論 文 の 内 容 の 要 旨

論文題目　Mathematical Approaches to Interaction among Congestion at Multiple Junctions in Transportation Network

（交通ネットワークにおける複数交差点の渋滞の相互作用に対する数理科学的アプローチ）

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Nowadays, there are many scenes we can see group motions of things or peoples which can move, that is, the collective behaviors of self-driven particles, for instance, actions or motions of cars or pedestrians in the traffic network, logistics in the transportation network, motions of auto-guided vehicles (AGVs) or automated robots in the manufacturing factories, and so on. There is a growing trend of research these kinds of behaviors in the context of physics and mechanics of collective behaviors. These kinds of behaviors are usually shown in the network. In general cases, network has junctions that are aggregation or branching points. The behavior of a singular junction is well known, but the behavior of the interaction among multiple junctions in network is not so well known. In this thesis, we propose the theoretical approach to investigate this interaction and method as a key of prediction of congestion in the network.

Self-driven particles always move in the determined rule, which is physically discussed as motions of particles in the non-equilibrium state. It is also known that transport phenomena in chemistry and biology have the same properties. In this research, we construct the mathematical model of networks with multiple junctions of transport phenomena based on Totally Asymmetric Simple Exclusion Process (abbreviated TASEP) which is used not only in the mechanism of translation of genes and the transport phenomena of proteins but also in many natural phenomena. TASEP is one of the stochastic cellular automata models and used as a model of motions of particles in the lattice of cells lying in the 1-dimensional line. This model is a `simple exclusion' stochastic Markov process where more than one

particle cannot enter the same cell and appropriate to express the behavior of self-driven particles.

Generally, aggregation causes velocities of particles to be smaller and branching makes those to be larger. In order to investigate the correlation between aggregation and branching, we construct a network of TASEPs including one single-in-dual-out branching point and one dual-in-single-out aggregation point as the minimum module. This model has six parameters, input probabilities, output probabilities, and probability parameters of junctions. A probability parameter of each junction is given as a priority of the junction. In the theoretical analysis, assuming the mean-field approximation, we can confirm that the total system has various and complicated behaviors. We confirm that the validity of the analysis by the numerical simulations. For any parameter combinations, as sufficiently long time passed, the model is considered to reach the steady state, which enables us to analyze theoretically the model by flow conservation law near junctions in the steady state, phase combinations and time-averaged density and flow of independent TASEP segments of each route and path in the network model. This approach can be extended to general aggregation and branching not only for single-in-dual-out branching and one dual-in-single-out aggregation, and also applied to arbitrary TASEP network because of independency of the shape of network.

Additionally, `the condition that propagation of congestion occurs at the aggregation point reaches the upstream of the branching point' is strictly derived. This condition is a condition theoretically derived from the interaction between aggregation and branching, and also the new theoretical success of this research. This condition is dependent solely on the input probability of the upstream of the branching, the output probability of the downstream of the aggregation, and priority parameters of junctions. This condition is named the `critical ellipse' named after the fact that one takes the aggregation parameter and the branching parameter as the horizontal and the vertical axes, the condition is illustrated as an ellipse. Critical ellipse plays a pivotal role in escape or control of congestion for this is the saturated congestion in the network from the upstream of branching to the downstream of aggregation. Finally, we can theoretically derive the conditions of propagation of congestion through multiple junctions, and expect applications for congestion problems of real networks.

Finally, in order to investigate interactions of multiple critical ellipses, we consider a ladder-shaped network (abbreviated `ladder-network' ) of TASEP and extend the results

of the small module network. In the general case of ladder-network of TASEP, it is difficult to calculate all critical ellipses because of the complexity of interactions in the network. Critical ellipses are calculated from downstream to upstream, similarly as the behaviors of decomposed segments are calculated from downstream to upstream. The saturated congested flow of the next-leading path of a considered path makes large the region of the above critical ellipse of the considered path. This phenomenon seems to be the backward propagation of congested flow of saturation in the network.

As the special case of ladder-network, where directions of each neighboring pair of paths are different, we can calculate all critical ellipses. We call this case the `zigzagged' ladder-network. We prove the backward propagation of saturated congestion and show the algorithm of calculation of critical ellipses. From the algorithm of calculation of critical ellipses in zigzagged TASEP ladder-network, The fact that The saturated congested flow of the next-leading path of a considered path makes large the region of the above critical ellipse of the considered path illustrates the backward propagation of saturated congested flow and expansion of saturated congestion.

In summary, except zigzagged lattice-network, for each decomposed segment, the effective input probabilities have to be calculated from upstream parameters and the behaviors of downstream segments, while the effective output probabilities have to be calculated from downstream parameters. It is not necessary for the case of zigzagged lattice-network to calculate the effective input probabilities are at most 1/2, which enables us to calculate all critical ellipses in the zigzagged lattice-network case.

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