# Discharge Plasma Fluctuations in Hall Thrusters

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**Abstract**

Characteristics of plasma fluctuations at frequencies of 10-100kHz in Hall thrusters were investigated since the fluctuations might affect not only its performance but also its lifetime. Measured fluctuation characteristics varied with magnetic flux density of the applied field and they were categorized into the four regimes. Since the electron mobility strongly depends on applied field strength, it was thought that electrons would play an important role on this oscillation. Therefore the oscillation model derived from ionization instability considering electron mobility was proposed. The predicted stable/unstable operating condition map agreed well with measured one.

## Nomenclature

| | |
|---|---|
| B | magnetic flux density |
| E | electric field strength |
| L | ionization zone length |
| N | number density |
| M | particle mass |
| T | temperature |
| $\mu$ | mobility |
| D | diffusion coefficient |
| S | cross-section of the channel |
| $\gamma$ | ionization coefficient |
| Q | particle flow rate |
| $\nu$ | collision frequency |
| $k_b$ | Boltzmann's constant |
| P | pressure |
| $\omega$ | frequency |
| e | electronic charge |
| m | mass flow rate |
| q | ionization degree |
| w | channel width |

### Subscripts

| | |
|---|---|
| e | electron |
| i | ion |
| n | neutral atom |
| 0 | anode side |
| 1 | exit side |

## 1. Introduction

A Hall thruster is one of the promising thrusters for satellite station keeping and orbit transfer, since they have a specific impulse of 1000-3000 sec and an efficiency of more than 50%. The ion beam density of a Hall thruster can be higher than that of an ion thruster because of the existence of electrons in the ion acceleration zone.

A Hall thruster is categorized into two different types: a linear type and a sheath type. One of a linear type Hall thruster is "Stationary Plasma Thruster" (SPT), which has been developed in Russia. The acceleration channel length is longer than the channel width and the channel walls are made of ceramics. On the other hand, "Thruster with Anode Layer"(TAL), having developed in Russia as well, is categorized into a sheath type Hall thruster. The channel length is shorter than the channel width. Walls are made of an electric conductor, being kept at the cathode potential. Owing to this channel design, the lifetime of a sheath type is believed to be longer than that of a linear type. However, it could work in a very limited range of operational condition compared with a linear type.

One of the problems in a Hall thruster is the discharge current oscillation, especially at frequencies of 10-100kHz, which seems to affect the performance and lifetime of thrusters. In our previous research, it was found that the discharge terminates due to this oscillation. There have been many studies about this range of the oscillation [1-3]. However they were not enough to adequately describe the mechanism of this instability. The aim of this study is to investigate the characteristics of the oscillation and to clarify oscillation mechanism.

## 2. Experimental Equipment

Figure 1 shows the cross-section of our laboratory-made 1kW class sheath type Hall thruster. It has a Guard ring of two stainless steel cylinders. The inner diameter is 48mm and outer diameters are 62mm and 72mm. The anode is located at 3mm upstream end of the channel. The solenoidal coil is set at the center of the thruster so as to apply radial magnetic fields in the channel. Magnetic flux density is maximum at the channel exit and minimum at the anode.

Figure 2 shows the cross-section of our laboratory-made 1kW class linear type Hall thruster. The acceleration channel is insulated with two BN cylinders. The inner and outer diameters are 50mm and 60mm, respectively. The channel length is 10mm. Magnetic flux density is almost uniform in the acceleration channel.

Xenon was used as the propellant gas. A filament cathode (φ0.27mm×500mm L×3, 20V×20A) was used as an electron source and neutralizer.

