論文の内容の要旨

論文題目　A Study of Degenerate Two-Body and Three-Body Coupled-Channels Scatterings -A New Formalism, Near-Thresholds Resonances and Its Implications For Exotic Hadrons

（縮退した２体-3 体チャネル結合散乱の解析 -新たな定式化、閾値近傍での共鳴状態そしてエキゾチックハドロンとの関連について）

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Since the monumental discovery of the $X(3872)$ in 2003 by the Belle collaboration group, a lot of candidates for the exotic hadron have been observed especially in the energy regions above the double open charm and bottom thresholds and it seems a new era for studies of exotic hadron spectroscopy had opened.

Those observed resonances which nowadays aggregately called the X families are embedded in various hadronic scattering states coupling to it. Some of those candidates for the exotic hadron lie very close to hadronic two-body as well as *three-body* thresholds. For example, the mass of $X(3872)$, 3871.69 ± 0.17 MeV is very close to $\bar{D}^{*0}D^0$ two-body threshold $\simeq 3871.8$ MeV and is *also* close to $D^{\pm}\bar{D}^0\pi^{\mp}$ three-body threshold $\simeq 3874.0$ MeV. Besides the X families that recently observed, some of other candidates for the exotic hadron such as the possible dibaryon state near ΔN threshold or possible strange dibaryon lying near $\bar{K}NN$ threshold seem to have the same circumstances. They lie in the energy regions where hadronic two-body and three-body (and more-body) thresholds reside close to each other.

This is due to the fact characteristic in QCD, that is, the typical QCD scale $\simeq 200$ [MeV] is comparable to pseudo scalar meson, especially pion whose mass is $\simeq 140$ [MeV]. The on-mass-shell pions and kaons are therefore easily created which results in multi-body hadronic thresholds lie close to each other. This is in contrast to, such as, electromagnetic interaction. In that case, typical energy differences between discrete energy eigenstates are of order ~ 1 [eV] or less while the lightest particle which interact electromagnetically is electron whose mass is $\simeq 0.5$ [MeV] which is far heavier compared to energy differences and this fact prevents generating on-mass-shell electrons. We therefore expect two-body and three-body coupled-channels system whose thresholds lie close to each other might contain interesting physics that is characteristic in QCD. It might also contribute for generating those exotics lying near hadronic two-body and three-body thresholds residing close to each other.

In this thesis, motivated by such current status of hadron spectroscopy, we consider two-body and three-body coupled-channels systems from a general perspective. To be more specific, we mainly focus on two-body and three-body coupled-channels system whose thresholds are degenerate and examine how poles of the S-matrix near the thresholds behaves since the observed resonances lie in such energy regions.

To achieve that, we adopt the so-called Feshbach projection procedure. It is a method applied to coupled-channels systems and effects induced by couplings to each channels are taken into account as effective interactions in the channel we focus on which is often referred to as P-channel. The channel whose effects are embedded as effective interactions in the P-channel is usually referred to as Q-channel. The effective interactions in the P-channel which are induced by the coupling to the Q-channel can be intuitively understood as a sum of interactions which make transition *once* to the Q-channel. More precisely, the effective interactions are a sum of processes that particles in the P-channel make transition to the Q-channel, then particles in the Q-channel fully interact with each other with elementary interactions then comes back to the P-channel in addition to elementary interactions in the P-channel. Here by elementary interactions, we mean the interactions which still remain even if the channels are decoupled.

The coupled-channels system we are interested in is that two-body and three-body channels are coupled. We regard three-body channel as the P-channel and two-body channel as the Q-channel. This is due to some cumbersomeness we encounter if we regard the two-body channel as P-channel and embed the effects induced by the coupling to the three-body channel as effective interactions in the two-body channel. If we do so, we need to deal with the fully interacting three-body Green function which is cumbersome to calculate. We therefore regard the three-body channel as P-channel and effects due to coupling to the two-body channel which we regard as Q-channel are embedded as effective interactions in the three-body channel. We then need to calculate the fully interacting two-body Green function whose interactions are the elementary one. The fully interacting two-body Green function is however, easier to calculate than the fully interacting three-body Green function. If we assume the elementary interactions are of the form represented in figure 1, the effective interactions in the three-body channel are represented as shown in figure 2. We can see that even in the absence of elementary three-body interactions

