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SEPARATION OF VIBRATION MODES OF MACHINE STRUCTURE USING A RESPONSE CURVE

応答曲線を用いた機械構造物の各次振動の分離法

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1. Introduction

Recently the response curve of machine structure is considered as an important factor on vibration control and isolation. If the response curve of real machine structure is approximately derived from the sum of each response curve of single degree-of-freedom system, the vibration is easily controlled under the treatment of independent mode shapes. Already some methods have been proposed to separate the vibration characteristics of multiple degree-of-freedom system[(1), (2)]. This study treats the separation of vibration modes of a structure on resilient supporting elements such as the structure analized by the authors on the natural frequency calculation[(3), (4)]. Because these compound systems have large resonant amplitude causing from supporting system at low frequency and have fairly small amplitude belonging to the structure itself, the separation of them is not easy.

First a method to separate vibration modes is descrived, using the test data for an example. Next the modes of U-type steel hanged by springs is separated from measured data. It is concluded that vibration modes of compound system and of course usual structure are separated by the proposed method.

2. Expression of Response Curve

In this study response curve are defined by a square of absolute displacement response per driving force, and phase curves are not treated. The same notation is used as Mitsui and Sato's report[(1)].

If a response curve with N peaks is assumed to be the sums of N curves of single degree-of-freedom system,

then

$$S_N=\sum_{j=1}^{N}\frac{D_j}{\left(1-\dfrac{p_i^2}{\omega_j^2}\right)^2+4\zeta_j^2\dfrac{p_i^2}{\omega_j^2}}$$

$$=\sum_{j=1}^{N}\frac{D_j\omega_j^4}{(\omega_j^2-p_i^2)^2+4\zeta_j^2\omega_j^2p_i^2} \tag{1}$$

where

$$D_j=\left(\frac{F}{k_j}\right)^2$$

and

S_N : a square of absolute displacement response per driving force

p_i : circular frequency of driving force

F : absolute driving force

ω_j : j-th undamped natural circular frequency

ζ_j : j-th equivalent damping coefficient

k_j : j-th equivalent spring constant

3. Separation of Vibration Modes by a Method of Small Exchange

M measured data of response curve are assumed as $R(p_i)\,(i=1, 2\ldots N)$. $D_j, \zeta_j (j=1, 2\ldots N)$ are decided to minimize the sum of M square differences between $R(p_i)$ and $S_N(p_i)$ given by Eq. (1). Evaluate function is as follows.

$$E=\sum_{i=1}^{M}\left\{S_N(p_i)-R(p_i)\right\}^2 \tag{2}$$

then

$$f_j=\frac{\partial E}{\partial D_j}=2\sum_{i=1}^{M}\left[\left\{S_N(p_i)-R(p_i)\right\}\cdot\frac{\omega_j^4}{(\omega_j^2-p_i^2)^2+4\zeta_j^2\omega_j^2p_i^2}\right]=0 \tag{3}$$

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$$g_j=\frac{\partial E}{\partial \zeta_j}=2\sum_{i=1}^{M}\left[\left\{S_N(p_i)-R(p_i)\right\}\cdot\frac{-8D_j\zeta_j\omega_j^6p_i^2}{\{(\omega_j^2-p_i^2)^2+4\zeta_j^2\omega_j^2p_j^2\}^2}\right]=0 \qquad (4)$$

D_j, ζ_j $(j=1, 2\ldots N)$ are sought definitely by the following process. At the first step D_j, ζ_j are assumed approximately. It is one of the appropriate methods to presume D_j, ζ_j by regarding each resonant point as the resonant peak of independent single degree-of-freedom system. Using ω_{jP} for the peak value of R, the undamped natural frequency is written as follows.

$$\omega_j=\omega_{jP}/\sqrt{1-2\zeta_j^2} \qquad (5)$$

At the next step f_j, g_j $(j=1, 2\ldots N)$ are calculated. If a calculated curve from the estimated D_j, ζ_j are the nearest curve to the experimental curve on evaluation by E, f_j, g_j will be zero. But for D_j, ζ_j are estimated values, f_j, g_j are not zero. Then D_j, ζ_j are changed to make f_j, g_j to zero by the following process. The small exchange $D_k \rightarrow D_k+\Delta D_k$ are given for $k=1$ and increments f_{j1}, g_j are obtained by calculation of f_j, g_j $(j=1, 2\ldots N)$. Similarly increments $f_{j2}, g_{j2}\ldots f_{jN}, g_{jN}$ for $k=2, 3\ldots N$ and $f_{jN+1}, g_{jN+1}\ldots f_{j2N}, g_{j2N}$ for $\zeta_k \rightarrow \zeta_k+\Delta\zeta_k$ $(k=1, 2\ldots N)$ can be calculated. If exchange rates $x_j\Delta D_j$ and $y_j\Delta\zeta_j$ are very small and linear relation holds between $\Delta D_j, \Delta\zeta_j$ and $\Delta f_{jk}, \Delta g_{jk}$, following $2N$th first order equation is derived.