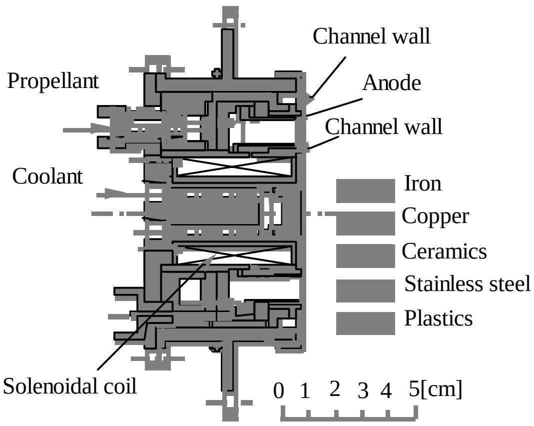


Fig.1 Cross-section of the sheath type Hall accelerator.

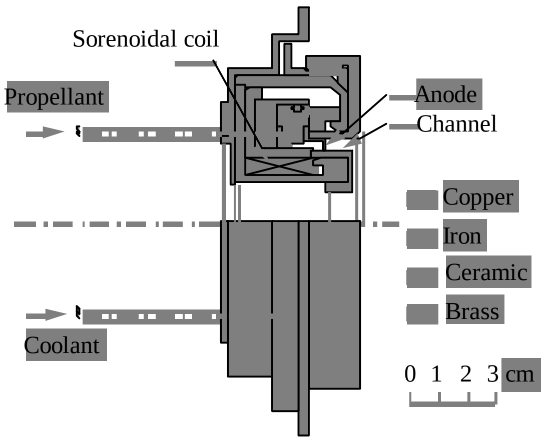


Fig.2 Cross-section of the linear type Hall accelerator.

All of the experiments were done at 2m diameters by 3m longs vacuum chamber. The pumping system consists of a diffusion pump, two rotary pumps and a mechanical booster pump. The pumping speed is 30000l/s and an ultimate base pressure is $4.5\times10^{-4}$ Pa. The operation pressure is under $5.3\times10^{-3}$ Pa.

## 3. Results and Discussion

### 3.1 Discharge characteristics

Figure 3 shows discharge characteristics. The current level in the sheath type is less than that in the linear type. This is thought due to the difference in electron conductivity. That is, electrons move to anode with classical diffusion in the sheath type, while in the linear type, with

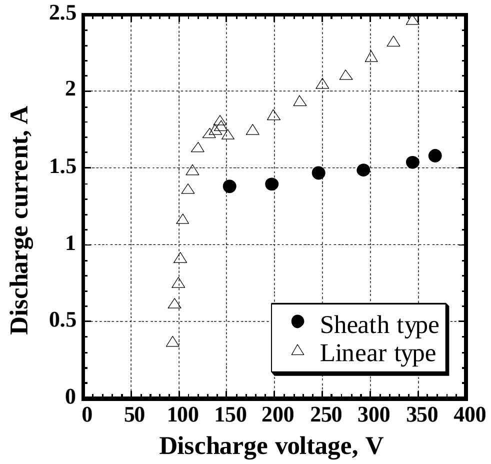


**Fig.3** Discharge voltage-current characteristics.
$\dot{m}$=1.36mg/s, $B$=0.02T (Sheath type)
$\dot{m}$=1.36mg/s, $B$=0.06T (Linear type)

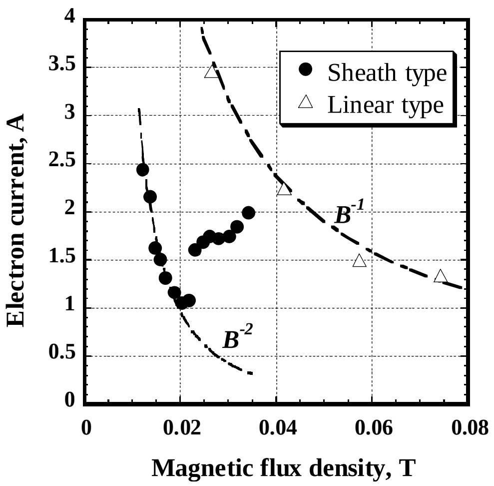


Fig.4 Electron current - magnetic flux density characteristics.
$V_d$=250V, m=2.72mg/s (Sheath type)
$V_d$=250V, m=2.04mg/s (Linear type)

magnetic flux density was low. On the other hand, anomalous diffusion. In the sheath type, there is no interaction between the wall and electrons since the wall is kept at the cathode potential and applied in the linear type, enormous diffusion occurs. As shown in Fig. 4, this assumption is certified by the fact that the electron current would scale as $B^{-2}$ in sheath type and as $B^{-1}$ in linear type.

