UDC 624.131.5
539.214

Dynamic Soil Reactions (Impedance Functions) Including The Effect of Dynamic Response of Surface Stratum (Part 1)

表層地盤の動反力係数（1）

Takanori HARADA*, Keizaburo KUBO* and Tuneo KATAYAMA*
原田隆典・久保慶三郎・片山恒雄

1. INTRODUCTION

Most of the foundations for machines, bridges, or nuclear reactor containment structures are usually embedded in soil. There are also many structures such as pipes and submerged tunnels which are entirely buried underground. These foundations and structures vary great deal in their proportions. A foundation proposed for a long-span suspension bridge has a height of 80 m, a cross-sectional area of 2800 m^2, and an embedded depth of about 25 m below the ground surface. On the contrary, typical machine foundation often have a height of 4 m and a plane area 20 m^2, embedded at a depth of less than 1 m. These foundations are frequently subjected to dynamic exitation due to operating machine, wind or earthquakes. The dynamic behavior of embedded foundations to such loads can be predicted if the dynamic loadings and the dynamic reactions of the soil acting to the foundations are known.

The use of the coefficient of subgrade reaction is one of the conventional ways to describe the soil reactions. Although this coefficient may be quite useful, it does not represent the frequency dependency of the soil reactions which arises from soil mass and energy dissipation through elastic wave. Novak et al. [1] defined the soil reactions to harmonic motion of an embedded cylinder in terms of linear viscoelasticity limited to the case that can be viewed as plane strain by extending the approach previously used by Baranov. Such a situation arises, e.g., when a rigid cylinder extending to infinity in an infinite medium undergoes uniform displacements in the direction of its exis or perpendicular to it. In the case of an infinite or a half-space medium, the dynamic (Complex) soil reaction is almost independent of frequency in the range of practically interest because the infinite or half-space medium does not have an eigenvalue and dose not exhibit the resonance phenomenon. By applying their complex soil reactions to rigid embedded small footings or elastic piles, Novak et al. [2,3,4,5] showed that the approach could give very reasonable results for the case investigated. However, it may not always be resonable to assume the plane strain condition for large foundations such as nuclear reactor containment structures or long-span bridge foundation structures. It seem also desirable to consider the response of the surface soil stratum for large foundations deeply embedded into the surface soil stratum.

This paper presents a relatively simple analytical approach which defines the dynamic soil reactions (similar to the so-called Winkler type model) by taking account of the dynamic response of surface stratum. The effect of the reponse of soil stratum is also assessed by comparing the results of the present solutions with those obtained for plane strain case. It should be noted that the following assumptions are adopted in the whole of the following analysis. 1. The soil stratum overlying a rigid bed rock is of a homogeneous and isotropic linearly elastic medium with frequency independent material damping of hysteretic type. 2. The foundation is perfectly rigid and of cylindrical crosssectional shape. 3. No separation is allowed between foundation and soil. 4. The vibration is harmonic.

2. SOIL REACTION IN VERTICAL VIBRATION

When the cylinder undergoes an axisymmetrical motion in the direction of its axis (Fig. 1), $u=v=0$ and the equations of motion of the surface soil stratum expressed in cylindrical coordinates reduce to (APPENDIX I)

$$\left(G+G'\frac{\partial}{\partial t}\right)\left(\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial r^2}\right)+\left(\lambda+2G+2G'\frac{\partial}{\partial t}\right)\frac{\partial^2 w}{\partial z^2}=\rho\frac{\partial^2 w}{\partial t^2} \qquad (1)$$

*Dept of Building and Civil Engineering, Institute of Industrial Science, University of Tokyo.

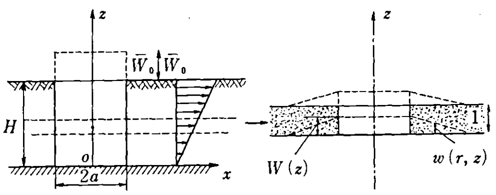

Fig. 1 Notation and Vertical Displacement of Cylinder

in which λ and G are lame's constants, G' is the viscosity modulus associated with G, and ρ is a mass density of the surface soil stratum. The viscosity is introduced only in the shear modulus G.

