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Dynamic Soil Reactions (Impedance Functions) Including The Effect of Dynamic Response of Surface Stratum (Part 2)
表層地盤の動反力係数 (2)

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(Continued from No. 9)

3. SOIL REACTION IN TORSION

The next axisymmetric case that is treated under the assumptions adopted is indicated in Fig. 4. It is assumed that all soil particles vibrate in the tangential direction of the cylinder and the motion is independent of θ.

Since both the vertical and radial componets of the motion vanish, $u=w=0$, and v is independent of θ, the equation of motion becomes

$$\left(G+G'\frac{\partial}{\partial t}\right)\left[\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial(rv)}{\partial r}\right\}+\frac{\partial^2 v}{\partial z^2}\right]=\rho\frac{\partial^2 v}{\partial t^2} \quad (16)$$

In a manner similar to the vertical case previously discussed, by expressing v as

$$v=R(r)\cdot\sum_n^\infty \sin(h_n\cdot z),\ n=1,\ 3,\ 5,\cdots \quad (17)$$

Eq. (16) becomes an ordinary differential equation in terms of R

$$\frac{r^2}{R}\frac{d^2R}{dr^2}+\frac{r}{R}\frac{dR}{dr}-(\beta_n r)^2=1 \quad (18)$$

where

$$\beta_n=\frac{\pi}{2H}\sqrt{\frac{(1+i2D)n^2-(\omega/\omega_g)^2}{(1+i2D)}} \quad (19)$$

The general solution to Eq. (18) is given by

$$R=A_n\cdot K_1(\beta_n r)+B_n\cdot I_1(\beta_n r) \quad (20)$$

As the function I_1 grows with r, $B_n=0$. Thus, the displacement and the circumferential shear stress take the following forms:

$$v=\sum_n^\infty A_n\cdot K_1(\beta_n r)\cdot\sin(h_n\cdot z) \quad (21)$$

$$\tau_{r\theta}=\left(G+G'\frac{\partial}{\partial t}\right)\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)$$
$$=-G(1+i2D)\sum_n^\infty A_n\left[\beta_n K_0(\beta_n r)+\frac{2}{r}K_1(\beta_n r)\right]\cdot\sin(h_n z) \quad (22)$$

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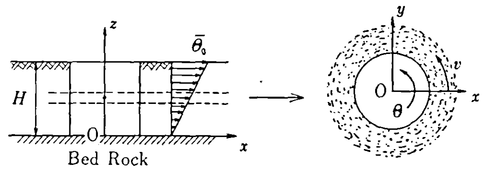

Fig. 4 Notation and Torsional Displacement of Cylinder

The moment around the cylinder axis is $\tau_{r\theta}|_{r=a}\cdot a$. Integrating this moment around the circumference of the cylinder yields the local torsional soil reaction as in the case of vertical vibration:

$$T(z)=-\int_0^{2\pi}\tau_{r\theta}|_{r=a}\cdot a\cdot a\cdot d\theta$$
$$=2\pi Ga^2(1+i2D)\sum_n^\infty A_n\left[\beta_n K_0(\beta_n a)+\frac{2}{a}K_1(\beta_n a)\right]\cdot\sin(h_n\cdot z) \quad (23)$$

Let the dynamic displacement of the cylinder be expressed in a form similar to the previous case:

$$\theta=\frac{z}{H}\cdot\bar{\theta}_0=\frac{8\bar{\theta}_0}{\pi^2}\sum_n^N a_n \quad (24)$$

By equating $v_{r=a}$ calculated by Eq. (21) and the quantity a.θ in which θ is determined from Eq. (24), the constant A_n is obtained. Thus, the local torsional soil reaction can be expressed as follows:

$$T(z)=\frac{16Ga^2\bar{\theta}_0}{\pi}(1+i2D)\sum_n^N \eta_n\cdot a_n \quad (25)$$

where

$$\eta_n=2.0+\beta_n\cdot a\cdot\frac{K_0(\beta_n a)}{K_1(\beta_n a)}$$

As in the vertical case, the local dynamic stiffness can be expressed in the form:

$$K_\theta(z)=\frac{T(z)}{\frac{z}{H}\cdot\bar{\theta}_0}=2\pi Ga^2(1+i2D)K_\theta'(z) \quad (26)$$