### 3.2 Oscillation characteristics

Before examining the experimental result, let us define the amplitude of oscillation as

$$A = \frac{R.M.S}{\bar{x}} = \frac{1}{\bar{x}}\sqrt{\frac{\sum (x_n - \bar{x})^2}{N}}, \quad (\bar{x} = \frac{\sum X_n}{N}) \qquad (1)$$

Figure 5 shows the relation between oscillation characteristics of the sheath and magnetic flux density. Oscillation characteristics vary sensitively with the magnetic flux density and can be categorized into four regimes. As the regime changes, the relation between the electron current and magnetic flux density changes. Since electron mobility depends strongly on the magnetic flux density, electron mobility would play an important role on this fluctuation.

Figure 6 shows current trace in Regime 1. In Regime 1, oscillation amplitude is large but regular, and oscillation frequency is constant. This is seen at low magnetic flux density. Figure 7 shows current trace in Regime 2, the oscillation amplitude of this regime is less than 0.05. The electron current scales as $B^{-2}$. As shown in Fig. 8, Regime 2 appears in a stronger magnetic flux density with the mass flow rate increasing. Considering Electron mobility is proportional to $N_n$ and $B^{-2}$, this result may show that operation will be stable if electron mobility is in some range.

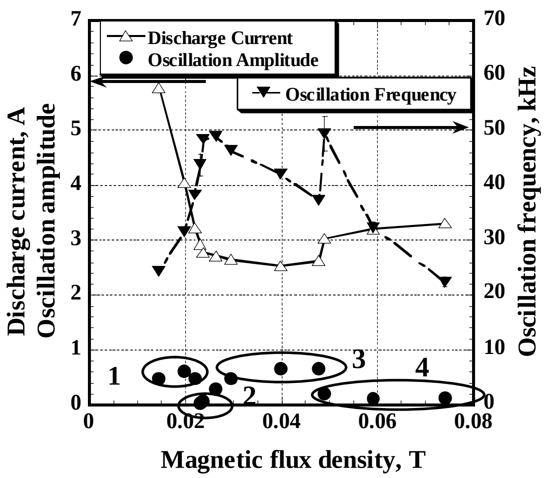

Fig. 5 Relation between oscillation characteristics and magnetic flux density.

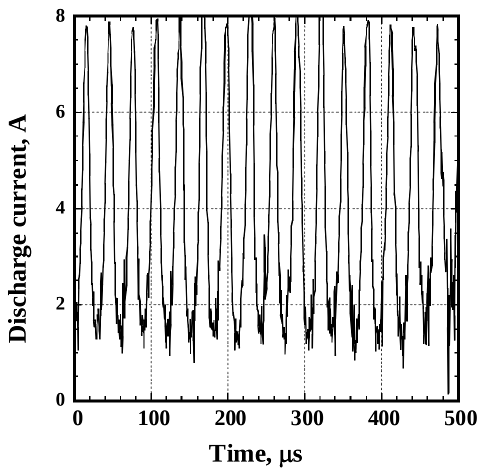

Fig. 6 Current trace in Regime 1.

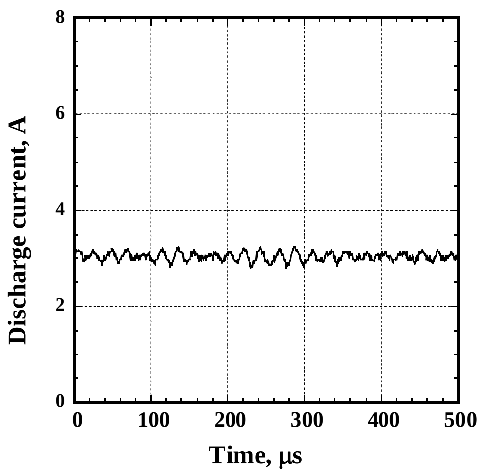

Fig. 7 Current trace in Regime 2.