$$\begin{pmatrix} \Delta f_{11}, \Delta f_{12} & & \\ \Delta g_{11}, \Delta g_{12} & & \\ & & \\ \cdots\cdots\cdots & & \\ & & \\ \Delta g_{N1}, \cdots & & \Delta g_{N2N} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ y_1 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} -f_1 \\ -g_1 \\ \vdots \\ \\ \\ -g_N \end{pmatrix} \qquad (6)$$

Then increments $D_j \rightarrow D_j+x_j\Delta D_j$, $\zeta_j \rightarrow \zeta_j+y_j\Delta\zeta_j$ $(j=1, 2\ldots N)$ will make f_j, g_j to zero.

But because of imperfect linearity between ΔD_j, $\Delta\zeta_j$ and $\Delta f_{jk}, \Delta g_{jk}$, (3), (4) do not lead to zero even if $D_j+x_j\Delta D_j$ and $\zeta_j+y_j\Delta\zeta_j$ are substituted in (3), (4). The above mentioned process is reiterated to compensate the nonlinearity until the value of f_j, g_j converge. D_j, ζ_j may also converge to some values at the time.

4. Separation of Test Data

The availability of the method descrived in Section 3 is studied with test data shown as true values in Table 1 and as response curve in Fig. 1.

Table 1 Vibration Characteristics of Test Data

	True	Initial	5th Iteration
D_1	30.0	16.2	30.00
D_2	30.0	19.2	30.00
D_3	25.0	14.0	24.99
D_4	5.0	2.7	5.00
D_5	5.0	2.3	5.00
ζ_1	0.08	0.055	0.080
ζ_2	0.10	0.074	0.100
ζ_3	0.10	0.071	0.100
ζ_4	0.04	0.028	0.040
ζ_5	0.05	0.034	0.050
ω_1	10.0	9.936	10.00
ω_2	15.0	14.849	15.00
ω_3	22.0	21.779	22.00
ω_4	30.0	29.952	30.00
ω_5	40.0	39.900	40.00

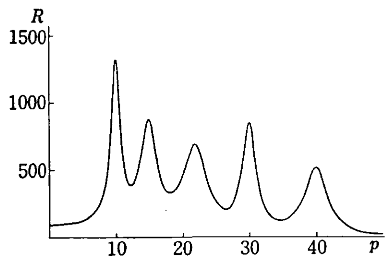

Fig. 1 Response curve of test data

They are refered to the report (1). First estimated values of ζ_j are taken $\frac{1}{\sqrt{2}}$ times as large as the values calculated by the usual bandwidth method, D_j are designated as $D_j=4\zeta_j^2R_{j\max}$ when the maximum valuse of R are given by $R_{j\max}$. These values are shown as the initial values in Fig. 1. 501 points from $p=0.0$ to $p=50.0$ with increments of 0.1 are regarded as the measured values, and D_j, ζ_j may be obtained with the method of small exchange from parameters $N=5$, $M=501$. Some examples of converging process are shown in Fig. 2. In this case calculated values converge to the true value with five times iteration. Though the initial values are 50～70%

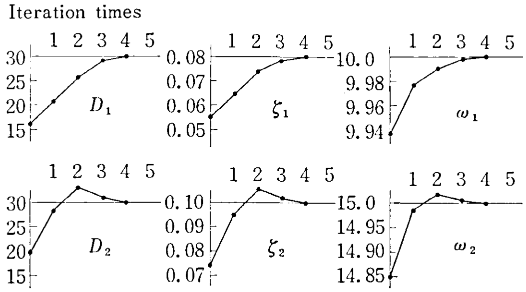

Fig. 2 Converging process of test data

of true values, iteration values converge accurately to the true value.

