# ON THE OVERTURNING OF COLUMNS.

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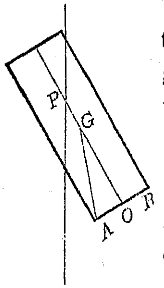
The acceleration formula of Prof. C. D. West $f=\frac{x}{y}g$, where $y$ is the height of the centre of gravity of the column, and $x$ half the basal dimension, seems, as tested in experiments with columns of moderate dimensions, such as tomb stones, to give practically satisfactory results. It applicability, however, ceases when the amplitude of motion becomes very small. A motion whose range is very small, even though the maximum acceleration be very great, would never be able to overturn large columns. For instance, if ampl. = 1 mm., period = 0.1 sec., then the maximum acceleration is 3,950 mm. per sec. per sec. But this motion, as may be easily tested, cannot overturn a column whose dimensions are, say, 2 in. in base and 8 in. in height. That amplitude plays a very important part in the overturning of columns may be seen from the following considerations.

We shall distinguish two cases of motion, in one of which the period is very short, and in the other where it is comparatively long.

First Case.—When the period is very short, the column would be overturned towards the direction from which the impulse comes. Since the period is very short, the column will behave as if a sudden blow or displacement were applied to its base, *i.e.*, will tend to rotate about its centre of percussion with respect to the edge about which the column will actually turn. The height of the centre of percussion P, for a column of rectangular section whose height is $2y$, and whose breadth is $2x$, is

$$\frac{4}{3}(x^2+y^2) \div y, \text{ or } \frac{4(x^2+y^2)}{3y},$$




The column will be overturned when the rotation of it is so great as to bring the centre of gravity $G$ vertically over the edge $A$, that is when

$$\frac{2a}{OP} = \frac{OA}{OG} \text{ approximately,}$$

$2a$ being the range of motion or displacement communicated to the edge $A$; or

$$(1)\ldots\ldots\ 2a = \frac{OA}{OG} \cdot OP = \frac{x}{y} \cdot \frac{4(x^2+y^2)}{3y} = \frac{4x(x^2+y^2)}{3y^2}$$

This equation will give the lowest limit of the range of motion, when the period is very short, necessary for overturning a rectangular column of height $2y$ and of breadth $2x$. We here give a few applications :—

| | $2x$ | $2y$ | $2a$ |
|---|---|---|---|
| (*a*) | 1 in. | 4 in. | 0.71 in. |
| (*b*) | 2 | 8 | 1.4 |
| (2)...... (*c*) | 3 | 12 | 2.1 |
| (*d*) | 6 | 24 | 4.3 |
| (*e*) | 12 | 48 | 8.5 |
| etc. | | etc. | etc. |

From equation (1), the range of motion $2a$, is seen, for columns whose height and breadth have a constant ratio, to be proportional to these dimensions. Columns with the same ratio of height and basal dimensions as in the above group (2), and of the absolute dimensions between (*d*) and (*e*), would represent the case at Nagoya on the occasion of the great earthquake. The range of motion at that city might have been not very much different from 8 inches.

As eqnation (1) shows, the range of motion necessary for overturning a column increases in proportion to its dimensions. This seems, judging from experiments tried with small columns, to be extremely probable. If the basal dimension of a column be 20 feet and its height 100 feet, equation (1) will give $2a=14$ feet. This may, in a measure, explain why the five-storied pagodas have not yet been overthrown even in such severe shocks as the Yedo Earthquake of Ansei 2nd year (1854) or the recent Gifu-Nagoya Earthquake. Again, we know that the upper broken portions of brick chimneys are often rotated, but not thrown down. Such may partly be due to the range of motion not being sufficiently great.

SECOND CASE.—Let the period of motion be comparatively long, say 1 or 2 seconds. Then, supposing the motion to be simple harmonic, the column, when of proper dimensions, would not be overthrown at the start (as in the preceding case), but will move together with the ground and acquire its velocity. When the ground particles reach one extremity of motion, and its velocity becomes zero, the column would still retain its acquired velocity and be overturned in the forward direction as the ground begins to move backwards. This is precisely what takes place in actual trials. Under this supposition, the behaviour of the column at the instant of overturning will be the same as when a certain velocity is suddenly communicated to the body.

If $2y$, $2x$ be respectively the height and breadth of a column, then the velocity, which, when suddenly communicated to the column, is just able to overturn it, that is to rotate the column so far as to bring the centre of gravity just vertically over the edge about which it turns, is

$$(3) \ldots V = \sqrt{\frac{8g\sqrt{x^2+y^2}\,(1-\cos\phi)}{3\cos^2\phi}}; \text{ or } \sqrt{\frac{8gy\,(1-\cos\phi)}{3\cos^3\phi}}$$

$\phi$ being the angle between the diagonal and a vertical side of the column. The following are a few illustrations.

| $2x$. | $2y$. | $V$. | $2a$. |
|---|---|---|---|
| 1 in. | 4 in. | 8.2 in. per sec. | 2.6 |
| 2 | 8 | 11.6 per sec. | 3.7 |
| 3 | 12 | 14.2 per sec. | 4.5 |
| 6 | 14 | 20.1 per sec. | 6.4 |
| 12 | 48 | 28.4 per sec. | 9.0 in. |
| etc. | etc. | | |

The values of $V$ for columns having the same $\phi$ varying as the square roof of $y$ or $x$.

Now $V$ may be looked upon as being equivalent to the maximum velocity of the particle in its simple harmonic motion, namely,

$$V = \frac{2 \pi a}{T}$$

Hence, if we know the period of motion, we can approximately determine the range of motion. The figures in the last column of the above table have been calculated from this formula, $2a$ varying as $V$. The period of motion at Nagoya was probably not far different from 1 second. The estimated range of motion for Nagoya is thus seen to be nearly the same, whether we suppose the motion to be very short in period or, what is more likely, to be comparatively long.

In my own belief, the "accelerations" of the motion of ground as determined from the observations of overturned columns seem to be practically correct, and the results of the calculations here indicated are only to be looked upon as checks.