NOTE ON THE DEVELOPEMENT AND INTERPRETATION OF THE RECORD WHICH A BRACKET MACHINE GIVES OF AN EARTHQUAKE.

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Figures 1 and 2 are for the purpose of reference in the geometrical demonstration for an out and in stroke respectively, and are not to scale, nor in any way intended to represent either a record of any one earthquake or that of the typical earthquake.

Our data are the path of the pointer traced upon the plate by the unknown motion of the pointer relative to the earth, the known constant angular velocity of the mutual rotation of the plate and the earth, the length of the pointer, and the distance from the centre of plate to the hinge of pointer.

Which data are sufficient to determine the direction and velocity of the draw point relative to the earth while it was at any given point of the path traced on the plate as will be shewn. The other end of pointer being hinged to the earth it is evident that the path of the draw point is a circular arc (very small in the typical record). The data also give us the length of the arc, and point for point on it, corresponding to any out or in stroke of the traced path. Hence the motion of the draw point in its circular path relative to the earth is determinate at each point. Hence the character of the motion of the draw point relative to the earth, in the small circular arc is an oscillating motion in a sensibly straight line, whose velocity at each point is determinate. To assume that this motion is S. H. is to beg the question. And the object of this paper is to point out that for the first two or three large strokes (which after all are the record of the earthquake shock) it is *impossible* to assume the motion S. H. For the remainder or tail of record it is always *possible* to assume the motion S. H., and it is just to do so, as it is in all probablity the record of the oscillation of the stand in its own natural period. It may be interesting to measure this, but the thing of greatest interest to this Society is to be sure of the *nature* of the actual shock, and then to measure it. I do not say that the actual shock does not produce a comp. S. H. motion or that it is unreasonable to assume it does. On the other hand I think it is probably S. H. and so conclude, that since the Bracket Machine's record indicates that it is not S. H. as just stated, that therefore the machine is imperfect in practice and worse than useless as it gives misleading results.

Construction figs. 1 or 2. From 0 the centre of plate draw the circle *abc* with radius equal to the distance to hinge of pointer. From B, one end of a wave, with B*b* equal to length of pointer make an arc cutting *abc* in *b* and join B*b*, similarly lay down C*c* equal to the length of pointer. Then it is evident, that the inclination of B*b* and C*c** is the angle of the semi-oscillation made by the pointer relative to the earth, while the tracing point travelled over the portion BAC of its path. Shift *c*C parallel to itself, till *c* coincides with *b*,* then the distance between the extremities C and B is the double amplitude of that semi-oscillation. Choose A any point on

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* After the paper was read it was pointed out separately by Professors West and Paul that from the inclination of the dotted pointers in fig. 1 the angle *boa* must be subtracted and to that inclination in fig. 2 the angle *b′o a′* must be added to give the angle of the semi oscillation made by the pointer relative to the earth. Since however this additional angle is discribed at constant rate, it in no way invalidates the fact that A is the point on path corresponding to the middle point of circular arc if A*a* in direction is midway between the directions of the dotted pointers. Of course this correction must be made on the angle to get the double amplitude of the oscillation. Mr. West demonstrated to me by the machine itself that the double amplitude is correctly and quickly got by taking the distance between the concentric circles from O through B and C.

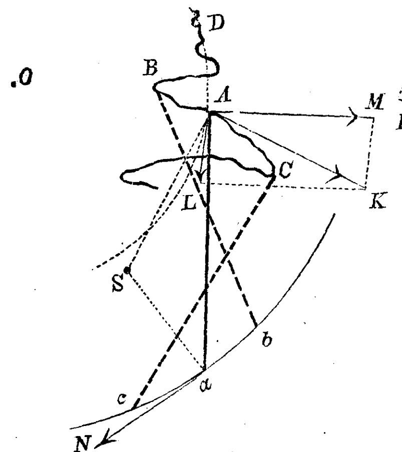

Fig. 1.

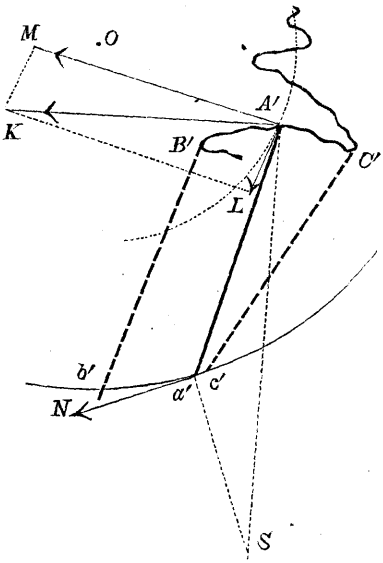

Fig. 2.

