows:

$$U^i = \begin{cases} U^i(a^L, a^F, b^L) & \text{if } e^i = a \text{ and } e^j = b \\ U^i(a^F, a^L, b^F) & \text{if } e^i = b \text{ and } e^j = a \\ U^i(a^C, a^C, b^C) & \text{if } e^i = e^j = a \\ U^i(a^B, a^B, b^B) & \text{if } e^i = e^j = b. \end{cases}$$

Proposition 5.2 states that both firms choose output first if they can commit to a timing.

Proposition 5.2: *There is a unique equilibrium, in which* $(a^i, b^i, a^j, b^j) = (a^C, b^C, a^C, b^C)$.

Proof: If we solve the game by backward induction, we have that duopolists are virtually faced with the game given by the following payoff matrix:

		Firm 2	
		$e^2 = a$	$e^2 = b$
Firm 1	$e^1 = a$	Π^C, Π^C	Π^L, Π^F
	$e^1 = b$	Π^F, Π^L	Π^B, Π^B

where $\Pi^C \equiv U^i(a^C, a^C, b^C)$, $\Pi^L \equiv U^i(a^L, a^F, b^L)$, $\Pi^F \equiv U^i(a^F, a^L, b^F)$, and $\Pi^B \equiv U^i(a^B, a^B, b^B)$.

Recall that $\Pi^C > \Pi^B$ by Result 5,1, and $\Pi^L > \Pi^C > \Pi^F$ by Result 5.2. Hence $\Pi^C > \Pi^F$ and $\Pi^L > \Pi^B$, so that the unique subgame perfect equilibrium entails $(e^1, e^2) = (a, a)$. Q.E.D.

5.6 Concluding remarks

In many economic situations, it is reasonable to assume that each firm has something say not only over its choice of action, but also over the timing of that choice. Therefore, it is important to check the robustness of results derived from models in which firms are restricted to a specified timing. In this chapter, we consider the endogenous timing in the two-stage game discussed in Brander and Spencer. We find that there is no equilibrium in which both firms choose cost-reducing investment before output in the endogenous timing game, while there is an equilibrium where both firms choose output first. Each firm has a strong incentive to choose output first and commit to a large output if it is possible. Hence it tries to increases its capacity first. A retailer has an incentive to buy a whole product line out in order to commit itself to large sales; thereafter reduces distribution costs. Furthermore, firms may try to change the time structure even with additional costs since committing to deciding output before cost-reducing investment increases their profits in the strategic substitutes cases. These cases remain to be considered in more detail in future research.

Appendix A

Appendix of Chapter 2

In Appendix A, we will prove all lemmas and propositions presented in Chapter 2. First, we will prove Lemmas 2.1–2.3 which state the equilibrium outcome in the third-stage subgames given $q^i(2)$ and $q^j(2)$. Let $B^i(x^j : q^i(2))$ denote firm i's reaction function after committing to $q^i(2)$, i.e.,

$$B^i(x^j : q^i(2)) = \arg \max_{\{x^i \geq q^i(2)\}} (V^i(x^i, x^j) - c^i x^i).$$

Since $V^i_{11} < 0$ and $x^i \geq q^i(2)$, we have that $B^i(x^j : q^i(2)) = \max(R^i(x^j), q^i(2))$, i.e., in equilibrium,

$$x^i = \max(R^i(x^j), q^i(2)) \quad (i \neq j,\ i = 1, 2). \tag{A.1}$$

Proof of Lemma 2.1

(By contradiction) Suppose that $q^i(2) \geq C^i$ and $x^i(3) > 0$. Suppose that $q^j(2) + x^j(3) = x^j$ on the equilibrium path. From (A.1), we have that $x^i = R^i(x^j)$ because $x^i > q^i(2)$. In this case, we have that $x^j < C^j$ because $x^i = R^i(x^j) > C^i = R^i(C^j)$ and $R^i(x^j)$ is decreasing. Since $x^j < C^j$, the stability condition ensures that $R^j(R^i(x^j)) > x^j$, but this contradicts (A.1). Q.E.D.

Proof of Lemma 2.2

First, consider the case in which $q^i(2) = C^i$ or $q^j(2) = C^j$. Suppose that $q^i(2) = C^i$ and

$q^j(2) \leq C^j$. Lemma 2.1 states that $x^i(3) = 0$. Given that $x^i = C^i$, firm j's unique best response is $x^j(3) = C^j - q^j(2)$.

Next consider the case in which $(q^i(2), q^j(2)) < (C^i, C^j)$. Suppose that $x^i < C^i$ on the equilibrium path. Since $x^j = \max(q^j(2), R^j(x^i))$ and $R^j(x^i)$ is decreasing, we have that $x^j \geq R^j(x^i) > R^j(C^i) = C^j \geq q^j(2)$. Hence, $x^j = R^j(x^i)$. Since $x^i < C^i$, the stability condition ensures that $R^i(R^j(x^i)) > x^i$, but this contradicts (A.1).

Suppose that $x^i > C^i$ on the equilibrium path. From (A.1), we have that $x^i = R^i(x^j)$ because $x^i > q^i(2)$. Since $x^j \geq R^j(x^i)$ and R^i is decreasing, we have that $C^i < x^i = R^i(x^j) \leq R^i(R^j(x^i))$, but this contradicts the stability condition. Q.E.D.

Proof of Lemma 2.3

Lemma 2.1 states that $x^j = q^j(2)$. Therefore, firm i chooses its total output x^i equal to $\max(q^i(2), R^i(x^j))$. Q.E.D.

Proof of Proposition 2.1

(By contradiction) Suppose that both $x^1(2)$ and $x^2(2)$ are positive. First, we show that in equilibrium $x^i(2) > C^i$ or $x^j(2) > C^j$ (or both). Suppose that $x^i(2) \leq C^i$ and $x^j(2) \leq C^j$. Lemma 2.2 states that $x^j = C^j$ if $x^i(2) \leq C^i$. Thus, if firm i deviates from the equilibrium strategy and chooses $x^i(2) = 0$, the deviation reduces the inventory cost ε without increasing x^j. Accordingly, choosing $C^i \geq x^i(2) > 0$ is not firm i's best response, a contradiction. Hence we can assume that $x^i(2) > C^i$.

From Lemma 2.1, we have that x^i equals $x^i(2)$ regardless of $x^j(2)$. If firm j deviates from the equilibrium strategy and chooses $x^j(2) = 0$, the deviation reduces the inventory cost ε without increasing x^i. Thus, choosing $x^j(2) > 0$ is not firm j's equilibrium strategy, a contradiction. Q.E.D.

Proof of Proposition 2.2

If part: First we show that there is a subgame perfect equilibrium which entails (L^1, F^2).

We construct a particular profile of strategies which yields a subgame perfect equilibrium. We show that firm 1's strategy below is its best response given firm 2's strategy below and *vice versa*. Suppose that the strategies of firm 1 and firm 2 are as follows:

$$x^1(2) = L^1, x^1(3) = \begin{cases} 0 & \text{if } x^1(2) \geq C^1 \\ \max(R^1(x^2(2)) - x^1(2), 0) & \text{if } x^1(2) < C^1 \text{ and } x^2(2) \geq C^2 \\ C^1 - x^1(2) & \text{otherwise.} \end{cases}$$

$$x^2(2) = 0, x^2(3) = \begin{cases} 0 & \text{if } x^2(2) \geq C^2 \\ \max(R^2(x^1(2)) - x^2(2), 0) & \text{if } x^2(2) < C^2 \text{ and } x^1(2) \geq C^1 \\ C^2 - x^2(2) & \text{otherwise.} \end{cases}$$

To compute a subgame perfect Nash equilibrium, the game is solved by backward induction. First we consider the last-stage strategies. Concerning $x^1(3)$ and $x^2(3)$, from Lemmas 2.1–2.3 we have that the above strategies of both firms become equilibria in all subgames.

Next we consider the first-stage action $x^1(2)$ and $x^2(2)$. Given that $x^1(2) = L^1$, we have that $x^1 = L^1$ regardless of $x^2(2)$ from Lemma 2.1. In order to reduce the inventory cost, firm 2 chooses $x^2(2) = 0$ and $x^2(3) = R^2(L^1) = F^2$.