By expressing w for harmonic motion with frequency ω as

$$w=R(r)\cdot Z(z)\cdot e^{i\omega t} \tag{2}$$

Eq. (1) can be split into the following two ordinary differential equations:

$$\frac{d^2R}{dr^2}+\frac{1}{r}\frac{dR}{dr}-\alpha_n^2\cdot R=0 \tag{3}$$

$$\frac{d^2Z}{dz^2}+\left(\frac{n\pi}{2H}\right)^2\cdot z=0 \tag{4}$$

where constant α_n is given by the following equation

$$\alpha_n=\frac{\pi}{2H}\sqrt{\frac{\{v_p/v_s)^2+i2D\}n^2-(\omega/\omega_g)^2}{(1+i2D)}} \tag{5}$$

in which

$$v_p=\sqrt{\frac{\lambda+2G}{\rho}}\ ,\quad v_s=\sqrt{\frac{G}{\rho}}\ ,\quad \omega_g=\frac{\pi v_s}{2H}$$

$$2D=\tan\delta=\frac{G'\omega}{G}$$

The damping property of the surface soil is represented by $2D$ or $\tan\delta$. Experiments have shown that the energy loss in soil is a function of the strain induced in the soil, and unlike viscous damping, it is to large extent independent of frequency. The value of $\tan\delta$ is usually less than 0.05 at small strain, and it may be as high as 0.3–0.5 for the larger strains associated with high intensity seismic motion [6,7].

The general solutions of Eqs. (3) and (4) are

$$\left.\begin{aligned} R(r)&=A_nK_0(\alpha_n r)+B_nI_0(\alpha_n r)\\ Z(z)&=C_n\sin\left(\frac{n\pi}{2H}z\right)+D_n\cos\left(\frac{n\pi}{2H}z\right)\end{aligned}\right\} \tag{6}$$

where $K_m(x)$ and $I_m(x)$ are the modified Bessel functions of order m of the first and second kind with argument x, respectively, and A_n, B_n, C_n and D_n are integration constants to be determined by boundary conditions.

The boundary conditions of the surface soil stratum overlying a rigid bed rock as is shown in Fig. 1 are

(a) Zero normal and shear stresses on the free surface.

(b) Zero displacements at the bed rock.

(c) Zero stresses and zero displacements at infinite horizontal distance.

Boundary condition (c) leads to $B_n=0$. Conditions (a) and (b) give $D_n=0$ and $n=1, 3, 5, \cdots$. Then, the displacement w in Eq. (2) can be written as

$$w=\sum_n^{\infty}A_nK_0(\alpha_n r)\cdot\sin(h_n z) \tag{7}$$

where A_n is a constant determined from the boundary condition between cylinder and soil, constant α_n is evaluated by Eq. (5), and $h_n=n\pi/(2H)$ with $n=1,3,5,\ldots$. For simplicity, the term $e^{i\omega t}$ was omitted in Eq. (7).

The shear stress τ_{rz} is given by

$$\tau_{rz}=\left(G+G'\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial r}$$
$$=-G(1+i2D)\sum_n^{\infty}A_n\cdot\alpha_n\cdot K_1(\alpha_n r)\cdot\sin(h_n z) \tag{8}$$

The local vertical soil reaction, $F_w(z)$, can be obtained by integrating the above stress over the circumference of the cylinder. Thus,

$$F_w(z)=-\int_0^{2\pi}\tau_{rz}|_{r=a}\cdot a\cdot d\theta$$
$$=2\pi Ga(1+i2D)\sum_n^{\infty}A_n\cdot\alpha_n\cdot K_1(\alpha_n r)\cdot\sin(h_n z) \tag{9}$$

Since the displacement in the surface stratum is expressed in terms of *Sine* series and the perfect bond is assumed between the cylinder and the soil, the dynamic displacement of the cylinder may be assumed in the following form:

$$W(z)=\frac{z}{H}\cdot\overline{W}_0=\frac{8\overline{W}_0}{\pi^2}\sum_n^N a_n \tag{10}$$

in which

$$a_n=\frac{(-1)^{\frac{n-1}{2}}}{n^2}\cdot\sin(h_n z),\quad n=1,\ 3,\ 5,\cdots N.$$

where $\overline{W}_0$ is the dynamic vertical displacement amplitude at the free surface $z=H$. Constant A_n can be determined by equating Eq. (7) to Eq. (10) at the circumference of the cylinder $r=a$. Thus, the local vertical soil reaction defined in Eq. (9) is expressed in the form:

