論文審査の結果の要旨

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The fast-paced world we live in has faced a transition where the financial system has changed from being relatively self-contained entity with little need for complexity, to a place where interconnection is the rule and sophisticated tools are increasingly present throughout the industry. Those financial instruments among which the exotic derivatives belong have built-in complexity that is a specific answer to an investor very specific need. When investors needed a tool to hedge risk bankruptcy for some risky loans, credit default swap were introduced; weather derivatives were introduced as a way to protect one against extreme weather events.

Complexity in the environment of financial derivatives is a side effect of the ongoing adjustment between investors' challenges and industry propositions. In this thesis we are interested in studying the pricing problem for complex derivatives: exotic options. We tackle this problem from the theoretical ground up to its efficient implementation using an interdisciplinary approach. It is usually the case that theoretical publications do not deal with the implementation of the proposed formula, and alternatively, implementation studies often limit their scope to the straightforward vanilla derivatives as a benchmark. The contributions of this thesis lie in both the theoretical and implementation space, and pursue this endeavor in an interdisciplinary fashion.

In the theoretical part, we demonstrate how particularly suitable are quantum mechanics path integrals to study path dependent options. Our first contribution is an improvement and extension on an existing exotic option pricing formula using path integrals. Our second theoretical contribution is the design of a new type of exotic option that we price using path integrals, demonstrating their power and flexibility.

Then the pricing problem in its computational and implementation complexity is studied and two original contributions are made: first a case study of heterogeneous CPU/GPU designs is done when price is available in a series form. Second, for the studies where new formula are introduced and must be benchmarked, we propose a novel GPU algorithm to speed up Monte Carlo simulations.

In the second chapter, the required background from probability theory is introduced. The probability space in which stochastic processes belong is discussed.

Stochastic processes are introduced as a suitable tool to describe randomness in financial assets dynamics. The Wiener process (also known as Brownian motion) is described along with the stochastic integration in the Ito interpretation. The defining properties of Ito calculus that will come handy later on are also showcased.

In the third chapter, an introduction to financial markets is given that concentrates on exotic derivatives. The basic working and segmentation of the financial market is explained, then after introducing the concept of financial derivatives we focus our attention on describing the features of an exotic option contract. The Black-Scholes model of a financial market is derived and introduced along with its philosophy, our work in this thesis is done inside this model entirely therefore some great care is given to disclose its limitation along with its appeal.

Since our original theoretical contributions both use path integrals as the main mathematical tool to price exotic options, the fourth chapter is devoted to presenting path integrals. We give a tour of the theory from the Wiener path integral and its connection with the diffusion process of Brownian motion, to the Feynman path integral and its historical role in providing another representation of quantum mechanics. Also we recall how path integrals are connected to the option pricing problem, the historical background and the parallel work by other authors in this field.

In the fifth chapter we give the first original result of this thesis: we describe the problem of pricing a contract whose averaged value is contingent on a correlated asset never crossing a threshold, the so-called outside barrier Asian option. We explicitly draw the connection with the problem of a Brownian particle facing an infinite wall in physics, and use it to solve the pricing problem demonstrating improvements over existing results. Then we propose an altogether original result where the barrier condition is imposed both above and below the initial conditions, the so-called double outside barrier Asian option. Building on the physics theory of a particle in a box, we solve the problem and study its accuracy. We finally propose an interpolating scheme to reduce the loss of accuracy due to infinite series truncation.

The sixth chapter is devolved to our second original study in the theoretical component of this thesis. We study the problem of a contract that is worthless if the price process spends more than a prescribed amount of time in a certain region of the price space. This complex condition is often termed Parisian condition. We start to describe this problem using the path integral framework and reach the point where we actually describe the motion of particle facing a potential step term. We use the Feynman-Kac formula to solve the problem and obtain a propagator. We use the propagator newly derived to study a more complex case where the payoff of the contract is written on the average value of the price process, and call this original product "Wasabi option". Facing a problem whose complexity may lead to implementation issue, we propose a result on a simplified version of the problem and study its accuracy.

We end this chapter by studying a quick fix to improve the accuracy of the derived result.

From the seventh chapter and moving forward, the implementation part of this thesis takes place. General Purpose Graphic Processing Units (GPGPU) as a computation framework for distributed computation is introduced and one of its most popular software suites CUDA is also briefly discussed. The relevant terminology and concepts are presented along with some code snapshots in order to be as practical as required for our need. A high level description of the GPU hardware is also introduced in order to make future discussions about optimization easier to grasp for the most general audience.

The eighth chapter is an original contribution we propose in the topic of heterogeneous computing. This chapter is a case study we conducted on the various heterogeneous CPU/GPU designs available for a distributed computation when the target is a basket option and price is available under series form. The option pricing problem is introduced following the recently proposed Fourier cosine methods for option pricing. We explicitly give the most general formula for n assets, but limit ourselves to a low dimension (two-asset) problem. We focused on the following challenges: what is the most efficient way to split the computational load in an online fashion, and how to easily optimize a familiar quadrature scheme on GPU. Empirical evidence is put forward for a heterogeneous implementation with distribution along the data axis rather than the functional one. We also put evidence forward that the trick we propose to decide the split-ratio on the fly is seen to be close to the optimal one.

We conclude our original contributions in the implementation part of this thesis by introducing in the ninth chapter a method to speed up the Monte Carlo simulation that we call "Shuffled Monte Carlo (SMC)". The relevance of Monte Carlo in the option pricing theory is undisputed, whether to benchmark newly derived formula or simply for problems that admit no analytical solutions. We acknowledge this fact by aiming to propose a GPGPU technique to reduce the computation time of Monte Carlo simulation by reusing some random numbers. We conduct some tests in order to select a viable pattern of numbers reuse that limit the bias introduced in the pure Monte Carlo scheme. After this selection, we benchmark our new SMC implementation against traditional Monte Carlo on both vanilla and exotic products admitting closed form solutions. After putting forward enough evidence that our technique does not break Monte Carlo convergence speed we finally display the actual speed up achieved by SMC, up to half for the most intensive reuse.

The conclusion chapter summarizes the relevance of our contributions to the field of econophysics and more generally to computational finance. We confront our initial expectation and target with what was actually achieved during this thesis.

本論文の内容において、論文提出者が主体となって理論解析、計算コードのプログラミングと実装を行ったものであり、学術上の独創性と有用性は十分である。本論文は博士の学位論文として合格と認められる。

したがって、論文提出者に博士（環境学）の学位を授与できると認める。

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