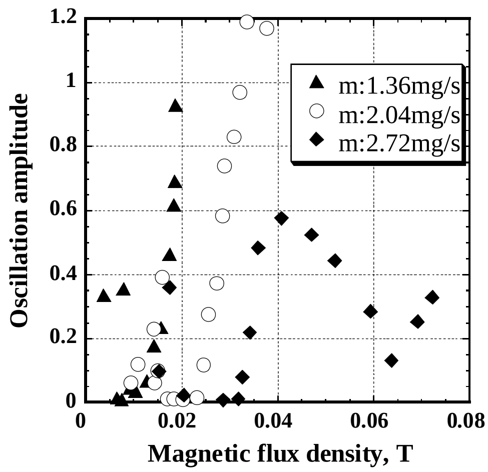

Fig.8 Relation between oscillation characteristics and magnetic flux density.

As shown in Fig. 9, the oscillation in Regime 3 is the fiercest oscillation. The oscillation amplitude increases with the magnetic flux density. Beyond the critical magnetic flux density, the discharge terminates or the transition to Regime 4 happens. The electron current does not scale as $B^{-2}$ but almost constant. This is because electron current would be affected by the plasma fluctuation.

In regime 4, the oscillation frequency and amplitude vary in time as shown in Fig. 10. This regime appears when discharge current increases discontinuously. In my opinion, the transition from regime 3 to regime 4 might happen when the electron diffusion mechanism changes from classical diffusion to enormous diffusion. All the oscillations observed in the linear type thrusters are categorized into this regime because of the similarity of the waveform of linear type shown in Fig. 11 and that of sheath type in Regime 4 shown in Fig.10. This similarity could be due to the similarity of electron mobility.

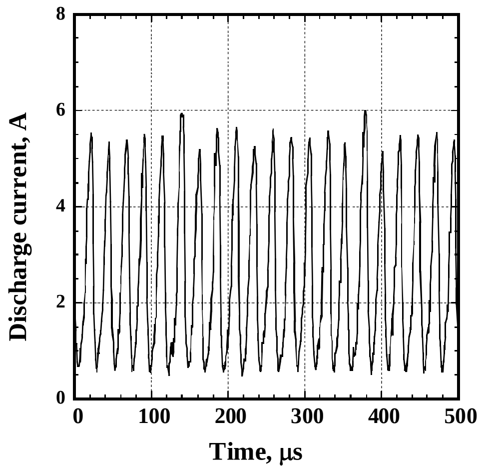


Fig. 9 Current trace in Regime 3.

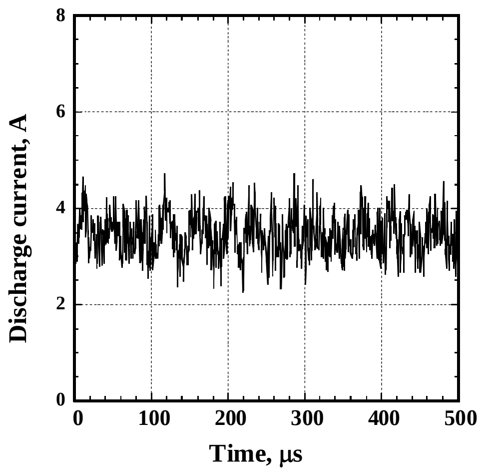


Fig. 10 Current trace in Regime 4.

