

# Fracture Behavior Analysis of Structures Using A New Efficient and Simple Technique

構造物の破壊現象を簡単なモデルで高精度に解析する手法の提案

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## 1. INTRODUCTION

A new method for nonlinear analysis of reinforced concrete structures is proposed. The concrete is modeled as an assembly of distinct elements made by dividing the concrete virtually. These elements are connected by distributed springs in both normal and tangential directions. The reinforcement bars are modeled as continuous springs connecting elements together. Local failure of concrete is modeled by failure of springs connecting elements when reaching critical principal stress. We developed the element formulation and the computer code<sup>1)</sup>. In this paper, we introduce one numerical result comparing with experiment. The result shows good agreement in determining the failure load, the load-deflection relations, prediction of crack initiation, crack location and crack propagation.

## 2. ELEMENT FORMULATION

The two elements shown in **Fig. 1** are assumed to be connected by pairs of normal and shear springs located at contact points which are distributed around the element edges. Each pair of springs totally represent stresses and deformations of a certain area of the studied elements. The total stiffness matrix is determined by summing the stiffness matrices of individual springs around each element. In the 2-dimensional model, three degrees of freedom are considered for each element and deformations are assumed to be small. The stiffness matrix is developed for an arbitrary contact point with one pair of normal and shear springs. Two types of springs were defined. The first is concrete-spring while the other is reinforcement-spring.

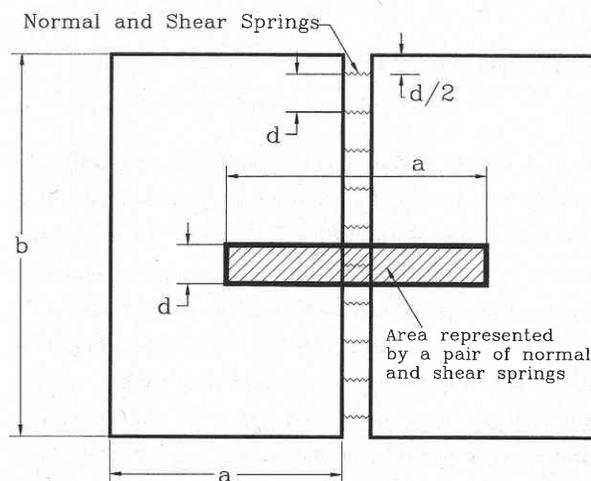


Fig. 1 Spring distributions and area of influence of each pair of springs

In this formulation, the element stiffness matrix depends on the contact point location and the stiffness of normal and shear springs which are determined according to the spring type and the stress and strain at the contact point location. Principal stresses are calculated at each contact point as shown in **Fig. 2**. Normal and shear stresses ( $\sigma_1$  and  $\tau$ ) are calculated from attached normal and shear springs while stress  $\sigma_2$  is calculated from normal stresses at points "B" and "C". Failure of springs is modeled by assuming zero stiffness for the spring being considered. Consequently, the developed stiffness matrix is an average stiffness matrix for the element according to the stress situation around the element.

## 3. EFFECTS OF ELEMENT SIZE AND THE NUMBER OF SPRINGS

To illustrate the effects of element size, a series of analyses

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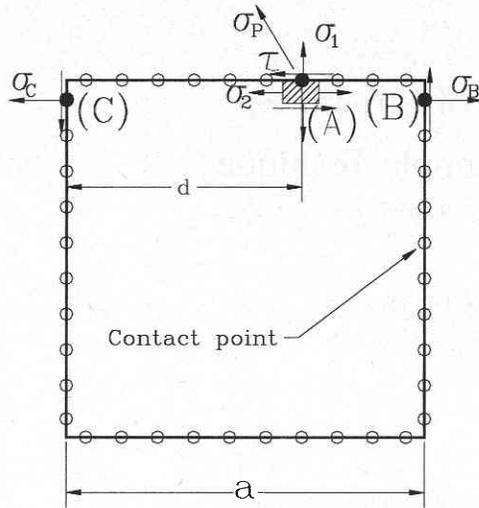


