研 究 速 報

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ON THE CRITICAL SIZE OF DROP DETACHMENT DURING THE PROCESS OF DROPWISE CONDENSATION (I)

滴状凝縮過程における離脱液滴径について (I)

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1. Iutroduction

The critical size of drop detachment during dropwise condensation is one of the most important factors that control the process. The authors have initiated an experimental investigation in which it has been attempted to find out the effect of reducing the critical size of drop detachment upon the heat transfer rate. As the means to reduce the critical size the authors have employed (1) the shear stress by vapor flow and also (2) the centrifugal acceleration field produced by rotation[1].

As is widely understood, the critical size of drop detachment is a quantity determined from the balance between the adhesive force due to surface tension and the external forces such as gravity, vapor shear stress, etc. Fatica and Katz[2] and Sugawara and Michiyoshi[3] have respectively derived an equation giving the critical sizes of the drops on inclined surfaces under the action of normal gravitational force. However, their results show considerable difference when compared with those values obtained by the observation of the actual dropwise condensation process. For example, the critical diameter measured by the present authors[4] for the condensation of steam under one atmospheric pressure is about 2 mm when the inclination of the condensing surface is 90°, while the ones calculated by Fatica *et al.*[2] and Sugawara *et al.*[3] are about 4 mm.

This discrepancy may be partly because the former value is obtained during actual condensation, while the latters are for the stationary, non-condensing drops. Since the drops are always moving around during dropwise condensation as the consequence of repeated coalescences, it is very likely that the actual critical size is much smaller than is calculated from the static balance of forces. However, the present authors have had some doubt on the analyses carried out by Fatica *et al.* and Sugawara *et al.* In both of the analyses, the balance of forces were considered only for the thin semicircular slab at the plane of symmetry of the drop (Fig. 1), and the contribution from the rest of the drop was ignored. There was given no evidence at all as to whether this was really an allowable approximation.

Thus, the present authors took it very important to establish more exact theory on the critical size of drop departure.

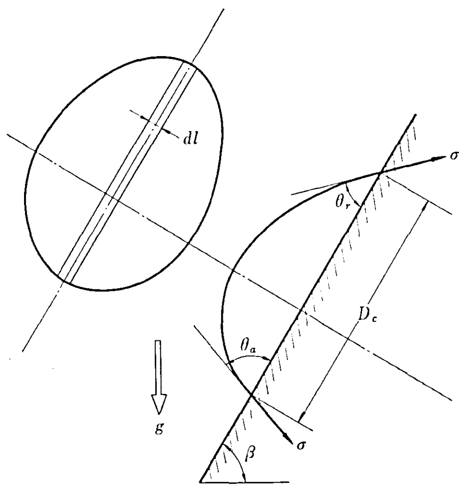

Fig. 1

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2. General Consideration upon the Profile of a Drop on a Horizontal Plate

Similar to the number of other physico-chemical phenomena occurring in the universe, the shape of a drop at rest under the action of several kinds of forces is thought to obey the general principle: the law of minimum energy. Let us first consider a liquid drop placed underneath a horizontal plate (Fig. 2). It is assumed that the drop is non-spreading, and the atmosphere sorrounding the drop consists of the pure vapor alone of the liquid.

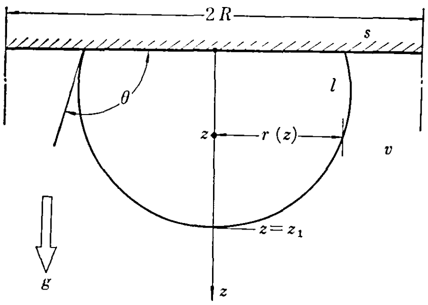

Fig. 2

In may easily be seen that in this system there exist three kinds of interfaces; liquid-vapor (lv), liquid-solid (ls) and vapor-solid (vs). Each of these interfaces has its own interfacial energy. The energy per unit area is denoted here σ_{ij} where i and j represent two of the three phases (v, l and s).