$$F_w(z)=\frac{16G\overline{W}_0}{\pi}(1+i2D)\sum_n^N\delta_n\cdot a_n \tag{11}$$

where

$$\delta_n=\alpha_n\cdot a\cdot\frac{K_1(\alpha_n a)}{K_0(\alpha_n a)}$$

The local dynamic stiffness of the surface soil stratum to a unit vertical displacement may be defined as follows by using Eqs. (10) and (11):

$$K_w(z)=\frac{F_w(z)}{\frac{z}{H}\cdot\overline{W}_0}=\frac{(\text{Local Soil Reaction})}{(\text{Local Vertical Displace. of Cylinder})}$$
$$=2G\pi(1+i\,2D)\cdot K'_w(z) \qquad (12)$$

where

$$K'_w(z)=\sum_n^N\delta_n\cdot a_n/\sum_n^N a_n \qquad (13)$$

Figure 2 shows the vertical distribution of $K'_w(z)$ normalized by the static value at $z=H$ for the case, $a/H=0.25, v_s/v_p=0.33$ and zero internal damping. From Fig. 2, it can be seen that the vertical distribution of $K'_w(z)$ is almost constant except for the small parts near the free surface and the bottom of the surface soil. The reason for the sharp increase and decrease of local soil stiffness near the free surface and the bottom may be due to the ignorance of horizontal displacement of the surface soil and the assumption on dynamic motion of cylinder that the vertical motion is zero at the bottom $z=0$.

From these results, it may be resonable to define the dynamic stiffness of the surface soil related to a unit of length of the cylinder by the following equation by averaging Eq. (12) over the depth:

$$K_w\cdot\frac{\overline{W}_0}{2}=\frac{1}{H}\int_0^H F_w(z)dz$$

from which the following expression is obtained

$$K_w=\frac{64G}{\pi^2}(1+i\,2D)\sum_n^N\delta_n\cdot\frac{(-1)^{\frac{n-1}{2}}}{n^3} \qquad (14)$$

This expression for dynamic stiffness (Impedance Function) to a unit harmonic vertical motion of a unit length of the cylinder can be rewritten as

$$K_w=G\,[s_{w1}(\omega/\omega_g,\ \tan\delta,\ v_s/v_p,\ a/H)$$
$$+is_{w2}(\omega/\omega_g,\ \tan\delta,\ v_s/v_p,\ a/H)] \qquad (15)$$

where both s_{w1} and s_{w2} are the dimensionless dynamic stiffness with real number. Equation (14) or (15) indicates that the dynamic soil reaction is a function of the excitation frequency, the shear modulus, the material damping, the Poisson's ratio and the a/H ratio. It should be noted that the soil reaction for plane strain case (APPENDIX II) dose not include the parameter, a/H, which characterizes the effect of the ratio between the radius of the cylinder and the surface soil depth on the dynamic response of the foundations.

The quantities s_{w1} and s_{w2} were normalized by their corresponding static values. Their variations with frequency and the a/H ratio are shown in Fig. 3 for the case of internal damping, $\tan\delta=0.01$. The real part of K_w, s_{w1}, represents the stiffness and the imaginary part of K_w, s_{w2}, stands for damping including both the material and the radiational damping. For comparison, the dynamic stiffness for plane strain case (the approximate solution by Novak et al. [3], APPENDIX II) is also shown in Fig. 3.

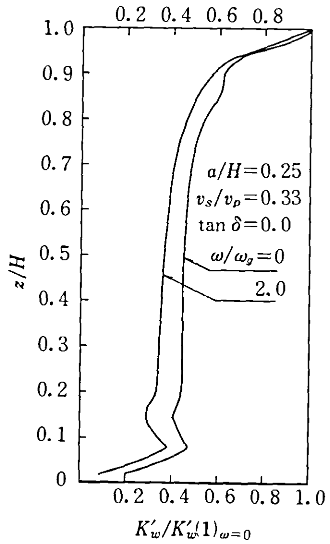

Fig. 2 Vertical Distriburtion of Local Dynamic Stiffness for Vertical Vibration (Eq. 13)