5. Separation of U-type Steel Hanged Down by Springs

U-type steel is hanged down by two springs (I) at the each end and driven to horizontal direction. The experimental response curve is obtained from measured horizontal acceleration curve as shown by the dotted line in Fig 3. The magnified curve is shown at the higher frequency than 100 Hz because of extremely small vibration level. The resonance point at 8.7 Hz is caused by the spring system, and three resonance points at the higher frequency than 100 Hz are caused by the natural vibration of steel or local vibration.

Four single degree-of-freedom systems are separated from the curve by a method of small exchange at a time with 80 measured data. Initial values and 23th iteration values are shown in Table 2. The response curve is conversely

Table 2 Vibration Characteristics of a Steel with Spring (I)

	Initial	23th Iteration
D_1	6430.0	6343.3
D_2	0.00034	0.00004
D_3	0.00040	0.00059
D_4	0.00235	0.00463
ζ_1	0.1250	0.1259
ζ_2	0.00755	0.00167
ζ_3	0.00284	0.00410
ζ_4	0.00523	0.00766
$\omega_1/2\pi$	8.686	8.827
$\omega_2/2\pi$	112.53	112.53
$\omega_3/2\pi$	118.41	118.41
$\omega_4/2\pi$	137.63	137.64

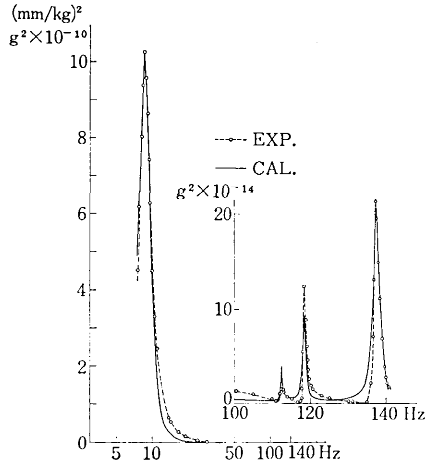

Fig. 3 Response curve of a steel with spring (I)

generated from these iteration values using equation (1) as shown by the solid line in Fig. 3. Though there are little differences at the foot of the curve between experimental data and calculated results, good agreement is obtained at the peaks. Those little differences may be caused by the errors of approximation to separate the sum of single degree-of-freedom system or low reliability of data at the region of small vibration level.

When a steel is hanged down by springs (II) whose constant is nearly twice the spring (I), the response curve changes as shown in Fig. 4. 5th iteration values are obtained with 42 measured data from estimated initial values as shown in Table 3. The generated response curve are shown as a solid line in Fig. 4. In this case the natural vibration of the steel is appeared only at 133.2 Hz. Therefore second and third little peaks appeared in the case of spring (I) may be caused other

Table 3 Vibration Characteristics of a Steel with Spring (II)

	Initial	5th Iteration
D_1	1120.0	1342.6
D_2	0.00405	0.00430
ζ_1	0.06560	0.07038
ζ_2	0.00480	0.00502
$\omega_1/2\pi$	11.810	11.869
$\omega_2/2\pi$	133.20	133.20

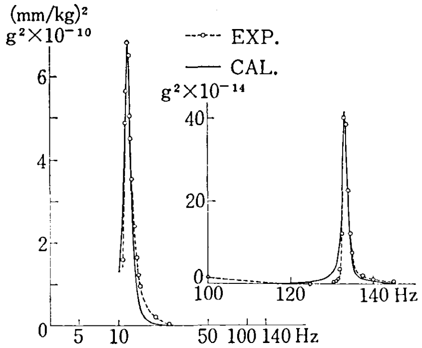

Fig. 4 Response curve of a steel with spring (II)

than by the natural frequency of the steel. The converged values of D_1, D_4, ζ_1 of spring (I) and D_1, D_2, ζ_1 of spring (II) may be reasonable in comparison with each other. But ζ_4 of spring (I) is slightly large. This may be caused by the effect of low level measured data. In spite of 1000 times difference of vibration level between low and high frequency resonance peak, separation is done to the sums of single degree-of-freedom system at a time.

With this method the effect of spring and local vibration are eliminated and the vibration characteristics of the structure itself may be extracted.

6. Conclusion

A method descrived in this report is characterized at the points where response curve is calculated to minimize the total sum of square errors from selected data and the accuracy of the curve can be easily improved by increasing the data at the important frequency range. The converged iteration values by this method are essentially stable to the initial values, then true values are obtained from rough initial values. It may be done with same process as this report to evaluate the error with weight function and to attach importance to the higher frequency range using the acceleration response curve.

Phase curves may be treated with slightly extended method.

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