ON THE OVERTURNING AND FRACTURING OF BRICK AND OTHER COLUMNS BY HORIZONTALLY APPLIED MOTION.

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The following paper gives an account of a series of experiments carried out with the object of determining the accelerations necessary to overturn or fracture columns of various descriptions standing freely, or fixed upon a truck which was moved horizontally back and forth through a small range of motion. As the object of these experiments was to furnish those who have to build in *earthquake countries* with data respecting the quantity of motion certain forms of structure might be expected to withstand, the range of back and forth motion employed was from a half to four inches,—quantities which are comparable with the greatest extent of earthquake movement of which we have any sure measurements.

As examples of such records we quote the following:—

1.—For the Neapolitan Earthquake of 1857 from observation on cracks in buildings and other phenomena, Mr. Mallet estimated amplitudes of motion of from 2.5 to 4.7 inches. From projection phenomena and the dimensions of bodies which were overturned, the same investigation determined maximum velocities, the average of which may be taken at 12 feet per second. The fact that these data lead to the conclusion that the period of a wave would be about 0,125 seconds, whereas we know from observations in Japan that period increases with amplitude, and that waves with amplitudes of even one inch have invariably a period of at least one second, we are compelled to accept Mr. Mallet's conclusions with caution. Yet until absolute measurements of earthquake motion were made in Japan Mr. Mallet's investigations respecting the amplitude and period of earthquake vibrations were by far the best to be obtained.

2.—February 22nd, 1880. On hard ground in Tokio a range of 21 millimetres was recorded. From the measurement of many earthquakes on similar ground we may safely conclude that the frequency of the vibrations did not exceed one per second. This indicates a maximum velocity of 60 mm. per sec. and a maximum acceleration of 360 mm. per sec. per. sec. A few chimneys fell, tiles were projected from the eaves of buildings, and one or two walls were slighty cracked. In Yokohama the range of horizontal motion was from 15 mm. ($\frac{5}{8}$ in.) to 50 mm. (2 in.). Many brick chimneys fell, tiles were shaken loose, some buildings were unroofed, grave stones were rotated, walls were cracked, and many bodies, like tiles, &c., and coping stones, were projected.

October 15th, 1884.—In Tokio, on *soft* ground, the greatest range of motion was 43 mm. and the period 2 seconds. This indicates a maximum velocity of 68 mm. per sec. and a maximum acceleration of 210 mm. per sec. per sec. One or two chimneys fell and a few walls were cracked.

January 15th, 1887.—The observations at three places in Tokio, the first of which is on soft ground, and the latter on moderately hard ground, were as follows:

	Range of motion in Millimetre.	Period in Seconds.	Maximum Velocity.	Maximum Acceleration.	Vertical Motion.	Period of Vertical Motion.
Hitotsu-bashi	21	2.5	26	66	1.8	0.9
Hongo	7.3	2.0	12	36	1.3	1.0
Chiri Kioku	19.2	2.3	24	64	5.5	0.8

In Tokio a few brick walls were cracked slightly.

In Yokohama, about 10 miles nearer to the origin of the disturbance where a horizontal motion of 35 mm. was recorded, many chimneys fell and buildings were shattered.

The conclusion is that when there is an *earth* movement of 18 mm. (¾ in.) or over, the period is usually sufficiently short to result in accelerations causing destruction, and ranges of motion used in the experiments may be described as comparable with the motions that structures may have to withstand in earthquake countries.

Earthquakes have undoubtedly occurred where movements greater than four inches have been experienced, but measurements of these movements are not obtainable. Eye-witnesses testify to the fact that the ground has thrown out wave-like undulations, and buildings therefore have not simply been subjected to horizontal stresses but have been tipped and rocked. Such disturbances are, moreover, extremely rare, and even when they do occur the areas where the motions have exceeded the limits discussed in the following paper have been small.

The reasons why the effects due to the vertical component of motion have been overlooked are, first, the difficulties of experiment, and secondly the fact that in all earthquakes recorded in Japan the vertical component is invariably very small as compared with the horizontal movement. In the case of the earthquake of January 15th, 1887, just given, it will be seen that the range of motion for the vertical component is to that of the horizontal component in the ratio of 1 to 10; at the most 1 to 4, the latter being unusually large.

The only other experiments bearing on the oscillations necessary to overturn bodies of various dimensions are those given by one of the present authors in a paper on Seismic Experiments in Vol. VIII. of the Transactions of the Seismological Society.[1] These experiments, which only refer to ex-

(1) Seismic Experiments, by John Milne. Trans. Seis., Soc., Vol. VII., pp. 1-82.

ceedingly small columns of wood, are again referred to in the following papers.

Theoretical investigations, many of which are due to the Rev. Samuel Haughton, F.R.S., respecting overturning, fracturing, and projection, are given by Mr. Mallet in his classical work on the Neapolitan Earthquake[1].

The overturning and rocking of columns has been treated by Messrs. Milne,[2] Gray,[3] Perry[4] and West.[5] The effects produced by earthquakes of known dimensions in causing overturning, fracture, projection, rotation, &c., may be found by reference to the descriptions of earthquakes given in the Transactions of the Seismological Society. Mr. Mallet and other investigators, who worked prior to the establishment of the Seismological Society, used the destructive phenomena of earthquakes to determine the dimensions of the earthquakes. The following experiments show how far the hypothesis then employed can be regarded as correct.

For assistance in carrying out the experiments, the authors' thanks are especially due to Mr. D. Larrieu, representative of Decauville & Co., who put at their disposal the truck and rails on which the experiments were made; Mr. K. Tatsuno, Professor of Architecture, who designed and built the walls and columns; the Authorities of the Imperial College of Engineering, who furnished the workshop and workmen; Mr. Y. Yamagawa, who superintended and furnished the electrical appliances; and finally to their colleagues who from time to time rendered valuable assistance.

The method of conducting the experiments will be understood by reference to Fig. 1 A is a steel-framed truck, with a wooden floor, measuring 3'.6" by 2'.8", the gauge of the rails on which it moved being 20". The back and forth

[1] The Neapolitan Earthquake of 1857, by Robert Mallet, F.R.S., &c. 2 vols.
[2] Trans. Seis. Soc. Vol. III. p. 44—48.
[3] Trans. Seis. Soc. Vol. III. p. 48—49.
[4] Trans. Seis. Soc. Vol. III. p. 103—106.
[5] Trans. Seis. Soc. Vol. VIII. p. 35—36.