Given that $x^2(2) = 0$, firm i's payoff is as follows:

$$U^1 = \begin{cases} V^1(x^1(2), R^2(x^1(2))) - c^1x^1(2) - \varepsilon & \text{if } x^1(2) \geq C^1 \\ V^1(C^1, C^2) - c^1C^1 - \varepsilon & \text{if } 0 < x^1(2) < C^1 \\ V^1(C^1, C^2) - c^1C^1 & \text{if } x^1(2) = 0. \end{cases}$$

Since $L^1 = \arg\max_{\{x^1 \geq 0\}}(V^i(x^1, R^2(x^1)) - c^1x^1)$, firm 1's best response is $x^1(2) = L^1$ if and only if Assumption 2.9 is satisfied.

Only if part: Next we show that there is no other equilibrium outcomes than two Stackelberg outcomes. Proposition 2.1 states that $x^1(2) = 0$ or $x^2(2) = 0$. The proof of (i) in Proposition 2.2 shows that given that $x^j(2) = 0$, firm i's best response is $x^i(2) = L^i$. Q.E.D.

Here we prove one supplementary lemma in order to prove Proposition 2.3.

Lemma 2.4: *Suppose that Assumptions 2.1–2.4, 2.6 and 2.9–2.10 are satisfied. Consider* *the game where firm i is a Stackelberg leader producing in period 2 only and firm j is a follower producing in period 3 only. Then the Stackelberg leader's output is larger than the Cournot output C^i.*

Proof. The proof is by contradiction. We denote the output of the Stackelberg leader (firm i) by X^i and $R^j(X^i)$ by X^j. Without loss of generality, let $i = 1$ and $j = 2$.

We prove Lemma 2.4 by contradiction. Suppose that $X^1 \leq C^1$. Since $C^1 = R^1(C^2)$, we have $U^1(C^1, C^2, 0, 0) \geq U^1(X^1, C^2, 0, 0)$. Since R^2 is decreasing and $X^1 \leq C^1$, we have $X^2 = R^2(X^1) \geq R^2(C^1) = C^2$. Since V^1 is decreasing in x^2 given x^1, we have $U^1(X^1, C^2, 0, 0) \geq U^1(X^1, X^2, 0, 0)$. From Assumption 2.10, we have that $U^1(X^1, X^2, 0, 0) \geq U^1(X^1, X^2, 0, X^1)$. Thus, $U^i(C^1, C^2, 0, 0) \geq U(X^1, C^2, 0, 0) > U(X^1, X^2, 0, 0) > U(X^1, X^2, 0, X^1)$, so that $U^1(C^1, C^2, 0, 0) \geq U^1(X^1, X^2, 0, X^1)$. This contradicts Assumption 2.11. Q.E.D.

Proof of Proposition 2.3

(i) Proposition 2.1 holds true in this case. (The proof is essentially the same as of Proposition 2.1.) Thus without loss of generality we can assume that $q^2(2) = 0$ in equilibrium. Given that $q^2(2) = 0$, we have that firm 1's best response is choosing $x^1(2) = X^1$ under Assumption 2.10. From Lemmas 2.1 and 2.4, we have that firm 1 never produces in period 3.

(ii) In equilibrium $x^1 = X^1 > C^1$ or $x^2 = X^2 > C^2$, thus, $(x^1, x^2) = (C^1, C^2)$ never becomes an equilibrium outcome. Q.E.D.

Proof of Proposition 2.4

if part Suppose that $(x^{1*}, x^{2*}) \in E$. Without loss of generality we assume that $x^{1*} \geq C^1$. We construct a particular profile of strategies which yields a subgame perfect equilibrium. This is only one example; other strategies also yield the same equilibrium outcome (x^{1*}, x^{2*}). We show that firm 1's strategy below is its best response given firm 2's strategy below and *vice versa.* Suppose that firm 1's and firm 2's strategies are as follows:

$$x^1(1) = x^{1*},$$

$$x^1(2) = \begin{cases} 0 & \text{if } x^1(1) \geq L^1 \\ 0 & \text{if } L^1 > x^1(1) \geq C^1 \text{ and } x^2(1) \geq R^2(x^1(1)) \\ \min(R^{2^{-1}}(x^2(1)), L^1) - x^1(1) & \text{if } L^1 > x^1(1) \geq C^1 \text{ and } x^2(1) < R^2(x^1(1)) \\ \min(R^{2^{-1}}(x^2(1)), L^1) - x^1(1) & \text{if } 0 < x^1(1) < C^1 \text{ and } x^2(1) < C^2 \\ 0 & \text{if } 0 < x^1(1) < C^1 \text{ and } x^2(1) \geq C^2 \\ 0 & \text{if } x^1(1) = 0. \end{cases}$$

$$x^1(3) = \begin{cases} 0 & \text{if } q^1(2) \geq C^1 \\ \max(R^1(q^2(2)) - q^1(2), 0) & \text{if } q^1(2) < C^1 \text{ and } q^2(2) \geq C^2 \\ C^1 - q^1(2) & \text{otherwise} \end{cases}$$

where $R^{2^{-1}}(x^2)$ is an inverse function of $R^2(x^1)$.

$$x^2(1) = x^{2*},$$

$$x^2(2) = \begin{cases} 0 & \text{if } x^1(1) \geq C^1 \\ 0 & \text{if } 0 < x^1(1) < C^1 \text{ and } x^2(1) < C^2 \\ \max(R^{1^{-1}}(x^1(1)) - x^2(1), 0) & \text{if } F^1 < x^1(1) < C^1 \text{ and } x^2(1) \geq C^2 \\ \max(L^2 - x^2(1), 0) & \text{if } 0 < x^1(1) \leq F^1 \text{ and } x^2(1) \geq C^2 \\ \max(L^2 - x^2(1), 0) & \text{if } x^1 = 0. \end{cases}$$

$$x^2(3) = \begin{cases} 0 & \text{if } q^2(2) \geq C^2 \\ \max(R^2(q^1(2)) - q^2(2), 0) & \text{if } q^2(2) < C^2 \text{ and } q^1(2) \geq C^1 \\ C^2 - q^2(2) & \text{otherwise} \end{cases}$$

where $R^{1^{-1}}(x^1)$ is an inverse function of $R^1(x^2)$.

To compute a subgame perfect Nash equilibrium, the game is solved by backward induction. Concerning the third-period strategies, Lemmas 2.1–2.3 show that the above strategies of both firms become equilibria in all subgames.

Consider the second-period strategies. We show that given $x^1(1), x^2(1)$, firm 1's second-period strategy above is its best response to the firm j's strategy above and *vice versa.*

Suppose that $x^1(1) \geq L^1$. Given firm 2's strategy above, $x^2(2) = 0$. By definition of L^1, firm 1 has no incentive to increase its output. Therefore, firm 1's best response is

$x^1(2) = 0$. Given the above strategy of firm 1, $x^1(2)$ and $x^1(3)$ equal zero regardless of $x^2(2)$. Therefore firm 2's second-period production has no strategic value. Under these conditions, $x^2(2) = 0$ is one of firm 2's best responses. The same principles apply to the case where $L^1 > x^1(1) \geq C^1$ and $x^2(1) \geq R^2(x^1(1))$.

Suppose that $L^1 > x^1(1) \geq C^1$ and $x^2(1) < R^2(x^1(1))$. Given firm 2's strategy above, $x^2(2) = 0$. Since $x^1(1) > 0$, we have that I^1 does not depend on $x^1(2)$. Given firm 2's strategy above, firm 2's total output x^2 is $\max(R^2(q^1(2)), x^2(1))$. Under these conditions, firm 1's payoff U^1 becomes $V^1(q^1(2), \max(R^2(q^1(2)), x^2(1))) - c^1q^1(2) - \delta$. This payoff is increasing in $q^1(2)$ if and only if $q^1(2) \leq \min(L^1, R^{2^{-1}}(x^2(1)))$. Hence, firm 1's best response is choosing $q^1(2) = \min(L^1, R^{2^{-1}}(x^2(1)))$. This strategy of firm 1 is the same as described above. Given this strategy of firm 1, $x^1(3)$ equals 0 regardless of $x^2(2)$. Therefore, $x^2(2) = 0$ is one of firm 2's best responses. The same principles apply to the case where $0 < x^1(1) < C^1$ and $x^2(1) < C^2$.