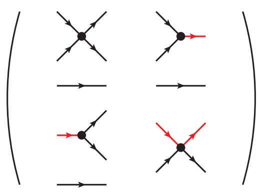

Figure 1: Elementary interactions

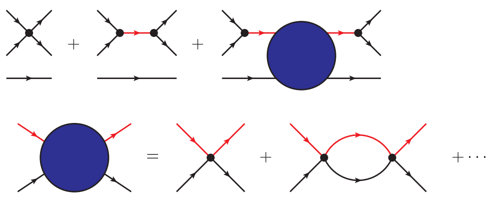

Figure 2: Effective interactions

in the three-body channel, coupling to the two-body channel induces the effective three-body interaction.

We have now reformulated the two-body and three-body coupled-channels problem as an effective three-body problem by leveraging the Feshbach projection procedure. Our next task is to solve the three-body coupled-channels scattering equations to search for the S-matrix poles. The scattering equations which three-body transition amplitudes satisfy are known as the (Faddeev-)AGS equations. In the absence of the three-body interactions, the (Faddeev-)AGS equations are represented in a diagrammatic form shown in figure 3. Written in a diagrammatic form, a rule how it sums up each interactions is obvious. It first sums each two-body interactions to give two-body T-matrices we denote as $t_k(E)$ and then sum those T-matrices up mixing with each other. The first term in figure 3 is a matrix which prevents the same two-body T-matrices to

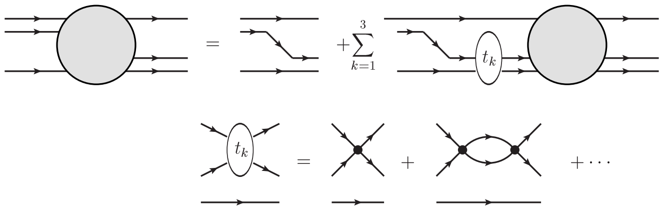

Figure 3: The (Faddeev-)AGS equations

appear in a row. The analogue (Faddeev-)AGS equations which contain three-body interactions have the same structure. It sums up two-body as well as three-body interactions to give two-body and three-body T-matrices and sums them up mixing with each other taking care of overcounting the two-body T-matrices.

We can now search poles of the S-matrix by solving the (Faddeev-)AGS equations whose two-body and three-body interactions are replaced by the effective one. To that end, we solve the eigenvalue equation of the kernel of the (Faddeev-)AGS equations instead of solving the equation itself. In the following, we show how it is done. The (Faddeev-)AGS equations are formally written as follows

$$X(E) = Z(E) + Z(E)\,T(E)\,X(E) \tag{1}$$

The eigenvalue equation of the kernel of it is,

$$Z(E)\,T(E)\,|n\rangle = \eta_n(E)\,|n\rangle \tag{2}$$

The formal solution of the (Faddeev-)AGS equation (1) is then written as

$$X(E) = \frac{1}{1 - Z(E)\,T(E)} Z(E) = \sum_n \frac{|n\rangle\langle n|}{1 - \eta_n(E)} Z(E) \tag{3}$$

Obviously, the S-matrix poles E_p are obtained as energies that satisfy $\eta_n(E_p) = 1$.

Throughout the thesis, we denote three particles in the three-body channel as $\phi_1\phi_2\phi_3$ and two-particles in the two-body channel as $\psi_3\phi_3$ and assume that ψ_3 couples $\phi_1\phi_2$. The physical mass of ψ_3 is then shifted from the bare one due to the self-energy correction. The transition amplitudes we obtain by solving the (Faddeev-)AGS equations of course have physical singularities, that is, they have branch cut starting from physical $\psi_3\phi_3$ two-body threshold in addition to branch cut starting from $\phi_1\phi_2\phi_3$ three-body threshold. However, the kernel of the equations and so the eigenvalue $\eta_n(E)$ does not if we naívely write down the eigenvalue equations. We then face the problem of *unphysical singularities*. We often encounter such situations in quantum field theory and we usually treat it by the mass and the field renormalization. Once the renormalization is done, no unphysical quantities appear in the theory. It is however, necessary to add counterterms every time the self-energies appear as a result. We proceed in the same manner, that is, we decompose the mass term in the Hamiltonian into the physical one and the counterterm and regard it as constant interaction.