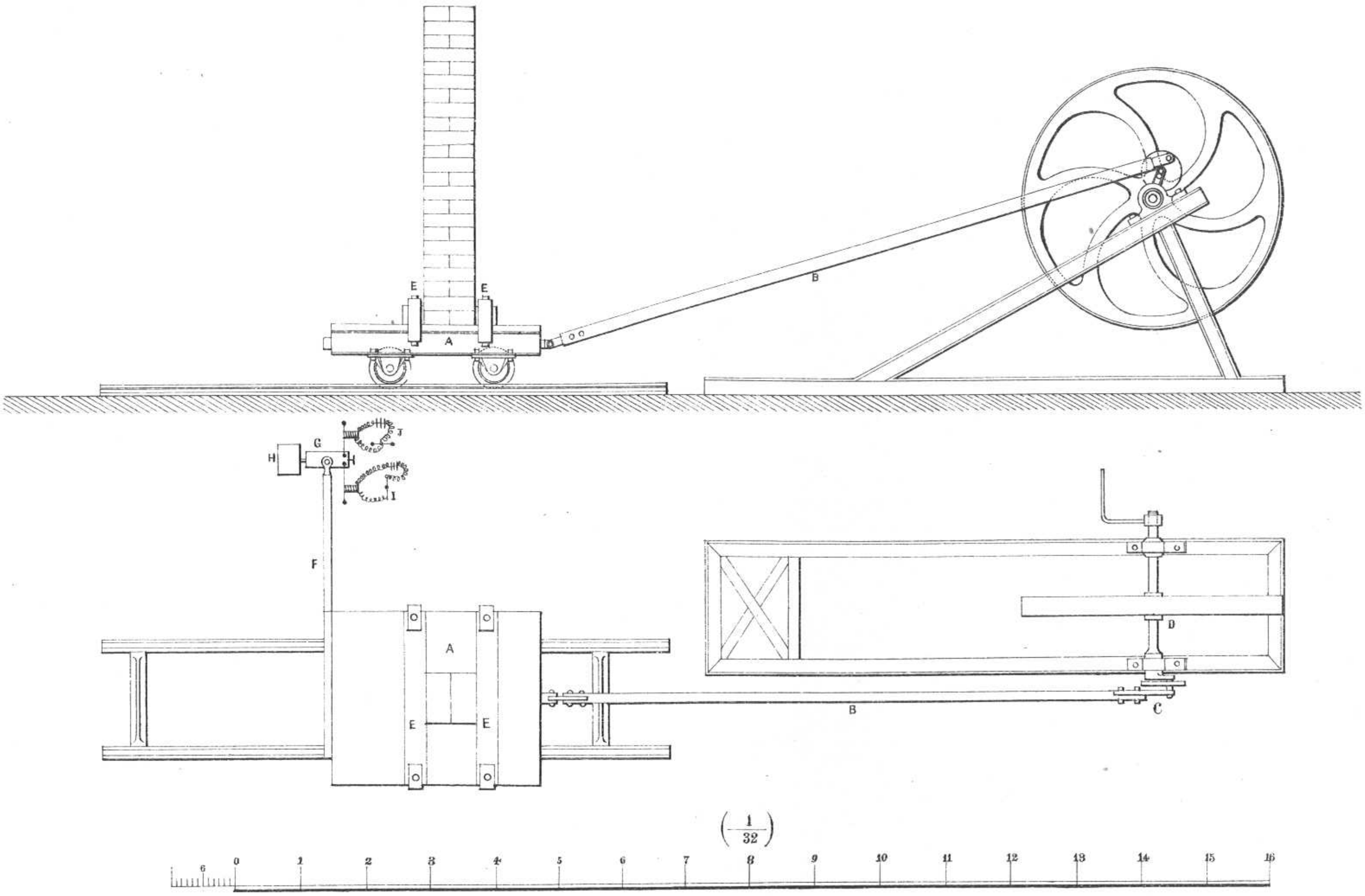
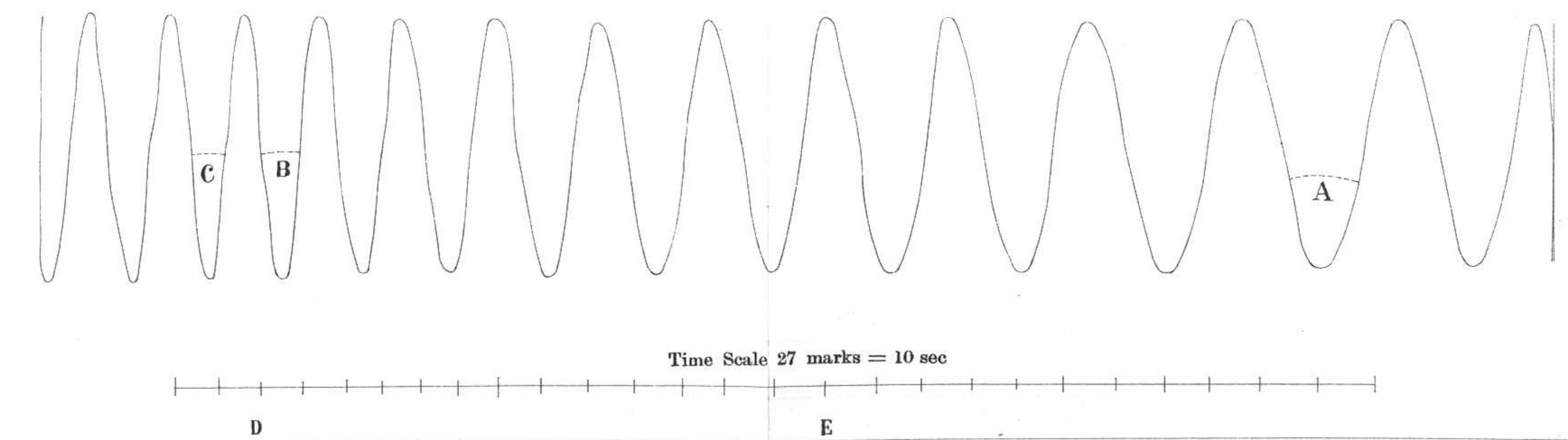


Fig. 1



Overturning of a Brick opposite to mark D $\frac{1}{32}$

Fig. 2

movement of this was given by a connecting rod B, about 10 ft. long connected by the crank C to the shaft D, which was turned by hand, there being a large fly-wheel to insure regularity of motion. Columns or walls to be fractured were fixed upon the truck A by blocks of wood EE which were first brought together horizontally by cramps and then bolted down as shewn in plan and elevation. It will be observed that the range of motion of the truck can be altered by the slot and crank arrangement shown at C. The back and forth motion of the truck A. was recorded by means of a pencil at the end of the arm F. writing on a band of paper passing over the drum driven by the clock H. The speed of the paper was recorded by the arrangement shown at I, consisting of a small pendulum swinging across a pool of mercury and completing the circuit of a battery and an electro magnet. At each completion of the circuit, which occurred at intervals of about $\frac{1}{3}$ second, the electro magnet deflected a lever carrying a pencil resting on the paper carried by G. At J there was a second battery, electro-magnet, lever, and pencil. This circuit was closed by a key at the moment of overturning or fracturing and a mark was made on the paper opposite the particular vibration which was taking place when such result occurred.

Fig 2 represents the diagram obtained when overturning a brick standing freely on its end with its flat side facing the direction of motion. It will be seen that the back and forth motion commenced gently, the wave A being described in 1.4 seconds, this interval being determined by reference to the time scale, 27 ticks on which, corresponding to 27 swings of the pendulum, being described in 10 seconds. When B was described, which by measurement has a period of 0.71 seconds, the brick was overturned, the overturning point being shown by the tick at D in the line DE. The amplitude or half range of motion at B or C is 18.7 millimeters. On the assumption of a simple harmonic motion, calling the period T and the amplitude a, from the formula

$$V = \frac{2\pi a}{T}$$

the maximum velocity may be calculated. Other quantities which follow from the above are $\frac{V^2}{a}$ or maximum acceleration, $V/\frac{T}{4}$ or mean time acceleration.

CALCULATIONS EMPLOYED.

For various reasons, amongst which the following are the principal, it seems impossible to absolutely determine the quantity of motion necessary to overturn a body of given dimensions.

1.—The body may be set in motion and by rocking with a definite period and amplitude when it receives the final impulse which may determine its overthrow.

2.—Bodies like columns standing on end have a period of oscillation varying with the arc through which they rock.

3.—An earthquake seldom if ever consists of a single sudden movement, but of a series of movements which continually vary in amplitude and period.

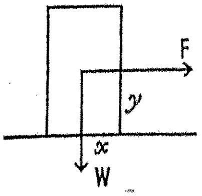
4.—Although an earthquake may consist of a series of movements which are recorded as a series of distinct waves, it often happens that such waves are accompanied by smaller superimposed waves.

The second and third reasons would lead to the conclusion that a body might be overturned by movements of exceedingly small amplitude, provided that the periods of these movements decreased at the same rate that the period of the oscillating body decreased.

Although the effects of earthquakes may be accelerated or retarded by the above mentioned phenomena, the destruction chiefly occurs with the larger movements; therefore by only considering these, although the analysis is imperfect, the results obtained are sufficiently near the truth to carry with them a practical significance.

Overturning.

Our colleague, Prof. C. D. West, treats the subject as follows:



Let M be mass of a column resting on the ground undergoing an acceleration of f feet per sec.

Let y be the height of the centre of gravity of the columns, and x the horizontal distance of the centre of gravity from the edge about which it may turn.

The inertia of the column is equivalent to a force

$$F=Mf$$

The overturning moment

$$Fy=Mfy$$

This is opposed by the moment of the weight or

$$Wx=Mgx=Mfy$$

whence $$f=g\frac{x}{y}$$

If f exceeds this value the column *may* go over, if less it *may* stand.

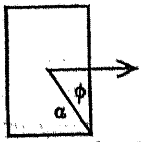
If the column is practically at rest when it receives the acceleration f it ought to fall, but if it is in a state of oscillation its fall may be hastened or retarded.

It will be shown that the quantity $g\frac{x}{y}$ which only depends on the dimensions of a body, often closely agrees with the observed quantity $\frac{V^2}{a}$ or maximum acceleration.

When V and T are small, $g\frac{x}{y}$ is nearer to the observed quantity $\frac{V}{\frac{T}{4}}$ or mean-time acceleration.

Mr. Mallet, by equating the statical work done in raising the centre of gravity of a body up to the edge over which it falls as equal to the kinetic energy of rotation, obtains the formula:—

$$v^2=2g\times\frac{k^2}{a}\times\frac{1-\cos\phi}{\cos^2\phi}$$



when ϕ=the angle formed between the vertical edge of the body and the line joining its centre of gravity and the edge about which it turns

a=the distance of the centre of gravity from that edge,

k=the radius of gyration about the same edge. For solid rectangular solids $k^2 = \frac{4a^2}{3}$.

g=acceleration due to gravity,

v=the velocity when suddenly applied horizontally to the centre of gravity of the body is able to bring it vertically over the edge about which the body rotates.

For a rectangular block from the above,

$$v^2 = 2g \frac{4\,a\,(1 - \cos\phi)}{3\cos^2\phi}$$

$$\therefore \quad v = \frac{2}{\cos\phi}\sqrt{\frac{2\,g\,a}{3}(1 - \cos\phi)}$$

The formula is only true on the assumption of a *sudden* impulse, whose action has practically ceased before the block has begun to move. In this case the value of the impulsive couple or moment of the impulse about the fixed edge is

$$mva\cos\phi$$

where m is the mass of the block.

The impulse, however, which acts on the block evidently depends upon the accumulated acceleration for the half period of motion to which the body is subjected.

Taking simple harmonic motion we have for displacement:—

$$x = a \sin \frac{2\pi}{T} t$$

where a = amplitude and T = period.

The acceleration is then :—

$$x'' = -a \frac{4\pi^2}{T^2} \sin \frac{2\pi}{T} t$$

And the total impulse for half period is

$$I = 2\int_0^{T/4} m\, x''\, dt$$

$$= m \times \frac{4\pi a}{T}$$

$$= m \times \text{twice the maximum velocity.}$$

Overturning Acceleration.