Suppose that $0 < x^1(1) < C^1$ and $x^2(1) \geq C^2$. From Lemma 2.1 we have that $x^2(3)$ equals 0 regardless of $x^1(2)$ and $x^2(2)$. Accordingly, firm 1's second-stage production $x^1(2)$ has no strategic value; thus $x^1(2) = 0$ is one of firm 1's best responses. Since $x^2(1) > 0$, we have that I^2 does not depend on $x^2(2)$. Given that $x^1(2) = 0$, firm 2 can enjoy the first-mover advantage and its best response is

$$x^2(2) = \begin{cases} \max(L^2 - x^2(1), 0) & \text{if } x^1(1) < F^1 \\ \max(R^{1^{-1}}(x^1(1)) - x^2(1), 0) & \text{if } x^1(1) \geq F^1. \end{cases}$$

This strategy of firm 2 is the same as described above.

Suppose that $x^1(1) = 0$. Given firm 1's strategy above, x^1 is $\min(R^1(q^2(2)), C^1)$. If $x^2(1) \geq L^2$, firm 2's dominant strategy is $x^2(1) = 0$. Suppose that $0 < x^2(1) < L^2$. In this case I^2 does not depend on $x^2(2)$. By definition of L^2, firm 2's best response is $x^2(2) = L^2 - x^2(1)$. Suppose that $x^2(1) = 0$. In this case, choosing $x^2(2)$ such that $0 < x^2(2) < L^2 - x^2(1)$ is inferior to $x^2(2) = L^2 - x^2(1)$ because a small $x^2(2)$ increases

$x^1(3)$ while positive $x^2(2)$ does not reduce the inventory cost. If $x^2(2) = 0$, its payoff is $V^2(C^2, C^1) - c^2C^2$ and if $x^2(2) = L^2$, its payoff is $V^2(L^2, F^1) - c^2L^2 - \varepsilon$. Under (2.3), firm 2's best response is $x^2(2) = L^2$ since $\varepsilon \leq \delta$. Given firm 2's strategy above, we have that $x^2 = \max(L^2, x^2(1))$ and x^2 does not depend on $x^1(2)$. Therefore, firm 1's best response is $x^1(2) = 0$.

Finally, consider the first-period action. Given firm 2's strategy above, $x^2 = x^{2*} \leq C^2$ if $x^1(1) > 0$. In this case, firm i's payoff is $V^1(x^{1*}, x^{2*}) - c^1x^{1*} - \delta$. If firm 1 chooses $x^1(1) = 0$, its payoff is $V^1(F^1, L^1) - c^1F^1$. Since $(x^{1*}, x^{2*}) \in E \subset R$ and $L^1 \geq x^{1*} \geq C^1$, we have that $V^1(x^{1*}, x^{2*}) - c^1x^{1*} \geq V^1(C^1, C^2) - c^1C^1$. Therefore, (2.4) ensures that $x^1(1) = 0$ is not firm 1's best response. If $x^1(1) > 0$ and $q^1(2) = x^{1*}$, then we have that $x^2 = x^{2*}$ regardless of $x^1(1)$. Therefore, $x^1(1) = x^{1*}$ is one of firm 1's best responses.[1]

Given firm 1's strategy above,

$$x^1 = \begin{cases} x^{1*} & \text{if } x^2(1) \geq R^2(x^{1*}) = x^{2*} \\ R^{2^{-1}}(x^2(1)) & \text{if } F^2 < x^2(1) < R^2(x^{1*}) = x^{2*} \\ L^1 & \text{if } x^2(1) \leq F^2. \end{cases}$$

Choosing $F^2 < x^2(1) < x^{2*}$ is not firm 2's best response because it increases x^1 resulting in reducing firm 2's payoff. Choosing $0 < x^2(1) \leq F^2$ is inferior to choosing $x^2(1) = 0$ because it does not decrease x^1 and increases its inventory cost. By definition of $\bar{x}$, choosing $x^2(1) = 0$ is inferior to choosing $x^2(1) = x^{2*}$. Since $x^{2*} = R^2(x^{1*})$, we have that choosing $x^2(1) > x^{2*}$ is not firm 2's best response to $x^1(1) = x^{1*}$.

Next, we show that $(x^1, x^2) = (L^1, F^2)$ is a subgame perfect equilibrium outcome. Suppose that the strategies of both firms are the same as described above except that $x^1(1) = L^1$ and $x^2(1) = 0$. Firm 1's total output $x^1 = L^1$ regardless of firm 2's behavior. Therefore, firm 2's best response is $x^2(1) = 0$. Given that $x^2(1) = 0$, firm 1 chooses $x^1(1) =$

[1] We can construct equilibrium strategies which make $x^1(1) = x^{1*}$ to be firm 1's unique best response. Suppose that firm 2 expands its output if $x^1(1) < x^{1*}$ and $x^2(1) = x^{2*}$. This strategy of firm 2 is optimal if $x^1(2) = 0$, which is firm 1's best response given this strategy of firm 2. Under these conditions, $x^1(1) = x^{1*}$ becomes firm 1's unique best response.

L^1 to enjoy the first-mover advantage.

only if part Next, we show that (x^{1*}, x^{2*}) is a subgame perfect equilibrium only if $(x^{1*}, x^{2*}) \in E \cup \{(L^1, F^2), (F^1, L^2)\}$. First we show that if (x^1, x^2) is a subgame perfect equilibrium, then $(x^1, x^2) \in R$. We prove it by contradiction.

Suppose otherwise. Suppose that $R^1(x^2) > x^1$ or $R^2(x^1) > x^2$ in equilibrium. If $R^i(x^j) > x^i$, firm i could increase its payoff by expanding $x^i(3)$, a contradiction. Therefore, (x^1, x^2) is never below R.

Suppose that (x^1, x^2) is above R, i.e., $R^1(x^2) < x^1$ and $R^2(x^1) < x^2$. Without loss of generality, we assume that $x^1 > C^1$. From (A.1), we have that $x^1 = q^1(2) > C^1$ and $x^2 = q^2(1)$. From Lemma 2.1, we have that $x^1(3) = 0$ regardless of $q^2(2)$. Therefore $q^2(2)$ must be $x^2(1)$, because otherwise firm 2 would increase its payoff by decreasing $x^2(2)$. Given that $x^2(1) = x^2$, firm 2's output does not depend on $x^1(1)$ as long as $x^1(1) \geq R^{2^{-1}}(x^2)$. Thus, firm 1's best response is $x^1 = x^1(1) = \min(R^{2^{-1}}(x^2), L^1)$. Here we suppose that $R^{2^{-1}}(x^2) \leq L^1$, i.e., $x^2 \geq F^2$. Then x^1 equals $R^{2^{-1}}(x^2)$. By definition of $R^{2^{-1}}$, we have that x^2 equals $R^2(x^1)$, a contradiction. Suppose that $R^{2^{-1}}(x^2) > L^1$, i.e., $x^2 < F^2$. Then x^1 equals L^1. Since F^2 equals $R^2(L^1)$, we have $x^2 < R^2(x^1)$. This contradicts (A.1). Accordingly, x^2 equals $R^2(x^1)$. Under these conditions, we have that $(x^1, x^2) \in R$.

Finally, we show that $x^1 \leq \bar{x^1}$ or $x^1 = L^1$. We prove it by contradiction. Suppose that $x^1 > L^1$. Since $(x^1, x^2) \in R$, we have $x^2 = R^2(x^1) < F^2$. Given that $x^2(1) < F^2$, if firm 1 chooses $x^1(1) = L^1$, then firm 2 chooses $x^2 = R^2(L^1) = F^2$. By definition of L^1, the above deviation from the equilibrium strategy by firm 1 increases firm 1's payoff, a contradiction.