In the case of Fig. 2, if it is assumed that the drop shape is symmetrical around z-axis, the interfacial energy of lv-surface is

$$E_{lv}=2\pi\sigma_{lv}\int_0^{z_1} r(z)\sqrt{1+r'^2}\,dz \qquad (1)$$

where $r'=dr/dz$. The energy of ls-interface is

$$E_{ls}=\pi\sigma_{ls}\{r(0)\}^2 \qquad (2)$$

The energy of vs-interface is apparently dependent upon the control area. If the area inside a circle of radius R is considered

$$E_{vs}=\pi R^2\sigma_{vs}-\pi\{r(0)\}^2\sigma_{vs} \qquad (3a)$$

However, the first term to the right side of Eq. (3) has no concern with the determination of the profile of the drop, and it can be replaced by a constant, C. Then

$$E_{vs}=-\pi\{r(0)\}^2\sigma_{vs}+C \qquad (3b)$$

On the other hand, the potential energy stored by the drop due to the action of external forces (in this case, the gravitational potential energy) is

$$U=-\pi\rho g\int_0^{z_1} z\{r(z)\}^2dz \qquad (4)$$

where ρ is the liquid density and g is the acceleration of gravity.

Thus, the total energy of this system becomes

$$E=E_{lv}+E_{ls}+E_{vs}+U$$
$$=2\pi\sigma_{lv}\int_0^{z_1} r\sqrt{1+r'^2}\,dz$$
$$+\pi\{r(0)\}^2(\sigma_{ls}-\sigma_{vs})-\pi\rho g\int_0^{z_1} zr^2dz+C \qquad (5)$$

In addition to the above, there is a constraining condition that the mass M of the drop is invariable, and M is expressed as

$$M=\pi\rho\int_0^{z_1} r^2dz \qquad (6)$$

According to the law of minimum energy, the profile of the drop should be so determined that the total energy E may be minimized under the condition of constant M. This is a typical variational problem and can be written as below:

$$\delta E=0 \qquad (7a)$$
$$M=\text{const.} \qquad (7b)$$

Or, if we introduce the so-called Lagrange multiplier λ, Eqs. (7) become

$$\delta H=0 \qquad (8a)$$
$$H\equiv E+\lambda M \qquad (8b)$$

Subtitution of Eqs. (5) and (6) into Eq. (8a) yields

$$\delta\left[\int_0^{z_1}(2\sigma_{lv}r\sqrt{1+r'^2}-\rho gzr^2+\rho\lambda r^2)dz + \{r(0)\}^2(\sigma_{ls}-\sigma_{vs})\right]=0 \qquad (9)$$

It is evident that at the bottom end of the drop $z=z_1$

$$r(z_1)=0 \qquad (10)$$

On the other hand, on the contacting surface $z=0$, the periphery of the drop is movable, and the natural boundary condition as below is applied[5].

$$-\left[\frac{\partial}{\partial r'}(2\sigma_{lv} r\sqrt{1+r'^2}-\rho g z r^2+\rho\lambda r^2)\right]_{z=0}\delta r(0)$$

$$+2(\sigma_{ls}-\sigma_{vs})r(0)\delta r(0)=0 \qquad (11)$$

or

$$\left[-2\sigma_{lv}\frac{rr'}{\sqrt{1+r'^2}}+2r(\sigma_{ls}-\sigma_{vs})\right]_{z=0}\delta r(0)=0 \qquad (12)$$

Since it can be assumed physically $r(0)\fallingdotseq 0$ and $\delta r(0)\fallingdotseq 0$

$$\sigma_{lv}\frac{r'(0)}{\sqrt{1+\{r'(0)\}^2}}=\sigma_{ls}-\sigma_{vs} \qquad (13)$$

Thus

$$r'(0)=\sqrt{\frac{(\sigma_{ls}-\sigma_{vs})^2}{\sigma_{lv}{}^2-(\sigma_{ls}-\sigma_{vs})^2}} \qquad (14)$$

A little consideration would reveal a fact that the right-hand side of Eq. (14) is equal to a cotangent of the contact angle θ, which seems physically quite reasonable. One more important result here is that this angle of contact is independent of the gravitational force. Thus, the boundary conditions are written as below:

$$r'(0)=\cot\theta \qquad (15a)$$

$$r(z_1)=0 \qquad (15b)$$

The authors solved this variational problem by a numerical method and found out that the solutions were identical with the result of a classical analysis of Bashforth[6].

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