ON A METHOD OF COMPENSATING A PENDULUM SO AS TO MAKE IT ASTATIC,

BY

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The usual practice in pendulum machines for earthquake registration, is to make them so long that they behave at each instant very much as a stationary mass would. In these instruments, however there is always a certain amount of periodic vibration, the amplitude of which may be considerable. I propose, in the present note, a mode of rendering the period of a short pendulum very long and hence of placing it practically in the same state as if its length were great. The method which suggests itself most readily, is to make the bob of the pendulum for the most part liquid contained in a spherical bowl, the centre of the sphere would require, in that case, to be at the point of suspension if the bowl had no weight but owing to the weight of the bowl, it will in practice require to be above the point of suspension. This is perhaps the simplest method that can be got for the purpose. A method which seems to me well suited to this purpose, is to make the pendulum rigid, pivot it at the point of suspension like a universal joint, such as a sharp point or a double knife edge and to fix on the bob a circular trough of liquid. This trough may have any desired shape of cross section but must in all cases have its centre in the axis of the pendulum and its plane at right angles to that axis. Suppose that the edges of the trough are formed by portions of the surfaces of two concentric spheres, the centre being in the axis of the pendulum and in the medial plane of the trough.

Let $r =$ internal radius of the trough
$R =$ external ,, ,, ,, ,,
$w =$ total weight of bob
$\varphi =$ angle of deflection at any instant
$l =$ length of pendulum

Then the couple tending to turn the pendulum back to its original position is, supposing the whole rigid

$$w\, l \sin \varphi$$

Again the couple due to displacement of liquid, in actual case, is equal to

$$a_1\, \delta\, \varphi \int_r^{R} \int_0^{\frac{\pi}{2}} r^3\, dr . \sin^2 \theta\, d\, \theta$$

where δ is the density of the liquid. The integral of this is evidently

$$\tfrac{1}{2}\, \pi\, \delta\, \varphi\, (R^4 - r^4)$$

Now for perfect astaticism we must have

$$w\, l \sin \varphi = \tfrac{1}{2}\, \pi\, \delta\, \varphi\, (R^4 - r^4)$$

which can only be case when $\sin \varphi = \varphi$ or for very small deflections. It is to be noticed here also that as φ increases faster than $\sin \varphi$ the tendency is towards instability. The equation indicates what must be the variation of the shape, namely, that the surfaces should have a greater curvature. The curvature should be such that φ becomes $\sin \varphi$.

As a practical example it will be found that when
$l = 30$ centimetres
$r = 10$ centimetres
$w = 2000$ grammes
$\delta = 13.6$ grammes per cubic centimetre

R is equal to 10.6 centimetres nearly.

Several other methods of obtaining this compensation have suggested themselves as for instance, a ball rolling in a hollow curved surface such as a sphere whose radius is longer than the pendulum. Another method would be to attach a vertical spiral spring to a point in the axis of the pendulum a little below the point of suspension and to a fixed point above it, so that when the pendulum is deflected it would introduce a couple. This method possesses the advantage of simplicity and ease of adjustment but requires a strong spring and inverted knife edges. Several other methods might perhaps be suggested but enough has been given to show that compensation is practicable.

DISCUSSION.

Mr. Ewing said that a method of reducing a pendulum to a state of neutral equilibrium had occurred to him, different from any of those suggested by Mr. Gray. Let the bob of the pendulum touch the top of a short vertical lever pivotted below the bob so that the top of the lever should be free to accompany the pendulum in all its (small) displacements. Let a point on this lever anywhere between the pivot and the bob of the pendulum be connected by a stretched spiral spring to a point fixed to the ground vertically below the pivot of the lever, and at some distance below it. Then when the pendulum was deflected to any side, this spring would introduce a force tending to increase the deflection. By properly arranging the length and tension of the spring with reference to the length of the lever, the position of attachment of the spring to it, and the length and mass of the pendulum, it would be easy to make this force just sufficient to hold the pendulum deflected against gravity, after any small displacement of the bob had occurred. In other words, a condition of sensibly neutral equilibrium could be secured, throughout a range of motion sufficiently extensive for seismometrical purposes.