Here we have (1) $f = \frac{x}{y} g$ or (2) $f y = x g$.

If, therefore, we have a series of columns of different heights but of the same width, equation (2) gives a relation between the heights or lengths and the accelerations necessary for overturning. Supposing x constant (2) is represented by a *rectangular hyperbola* with y and f as coördinates. The theoretical curves in Figs. 3 and 4 have been constructed from equation (2) by putting $x = 1$.

If the columns have the same height but different widths, equation (1) may be represented as a *straight line* through the origin with x and f as coördinates.

Bodies Overturned or Fractured.

In the following list the bodies overturned or fractured are enumerated following the order in which the experiments were made. In describing the experiments classifications have been made according to the nature of the experiments.

No. of Experiments.	Nature of Bodies.	Dimensions.	Remarks.
1.	Deal Box. ½" wood.	14½" × 10½" × 23½"	On end. Flat side on. Overturned.
2.	Deal Box. ½" wood.	14½" × 10½" × 23½"	On end. Flat side on. Overturned.
3. 4. 5.	A block of wood.	2½" × 6½" × 1'.11½"	On end. Flat side on. Overturned.
6. 7.	Block of Pear Wood.	11½" × 3½" × 19"	On end. Flat side on. Overturned.
8. 9.	Block of Deal.	2" × 1½" × 18"	On end. Flat side on. Overturned.
10. 11.	A brick.	9" × 4½" × 2½"	On end. Flat side on. Overturned.
12. 13.	A brick.	9" × 4½" × 2½"	On end. Edge on. Overturned.
14.	One brick on edge of another.	2½" × 9" × 9"	On edge. Flat side on. Overturned.

No. of Experiments.	Nature of Bodies.	Dimensions.	Remarks.
15. 16.	Block of Pear Wood.	$11\frac{1}{8}'' \times 3\frac{7}{8}'' \times 19''$	On end. Flat side on. Overturned.
17. 18.	A brick.	$9'' \times 4\frac{1}{8}'' \times 2\frac{1}{4}''$	On end. Flat side on. Overturned.
19.	$12\frac{1}{2}$ bricks as a column, mortar joints.	$4'' \times 4'' \times 32''$	On end. Broke at 2nd joint.
20.	14 bricks as a column, mortar joints.	$4'' \times 8\frac{1}{2}'' \times 3.'1\frac{1}{2}''$	On end. Flat side on. Broke at 3rd joint.
21. 22.	11 bricks as a column.	$4'' \times 8\frac{1}{2}'' \times 2.'5\frac{3}{4}''$	On end. Flat side on. Broke at 2nd joint.
23.	20 bricks as a column.	$4'' \times 8\frac{1}{2}'' \times 4.'4\frac{3}{4}''$	On end. Flat side on. Broke at 2nd joint.
24.	Square brick column 23 courses.	$8\frac{3}{4}'' \times 9'' \times 5.'0\frac{3}{4}''$	On end. Broke at 2nd joint.
25.	Square brick column 20 courses.	$8\frac{3}{4}'' \times 9'' \times 4.'6\frac{1}{2}''$	On end. Broke at 2nd brick.
26.	Square brick column 18 courses.	$8\frac{3}{4}'' \times 9'' \times 4.'1\frac{1}{2}''$	On end. Broke at 2nd brick.
27.	Square brick column 16 courses.	$8\frac{1}{2}'' \times 9'' \times 3.'7\frac{1}{2}''$	On end. Broke at 5th brick—a bad joint.
28.	Square truncated pyramid, 15 courses.	$5\frac{1}{4}'' \times 5\frac{1}{4}''$ at top, $9'' \times 9''$ at bottom $\times 3.'5\frac{3}{4}''$	On end. Broke at 2nd joint.
29.	20 bricks as a column.	$4\frac{1}{4}'' \times 9'' \times 4.'6\frac{3}{4}''$	On end. Edge on. Broke at 2nd joint.
30.	18 bricks as a column.	$4\frac{1}{4}'' \times 9'' \times 4.'1\frac{1}{2}''$	On end. Edge on. Broke at 4th joint.
31. 32.	14 bricks as a column.	$4\frac{1}{4} \times 9'' \times 3.'2\frac{1}{2}''$	On end. Flat side on. Broke at 4th joint.
33.	Square brick column 20 courses.	$9'' \times 9'' \times 4.'6\frac{1}{2}''$	On end. Broke at 3rd joint.
34.	Square brick column 17 courses.	$9'' \times 9'' \times 3.'10\frac{3}{4}''$	On end. Broke at 2nd joint.
35.	Column of cement.	$2'' \times 2'' \times 2.'1''$	On end. Did not break.
36.	Column of cement with an iron cape of $3\frac{1}{4}$lbs.	$2'' \times 2'' \times 2.'1''$	On end. Did not break.
37.	Same as above with a cap of 11lbs.	$2'' \times 2'' \times 2.'1''$	On end. Did not break.
38.	A small brick pyramid.	$1\frac{1}{4}'' \times 1\frac{1}{4}''$ on top, $3\frac{1}{4}'' \times 3\frac{1}{4}''$ bottom $\times 1.'6''$	On end. Broke at 5th joint, which was bad.

No. of Experiments	Nature of Bodies.	Dimensions.	Remarks.
39.	Concrete column. 1 part cement, 6 fine gravel.	$2'' \times 2'' \times 22''$	On end. Did not break.
40.	A block of wood.	$3\frac{3}{4}'' \times 3\frac{3}{4}'' \times 12\frac{1}{8}''$	On end. Overturned.
41.	A block of wood.	$3\frac{3}{4}'' \times 4\frac{1}{4}'' \times 18\frac{3}{4}''$	On end. Flatside on. Overturned.
42.	Column of Cement. 1 part cement, 6 fine gravel.	$2'' \times 2'' \times 20''$	
43.	A brick.	$2\frac{1}{4}'' \times 4\frac{1}{4} \times 8\frac{1}{8}''$	On end. Fladside on. Overturned.
44.	Column of cement. 1 part cement, 6 fine gravel.	$2'' \times 2'' \times 18''$	On end. Broke at end of motion by jumping of truck.
45.	A brick supported on one side.	$2\frac{1}{4}'' \times 4\frac{1}{4}'' \times 8\frac{1}{8}''$	On end. Flatside on. Overturned.
46.	A brick.	$2\frac{1}{4}'' \times 4\frac{1}{4}'' \times 8\frac{1}{8}''$	On end. Flatside on. Overturned.
47.	A brick.	$2\frac{1}{4}'' \times 4\frac{1}{4}'' \times 8\frac{1}{8}''$	On end. Flatside on. Overturned.
48.	Column of cement. Fine 1 part cement, 6 gravel.	$2'' \times 2'' \times 1.'6''$	Connecting rod broke, car jumped and column broke.
49.	A block of wood.	$3\frac{3}{4}'' \times 3\frac{3}{4}'' \times 12\frac{1}{8}''$	On end. Overturned.
50.	A block of wood.	$3\frac{3}{4}'' \times 4\frac{1}{4}'' \times 18\frac{3}{4}''$	On end. Overturned.
51.	Column of cement. 1 part cement, 6 fine gravel.	$4'' \times 4'' \times 5.'0''$	On end. Broke 10″ from base.
52.	Column of cement. 1 part cement, 6 fine gravel.	$4'' \times 4'' \times 4.'10''$	On end. Broke 16″ from base.
53.	Column of cement. 1 part cement, 6 fine gravel.	$4'' \times 4'' \times 2'$	Did not break.

In addition to the above, there were 9 square columns, each 1 *sun* square and from 2 to 10 *sun* in length; also a series of cylindrical columns of similar lengths, but 1 *sun* in diameter (1 *sun*=30.3mm.) Each of these, which were made of deal, were overturned several times. The brick columns had mortar joints and were from 25 to 30 days old. The square columns were composed of courses of headers and stretchers. The concrete and cement columns were also from 25 to 30 days old.

Overturning of the Nine Small Square Columns.

In the following table the dimensions of the columns are given in Japanese *sun* (1 *sun* = 30.3 mm.) The amplitude or half range of motion a is given in millimeters. The period T is given in seconds. The maximum velocity V is calculated from the formula $V = \frac{2 \pi a}{T}$. Twice the maximum velocity or 2 V, ought theoretically to show some relation to Mallet's v.

The mean time acceleration is $V/\frac{T}{4}$. The maximum acceleration is V^2/a, quantities that may be compared with the quantities in the column headed West's $f = g\frac{x}{y}$.