in which

$$K'_\theta(z)=\sum_n^N \eta_n \cdot a_n \Big/ \sum_n^N a_n \tag{27}$$

Figure 5 shows the vertical distribution of $K'_\theta(z)$ normalized by the static value at $z=H$. The variation shown in Fig. 5 is very similar to that of the vertical case, although the increase and the decrease near the free surface and the bottom is not so big as in the case of vertical reaction. Thus, the dynamic stiffness can be defined as follows:

$$K_\theta \cdot \frac{\overline{\Theta}_0}{2}=\frac{1}{H}\int_0^H T(z)dz$$

which yields the expression of K_θ as follows:

$$K_\theta=\frac{64Ga^2}{\pi^2}(1+i2D)\sum_n^N \eta_n \cdot \frac{(-1)^{\frac{n-1}{2}}}{n^3} \tag{28}$$

The above expression for the dynamic stiffness to a unit harmonic torsional motion of a unit length of the cylinder can be rewritten as

$$K_\theta=Ga^2[s_{\theta 1}(\omega/\omega_g,\ \tan\delta,\ v_s/v_p, a/H) + is_{\theta 2}(\omega/\omega_g,\ \tan\delta,\ v_s/v_p,\ a/H) \tag{29}$$

where $s_{\theta 1}$ and $s_{\theta 2}$ are both real. The variations of the dimensionless dynamic stiffness $s_{\theta 1}$ and $s_{\theta 2}$ normalized by their corresponding static values are shown in Fig. 6. The approximate solution by Novak et al. for plane strain case are also shown in Fig. 6.

Although the discrepancies between the two solutions in range below and above the fundamental hrizontal frequency ω_g are similar as observed in the vertical case, the variation with frequency is relatively smooth and more or less resembles the plane strain solution over the whole frequency range. Particularly, as the thickness of the surface soil stratum becomes larger, i.e. a/H becomes smaller, the solution by the present method tends to become more similar result to the plane strain solution.

4. SOIL REACTION IN HORIZONTAL VIBRATION

In horizontal vibration, the vertical displacement $w=0$ (Fig. 7) and the equations of motion of the surface soil stratum can be written as

$$\left.\begin{aligned}(\lambda+2G)\frac{\partial\Delta}{\partial r}-\left(G+G'\frac{\partial}{\partial t}\right)\left[\frac{1}{r}\frac{\partial\omega_z}{\partial\theta}-\frac{\partial\omega_\theta}{\partial z}\right]&=\rho\frac{\partial^2 u}{\partial t^2}\\(\lambda+2G)\frac{1}{r}\frac{\partial\Delta}{\partial r}-\left(G+G'\frac{\partial}{\partial t}\right)\left(\frac{\partial\omega_r}{\partial z}-\frac{\partial\omega_z}{\partial r}\right)&=\rho\frac{\partial^2 v}{\partial t^2}\end{aligned}\right\} \tag{30}$$

in which

$$\Delta=\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial\theta}$$

$$\omega_r=-\frac{\partial v}{\partial z}$$

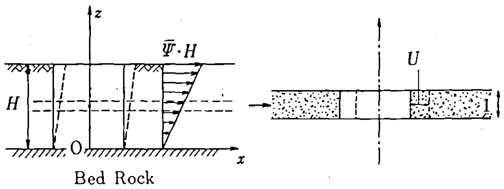

Fig. 7　Notation and Horizontal Displacement of Cylinder

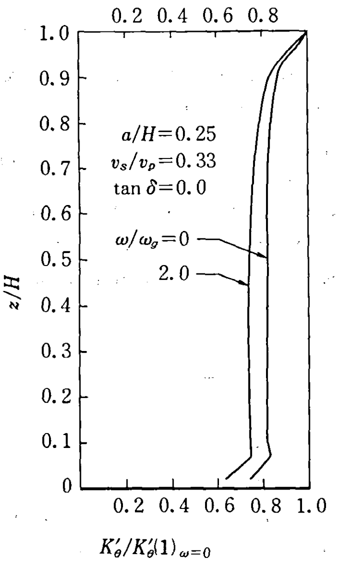

Fig. 5　Vertical Distribution of Local Dynamic Stiffness for Torsion (Eq. (27))