Suppose that $x^1 \in (\bar{x^1}, L^1)$. Then $x^2(1)$ equals $R^{2^{-1}}(x^1)$ because otherwise, firm 1 would increase its output in the first stage in order to enjoy the first-mover advantage. If firm 2 deviates from the equilibrium strategy and chooses $x^2(1) = x^2(2) = 0$, its payoff becomes at worst $V^2(F^2, L^1) - c^2F^2$. By definition of $\bar{x^1}$ (Definition 2.5), the above deviation from the equilibrium strategy by firm 2 increases firm 2's payoff, a contradiction. Q.E.D.

Proof of Corollary

Since $V_2^j < 0$, we have that $V^j(R^j(\bar{x}^i), \bar{x}^i) - c^i R^j(\bar{x}^i)$ is decreasing in $\bar{x}^i$. Therefore, from Definition 2.5, we have that $\bar{x}^i$ is decreasing in δ. Under these conditions, (2.4) ensures that $\bar{x}_i > C_i$ $(i = 1, 2)$; thus we have that $(C_1, C_2) \in E$. Q.E.D.

Appendix B

Appendix of Chapter 3

In this Appendix B, we will prove all propositions presented in Chapter 3. In order to prove them, we prove some supplementary lemmas.

To compute a subgame perfect Nash equilibrium, the game is solved by backward induction. We discuss the equilibrium outcomes in the second-stage subgames given $p_i(1)$ and $p_j(1)$, and thereafter we discuss the equilibrium outcome in the full game.

Let $S_i(x_j : p_i(1))$ denote firm i's reaction function after committing to $p_i(1)$. Since $\partial^2 U_i / \partial p_i^2 < 0$ (Assumption 3.3) and $p_i \leq p_i(1)$, we have

$$S_i(p_j : p_i(1)) = \min(R_i(p_j), p_i(1)) \qquad (i = 1, 2). \tag{B.1}$$

The following Lemmas 3.3–3.5 describe the equilibrium outcome in the second stage subgames. First, we prove Lemmas 3.1 and 3.2, and then use these to prove Lemmas 3.3–3.5.

Lemma 3.1: *Suppose that Assumptions 3.1–3.4 are satisfied. Suppose that* $(p_1, p_2) \neq (B_1, B_2)$. *Then* $p_1 < R_1(p_2)$ *and/or* $p_2 < R_2(p_1)$.

Proof: The proof is by contradiction. From (B.1), we have $p_i \leq R_i(p_j)$. Suppose that $(p_1, p_2) \neq (B_1, B_2)$, and $(p_1, p_2) = (R_1(p_2), R_2(p_1))$. Then (p_1, p_2) is one of equilibrium outcomes in the original Bertrand game discussed in Section 3.1. This contradicts the assumption of the uniqueness of Bertrand equilibrium, which is ensured by Assumption 3.4.

Q.E.D.

Lemma 8.2: *Suppose that Assumptions 3.1–3.3 are satisfied. Suppose that $(p_1, p_2) \neq (\hat{P}_1, \hat{P}_2)$. Then $p_1 = p_1(1) < \bar{P}_1(p_2)$ and/or $p_2 = p_2(1) < \bar{P}_2(p_1)$.*

Proof: We can derive straightforwardly from Lemma 8.1 and (B.1). Q.E.D.

Now, we derive the equilibrium outcomes in the second stage of [illegible] given $p_1(1)$ and $p_2(1)$.

Lemma 8.3: *Suppose that Assumptions 3.1–3.3 are satisfied. If $(p_1(1), p_2(1)) \geq (\hat{P}_1, \hat{P}_2)$, then $(p_1, p_2) = (\hat{P}_1, \hat{P}_2)$.*

Proof: The proof is by contradiction. Suppose that $(p_1, p_2) \neq (\hat{P}_1, \hat{P}_2)$. From Lemmas 8.1 and 8.2, we have $p_1 = p_1(1) < \bar{P}_1(p_2)$ and/or $p_2 = p_2(1) < \bar{P}_2(p_1)$. Without loss of generality, we can assume that $p_2 = p_2(1) < \bar{P}_2(p_1)$. From (B.1), we have $p_1 = \max\{\bar{P}_1(p_2(1)), p_1(1)\}$.

case (1) and case (3): strategic complements cases.

Suppose that $\bar{P}_i$ $(i = 1, 2)$ is increasing. From (B.1), we have $p_1 \geq \bar{P}_1(p_2(1))$. Since $\bar{P}_2$ is increasing, we have $\bar{P}_2(p_1) \geq \bar{P}_2(\bar{P}_1(p_2(1)))$. Since $p_2(1) < \bar{P}_2(p_1)$, we have $p_2(1) < \bar{P}_2(\bar{P}_1(p_2(1)))$. From the stability condition (3.4), we have $p_2(1) \geq \bar{P}_2(\bar{P}_1(p_2(1)))$ because $p_2 = p_2(1) \geq \hat{P}_2$, a contradiction.

case (2) and case (4): strategic substitutes cases.

Suppose that $\bar{P}_i$ $(i = 1, 2)$ is decreasing. Since $\bar{P}_2$ is decreasing and $p_1 = p_1(1) > \hat{P}_1$, we have $\bar{P}_2(p_1(1)) < \bar{P}_2(\hat{P}_1) = \hat{P}_2 \leq p_2(1)$. From (B.1), we have $p_2 = \bar{P}_2(p_1(1))$. Since $p_2 = p_2(1) < \bar{P}_2(p_1)$ and $p_1 = \bar{P}_1(p_2(1))$, we have $p_2(1) < \bar{P}_2(\bar{P}_1(p_2(1)))$. From the stability condition (3.4), we have $p_2(1) \geq \bar{P}_2(\bar{P}_1(p_2(1)))$ because $p_2 = p_2(1) \geq \hat{P}_2$, a contradiction.

Q.E.D.

Lemma 8.4: *Suppose that Assumptions 3.1–3.3 are satisfied. If $p_1(1) < \hat{P}_1$ and $p_2(1) \geq \hat{P}_2$, then $p_2 = p_2(1)$ and $p_1(1) = \max\{\bar{P}_1(p_2(1)), p_1(1)\}$. Furthermore, if we assume $\theta_i^* > 0$, then*

Q.E.D.

Lemma 3.2: *Suppose that Assumptions 3.1–3.4 are satisfied. Suppose that* $(p_1, p_2) \neq (B_1, B_2)$. *Then* $p_1 = p_1(1) < R_1(p_2)$ *and/or* $p_2 = p_2(1) < R_2(p_1)$.

Proof: We can derive straightforwardly from Lemma 3.1 and (B.1). Q.E.D.

Next, we discuss the equilibrium outcomes in the second stage subgames given $p_1(1)$ and $p_2(1)$.

Lemma 3.3: *Suppose that Assumptions 3.1–3.4 are satisfied. If* $(p_1(1), p_2(1)) \geq (B_1, B_2)$, *then* $(p_1, p_1) = (B_1, B_2)$.

Proof: The proof is by contradiction. Suppose that $(p_i, p_j) \neq (B_i, B_j)$. From Lemmas 3.1 and 3.2, we have $p_i = p_i(1) < R_i(p_j)$ and/or $p_j = p_j(1) < R_j(p_i)$. Without loss of generality, we can assume that $p_1 = p_1(1) < R_1(p_1)$. From (B.1), we have $p_2 = \min(R_2(p_1(1)), p_2(1))$.

case (1) and case (3): strategic complements cases:

Suppose that R_i $(i = 1, 2)$ is increasing. From (B.1), we have $p_2 \leq R_2(p_1(1))$. Since R_1 is increasing, we have $R_1(p_2) \leq R_1(R_2(p_1(1)))$. Since $p_1(1) < R_1(p_2)$, we have $p_1(1) < R_1(R_2(p_1(1)))$. From the stability condition (3.4), we have $p_1(1) \geq R_1(R_2(p_1(1)))$ because $p_1 = p_1(1) \geq B_1$, a contradiction.

case (2) and case (4): strategic substitutes cases:

Suppose that R_i $(i = 1, 2)$ is decreasing. Since R_2 is decreasing and $p_1 = p_1(1) > B_1$, we have $R_2(p_1(1)) < R_2(B_1) = B_2 < p_2(1)$. From (B.1), we have $p_2 = R_2(p_1(1))$. Since $p_1 = p_1(1) < R_1(p_2)$ and $p_2 = R_2(p_1(1))$, we have $p_1(1) < R_1(R_2(p_1(1)))$. From the stability condition (3.4), we have $p_1(1) \geq R_1(R_2(p_1(1)))$ because $p_1 = p_1(1) \geq B_1$, a contradiction. Q.E.D.