Dimension.			Quantities Observed.					Quantities Calculated.	
No.	Cross Section.	Height.	a.	T.	V.	$V/\frac{T}{4}$.	V^2/a.	West's f.	Mallet's v.
9	1	10	25	1.4	113	322	506	980	141
—	—	—	26	1.6	102	256	398	—	—
—	—	—	10	1.0	101	400	632	—	—
—	—	—	38.5	1.6	156	400	628	—	—
—	—	—	38	1.6	147	360	565	—	—
			Average		124	348	546		
8	1	9	26	.98	168	688	1080	1090	150
—	—	—	26.5	1.2	138	460	720	—	—
—	—	—	16	.94	107	452	710	—	—
—	—	—	39	1.2	203	670	1050	—	—
			Average		154	568	890		
7	1	8	26	.98	168	688	1080	1230	158
—	—	—	27	.93	183	790	1250	—	—
—	—	—	16	.74	136	730	1150	—	—
—	—	—	39.5	.99	252	1020	1610	—	—
—	—	—	39	1.0	246	988	1550	—	—
			Average		197	843	1410		
6	1	7	27	.85	200	932	1470	1400	170
—	—	—	27	.86	197	920	1440	—	—
—	—	—	39	1.07	228	850	1340	—	—
			Average		208	900	1420		
5	1	6	40	1.07	234	880	1370	1640	184
—	—	—	39.5	1.03	241	936	1470	—	—
			Average		238	908	1420	—	...
4	1	5	Records lost					1960	202

3	1	4	40.7	.85	300	1410	2220	2450	228
—	—	—	40.5	.91	280	1230	1940	—	—
—	—	—	40.5	.75	340	1820	2870	—	—
			Average		307	1490	2340		
2	1	3	41	.74	348	1880	2950	3270	267
—	—	—	41.5	.58	446	3050	4800	—	—
—	—	—	41.5	.77	339	1760	2770	—	—
			Average		378	2230	3510		
1	1	2	—	—	—	—	—	4900	342

Overturning of the Nine Small Cylindrical Columns.

Dimensions.				Quantities Observed.				Quantities Calculated.
No.	Cross Section.	Height.	a.	T.	V.	$V/\frac{T}{4}$	V^2/a	$f = g.\frac{x}{y}$
9	1	10	39	1.08	227	840	1320	980
—	—	—	39	1.27	194	610	960	—
			Average			213	730	1140
8	1	9	40	1.22	206	670	1060	1090
—	—	—	40	1.22	206	670	1060	—
			Average			206	670	1060
7	1	8	39.5	1.27	196	620	974	1230
6	1	7	39.5	1.24	200	650	1020	1400
5	1	6	40.5	.78	326	1680	2630	1640
—	—	—	39.5	1.27	196	620	972	—
			Average			261	1150	1800
4	1	5	41	1.12	230	820	1280	1960
—	—	—	41	.97	265	1090	1720	—
			Average			248	955	1500
3	1	4	41	.63	410	2600	4080	2450
—	—	—	41	.91	283	1240	1960	—
			Average			347	1920	3020
2	1	3	42	.53	497	3750	5890	3270
—	—	—	41	.77	333	1720	2700	—
			Average			415	2735	4300

From the above tables we see that the calculated quantity called "Mallet's v" has no relationship with the observed quantity 2 V.

The quantity called "West's f" or moment of overturning, is, however, closely related to the observed quantity V^2/a, especially for the shorter square columns. From theoretical considerations it might have been expected that the columns with relatively the smaller bases, that is the long columns, should have given results more nearly in agreement with West's f. The principal causestending to vitiate the experiments were :—

1. The bases of the columns may not have been absolutely flat and not accurately cut at right angles to the length of the columns.

2. The truck on which the experiments were made being designed to carry heavy weights, it had neither the smooth surface nor the even motion necessary for experimenting on columns so smooth as those which were employed.

That errors due to causes like these must have entered into the result may be inferred from the order in which the columns fell when standing together on the truck which was moving back and forth with increasing rapidity.

For cylindrical columns, one order was :—

7, 9, 6, 8, 5, 4, 3, 2.

For square columns, one order was :—

9 8 6 7 5 4 3 2

Notwithstanding the roughness of the experiments the tables show that the theoretically determined quantity called West's f, which depends upon the dimensions of a column, is closely connected with the maximum acceleration it experiences at the time it is overturned by a back and forth simple harmonic motion.

The same results as those given in the tables are shown graphically in the accompanying diagram.

The above results may be compared with experiments performed some years ago by one of the present authors in the

Physical Laboratory of the Imperial College of Engineering in Tokio. The details of these experiments may be found in Trans: Seis: Soc. Vol. VIII. p. 74. The results which where briefly as follows, are also represented graphically. The dimensions of the columns and amplitude of motion are given in inches, the remaining quantities being in feet.

No.	Diameter	Height	a	$V/\frac{T}{4}$	V^2/a	$f=g\frac{x}{y}$
1	1	8	1.13	4.61	7.24	4.01
2	1	6	1.38	5.64	8.85	5.36
3	1	4	1.50	6.15	9.65	8.05
4	1	3	2.25	9.23	14.49	10.7
5	1	2	2.75	11.29	17.73	16.1

OVERTURNING OF RECTANGULAR PARALLELOPIPEDS.

	Quantities Observed.					Quantities Calculated.	West's.	Mallet's.	
No.	A	T	V	$V/\frac{T}{4}$	$V/\frac{a}{4}$	2V	$f=g\frac{x}{y}$	v	
50	18.5	.60	193	1280	2000	386	1950	382	Block like 41
49	19	.61	194	1270	1990	388	2940	330	Block like 40
47	19.5	.64	190	1190	1870	380	2600	320	A brick.
46	18.7	.71	165	924	1450	330	2600	320	A brick.
43	49	1.1	260	900	1420	520	2600	320	A brick.
41	49	1.1	284	1050	1650	568	1950	382	Block like 30
40	50	.93	340	1470	2300	680	2940	326	Block like 40

For experiments 50, 49, 47, 46 where the amplitude is small and the period short, the quantities 2V and Mallet's v are fairly comparable,

In 49 and 40, 50 and 41, 47 and 43, V varies considerably, but $V^2/_a$ does not vary to so great an extent. $V^2/_a$ is less than f the difference being sometimes as much as 30 per cent.

OVERTURNING OF RECTANGULAR PARALLELOPIPEDS.

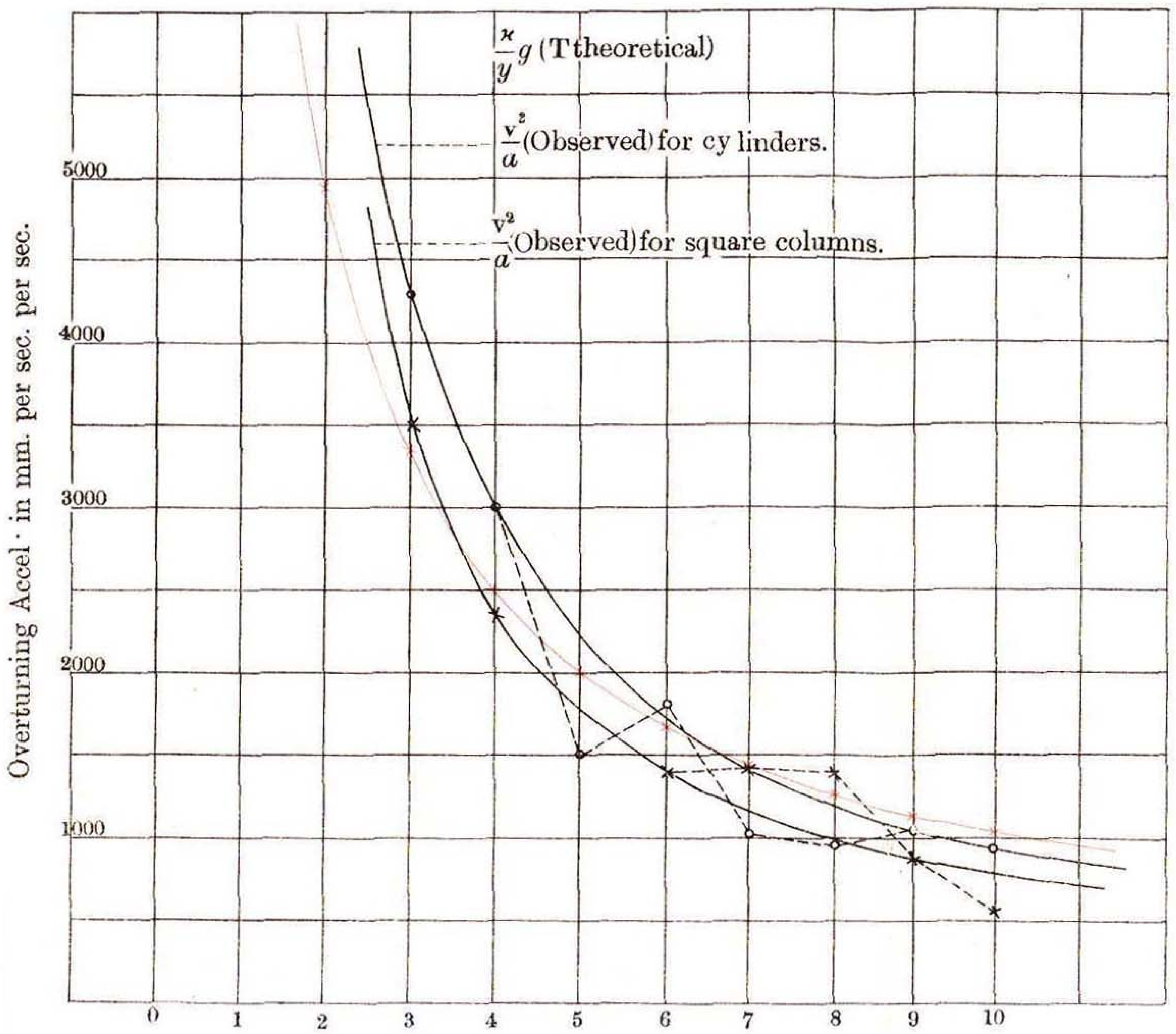
In the following tables c is half the width of a column in inches measured in the direction of the motion, and a is half the height, also in inches.