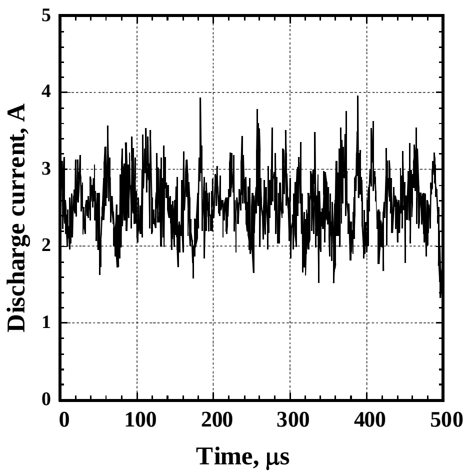


Fig. 11 Current trace in a linear type.

### 3.3 Oscillation Model

In our present model, an ionization zone is regarded as a homogeneous box region where ionization rates, neutral atom and plasma densities are homogeneous. The box length $L$ as shown in Fig.12.

The equation of continuity for neutral atoms

$$SL\frac{\partial n_n}{\partial t} = -\gamma\, n_n n_e SL + S(Q - n_{n1}V_n)\cdot \qquad (2)$$

The equation of continuity for electrons

$$SL\frac{\partial n_e}{\partial t} = \gamma\, n_n n_e SL + S(n_{e1}V_{e1} - n_{e0}V_{e0})\cdot \qquad (3)$$

These equations were solved analytically. First, each parameter is assumed to consist of a steady state part and a small disturbance part which is proportional to exp[- iωt], the fluctuation travels as the particles move. Since ωL/Ve<<1, hence it is ignored.

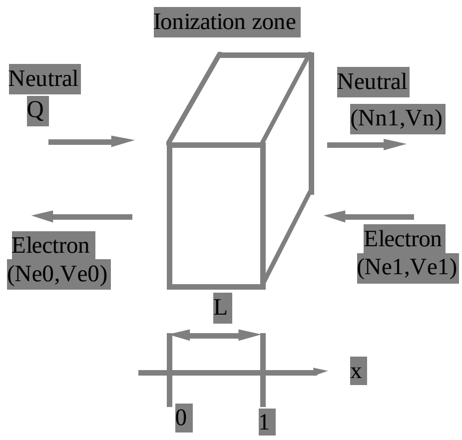


Fig. 12 Schema of ionization zone.

$$
\begin{aligned}
n_e &= N_e + n_e\, e^{-i\,\omega\, t} \\
n_n &= N_n + n_n\, e^{-i\,\omega\, t} \\
n_{e1} &= N_{e1} + n_e\, \exp[-i\omega(t + \tfrac{L}{2Ve})] \\
&\approx N_{e1} + n_e\, e^{-i\,\omega\, t} \\
n_{e0} &= N_{e0} + n_{e0}\, \exp[-i\omega(t + \tfrac{L}{2Ve})] \\
&\approx N_{e1} + n_e\, e^{-i\,\omega\, t} \\
n_{n1} &= N_{n1} + n_{n1}\, \exp[-i\,\omega\,(\,t - \tfrac{L}{2Vn})]
\end{aligned}
\qquad (4)
$$

Substituting Eq. (4) into Eqs. (2) and (3), the master equation is rewritten as,

$$
\begin{aligned}
&(i\omega - \gamma\, N_e + \frac{V_n}{L}\frac{N_{n1}}{N_n}\exp[\tfrac{i\,\omega\, L}{2V_n}]) \\
&(i\omega + \gamma\, N_n + \frac{V_{e1} - V_{e0}}{L}) + \gamma^2\, N_n N_e = 0
\end{aligned}
\quad . \qquad (5)
$$

To get a simple condition, exp[iωL/2Vn] is approximated as first Taylor expansion around 1. The instability condition of ionization oscillation Im[ω]>0 is written as inequality,

$$
(2.718\frac{N_{n1}}{N_{n0}} + 1)(\gamma\, N_n + \frac{(V_{e1-}V_{e0})}{L}) - \gamma\, N_e < 0\,\cdot \qquad (6)
$$

The condition of no oscillation is obtained as inequality,

$$
\begin{aligned}
&\{(2.718\frac{N_{n1}}{N_{n0}} + 1)(\gamma\, N_n + \frac{(V_{e1-}V_{e0})}{L}) - \gamma\, N_e\}^2 \\
&+ 4\{2.718\frac{N_{n1}}{N_{n0}} + 1\}\gamma\, N_e\,\frac{(V_{e1-}V_{e0})}{L} > 0
\end{aligned}
\quad . \qquad (7)
$$

To compare this model with the experimental result in sheath type thruster, which has clearer boundary than that of linear type, Eq. (7) was rewritten about operation parameter.