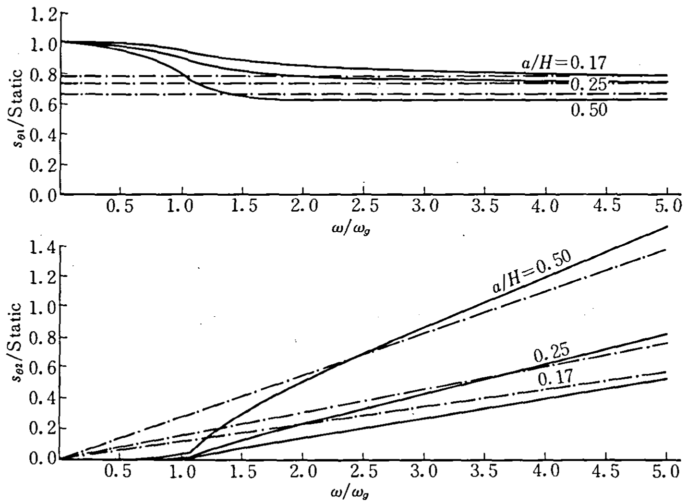

Fig. 6　Variations of Torsional Dimensionless Dynamic Stiffness with Frequency and the a/H Ratio (Eq. (28) or (29). Full Line for the Proposed Solution and Dushed Line for the Approximate Solution by Novak

$$\omega_\theta = \frac{\partial u}{\partial z}$$

$$\omega_z = \frac{1}{r}\frac{\partial(rv)}{\partial r} - \frac{1}{r}\frac{\partial u}{\partial \theta}$$

To solve the above equations, let the potential functions, ϕ_1, ϕ_2, be defined as follows:

$$\left.\begin{aligned} u &= \frac{\partial \phi_1}{\partial r} + \frac{1}{r}\frac{\partial \phi_2}{\partial \theta} \\ v &= \frac{1}{r}\frac{\partial \phi_1}{\partial \theta} - \frac{\partial \phi_2}{\partial r} \end{aligned}\right\} \qquad (31)$$

in which ϕ_1 and ϕ_2 are related to the longitudinal and shear wave, respectively. With these potential functions, Eq. (30) is decoupled as

$$\left.\begin{aligned} (\lambda+2G)\nabla^2\phi_1 &= \left[\rho\frac{\partial^2}{\partial t^2} - \left(G+G'\frac{\partial}{\partial t}\right)\frac{\partial^2}{\partial z^2}\right]\phi_1 \\ G\nabla^2\phi_2 &= \left[\rho\frac{\partial^2}{\partial t^2} - \left(G+G'\frac{\partial}{\partial t}\right)\frac{\partial^2}{\partial z^2}\right]\phi_2 \end{aligned}\right\} \qquad (32)$$

in which

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

Under the harmonic motion, one may put

$$\left.\begin{aligned} \phi_1 &= R_1(r)\cdot\cos\theta\cdot\sum_n^\infty \sin(h_n\cdot z) \\ \phi_2 &= R_2(r)\cdot\sin\theta\cdot\sum_n^\infty \sin(h_n\cdot z) \end{aligned}\right\} \qquad (33)$$

The above expressions for ϕ_1 and ϕ_2 satisfy the boundary conditions that the shear stresses vanish at the free surface $z=H$ and that the displacement is zero at the bed rock $z=0$. As in the case of torsion, substitution of Eq. (33) into Eq. (32) leads to two pairs of decoupled ordinary differential equations that yield the general solutions as follows:

$$\left.\begin{aligned} \phi_1 &= \sum_n^\infty [A_n K_1(\gamma_n\cdot r) + C_n I_1(\gamma_n\cdot r)]\cdot\cos\theta\cdot\sin(h_n z) \\ \phi_2 &= \sum_n^\infty [B_n K_1(\beta_n\cdot r) + D_n I_1(\beta_n\cdot r)]\cdot\sin\theta\cdot\sin(h_n z) \end{aligned}\right\} \qquad (34)$$

where

$$\gamma_n = \frac{\pi}{2H}\frac{v_s}{v_p}\cdot\xi_n, \quad \xi_n = \sqrt{(1+i2D)n^2 - (\omega/\omega_g)^2} \qquad (35)$$

The expression of β_n is given by Eq. (19).