DIMENSIONS			QUANTITIES OBSERVED.						QUANTITIES CALCULATED.		
No.	c.	d.	a.	T.	V.	$V/\frac{T}{4}$	V^2/a	2V.	West's f.	Mallet's v.	
1	5.3	11.8	81	.78	650	3320	5210	1300	4400	960	Deal Box.
3	1.3	11.8	73	2.6	230	460	722	460	1080	220	Block of wood.
4	1.3	11.8	73	1.9	241	504	790	482	—	—	
5	1.3	11.8	73	1.8	253	552	863	506	—	—	
6	2.0	9.5	74	1.3	350	1060	1670	700	2070	390	Block of wood.
7	2.0	9.5	74	1.3	370	1180	1850	740	—	—	
8	.94	9	74	1.7	274	640	1010	548	1090	182	Block of wood.
9	.94	9	73	2.0	231	460	720	462	—	—	
10	1.1	4.5	74	1.7	269	630	993	538	2400	320	A brick.
11	1.1	4.5	74	1.7	281	680	1070	562	—	—	
12	2.3	4.5	76	1.3	358	1100	1730	716	4900	680	A brick.
13	2.3	4.5	77	.78	620	3180	4990	1240	—	—	
14	1.1	4.5	73.5	1.7	279	680	1070	558	2400	320	Two bricks.
17	1.1	4.5	39	1.0	244	980	1530	488	2400	320	A brick like 10 and 11.
18	1.1	4.5	39	.85	289	1390	2100	578	—	—	
15	2.0	9.5	39	.93	265	1150	1800	530	2070	390	Block of wood like 6 and 7.
19	2.0	9.5	39	1.0	244	980	1530	488	—	—	

From the above table is will be seen that the observed quantity V^2/a or maximum acceleration in many instances is comparable with the calculated quantity f, the closest approximations to equality being, when the period of motion or T is small, and the greatest divergence when T is large. The difference between V^2/a and f is usually such that $V^2/a > f$. Where the period T is two seconds, which is a quantity to be expected in large earthquakes, f may be 30 per cent greater than the maximum acceleration at the time of overturning.

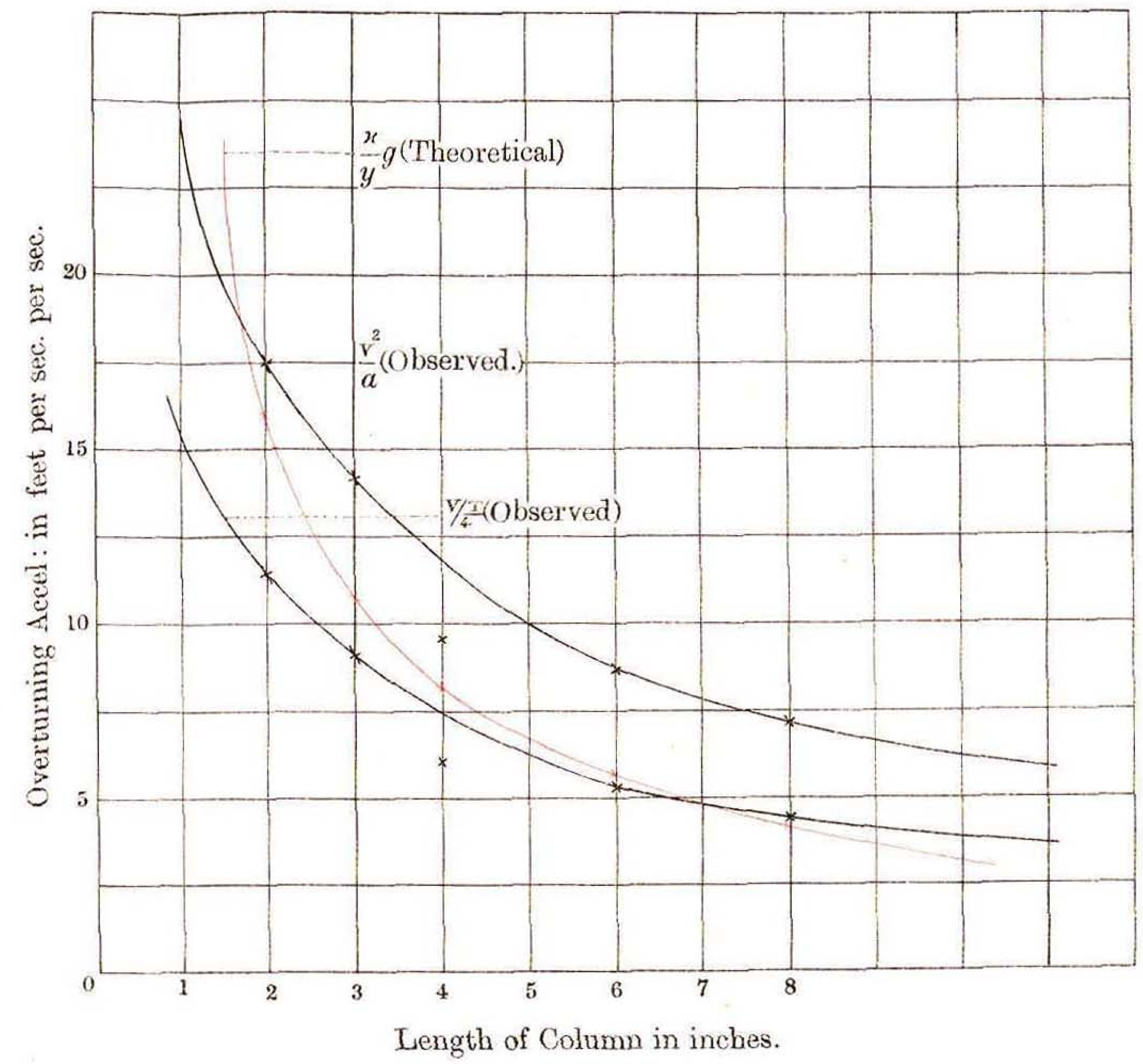
Overturning of Columns where the Ratio of Base to Height is Constant.

The absolute dimensions of columns may possibly have effect on the values of the acceleration necessary for overturn-



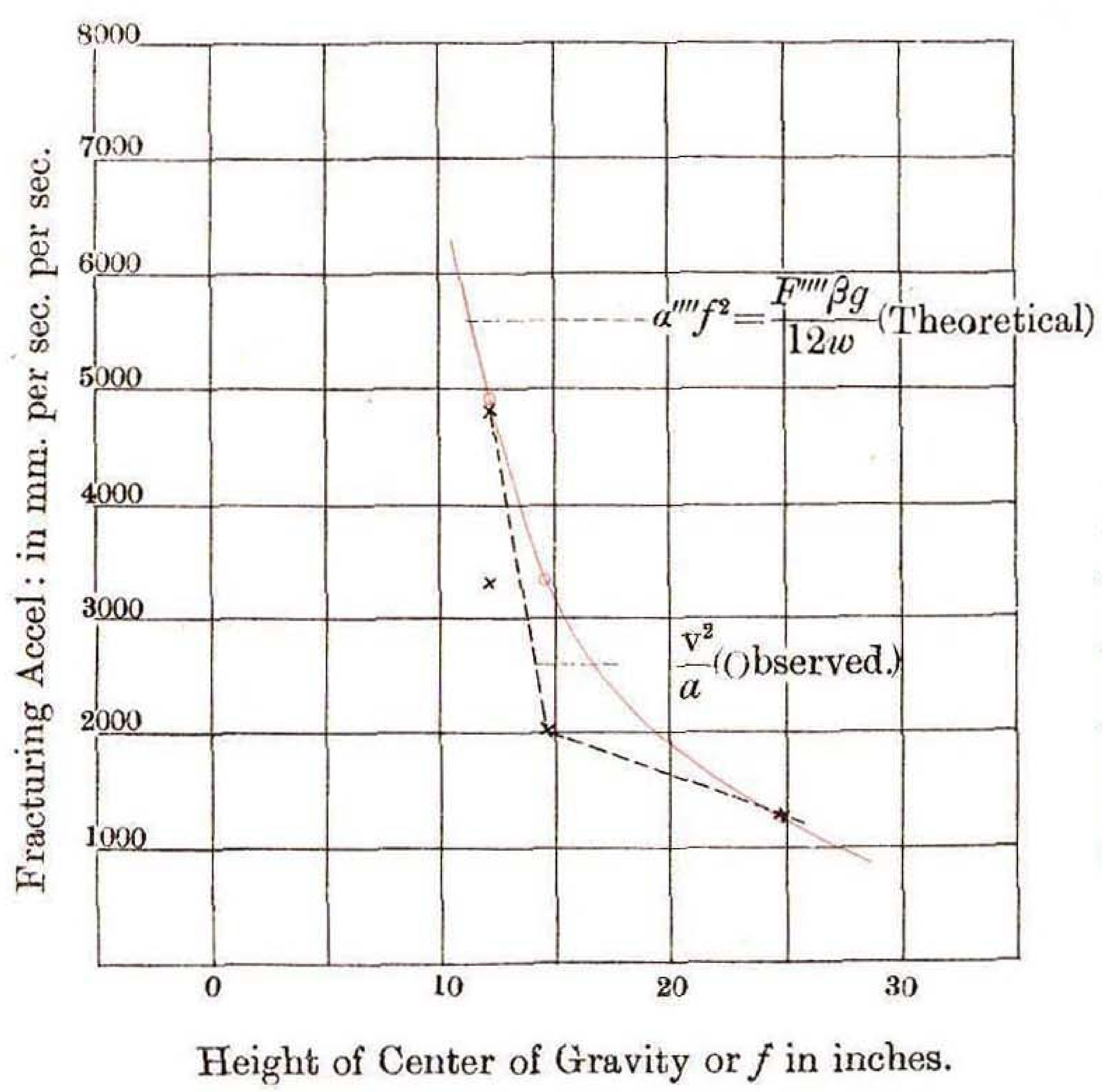
Overturning of Columns.