Adding the counterterm when the self-energy appears is straightforward in case of perturbative calculations. However, we solve the eigenvalue equation and so the (Faddeev-)AGS equations numerically, that is, non-perturbatively and not perturbatively. The problem is, the self-energies appear when the (Faddeev-)AGS equations are *iterated* in addition to those in the kernel of the equations. The program that we add counterterm when the self-energy appears is therefore no longer applicable in our case. We rather need to find a way to incorporate those counterterms which should be added to the iteratively appearing self-energies into what we can feed to the (Faddeev-)AGS equations, namely the kernel of the equations. We have invented the trick to actually do that by an appropriate reorganization of each terms that appear in the (Faddeev-)AGS equations and discuss it in detail in our thesis. The key point is, that the self-energies always appear as a set shown in figure 4 and we reorganize them accordingly so as

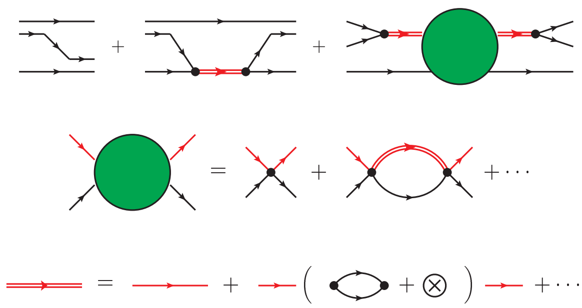

Figure 4: A set of self-energies

to keep the structure of the (Faddeev-)AGS equations the same.

We perform numerical analysis on how the S-matrix pole behaves near the threshold in case of two-body and three-body thresholds are degenerate specifying the model interactions. As we gradually increase the coupling constant in an attractive way, the S-matrix pole approaches the thresholds from the fourth quadrant of the unphysical complex energy sheet which might become a resonance if it lies close enough to the physical energy region. This behavior is in contrast to

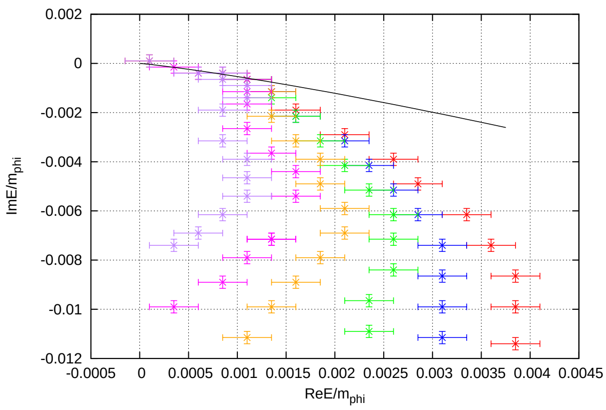

Figure 5: The S-matrix pole trajectories near the threshold

degenerate coupled-channels two-body system which contains two-body s-wave state. In that case, the pole approaches from the negative axis on the unphysical energy sheet and resonance does not appear. We can also see from figure 5 that poles lying very close to thresholds reside on an identical curve, that is, pole behaves universally near the thresholds.

To summarize, we formulated two-body and three-body coupled-channels problem as an effective three-body scattering problem in which effects induced by the coupling to the two-body channel is taken into account as effective interactions in the three-body channel. We then face the problem of unphysical singularities which can be resolved by an appropriate reorganization of the scattering processes. We showed that in contrast to degenerate multi-channel two-body system, a resonance might appear if the interactions are set to an appropriate values and if the pole lies very close to the thresholds, it behaves universally just like single-channel two-body scattering.