Neutral atom velocity is assumed constant as,

$$
V_n = \frac{1}{4}\sqrt{\frac{8k_b T}{\pi M}}\,\cdot \qquad (8)
$$

The equation of motion for electron,

$$
m_e N_e \frac{Dv}{Dt} = -eN_e E - \nabla P - m_e N_e \nu\, v\,\cdot \qquad (9)
$$

Considering a steady state and ν is sufficiently large, the left-hand side of the equation (9) could be zero. Electron travels with classical diffusion in a sheath type. Eq. (9) is rewritten as,

$$
\begin{aligned}
V_e &= -\mu_\perp E - D_\perp\frac{\nabla N_e}{N_e} = -\mu(E + \frac{kT_e}{e}\frac{\nabla N_e}{N_e}) \\
&= -\frac{m_e N_n < \sigma\, v >_{en}}{eB^2}(E + \frac{kT_e}{e}\frac{\nabla N_e}{N_e})
\end{aligned}
\quad . \qquad (10)
$$

From the experimental results, electron temperature is estimated as Te=0.012V/$\sqrt{B}$ .

According to ref. [4], the length of ionization zone of a sheath type is estimated as ,

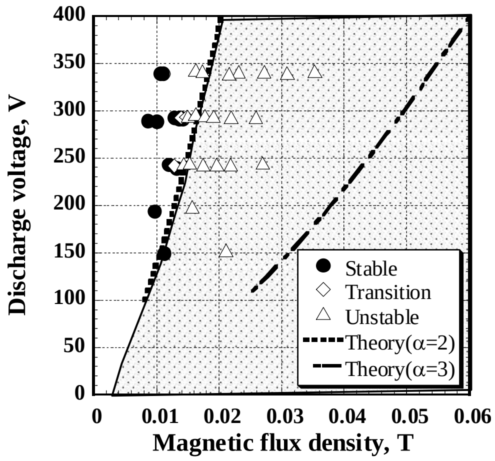


Fig. 13 stable/unstable Operational condition map.

$$
L = \alpha\, r_{Le} = \frac{\alpha}{B}\sqrt{\frac{8m_e}{\pi\, e}} \quad (\alpha : 2-3)\,\cdot \qquad (11)
$$

Electron density is estimated as below,

$$
N_e = \frac{I_e}{eSV_e}\,\cdot \qquad (12)
$$

Figure 13 shows the measured stable/unstable operating condition map of the sheath type The doted area indicates the predicted unstable operation zone. Triangles and squares represent Regime 3 and Regime 2, respectively. Unstable operation zone deduced by this theory roughly agrees with the experimental result.

Thus this model would be good expression of this oscillation mechanism.

From inequality (6), Direction of stable operation from this model would be

1. Enlarging ionization zone
2. Enlarging the difference of electron velocity between anode side and exit side, for example plasma potential fall more in exit side than anode side, or plasma density on the anode side enlarge et al.
3. Suppressing electron temperature.

## 4. Conclusion

Characteristics of plasma fluctuations in Hall thrusters were investigated. Measured fluctuation characteristics, especially oscillation amplitude, changed sensitively with magnetic flux density of the applied field and they were categorized into four regimes. Regime 2 in which operation was stable, enlarged with the increase of mass flow rate. Thus electron dynamics played an important role on this fluctuation since it depended strongly on the magnetic flux density. The ionization oscillation model considering this assumption was suggested. Predicted unstable operational condition agreed with experimental result in tendency. Thus Direction of stable operation from this model would be Enlarging ionization zone or Enlarging the difference of electron velocity between anode side and exit side.

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