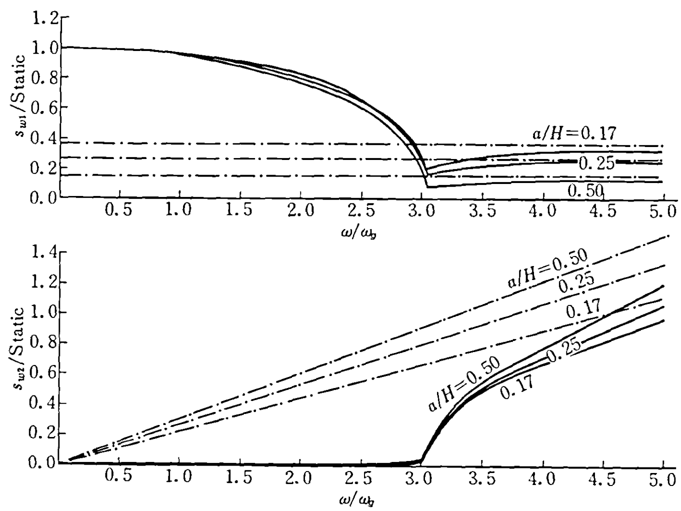

Fig. 3 Variations of Vertical Dimensionless Dynamic Stiffness with Frequency and the a/H Ratio (Eq. (14) or (15)). Full line for the Proposed Solution and Dushed line for the Approximate Solution by Novak

研究速報

There is a notable discrepancy between the plane strain solution and the proposed solution especially in the range below the fundamental vertical frequency of the surface stratum, $\omega_p=\pi v_p/(2H)$, which is assumed to be 3.0 ω_g in the case shown in Fig. 3 because $\omega_p=\pi v_p/(2H)=(v_p/v_s)\omega_g=(1/0.33)\omega_g=3.0\omega_g$, ω_g is the fundamental horizontal frequency of the surface stratum defined by $\pi v_s/(2H)$. In the range above the fundamental vertical frequency s_{w1} is of the same order as the value by the plane strain solution and s_{w2} is smaller than the value by plane strain solution. The differences both in the region below and above the fundamental vertical frequency ω_p are due to the dynamic response of the surface soil stratum. Below ω_p, the stiffness s_{w1} strongly depends on the frequency, and the damping s_{w2} is very low. The damping in this range is mostly caused by material damping and the radiation damping is absent. Above ω_p, s_{w2} rapidly increases linearly with frequency while s_{w1} is almost constant because a horizontally progressive wave only appears above this frequency.

Reference

1) Novak, M., Nogami, T., and Aboul-Ella, F., "Dynamic Soil Reactions for Plane Sfrain Case". Proc. of ASCE, Vol. 104, No. EM4, 1978, pp. 953–959
2) Beredugo, Y. O., and Novak, M., "Coupled Horizontal and Rocking Vibration of Embedded Footings". Canadian Geotechnical Journal, Vol. 9, 1972, pp. 477–497
3) Novak, M., and Beredugo, Y. O., "Vertical Vibration of Embedded Footings". Proc. of ASCE Vol. 98, No. 5M12, 1971, pp. 1291–1310
4) Novak, M., "Dynamic Stiffness and Damping of Piles". Canadian Geotechnical Journal, Vol. 11, 1974, pp. 574–598
5) Novak, M., and Aboul-Ella, F., "Impedance Functions of Piles in Layered Media". Proc. of ASCE, Vol. 104, No. EM6, 1978, pp. 643–661
6) Hardin, B. O., and Drenvich, V. P., "Shear Modulus and Damping in Soil, Measurement and Parameter Effects". Proc. of ASCE, Vol. 98, No. SM6, 1972, pp. 603–624
7) Veletsos, A. S., and Verbic, B., "Vibration of Viscoelastic Foundations". Earthquake Eng. and Struc. Dynamics, Vol. 2, 1973, pp. 82–102
8) Kolsky, H., "Stress Waves in Solids". Dover Publications, Inc., 1963, pp. 106–112
9) Isoda, K., and Ohno, U., editor "Handbook for Numerical Calculation by FORTRAN" (in Japanese). Korona Publication Campany

To Be Continued on No. 10

正　誤　表（9月号）

頁	段	行	種　別	正	誤
607		↑9	表 3	作動	差動
610			図3（右図）	診断書	診断所
619	右	↓21	数式	$N_{ij}=-\rho\omega(f_{ij}-f_{ij}^*)/2i$	$N_{ij}=-\rho\omega(f_{ij}-f_{ij}^*)/2$
628		↓5	表 1	−23.0	23.0