Fig. 2 Determination of principal stresses at the contact point location

were made for the laterally loaded cantilever shown in Fig. 3. Elastic analyses were performed using our proposed method for the different cases shown in Figs. 3, 4 and 5. The results were compared with those obtained from elastic theory of structures. The percentage of error in maximum displacement and the CPU time (CPU: DEC ALPHA 300 MHz) are shown in Fig. 3. To study the effects of the number of connecting springs, two different analyses were performed using 20 and 10 springs connecting each pair of adjacent element faces (Fig. 3). From the figure, it is evident that increasing the number of base elements leads to decreasing the error but increasing the CPU time. The error reduces to less than 1% when the number of elements at the base increases to 5 or more. Although the CPU time in case of 10 springs is almost half of that in case of 20 springs, its results congruent with those of 20 springs. Figs. 4 and 5 show the distribution of normal and shear stresses at the base of the studied columns for different number of base elements. From those figures, the followings should be noticed:

- Calculated normal stresses are very close to the theoretical values even in case of one element at the base
- Shear stress values are constant for the same element
- Shear stress values are far from the theoretical values in case of small number of elements at the base and close to the theoretical results in case of large number of elements.

This means that behavior in the case, where the effects of shear stress are minor, like case of slender frames, can be simulated by elements of relatively large size. On the other hand, in

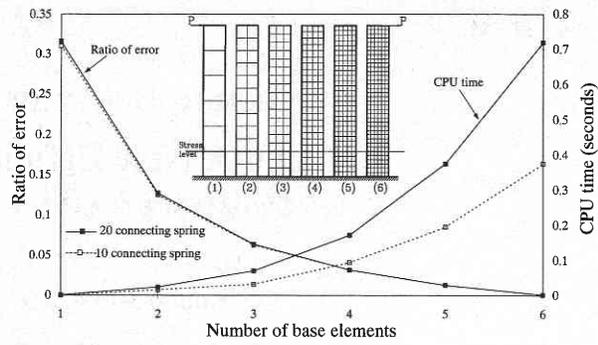


Fig. 3 Relation between the number of base elements, percentage of error and CPU time

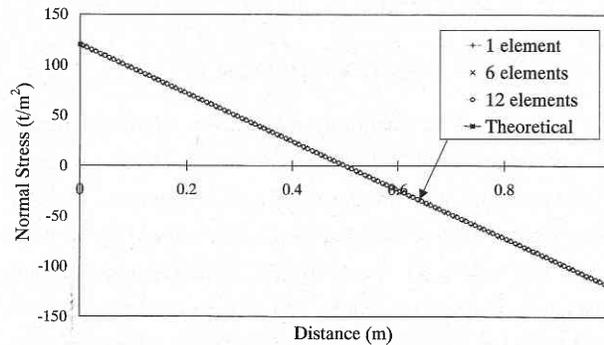


Fig. 4 Normal stress distribution at the column base

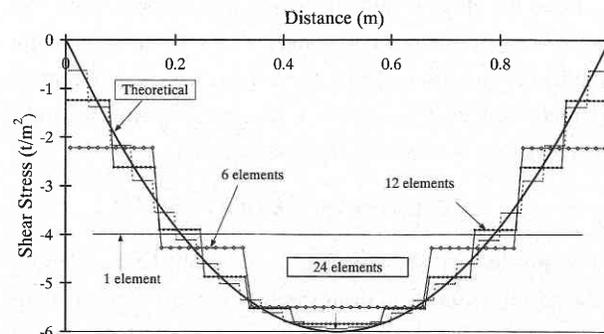


Fig. 5 Shear stress distribution at the column base

case of walls and deep beams, elements of small size should be used to follow the fracture behavior in the shear dominant zone.