Since the stresses and the displacements should decay with the horizontal distance r, the constants C_n and D_n must be zero. With the potential functions defined by Eq. (31), the displacements u and v can be obtained as

$$\left.\begin{aligned} u &= \frac{\partial \phi_1}{\partial r} + \frac{1}{r}\frac{\partial \phi_2}{\partial \theta} \\ &= \sum_n^\infty \Big[-A_n\Big\{\frac{1}{r}K_1(\gamma_n\cdot r) + \gamma_n\cdot K_0(\gamma_n\cdot r)\Big\} \\ &\quad + B_n\frac{1}{r}K_1(\beta_n\cdot r)\Big]\cdot\cos\theta\cdot\sin(h_n\cdot z) \\ v &= \frac{1}{r}\frac{\partial \phi_1}{\partial \theta} - \frac{\partial \phi_2}{\partial r} \\ &= \sum_n^\infty \Big[-\frac{1}{r}A_n K_1(\gamma_n\cdot r) + B_n\Big\{\frac{1}{r}K_1(\beta_n r) \\ &\quad + \beta_n K_0(\beta_n\cdot r)\Big\}\Big]\cdot\sin\theta\cdot\sin(h_n\cdot z) \end{aligned}\right\} \qquad (36)$$

The stresses are

$$\left.\begin{aligned} \sigma_r &= \lambda\left[\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{1}{r}\frac{\partial v}{\partial \theta}\right] + 2\left(G+G'\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial r} \\ &= \sum_n^\infty \Big[A_n\rho\xi_n^2\omega_g^2 K_1(\gamma_n r) \\ &\quad + 2G(1+i2D)A_n\Big\{\frac{2}{r^2}K_1(\gamma_n r) + \frac{\gamma_n}{r}K_0(\gamma_n r)\Big\} \\ &\quad - 2G(1+i2D)B_n\Big\{\frac{2}{r^2}K_1(\beta_n r) \\ &\quad + \frac{\beta_n}{r}K_0(\beta_n r)\Big\}\Big]\cdot\cos\theta\cdot\sin(h_n z) \\ \tau_{r\theta} &= \left(G+G'\frac{\partial}{\partial t}\right)\left(\frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta}\right) \\ &= \sum_n^\infty \Big[-B_n\rho\xi_n^2\omega_g^2 K_1(\beta_n r) \\ &\quad + 2G(1+i2D)A_n\Big\{\frac{2}{r^2}K_1(\gamma_n r) + \frac{\gamma_n}{r}K_0(\gamma_n r)\Big\} \\ &\quad - 2G(1+i2D)B_n\Big\{\frac{2}{r^2}K_1(\beta_n r) \\ &\quad + \frac{B_n}{r}K_0(\beta_n r)\Big\}\Big]\cdot\sin\theta\cdot\sin(h_n z) \end{aligned}\right\}$$

(37)

By using the stresses σ_r and $\tau_{r\theta}$ in Eq. (37), the local horizontal soil reaction to the motion of the cylinder is given by

$$\begin{aligned} F_u(z) &= -\int_0^{2\pi}(\sigma_r|_{r=a}\cdot\cos\theta - \tau_{r\theta}|_{r=a}\cdot\sin\theta)\cdot a\cdot d\theta \\ &= -\sum_n^\infty \rho\pi a\cdot\xi_n^2\cdot\omega_g^2[A_n K_1(\gamma_n a) \\ &\quad + B_n K_1(\beta_n a)]\cdot\sin(h_n\cdot z) \end{aligned} \qquad (38)$$

To determine the constants A_n and B_n, the dynamic displacement of the cylinder may be assumed as

$$\left.\begin{aligned} U_r &= z\cdot\overline{\Psi}\cdot\cos\theta = \frac{8H\cdot\overline{\Psi}}{\pi^2}\sum_n^N a_n\cdot\cos\theta \\ U_\theta &= z\cdot\overline{\Psi}\cdot\sin\theta = \frac{8H\cdot\overline{\Psi}}{\pi^2}\sum_n^N a_n\cdot\sin\theta \end{aligned}\right\} \qquad (39)$$

where U_r and U_θ are the horizontal displacement of the cylinder in r and θ direction, respectively, and $\overline{\Psi}$ is the rocking angle amplitude. Equating Eq. (36) and Eq. (39) at $r=a$ yields the constants A_n and B_n. Then, the expression for the local horizontal soil reaction becomes