Lemma 3.4: *Suppose that Assumptions 3.1–3.4 are satisfied. If* $p_i(1) < B_i$ *and* $p_j(1) \geq B_j$, *then* $p_i = p_i(1)$ *and* $p_j(1) = \min(R_j(p_i(1)), p_j(1))$. *Furthermore, if in addition* $R'_j > 0$, *then*

$p_j = R_j(p_i(1))$.

Proof: First, we prove that if $p_i(1) < B_i$ and $p_j(1) \geq B_j$, then $p_i = p_i(1)$. The proof is by contradiction. Suppose that $p_i \neq p_i(1)$. Then, from (B.1) we have $p_i = R_i(p_j) < p_i(1)$. In this case, from Lemma 3.2 we have $p_j = p_j(1) < R_j(p_i)$. Since $p_i = R_i(p_j)$ and $p_j = p_j(1) < R_j(p_i)$, we have $p_j(1) < R_j(R_i(p_j(1)))$. From the stability condition (3.4), we have $p_j(1) \geq R_j(R_i(p_j(1)))$ because $p_j(1) \geq B_j$, a contradiction.

Next, we show that if in addition $R'_j > 0$, then $p_j = R_j(p_i(1))$. Since R_j is increasing and $p_i = p_i(1) < B_i$, we have $R_j(p_j) < R_j(B_i) = B_j$. Since $p_j(1) \geq B_j$, we have $\min(R_j(p_i), p_j(1)) = R_j(p_i)$. From (B.1), we have $p_j = R_j(p_i(1))$. Q.E.D.

Lemma 3.5: *Suppose that Assumptions 3.1–3.4 are satisfied. If* $(p_i(1), p_j(1)) < (B_i, B_j)$, *then* $p_i = p_i(1)$ *and/or* $p_i = p_i(1)$. *Furthermore if in addition* $R'_i < 0$, *then* $p_i = p_i(1)$.

Proof: First, we prove that $p_i = p_i(1)$ and/or $p_i = p_i(1)$. Since $p_i \leq p_i(1) < B_i$, we have $p_i \neq B_i$. So from Lemma 3.2, we have $p_i = p_i(1)$ and/or $p_i = p_i(1)$.

Next, we consider the case of negative R'_i. We show that $p_i = p_i(1)$. The proof is by contradiction. Suppose that $p_i \neq p_i(1)$. Then, from (B.1) we have $p_i = R_i(p_j) < p_i(1)$. Since $R_i(p_j) = p_i < p_i(1) < B_i$, we have $R_i(p_j) = p_i < B_i$. Since R_i is decreasing, the inequality $R_i(p_j) \leq R_i(B_j) = B_i$ is satisfied only if $p_j \geq B_j$, but it is impossible because $p_j < p_j(1) < B_j$. Q.E.D.

Finally, we prove our main results, Propositions 3.1–3.3.

Proof of Proposition 3.1

case (1):

existence: We show that there is an equilibrium in which $(p_1, p_2) = (B_1, B_2)$.

From Lemmas 3.3–3.5, we have

$$p_j = \begin{cases} B_j & \text{if } p_i(1) \geq B_i \text{ and } p_j(1) \geq B_j \\ R_j(p_i(1)) & \text{if } p_i(1) < B_i \text{ and } p_j(1) \geq B_j \\ p_j(1) & \text{if } p_i(1) \geq B_i \text{ and } p_j(1) < B_j \\ \min(R_j(p_i(1)), p_j(1)) & \text{if } p_i(1) < B_i \text{ and } p_j(1) < B_j \end{cases} \tag{B.2}$$

We prove that choosing $p_i(1) \geq B_i$ is one of firm i's best responses to $p_j(1) \geq B_j$. Lemma 3.3 then ensures that $(B_1, B_2) \in E$.

Suppose that $p_j(1) \geq B_j$. If firm i also chooses $p_i(1) \geq B_i$, firm 2 chooses $p_j = B_j$. Otherwise firm 2 chooses $p_j < B_j$. Since firm i's payoff is increasing in p_j, its best response is choosing $p_i(1) \geq B_i$.

uniqueness: Next, we show the uniqueness of the equilibrium outcome. The proof is by contradiction. Suppose that there is an equilibrium which entails $(p_1, p_2) \neq (B_1, B_2)$. We have already shown that given $p_j(1) \geq B_j$, firm i's best response is $p_i(1) \geq B_i$ and the equilibrium outcome is of Bertrand-type. Therefore, $(p_1, p_2) \neq (B_1, B_2)$ only if $(p_1(1), p_2(1)) < (B_1, B_2)$. Suppose that there is an equilibrium in which $(p_1(1), p_2(1)) < (B_1, B_2)$. Then choosing $p_i(1) < B_i$ must be one of firm i's best responses to $p_j(1) < B_j$.

From Lemma 3.1, we can assume that $p_1 < R_1(p_2)$ without loss of generality. From (B.2), we know that p_2 is non-decreasing in $p_1(1)$. Since $p_1 < R_1(p_2)$ and firm 1's payoff is increasing in p_2, firm 1 can improve its payoff by raising $p_1(1)$, a contradiction.

case (4):

existence: We show that there is an equilibrium in which $(p_1, p_2) = (B_1, B_2)$. From Lemmas 3.3–3.5 we get

$$p_j = \begin{cases} B_j & \text{if } p_i(1) \geq B_i \text{ and } p_j(1) \geq B_j \\ \min(R_j(p_i(1)), p_j(1)) & \text{if } p_i(1) < B_i \text{ and } p_j(1) \geq B_j \\ p_j(1) & \text{if } p_j(1) < B_j \end{cases} \tag{B.3}$$

We prove that choosing $p_i(1) \geq B_i$ is one of firm i's best responses to $p_j(1) \geq B_j$. Lemma 3.3 then ensures that $(B_1, B_2) \in E$.

Suppose that $p_j(1) \geq B_j$. If firm i also chooses $p_i(1) \geq B_i$, firm j chooses $p_j = B_j$. If firm i chooses $p_i(1) < B_i$, firm j chooses $p_j = \min(R_j(p_i(1)), p_j(1))$. Since R_j is decreasing, if $p_i(1) < B_j$, then $R_j(p_i(1)) \geq R_j(B_i) = B_j$, so $p_j \geq B_j$. Since firm i's payoff is decreasing in p_j, its best response is choosing $p_i(1) \geq B_i$.

uniqueness: Next, we show the uniqueness of the equilibrium outcome. The proof is by contradiction. Suppose that there is an equilibrium which entails $(p_1, p_2) \neq (B_1, B_2)$.

We have already shown that given $p_j(1) \geq B_j$, firm i's best response is $p_i(1) \geq B_i$ and equilibrium outcome is of Bertrand-type. Therefore, $(p_1, p_2) \neq (B_1, B_2)$ only if $(p_1(1), p_2(1)) < (B_1, B_2)$. Suppose that there is an equilibrium in which $(p_1(1), p_2(1)) < (B_1, B_2)$. Then choosing $p_i(1) < B_i$ must be one of firm i's best responses to $p_j(1) < B_j$.

From Lemma 3.1, we can assume that $p_1 < R_1(p_2)$ without loss of generality. From (B.3), we know that p_2 is non-increasing in $p_1(1)$. Since $p_1 < R_1(p_2)$ and firm 1's payoff is decreasing in p_2, firm 1 can improve its payoff by raising $p_1(1)$, a contradiction. Q.E.D.

Before proving Propositions 3.2 and 3.3, we present another supplementary Lemma.