#### 4. CASE STUDY AND NUMERICAL SIMULATION

To verify the accuracy of the model, we compared various simulation results with experiments<sup>1)</sup>. As one example, we show the simulation results of a continuous deep beam subjected to monotonic loading. The studied deep beam shape and reinforcement are shown in Fig. 6. For more details, see the Reference (2). Analysis of such type of deep beams is difficult because of

the following reasons:

- Not alike simple beams, maximum bending moments and shear forces are at the same sections. This has great influence on the cracking behavior<sup>2)</sup>.
- No shear reinforcement is used.
- Existence of many types of cracks; bending cracks, diagonal tension cracks and compression shear cracks.

Half of the deep beam is modeled using 2,525 square elements. The number of distributed springs set between each two adjacent faces is 10. The load is applied in 500 increment.

The relation between load and displacement is shown in **Fig. 6**. From this figure, it is obvious that the results of displacement at the mid-span, beginning of cracking load and failure load are very similar to that obtained from the experiment.

**Figure 7** shows the deformed shape and crack pattern during the analysis. The crack location is very close to that obtained from the experiment.

From these figures, the followings can be noticed:

- Initial cracks, which are bending cracks, started in the middle of the beam and at the middle support.
- Propagation of bending cracks stops due to the existence of horizontal reinforcement at the middle of the beam.
- The width of bending cracks is maximum at locations far away from reinforcement.
- In plain concrete zones, the number of bending cracks is small but width is wide. In contrast, the number of bending cracks in reinforcement locations is large but width is small.
- Diagonal tension cracks begin to propagate from the tension zone and stop before compression zone
- Large displacement starts to appear after formation of diagonal cracks (see point (3) in **Fig. 6**)
- After propagation of diagonal tension cracks, the bending cracks tend to close.
- Formation of compression strut between the loading point and the support is very obvious.
- After reaching point (6), compression shear cracks appear at the intermediate support. Moreover, concrete failure wedge is formed between the intermediate supports and the loading points. This indicates that the shear resistance is damaged due to compression shear failure at the support and the loading points locations.
- Continuing analysis after compression shear failure and separation of concrete wedge is difficult due to the rigid body motion behavior of the failed zones. This emphasizes the