Fig. 3.



Overturning of Cylindrical Columns.

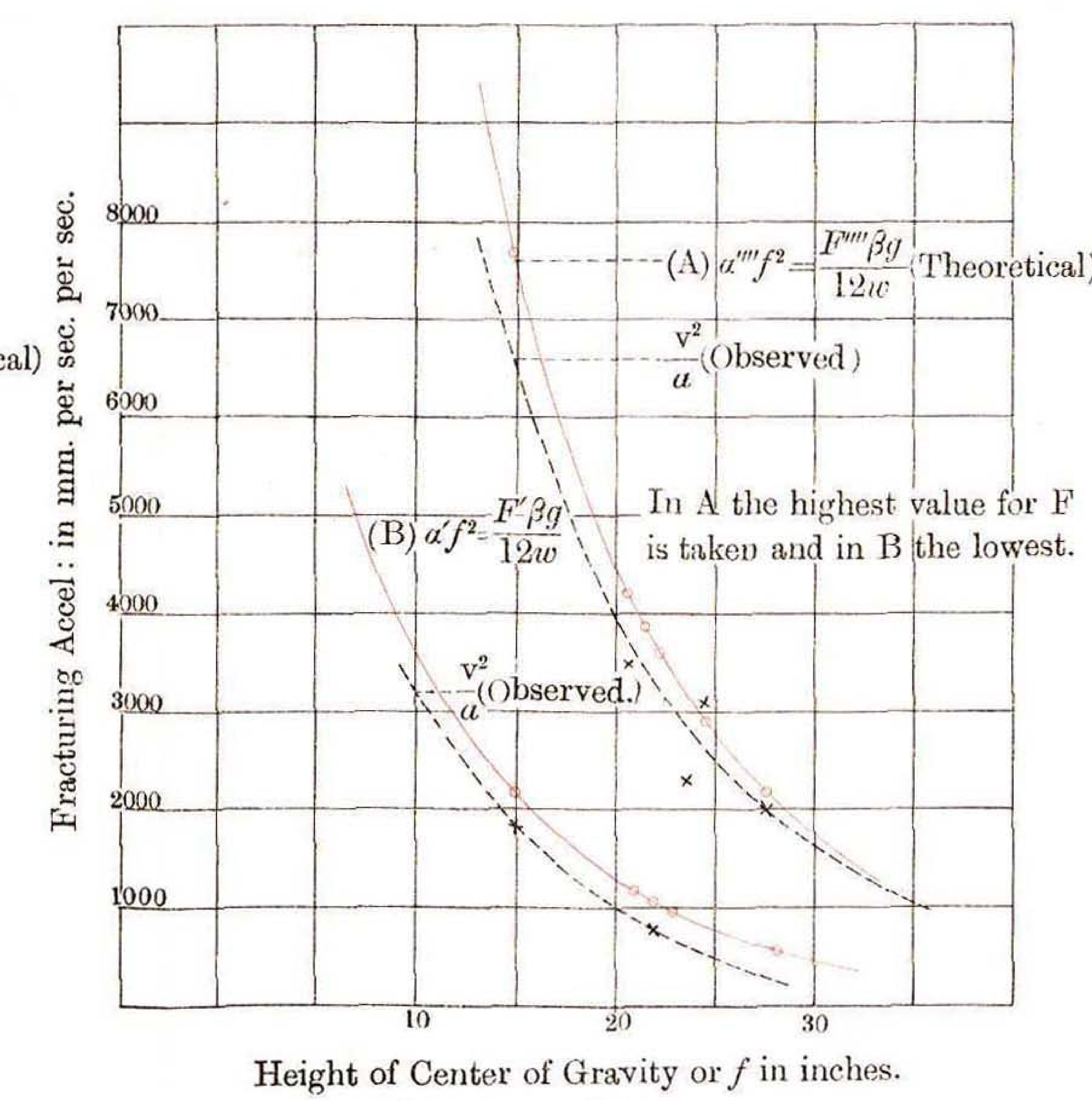
Fig. 4



Fracturing Columns.

Single layers of brick (Nos. 20, 21, 22, 23)

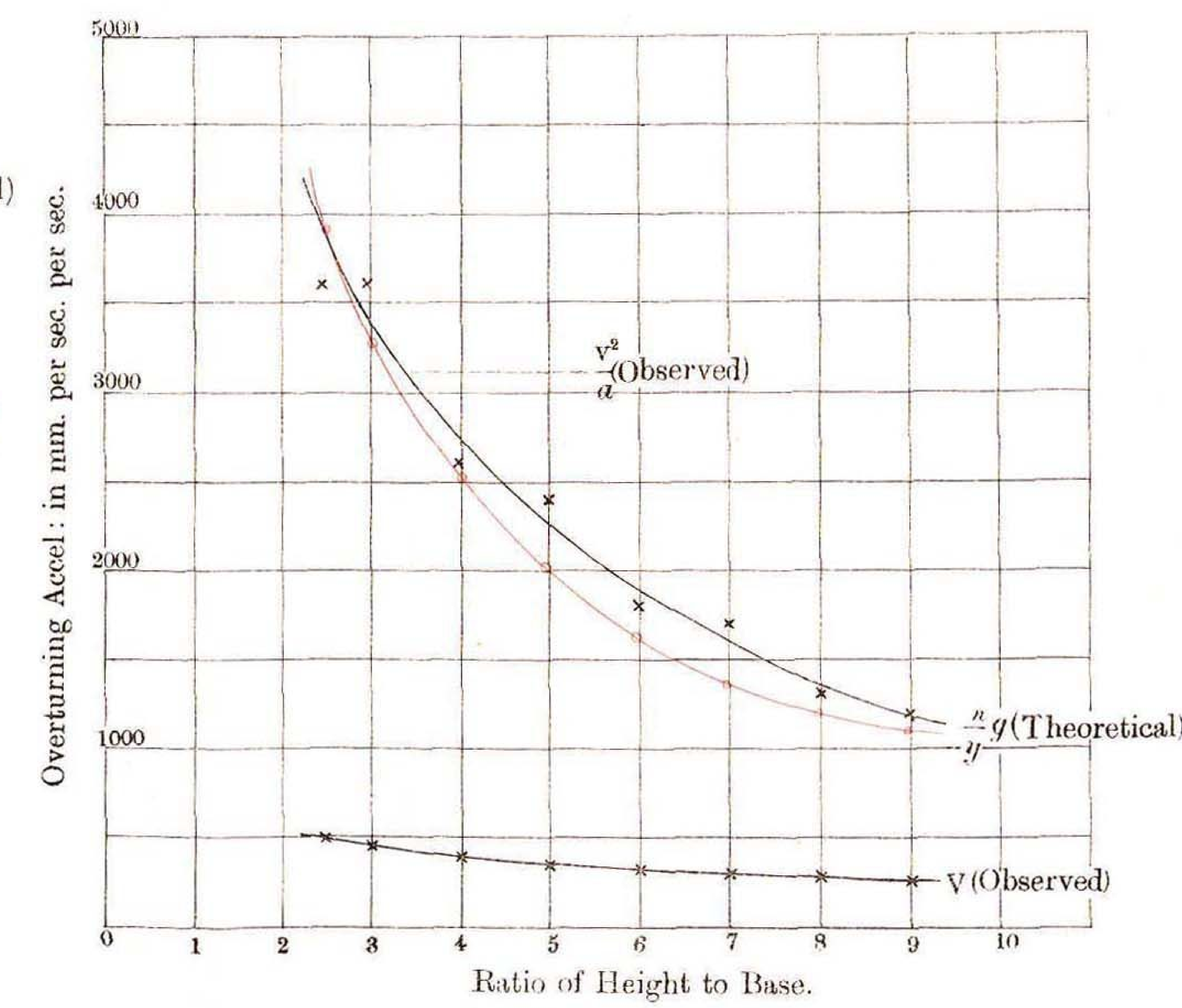
Fig. 5.



Fracturing Columns.

Square Columns (Nos. 24, 25, 26, 27, 33, 34)

Fig. 6.



Overturning of Columns.