Lemma 3.6: *Suppose that Assumptions 3.1–3.4 are satisfied. If $(p_1, p_2) \in E$, then $p_1 = R_1(p_2)$ and/or $p_2 = R_2(p_1)$.*

Proof: The proof is by contradiction. Suppose that $p_1 \neq R_1(p_2)$ and $p_2 \neq R_2(p_1)$. Since $p_i = \min(p_i(1), R_i(p_j))$, we have $p_i(1) = p_i < R_i(p_j)$ $(i = 1, 2)$. Then (p_1, p_2) lies below the inner envelope of reaction curves, so $p_1 < B_1$ and/or $p_2 < B_2$. Without loss of generality we assume $p_1(1) = p_1 < B_1$. From (B.1), we have $p_2 = p_2(1) < R_2(p_1(1))$. Since R_2 is continuous, there exists ε such that $p_2(1) < R_2(p_1(1) + \varepsilon)$. If firm 1 increases $p_1(1)$ by ε, p_1 becomes closer to $R_1(p_2)$ without changing p_2, so firm 1's payoff can be improved. Thus, the above deviation from the equilibrium strategy increases firm 1's payoff, a contradiction. Q.E.D.

Proof of Proposition 3.2

existence: We show that $(p_1, R_2(p_1)) \in E$ if $p_1 \in [L_1, B_1]$. We show that (p_1^*, p_2^*) is a subgame perfect equilibrium outcome if $p_2^* = R_2(p_1^*)$ and $p_1^* \in [L_1, B_1]$.

Suppose that $p_2^* = R_2(p_1^*)$ and $p_1^* \in [L_1, B_1]$. Note that $R_2(p_1^*) \in [B_2, F_2]$ because $p_1^* \in [L_1, B_1]$ and R_2 is decreasing. We construct a particular profile of strategies which entails $p_1^* \in [L_1, B_1]$ and $p_2^* = R_2(p_1^*)$. We show that firm 1's strategy below is its best response given firm 2's strategy below and *vice versa*. Suppose that firm 1's and firm 2's strategies are as follows:

$$p_1(1) = p_1^*, \; p_1 = \begin{cases} B_1 & \text{if } p_1(1) \geq B_1 \text{ and } p_2(1) \geq B_2 \\ \min(R_1(p_2(1)), p_1(1)) & \text{if } p_1(1) \geq B_1 \text{ and } p_2(1) < B_2 \\ p_1(1) & \text{if } p_1(1) < B_1 \end{cases}$$

$$p_2(1) = p_2^* = R_2(p_1^*), \; p_2 = \begin{cases} B_2 & \text{if } p_2(1) \geq B_2 \text{ and } p_1(1) \geq B_1 \\ \min(R_2(p_1(1)), p_2(1)) & \text{if } p_2(1) \geq B_2 \text{ and } p_1(1) < B_1 \\ p_2(1) & \text{if } p_2(1) < B_2 \end{cases}$$

To compute a subgame perfect Nash equilibrium, the game is solved by backward induction. Concerning the second-stage strategies, from Lemmas 3.3–3.5 we obtain that the above strategies of both firms construct Nash equilibria in all subgames.

Next, we consider the first-stage actions. Since $p_1(1) = p_1^* \leq B_1$, from Lemmas 3.4 and 3.5, we have $p_1 = p_1^*$ regardless of $p_2(1)$. Therefore, by definition of R_2, the above strategy is one of firm 2's best responses.

Given the above strategy of firm 2, from Lemmas 3.3 and 3.4 we get that p_2 becomes as follows:

$$p_2 = \begin{cases} B_2 & \text{if } p_1(1) \geq B_1 \\ \min(R_2(p_1(1)), p_2(1)) & \text{if } p_1(1) < B_1 \end{cases}$$

From result 1, we have $L_1 < B_1$. From the definition of L_1 and from Assumption 3.5, we obtain that firm 1's best response is choosing $p_1 = R_2^{-1}(p_2^*) = p_1^*$, where $R_2^{-1}(p_2)$ is the inverse function of $R_2(p_1)$. Thus, $(p_1^*, p_2^*) \in E$.

non-existence of other equilibrium outcomes: Here we prove that, if $(p_1, p_2) \notin \{(p_1, R_2(p_1)) \mid p_1 \in [L_1, B_1]\} \cup \{(R_1(p_2), p_1) \mid p_2 \in [L_2, B_2]\}$, then $(p_1, p_2) \notin E$.

From Lemma 3.6, without loss of generality we assume $p_2 = R_2(p_1)$ in equilibrium. We prove that $p_2 \in [B_2, F_2]$. The proof is by contradiction.

Suppose that $p_2 < B_2$. Since $p_2 = R_2(p_1)$, we have $R_1(p_2) = R_1(R_2(p_1))$. Since $p_2 = R_2(p_1) < B_2 = R_2(B_1)$ and R_2 is decreasing, p_1 must be larger than B_1. From the stability condition (3.4), we have that $R_1(R_2(p_1)) < p_1$ when $p_1 > B_1$. This contradicts (B.1).

Suppose that $p_2 > F_2$. Note that in this case $p_2(1) \geq p_2 > F_2 > B_2$. Since $p_2 \neq B_2$ and $p_2 = R_2(p_1)$, from Lemma 3.2 we have $p_1(1) = p_1$. Since $p_2 = R_2(p_1) > F_2 > B_2 = R_2(B_1)$ and R_2 is decreasing, $p_1(1)$ must be smaller than L_1. Suppose that firm 1 deviates from the equilibrium strategy and chooses $p_1(1) = L_1$. From Lemma 3.4 and 3.5, we have that $p_1(1) = p_1$ regardless of $p_2(1)$. If firm 1 chooses $p_1(1) = L_1$, firm 2 chooses $p_2 = R_2(L_1) = F_2$ because $p_2(1) > F_2$. From the definition of L_1 and from Assumption 3-5, we have $U_1(L_1, F_2) > U_1(p_1, R_2(p_1))\ \forall p_1 \neq L_1$. Therefore the above deviation from the equilibrium strategy increases firm 1's payoff, a contradiction. Q.E.D.

Proof of Proposition 3.3

existence: We show that $(L_1, F_2) \in E$. We construct a particular profile of strategies which yields a subgame perfect equilibrium in which $(p_1, p_2) = (L_1, F_2)$. The following strategies construct a subgame perfect equilibrium.

$$p_1(1) = L_1,\ p_1 = \begin{cases} B_1 & \text{if } p_1(1) \geq B_1 \text{ and } p_2(1) \geq B_2 \\ p_1(1) & \text{if } p_1(1) < B_1 \text{ and } p_2(1) \geq B_2 \\ R_1(p_2(1)) & \text{if } p_1(1) \geq B_1 \text{ and } p_2(1) < B_2 \\ \min(R_1(p_2(1)), p_1(1)) & \text{if } p_1(1) < B_1 \text{ and } p_2(1) < B_2 \end{cases}$$

$$p_2(1) = \infty,\ p_2 = \begin{cases} B_2 & \text{if } p_2(1) \geq B_2 \text{ and } p_1(1) \geq B_1 \\ p_2(1) & \text{if } p_2(1) < B_2 \text{ and } p_1(1) \geq B_1 \\ R_2(p_1(1)) & \text{if } p_2(1) \geq B_2 \text{ and } p_1(1) < B_1 \\ \min(R_2(p_1(1)), p_2(1)) & \text{if } p_2(1) < B_2 \text{ and } p_1(1) < B_1 \end{cases}$$

Since the proof is essentially the same as of Proposition 3.2, we omit it here.

non-existence of other equilibrium outcomes: We show that, if $(p_1, p_2) \neq (L_1, F_2)$ and $(p_1, p_2) \neq (F_1, L_2)$, then $(p_1, p_2) \notin E$. From Lemma 3.6, without loss of generality we assume $p_2 = R_2(p_1)$ in equilibrium. First, we show that $(B_1, B_2) \notin E$. Since $(p_1, p_2) \leq (p_1(1), p_2(1))$, (B_1, B_2) is an equilibrium outcome only if $(p_1(1), p_2(1)) \geq (B_1, B_2)$. Suppose that $p_2(1) \geq B_2$. In this equilibrium firm 1's payoff is $U_1(p_1, p_2) = U(B_1, B_2)$. Given the above strategy of firm 2, if firm 1 chooses $p_1(1) = L_1$, its payoff becomes $U_2(L_1, F_2)$. Note that $L_1 < B_1$ and $F_2 < B_2$. From the definition of L_1, it is larger than $U_1(B_1, B_2)$, a contradiction. Under these conditions, from Lemmas 3.1 and 3.2, we have $p_1 = p_1(1)$ on the equilibrium path.