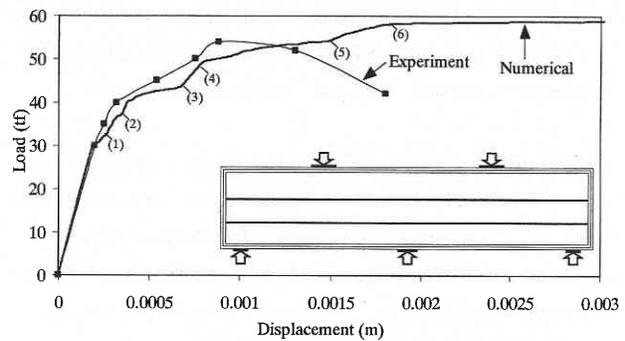


Fig. 6 Relation between load and mid-span displacement

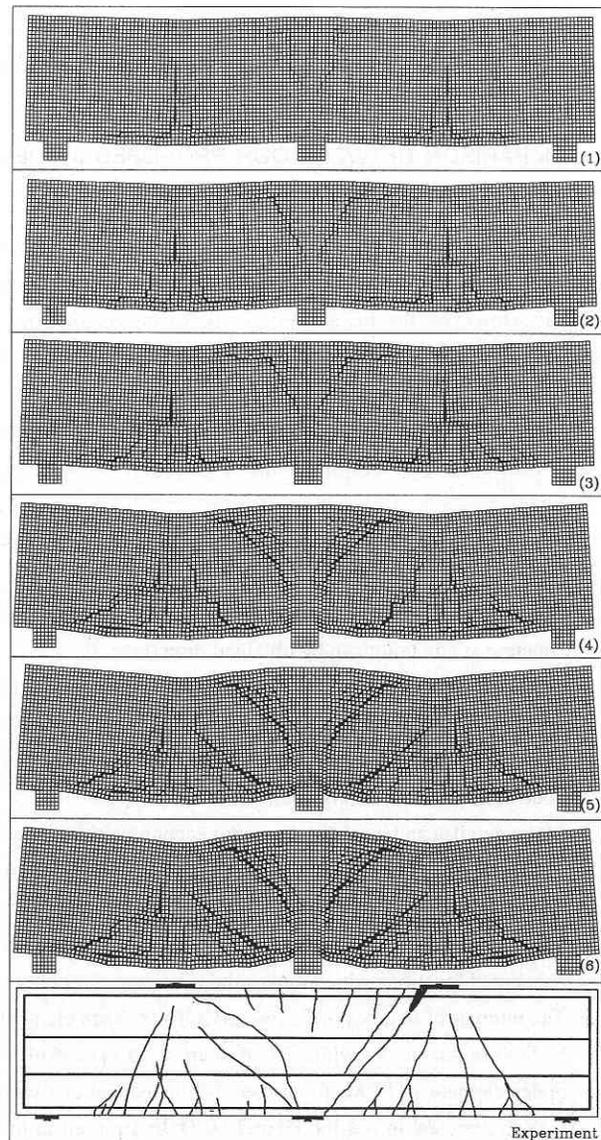


Fig. 7 Deformed shape and crack pattern (Scale factor = 50)

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importance of connecting this method with the EDEM<sup>3)</sup> to follow the post peak behavior of the structures till total collapse, in reasonable time and reliable accuracy.

It should be also noted that in the previous analysis using rigid elements, like RBSM<sup>4), 5), 6)</sup>, the results obtained depend mainly on the element discretization<sup>3)</sup>. This may be due to:

- The use of Mohr-Coloumb's failure criterion based on two components of stresses (not based on principal stresses),
- Poisson's ratio effect is not taken into account,
- The spring stiffness is not determined in a proper way to simulate the element deformation<sup>6)</sup>,
- The use of relatively large sized elements, and
- The use of relatively small number of springs between edges which leads to an inaccurate failure mechanism.

##### 5. COMPARISON BETWEEN OUR PROPOSED MODEL AND THE FINITE ELEMENT METHOD (FEM)

Many numerical techniques were developed for structural analysis. The FEM proved to be the best technique for structural analysis. However, the FE analysis includes many complications which don't exist in our model. The main advantages of our model are summarized as follows:

1. Material models used for concrete and steel do not change according to the reinforcement ratio like in case of the FEM.
2. In our model, reinforcement springs can be set exactly in the same location of reinforcement bar. Stress and strain, which are not average value of mesh, of reinforcement and concrete at any point can be obtained directly.
3. Analysis can be conducted before and after cracking without any change in material models used.
4. Modeling is much simpler than the FEM. No need to have node data and element connectivity data.
5. Many details can be easily taken into account without complications like stirrup location, stirrups diameter, concrete cover and loading plate width. With the FEM, consideration of these effects requires changing the mesh size at the concerned locations which makes the modeling difficult.
6. The number of degrees of freedom (DOF) for each element is three in our model while 16 DOF are used in case of 8-nodes element in FEM. In case of 3-dimensional analysis, 6 DOF are used in our model and 60 DOF are used in the

20-nodes block element in FEM.

7. Representation of large cracks in the FEM requires the use of joint elements<sup>7)</sup> or fictitious cracks techniques. The main disadvantage of these techniques is that the crack location should be decided before analysis. Moreover, fictitious crack techniques are used for limited number of cracks. In our model, there is no need to define the crack location before analysis. In addition, moving mesh techniques are very difficult to apply to reinforced concrete because of large number of cracks existing in the RC zones.
8. Crack propagation can not be followed easily using smeared cracks methods.

Although our model can give good results, the following difficulties still exist. Analysis can not be continued after total collapse or separation of part of the structure.

Research is conducted to the problem mentioned above. Results of the conducted research will be published in future papers.

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