Fig. 7.

ing them. To test this for ordinary columns, such as tombstones, we made the following eight sets of square wooden columns, in each of which the ratio of the base and height was the same, but the absolute dimensions were different:—

BASE. in. sq.	HEIGHT. Sun.	BASE. in. sq.	HEIGHT. Sun.
1	9	4	16
2	18	5	20
1	8	6	24
2	16	1	3
3	24	2	6
1	7	3	9
2	14	4	12
3	21	5	15
1	6	6	18
2	12	7	21
3	18	8	24
4	24	1	2.5
1	5	2	5
2	10	3	7.5
3	15	4	10
4	20	5	12.5
5	25	6	15
1	4	7	17.5
2	8	8	20
3	12	9	22.5

The blocks of the same group were put together on the truck, which was moved forward and backward either suddenly or else it was gradually worked into quick motion. It was found that almost always these blocks were practically thrown down at the same moment, so that we should think their absolute dimensions may be left out of consideration in the first approximation. From the following results, made for two different amplitudes, it will be seen that the acceleration necessary for overturning as actually observed, are generally somewhat greater than those calculated by the formula $f=\frac{x}{y}g$. The results are graphically represented in Fig. 7, in which (f) is the curve given by the latter formula, while (A) and (V) are those obtained from the average values of the maximum accelerations and velocities observed.

Overturning of Square Columns.

Number of Experiments.	Ampl. mm.	Period. sec.	Max. Vel. mm. per sec.	Max. Acc. mm. per sec.	Common ratio of base and height of blocks	$f=\frac{x}{y}g$ mm. per sec.
37	62	0.8	490	3800	1:2½	3900
38	62	0.85	460	3400	—	—
39	62	0.81	480	3700	—	—
Average			480	3600		
32	43	0.79	340	2700	1:3	3300
33	61	0.86	450	3300	—	—
34	61	0.71	540	4800	—	—
35	62	0.84	460	3500	—	—
Average			450	3600		
30	43	0.98	280	1800	1:4	2500
31	43	0.85	320	2300	—	—
1	68	1.00	430	2700	—	—
3	69	0.89	480	3400	—	—
Average			380	2600		
28	43	0.84	320	2400	1:5	2000
29	43	0.97	280	1800	—	—
5	68	0.94	450	3000	—	—
8	68	1.00	430	2700	—	—
10	68	1.20	370	2000	—	—
Average			370	2400		
26	43	1.00	270	1700	1:6	1600
27	43	0.95	290	1900	—	—
11	68	1.40	300	1400	—	—
13	68	1.10	380	2100	—	—
Average			310	1800		
24	43	1.10	250	1500	1:7	1400
25	43	0.95	280	1900	—	—
14	68	1.40	310	1400	—	—
15	68	1.20	350	1800	—	—
Average			300	1700		
22	43	1.20	230	1200	1:8	1200
23	43	1.10	240	1400	—	—
16	68	1.5	290	1200	—	—
17	68	1.4	300	1300	—	—
Average			270	1300		

20	43	1.3	210	1000	1:9	1100
21	42	1.1	230	1300	—	—
18	67	1.5	290	1200	—	—
19	67	1.5	280	1200	—	—
Average			250	1200		

Overturning of Brick Columns after Fracture.

After a column by the back and forth motion had been fractured it usually remained standing. By increasing the rapidity of motion this could be overturned, and it is to the overturning of these fractured portions to which these experiments refer.

Dimensions.			Quantities observed.						Quantities calculated.		
No.	c.	d.	a.	T.	V.	$V^{1}/_{4}$	$V^{2}/_{a}$	2V	Wests. I.	Mallets. v.	
23	2	24.5	12.6	.65	122	750	1180	244	797	232	Broke at 2nd joint.
24	4.3	27.7	27.3	.84	200	960	1520	400	1520	473	Broke at 2nd joint.
19	2	13.4	—	—	—	—	—	—	1460	315	Broke at 2nd joint.
20	2	14.6	—	—	—	—	—	—	1340	308	Broke at 3rd joint.
21	2	12.2	20.8	.5	260	2090	3290	520	1610	348	Broke at 2nd joint.
25	4.4	24.8	30	.69	273	1580	2480	546	1750	405	Broke at 2nd joint.
33	4.4	23.3	51	.93	343	1470	2310	686	1850	422	—
26	4.4	22.1	—	—	—	—	—	—	1950	546	Broke at 2nd joint.
27	4.4	15	30	.70	272	1550	2440	544	2880	674	Broke at 5th joint.
29	4.4	24.7	26	.58	280	1920	3020	560	1750	405	Broke at 2nd joint.
30	4.4	19.3	26	.55	298	2170	3410	596	2240	586	Broke at 4th joint.
32	2.1	13.9	—	—	—	—	—	—	1480	327	Broke at 4th joint.
34	4.5	20.7	52.5	.8	413	2070	3260	826	2130	578	Broke at 2nd joint.

In the above table the period or T is short, and Mallet's v, is in several instances fairly comparable with 2V. In Experiment 24, the maximum acceleration or V/a appears to be identica with West's f. ; but in the remainder there is usually a wide divergence, the latter quantity being usually the smaller.

Fracturing.

For a wall or column like body, assuming that the condition for fracture is that the overturning moment shall be equal to the moment of cohesion of the fractured surface at the base, Mallet deduces the following formula :—

$$V = g\,\frac{F_0 A}{W} \times \frac{k^2}{f\beta}$$

(see. "The Neapolitan Earthquake Vol. I p 141.)

Where

V = velocity of wave path.

f = distance of centre of gravity of portion broken off from the fractured base.

F_0 = force of cohesion, or force upon unit surface which when *suddenly* applied produces fracture.

k = radius of gyration of plane of fracture about its edge.

β = thickness of the column.

W = weight of portion broken off.

g = acceleration due to gravity.

Putting $k^2 = \frac{\beta^2}{3}$ the above formula becomes :—

$$V = g\,\frac{F.\,A.\,\beta.}{3 f W}$$

Now if F is the force of cohesion or force upon unit surface which, when *gradually* applied, is sufficient to produce fracture, F being double F_0, the acceleration to produce fracture or α may be writen :—

$$\alpha = \frac{1}{6}\,\frac{g\,F\,A\,\beta}{f w} \text{ (1)} \qquad (1)$$

(1). The relation between this formula and the one employed by Mallet will be seen from the following consideration. Let the column to be fractured be regarded as a beam which is bent by its own inertia, or the impressed force $m\,\alpha$. The bending moment, or M, is therefore equal to $m\,\alpha\,f$ where f as before is equal to the height of the centre of gravity above the fractured face.

If p be the longitudinal stress at a point distant y from the neutral surface

$$p = \frac{M}{I}\,y$$

where I is the moment of inertia of the rectangular cross section with respect to the line in the neutral surface and is

$$= \frac{\beta^2}{3.4}\,A$$

If w be the weight of the unit volume of brick so that $W = 2 f A w$, then from (1)

$$\alpha = \frac{F \beta g}{12 w f^2} \qquad (2)$$

$$\text{or } \alpha f^2 = \frac{F \beta g}{12 w} \qquad (3)$$

Hence, if we have a series of rectangular columns of the same width, equation (2) gives a relation between the heights

---

Farther the greatest value of y in the rectangular columns experimented upon is

$$\pm \frac{\beta}{2}$$

Substituting the values for y I and M we obtain a maximum value for p, or a quantity corresponding to the co-efficient of cohesion F

$$\text{whence} \qquad F = \pm \frac{6 m \alpha f}{A \beta}$$

$$\therefore \qquad \alpha = \frac{A \beta F}{6 m f} = \frac{A \beta F g}{6 w f}$$

In Mallet's formula the radius of gyration is with respect to the edge of the plane of fracture, whilst in determining α, the radius of gyration is with regard to the line in the neutral surface

(2). The relationship between this formula and the formula usually employed when discussing the transverse strength of a beam supported at its ends and loaded at its centre, will be seen from the following consideration.

If l=length of beam $= 2f$

β=depth of beam in inches.

A=area of cross section of beam in sq. inches.

k=the weight required to fracture a beam 1 foot long and 1 in. sq. supported at each end and loaded in the centre.

$m\alpha_0$=product of mass and acceleration causing breaking.

Since $\frac{k}{2}$ may be taken as the breaking co-efficient for the same beam fixed at one end and the weight uniformly distributed

the transverse breaking force

$$m\alpha_0 = \frac{A \beta k}{2 l}$$

$$\therefore \quad \alpha_0 = \frac{A \beta k}{2 l m} = \frac{A \beta k g}{2 l w} = \frac{A \beta k g}{4 f w}$$

Comparing this with the above formula $\alpha = \frac{1}{6} \frac{g y A \beta}{f w}$ and if we can assume $\alpha = \alpha_0$,

$$\text{then } k = \tfrac{2}{3} f$$

α, however, is the acceleration which is just sufficient to produce fracture of the beam where the stress is greatest, namely, at the concave or convex side of the beam, while α_0 is the acceleration necessary for complete cross breaking. For mortar joints α and α_0 may have values near each other, because when a part of such a point begins to break the entire joint may simultaneously give way.

The formula $\alpha = \frac{1}{6} \frac{g F A \beta}{f w}$ used in our calculations where f is the longitudinal strength may perhaps be admissable.

of the columns (*i.e.* the heights of the portions broken off) and the accelerations necessary for fracture. Supposing β to be constant, equation (2) is represented by a cubic curve between α and f as co-ordinates. The curve is symmetrical about the axes of α (α being always positive), and it has the co-ordinate axes for asymptotes.

The theoretical curves in Figs 5 and 6 have been traced from (3) by taking proper values for F.