Next, we show that in equilibrium $p_2 = F_2$. The proof is by contradiction. Suppose that $p_2 > F_2$. Then $p_2(1)$ must be larger than F_2. Given the above strategy of firm 2, if firm 1 chooses $p_1(1) = L_1$, its payoff becomes $U_2(L_1, F_2)$. From the definition of L_1, it is larger than $U_1(p_1, p_2)$ since $p_2 = R_2(p_1) \neq F_2$, a contradiction. Therefore, p_2 must be smaller than or equal to F_2.

Suppose that $p_2 \neq F_2$. Then we have that $p_2 < F_2$ in equilibrium. Since $p_2 = R_2(p_1)$, $U_1(p_1, p_2) = U_1(p_1, R_2(p_1))$. From the definition of L_1, we have $U_1(p_1, p_2) \leq U_1(L_1, F_2)$.

Suppose that $p_2(1) \geq F_2$. Then if firm 1 deviates from the equilibrium strategy and chooses $p_1(1) = L_1$, its payoff becomes $U_1(L_1, F_2)$, so the above deviation from the equilibrium strategy increases firm 1's payoff, a contradiction. Therefore $p_2(1)$ must be smaller than F_2. Since $p_2 = R_2(p_1) < F_2 = R_2(L_1)$ and R_2 is increasing, p_1 must be smaller than L_1. Since $p_1(1) = p_1$, we obtain that $p_1(1) < L_1$ in the equilibrium. Suppose that firm 1 deviates the equilibrium strategy and chooses $p_1(1) = L_1$. We denote the equilibrium outcome in the subgame after the above deviation by (p_1^*, p_2^*). Since the goods are complements, U_1 is decreasing in p_2. Since $p_2^* \leq p_2(1) < F_2$, we have $U_1(L_1, p_2^*) > U_1(L_1, F_2)$. Since p_1^* is $R_1(p_2^*)$ or L_1, we have $U_1(p_1^*, p_2^*) \geq U_1(L_1, p_2^*)$. Under these conditions, we have

$U_1(p_1^*, p_2^*) \geq U_1(L_1, p_2^*) > U_1(L_1, F_2) \geq U_1(p_1, p_2)$. Therefore, firm 1 can increase its payoff by the above deviation, a contradiction. Q.E.D.

Appendix C

Appendix of Chapter 4

In this Appendix C, we will prove Lemmas and Propositions presented in Chapter 4. As part of the proofs, we will present some supplementary lemmas.

Lemma C.1: *Suppose that Assumptions 4.2–4.6 are satisfied. Then R_i is decreasing.*

Proof: From simple calculations we have that $R_i' = \partial^2 U_i / \partial x_i \partial X_{\{-i\}}$. Under Assumption 4.3, we have that R_i is decreasing Q.E.D.

Next, we consider the two-stage games with exogenous sequencing formulated in Section 4.3.

Lemma C.2: *Suppose that Assumptions 4.1–4.8 are satisfied. Suppose that $|S| > 1$. Then $C(S, \tilde{X}) > C(S \backslash \{i\}, \tilde{X} + x)$ $\forall x > C(S, \tilde{X})$, $i \in N$, $\tilde{X} \in R_+$, and $S \subseteq N$.*

Proof: Denote $C(S, \tilde{X})$ by C^* and $C(S \backslash \{i\}, \tilde{X} + x)$ by C^{**}. From the definition of C, we have that

$$C^* = R(\tilde{X} + (|S| - 1)C^*) \text{ and } C^{**} = R(\tilde{X} + x + (|S| - 2)C^{**}). \tag{C.1}$$

The proof is by contradiction. Suppose that $x > C^*$ and

$$C^* \leq C^{**}. \tag{C.2}$$

From Lemma C.1 and (C.1), we have that (C.2) is satisfied only if

$$(|S|-1)C^* \geq x + (|S|-2)C^{**}. \tag{C.3}$$

Since $x > C^*$, (C.2) contradicts (C.3). Q.E.D.

Here we prove Lemmas 4.1 and 4.2 presented in Section 4.3.

Proof of Lemma 4.1:

In cases of duopoly, Dowrick [24] proved Lemma 2. Thus, we show that Lemma 2 holds true when $|S| \geq 3$.

Let us denote $C(S, \tilde{X})$ by C^*, $F(S, S\backslash\{i\}, \tilde{X})$ by F^*, and $L(S, S\backslash\{i\}, \tilde{X})$ by L^*. By definitions of C^*, F^*, and L^*, we have that

$$F^* = R((|S|-1)L^* + \tilde{X}), \textit{ and} \tag{C.4}$$

$$C^* = R((|S|-1)C^* + \tilde{X}). \tag{C.5}$$

From Assumption 4.3, we have that $U(R(X), X)$ is decreasing in X. Therefore, from (C.4) and (C.5) we have that Lemma 4.1 holds true if and only if $L^* > C^*$. Now we prove that $L^* > C^*$.

We prove it by contradiction. Suppose that $L^* \leq C^*$. From Lemma C.1, (C.4) and (C.5), we have that $L^* \leq C^*$ implies $F^* \geq C^*$. From the stability condition (Assumption 4.7) we have that

$$|R(y) - R(y')| \leq \frac{1}{2}|y - y'|. \tag{C.6}$$

Substituting $y = (|S|-1)C^* + \tilde{X}$ and $y' = (|S|-1)L^* + \tilde{X}$ into (C.6), we have that

$$F^* - C^* \leq \frac{|S|-1}{2}(C^* - L^*) \leq (C^* - L^*), \tag{C.7}$$

where we use (C.4) and (C.5), and the last inequality derived from that $|S| - 1/2 \geq 1$ for any $|S| \geq 3$. Note that $F^* - C^*, C^* - L^* > 0$ and $|S| \leq n$.

From (C.7) we have that

$$(|S|-2)(C^*-L^*) \geq F^*-C^*, i.e., (|S|-1)C^* \geq (|S|-2)L^*+F^*. \qquad \text{(C.8)}$$

From the definition of L^*, we have that

$$L^* \in \arg\max_{x_j} U\Big(x_j, \tilde{X}+(|S^*|-1)L^*+R(\tilde{X}+(|S^*|-1)L^*+x_j)\Big). \qquad \text{(C.9)}$$

By differentiating the payoff function of firm $j \in S^*$, we have that

$$\frac{dU_j}{dx_j} = \frac{\partial U_j}{\partial x_j} + \frac{\partial U_j}{\partial X_{-\{j\}}} R_i'. \qquad \text{(C.10)}$$

From the definition of C^*, we have that $\partial U_j/\partial x_j = 0$ at $(x_j, X_{-\{j\}}) = (C^*, \tilde{X}+(|S|-1)C^*)$. From Assumption 4.4 and the inequality (C.8), we have that $\partial U_j/\partial x_j \geq 0$ at $(x_j, X_{-\{j\}}) = (C^*, \tilde{X}+(|S|-2)L^*+F^*)$. Since we assume that $L^* \leq C^*$, from Assumption 4.5 we have that $\partial U_j/\partial x_j > 0$ at $(x_j, X_{-\{j\}}) = (L^*, \tilde{X}+(|S|-2)L^*+F^*)$.

From Assumption 4.3, we have that $\partial U_j/\partial X_{-\{j\}} < 0$. From Lemma C.1, we have that $\partial R_i/\partial x_j < 0$, thus we have that $dU_j/dx_j > 0$ at $(x_j, X_{-\{j\}}) = (L^*, \tilde{X}+(|S|-2)L^*+F^*)$. This contradicts (C.9). Q.E.D.

Proof of Lemma 4.2:

From the definition of L, we have that the following inequality is always satisfied:

$$U_i\Big(C(S,\tilde{X}), (|S|-1)C(S,\tilde{X})+\tilde{X}\Big) \leq U_i\Big(L(S,\{i\},\tilde{X}), (|S|-1)F(S,\{i\},\tilde{X})+\tilde{X}\Big).$$

The strict inequality is guaranteed by the strategic effect shown in Lemma C.2. Q.E.D.

We now prove the propositions presented in Section 4.4.