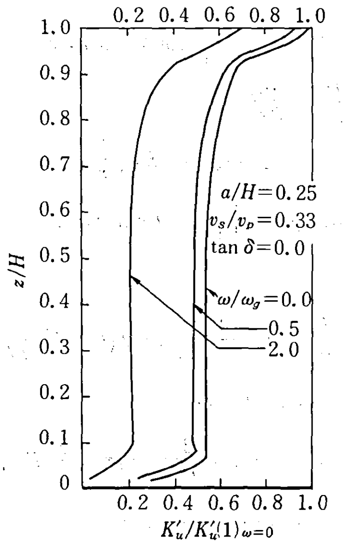

Fig. 8 Vertical Distribution of Local Dynamic Stiffness for Horizontal Vibration (Eq. (42))

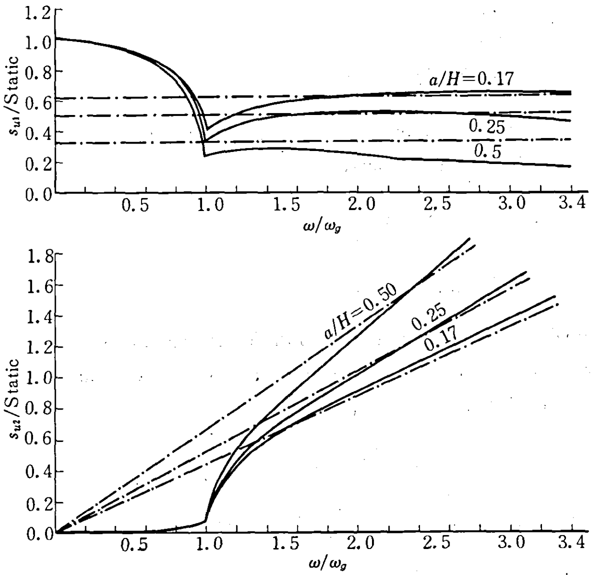

Fig. 9 Variations of Horizontal Dimensionless Dynamic Stiffness with Frequency and the a/H Ratio (Eq. (43) or (44)). Full Line for the Proposed Solution and Dushed Line for the Approximate Solution by Novak

$$F_u(z)=\frac{2G\pi a^2}{H}\cdot\overline{\Psi}\cdot\sum_n^N \Omega_n\cdot\xi_n^2\cdot a_n \tag{40}$$

where

$$\Omega_n=\frac{[4K_1(\gamma_n a)K_1(\beta_n a)+\beta_n aK_1(\gamma_n a)K_0(\beta_n a)+\gamma_n aK_1(\beta_n a)K_0(\gamma_n a)]}{[K_1(\gamma_n a)+\gamma_n aK_0(\gamma_n a)][K_1(\beta_n a)+\beta_n aK_0(\beta_n a)]-K_1(\gamma_n a)K_1(\beta_n a)}$$

The local dynamic stiffness can be now written as

$$K_u(z)=\frac{F_u(z)}{z\cdot\overline{\Psi}}=\frac{G\pi^3a^2}{4H^2}K_u'(z) \tag{41}$$

in which

$$K_u'(z)=\sum_n^N \Omega_n\cdot\xi_n^2\cdot a_n/\sum_n^N a_n \tag{42}$$

The vertical distribution of $K_u'(z)$ normalized by the static value at $z=H$ is shown in Fig. 8. The characteristics observed in Fig. 8 resemble those obtained for the previous cases. Thus, the dynamic stiffness to a unit length of the cylinder is defined by

$$K_u\cdot\frac{H}{2}\cdot\overline{\Psi}=\frac{1}{H}\int_0^H F_u(z)dz$$

The expression for the dynamic stiffness to a unit horizontal displacement of a unit length of the cylinder can be written as

$$K_u=\frac{8Ga^2}{H^2}\sum_n^N\frac{\Omega_n\cdot\xi_n^2}{n^3}(-1)^{\frac{n-1}{2}} \tag{43}$$

Separating the real and imaginary parts of Eq. (43) leads to the following expression:

$$K_u=G[s_{u1}(\omega/\omega_g,\tan\delta,v_s/v_p,a/H)+is_{u2}(\omega/\omega_g,\tan\delta,v_s/v_p,a/H)] \tag{44}$$

where s_{u1} and s_{u2} are both real. Figure 9 shows the variations with frequency of s_{u1} and s_{u2} normalized by the corresponding static values for several values of the a/H. The approximate solutions for the plane strain case are also shown in Fig. 9. The primary effect of the depth of surface soil stratum is seen in the abrupt changes of stiffness and damping near the fundamental horizontal frequency ω_g: The absence of the radiation damping below ω_g is also noted.

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(To Be Continued on No. 11)