If the columns have the same height but different widths, then (2) gives a relation between their widths and the accelerations necessary for fracture; now if f is constant, equation (2) represents a straight line between α and β as co-ordinates. *The fracturing acceleration is therefore simply proportional to the width or depth of the column.* The strength of a beam is, however, proportional to the square of its depth. The reason for this difference in the cases under consideration, and probably in actual earthquakes, is that the fracturing force is assumed to be the acceleration impressed throughout the mass of the column, and not a totally external force.

Pulling Stress.

1.—To separate 2 bricks united on flat faces = 125 lbs. = 3.67 lbs. per sq. in.

2.—To separate 2 bricks united on flat faces = 156.8 lbs. = 4.6 lbs. per sq. in.

3.—To separate 2 half bricks = 126 lbs. = 7.88 lbs. per sq. in.

4.—To separate 2 half bricks = 126 lbs. = 7.88 lbs. per sq. in.

5.—To separate 2 bricks united on flat faces = 378 lbs. = 11.1 lbs. per sq. in.

6.—To separate 4 bricks or two headers from two stretchers = 106.4 lbs. = 14.8 lbs. per sq. in.

The average for 1 and 2	= 4.1 lbs. = F'
The average for 3, 4 and 5	= 8.95 lbs. = F''
The average for 1, 2, 3, 4, and 5	= 7.03 lbs. = F'''
While 6	= 14.8 lbs. = F''''

α has been calculated for F', F'', F''' and F'''' giving values denoted by α', α'', α''' and α''''

For columns broken at a bad joint α' may apply, while for those which did not break easily, as for example No. 32, α'''' may apply.

Breaking of Brick Pyramids.

No.	β	A	g	F′	F″	F‴	F⁗	f	W	α'	α''	α'''	α''''
28	8.6	73.9	384	4.1	8.95	7.03	14.8	15.4	111	2490	5440	4270	8990

In the above, β, g and f are measured in inches.

A is measured in square inches.

F' &c. is measured in lbs. per square inch

α' &c. is measured in millimeters per sec. per sec.

By observation, fracture occurred when

$$a = 40 \text{ and } T = 0.52$$

whence $V = 480$, $V/\frac{T}{4} = 3700$ and $V^2/a = 5810$ a quantity approximating to α''

No. 38. A small brick pyramid which readily broke at a bad joint and was therefore not calculated.

Cement Columns.

No. 35. A cement column.

Here $\beta = 2$ in., $A = 4$ sq. in., $f = 10.6$ in., $W = 6.2$ lbs., $F = 400$; whence $\alpha = 79300$ mm. per sec. per sec., the theoretical value of acceleration to cause fracture at the base.

No. 36. The same column as No. 35, but with an iron cylinder of weight $W' = 3\frac{1}{4}$ lbs. as a cap.

$$\text{Here} \qquad \alpha = \frac{F\,A\,\beta\,g}{6f\,(W + 2\,W')}$$

$$= 38600 \text{ mm. per sec. per sec.}$$

No. 37. The same column as Nos. 35 and 36 but with a cap of 11 lbs.

Here $\alpha = 16000$ mm. per sec. per sec. is necessary to cause fracture.

In neither of the above three cases was the column broken.

RECTANGULAR BRICK COLUMNS.

No.	a	T	V	$V/\frac{T}{4}$	V^2/a	α'	α''	α'''	α''''	A	f	B
20	46	.96	305	1280	2000	980	2000	1570	3310	34	14.6	4
19	46	.86	340	1600	2500	1030	2240	1760	3700	16	13.4	4
21	20.8	.5	260	2090	3290	1340	2930	2300	4840	34	12.2	4
22	40	.57	440	3080	4800	1340	2930	2300	4840	34	12.2	4
23	12.6	.61	130	850	1340	330	730	570	1200	34	24.5	4
24	27.3	.73	240	1320	2020	570	1240	970	2050	72.3	27.7	8.5
25	30	.62	305	1980	3110	788	1720	1350	2850	78.8	24.8	8.75
33	52	.93	350	1500	2350	888	1940	1520	3200	78.8	23.3	8.75
26	30	1.2	155	510	795	985	2150	1690	3560	78.8	22.1	8.75
27	30	.79	238	1200	1880	2130	4640	3650	7680	78.8	15	8.75
29	27	.58	291	2000	3020	791	1730	1360	2860	37.2	24.7	8.75
30	27	.52	326	2500	3920	1390	3030	2330	4910	37.2	19.3	8.75
32	59	.54	691	5120	8080	1250	2730	2150	4530	38.3	13.9	4.25
34	56	.8	440	2210	3470	1160	2540	2000	4210	80	20.7	9

The indistinctness of the record for No. 26 makes the observed quantities a and T uncertain.

Generally V^2/a and α'''' are fairly comparable.

In 26 and 27 where V^2/a is more nearly equal to α', it may be that fracture occurred at a bad joint. No. 27 certainly showed a bad joint.

The average value of V^2/a for 24, 33 and 29 is 2460, which is about double the value for 23, which is 1340.

Column 23, it will be observed, is about half the width of the other three columns. The result, therefore, tends to confirm the view that fracturing acceleration is simply proportional to the width of a column.

ILLUSTRATIONS OF THE APPLICATION OF RESULTS.

I. An earthquake with a maximum range of motion of 4 inches, and with a period of 2 seconds, would imply a maximum acceleration of about 450 mm. per. sec. per sec., a quantity very much greater than anything recorded in Tokio. As a maximum acceleration to be expected we will increase this to 1000 mm. per. sec. per sec. and determine the height to which a brick column, two feet square, might be built above its foundation and just able to withstand this motion

Let x=height required

β=2 ft.

A=4 sq. feet.

F'=5 lbs.

F''=15 lbs.

w =the weight of one cubic inch of brickwork =.0608 lbs.

By substitution in the formula employed for fracturing we obtain

$$a = \frac{F\,A\,\beta\,g}{6\,f\,W} = \frac{F\,\beta\,g}{3\,x^2\,w}$$

$$\text{whence } x = \sqrt{\frac{F\,\beta\,g}{2\,a\,w}}$$

with value for F′, x=6ft, 8in.

with value for F″, x=11ft. 7in.

From the last equation, given a and β, we see that the value for x is proportional to the square root of F, or the force of cohesion.

II. As a second illustration of the application of the preceding results, we append the following discussion regarding the form which columns of given sections must have in order that they may be equally able to resist fracture when acted on by horizontal movements, at any horizontal section.

(1.) First take a column of square section. For the uniformity of strength of the column relatively to the inertia of the portion above any given horizontal section, a must be constant in the following equation

$$a = \frac{g\,F}{6} \cdot \frac{A\,\beta}{W\,f} = \frac{g\,F}{6\,w} \cdot \frac{A\,\beta}{V\,f}, \text{ (1)}$$

in which $W = Vw$, V being the volume and w the density.

(1) *We thank Mr. A. Inokuty of the Engineering College for having checked the following formula.*

Let x_1 = half of the dimension of any given section whose distance from the top is $= y_1$. Then

$$f = \int_0^{y_1} \frac{4\, x^2\ (y_1 - y)\ dy}{V}$$

$$\therefore \quad a = \frac{g\,F}{6\,w} \cdot \frac{4x_1^2 \cdot 2x_1}{V \int_0^{y_1} \frac{4x^2\,(y_1-y)\,dy}{V}} = \frac{g\,F}{3\,w} \cdot \frac{x_1^3}{\int_0^{y_1} x^2 (y_1-y)\,dy}$$

From the above equation, it can be shown that for a to be constant, there must exist between x_1 and y_1 the following relation:—

$$y_1^2 = \frac{10\,g\,F}{a\,w}\,x_1 \qquad (1)$$

which represents a parabola with its concavity turned outwards.

(2.) For a column of a circular section we have

$$a = \frac{\pi\,g\,F x_1^3}{4\,Wf} = \frac{\pi\,g\,F}{4\,w} \quad \frac{x_1^3}{\dfrac{V \int_0^{y_1} \pi x^2 (y_1-y)\,dy}{V}} = \frac{g\,F}{4\,w}\,\frac{x_1^3}{\int_0^{y_1} x^2\,(y_1-y)\,dy}$$

in which x_1 is the radius of any section. This leads to the relation:—

$$y^2 = 7\tfrac{1}{2} \cdot \frac{F\,g}{a\,w}\,x_1 \qquad (2)$$

(3.) Let the section be rectangular, and the dimension perpendicular to the direction of the motion be constant and $= b$.

$$a = \frac{g\,F}{6\,w}\ \frac{A\,\beta}{Vf} = \frac{g\,F}{6\,w}\ \frac{2\,x_1\,b.\,2\,x_1}{\dfrac{V\int_0^{y_1} 2\,x\,b(y_1-y)\,dy}{V}} = \frac{g\,F}{3\,w}\ \frac{x_1^2}{\int_0^{y_1} x(y_1-y)\,dy}$$

From which again follows the parabolic relation:—

$$y_1^2 = \frac{4\,g\,F}{a\,w}\,x_1 \qquad (3)$$

By comparing the formulæ (1), (2), and (3), we see that with the same given dimensions of base and height, the strongest column would be one with a square section.

To give an illustration, suppose

$$a = 1{,}000 \text{ mm. per sec. per sec.}$$
$$F = 5 \text{ lbs. per sq. inch.}$$
$$\text{and } w = 0.0608 \text{ lbs.}$$

Then supposing the section to be square, we have

$$y^2 = 8100\,x$$

The outline of the column is as represented in the accompanying diagram.