Proof of Proposition 4.1

We prove it by contradiction. Suppose that there is more than one follower in an equilibrium. Without loss of generality, we assume that followers produce in period $t' > 1$.

Suppose that one of the followers unilaterally deviates from the equilibrium strategy and produces in period $t' - 1$. From the definition of followers, all players other than followers produce before observing this deviation, so the deviation never affects the actions of leaders and intermediates. From Lemma 4.2, we have that the deviation strictly increases the deviator's payoff, a contradiction. Q.E.D.

Proof of Proposition 4.2

By definition of intermediate, intermediate never exists if m (the number of periods) is two. Thus we can restrict our attention to the cases where m is larger than two.

We prove the proposition by contradiction. Suppose that there is at least one intermediate in an equilibrium. From the definitions of intermediate, we have that at least one follower exists. From Proposition 4.1, we have that exactly one follower exists. Without loss of generality, we assume that the last intermediates produce in period t' $(m > t' > 1)$. Suppose that the follower unilaterally deviates from the equilibrium strategy and produces in period $t' - 1$. From the definition of the last intermediates and followers, all players other than followers and the last intermediates produce before observing this deviation, so this deviation affects the actions of last intermediates only. From Lemmas 4.1 and 4.2, we have that the deviation strictly increases the deviator's payoff, a contradiction. Q.E.D.

Proof of Proposition 4.3

(i) Suppose that each firm $i \in N$ produces $C(N, 0)$ in period 1. Obviously, the above strategies construct an equilibrium.

(ii) We show that there is an equilibrium in which firms $2, 3, ...n$ produce in period 1 and firm 1 produces in period $t > 1$. The deviation of firm 1 never affects the actions of other firms, so firm 1 has no incentive to deviate from the equilibrium strategy.

Suppose that firm $i \neq 1$ deviates from the equilibrium strategy. Note that the deviation never affects the actions of leaders. Suppose that firm i chooses period $t'(1 < t' < t.)$ Then the deviation gives it the same payoff as before the deviation. Suppose that firm i chooses period $t \geq t'$. Then firm 1 and i are faced with Cournot-type competition, or the Stackelberg duopoly where firm i is a follower. From Lemmas 4.1 and 4.2 (considering the case of $S = \{1,i\}$), we have that the deviation decreases firm i's payoff. Therefore no deviation by firm 2 strictly improves its payoff.

(iii) This is straightforwardly derived from Propositions 4.1 and 4.2. Q.E.D.

From here on we prove the propositions presented in Section 4.5. The following three lemmas are used to prove the remaining propositions.

Lemma C.3: *Suppose that Assumptions 4.9 and 4.10 are satisfied. If there is a pure strategy equilibrium, then there exists $\bar{\varepsilon}$ such that for every $0 < \varepsilon < \bar{\varepsilon}$, the number of followers is one and the follower produces in period m in the equilibrium.*

Proof: In cases of positive and sufficiently small ε, Proposition 4.1 holds true (the proof is almost the same, as for Proposition 4.1). We now prove that at least one firm produces in the last period (period m). Suppose that no follower exists in an equilibrium. Then all firms are leaders. Suppose that one firm unilaterally deviates from the equilibrium strategy. The deviation never affects the others' actions, so it improves the deviator's payoff by decreasing inventory costs, a contradiction. Thus exactly one follower exists. Obviously, the follower chooses the last period in order to economize on costs $\varepsilon I(e)$. Q.E.D.

Lemma C.4: *Suppose that Assumptions 4.9 and 4.10 are satisfied. If there is a pure strategy equilibrium, then there exists $\bar{\varepsilon}$ such that for every $0 < \varepsilon < \bar{\varepsilon}$, the number of leaders is n - 1 in the equilibrium.*

Proof: In cases of sufficiently small $|\varepsilon|$, Proposition 4.2 holds true (the proof is virtually the same as for Proposition 4.2). Thus, the number of intermediates is zero. From Lemma C.3, we have that the number of followers is one for positive and sufficiently small ε. Thus, if there is an equilibrium, the number of leaders is n - 1 in the equilibrium. Q.E.D.

Lemma C.5: *Suppose that Assumptions 4.9 and 4.10 are satisfied. Suppose that ε is negative. If there is an equilibrium, then no follower exists in the equilibrium.*

Proof: The proof is by contradiction. In cases of negative ε, Proposition 4.1 holds true (the proof is virtually the same as for Proposition 4.1). So we show that there is no equilibrium in which one follower exists. Suppose that in an equilibrium one follower exists. Without loss of generality, we assume that the follower produces in period $t' > 1$. Suppose that the follower deviates from the equilibrium strategy and produces in period $t' - 1$. From the definition of follower, all other firms produce before observing the deviation; thus the deviation never affects the actions of others. Under these conditions, the deviation increases the deviator's payoff, a contradiction. Q.E.D.

Finally we prove remaining propositions. They are derived straightforwardly from results proved above.

Proof of Proposition 4.4

First, we show that there is no equilibrium in which all firms produces in period 1. The proof is by contradiction. Suppose that there exists an equilibrium without followers. Given others' strategies, suppose that firm 1 deviates from the equilibrium strategy and produces in period 2. Since other firms produce their output before observing the deviation, this deviation never affects the actions of others. Thus, the deviation increases firm 1's payoff because of positive inventory costs, a contradiction.

Next, we prove that the number of leaders is at least n - 1 if inventory costs are sufficiently small. In the proof of Proposition 4.1, we show that a follower strictly increases its payoff by deviating if more than one follower exists. The same principle applies to the cases of small inventory costs.

Finally, we show the existence of equilibrium. Suppose that firm 1 produces in period 2 and the others produce in period 1. Because of positive inventory costs, the above strategy of firm 1 is is its best response. If one of leaders, firm 2, deviates from the equilibrium strategy and wait until period 2. Then firms 1 and 2 are faced with Cournot competition. From Lemma 4.2 we have that firm 2's payoff decreases by this deviation if inventory costs are sufficiently small. Q.E.D.

Proof of Proposition 4.5

From Lemmas C.3 and C.4, we have that if there is an equilibrium, the number of leaders is n - 1 and that one follower produces in the last period in the equilibrium. We show the non-existence of the equilibria with n - 1 leaders. We prove it by contradiction.

Suppose that there exists an equilibrium. In the equilibrium n - 1 firms choose period 1 and one firm chooses the last period. Suppose that one of the leaders deviates from the equilibrium strategy and produces the same level of output in period 2. The deviation never affects the actions of other leaders. Since the deviator chooses the same level of output, the follower who produces in period $m(\geq 3)$ also chooses the same level of output. Under these conditions, the above deviation never changes the actions of other firms; thus improves the payoff of the deviator by decreasing inventory costs, a contradiction. Q.E.D.

Proof of Proposition 4.6

From Lemma C.5, we have that if there is an equilibrium, in the equilibrium the number of leaders is n. We show the existence of the equilibria with n leaders. Suppose that all firms produces the Cournot output in period 1. Obviously, the strategies construct an equilibrium. Q.E.D.

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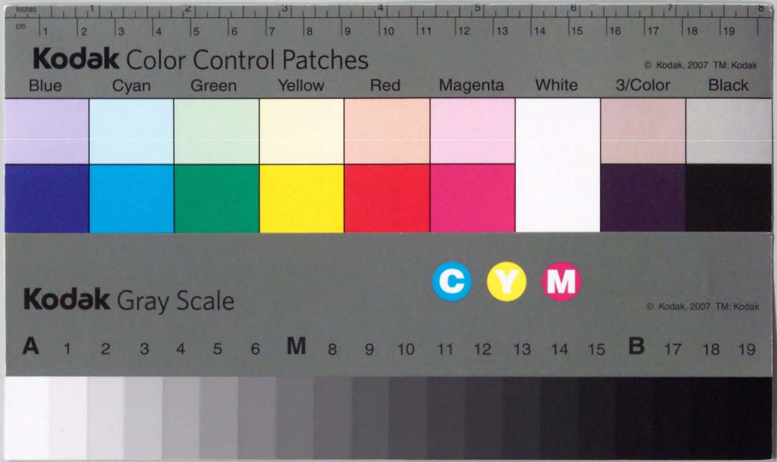
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