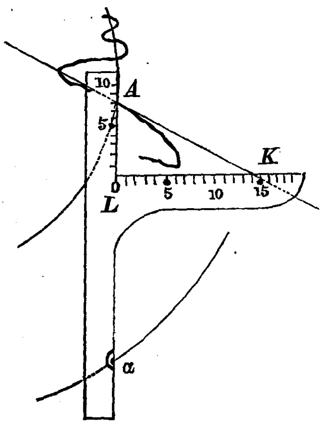

Fig. 3.

the path near the middle of BC and lay down A*a* equal to length of pointer as above.

If A*a* bisect the angle between B*b* and C*c*, then, while the tracing point was at A, the pointer A*a* was at the middle of the semi-oscillation, and consequently the point A was at the max. velocity for the semi-oscillation. After a trial or two A may be found to satisfy this condition.

While the pointer is in the position A*a*, by considering the plate to be at rest, we see that *a*N, the tangent at *a*, is the direction of the motion of *a* relative to the plate and that AK the tangent to the path at A is the direction of the motion of A relative to the plate. Hence S, where AS the normal to path of A meets the radius *a*S, is the instantaneous axis for the relative rotation of the pointer and plate. If *a*N be laid off to scale equal to the product of the known angular velocity of plate relative to earth into radius O*a*, it will represent the volocity of *a* relative to the plate and AK laid off, bearing the same ratio to *a*N as SA bears to S*a*, will represent the velocity of A relative to the plate.

Let E be the point on the earth below A, then AL laid off along the tangent to the circle through A and equal to the product of the known angular velocity into OA, will represent the velocity of E relative to plate.

On AK as a diagonal construct the parallelogram ALKM then will AM represent the velocity of A to E, that is, it will be the max. velocity of the tracing point during the semi-oscillation of the pointer relative to the earth, while the tracing point travelled over BAC. We have therefore the amplitude and the velocity of the point in the relative circle for the semi-oscillation of the tracing point relative to the earth, sensibly simple hamonic, from which we can calculate the max. acceleration at B or C the extremities of the amplitude by dividing the square of AM by the amplitude.

It follows of necessity that AM is at right angles to *a*A which is a check upon the construction, rendered essential, by the fact that this graphical solution depends upon the accuracy with which AK, the tangent to the path at A, is drawn.

By comparing figs. 1 and 2 it will be seen that the dotted pointers cross in the one and do not cross in the other. This will geometrically account for the fact that the successive out and in strokes of path are of markedly different lengths, observed alike on the actual record of an earthquake, and upon the path traced by the pointer when constrained to oscillate by a pendulum. What I mean is, that the fact of the difference of the lengths of the out and in stokes does not necessarily point to the out and in reciprocating motion of the point relative to earth as differing either in magnitude or velocity.

In the bracket machine the pointer A*a* is set sensibly along the tangent at the point of contact while the machine is at rest. If DA be the circle which it traces out when the plate revolves while the pointer remains at rest and if we find or assume that A, the intersection of the path with this circle, is the middle of the semi-oscillation, it follows that AL will lie along A*a* and AM or LK the velocity of tracing point relative to earth is readily found as shewn on fig. 3, thus.—On the template lay off LA to scale to represent the lineal velocity in the circle DA corresponding to the rotation of plate and lay off A*a* equal to length of pointer.—Place the template with A at the junction of path and circle and *a* on outer circle then read of LK intercepted by AK the tangent to path at A.

At the point A fig. 3 which is the junction of path and neutral circle, LK = LA cot AKL is the velocity of tracing point into cotangent of the slope of path to radius at A; so that if path cross neutral circle at right angles LK the vel. of oscillation is infinite, if path lie along neutral circle LK the vel. of oscillation is zero.

From figs. 1 and 2 it is evident, that the instantaneous axis for the rotation of the pointer relative to the plate will fall alternately inside and outside of the circle *cab*.

I examined the path traced by a pointer constrained to oscillate by means of a pendulum. First the pointer was placed as nearly as possible, by inspection, at right angles to the radius and the plate allowed to rotate slowly. The tangent to inward stroke appeared to be along the radius which we know is impossible. That is to make the construction with the data, the length of pointer and its angle to radius assumed to be set at a right angle, gave an impossible result. When the construction was made with the strictly measured data the instantaneous axis fell as in figs. 1 and 2 alternately for out and in stroke. Again when the pointer was placed obliquely, so as to make the angle L*Aa* about 70°, and the radius of the circle *bac* was found by laying off the length of pointer normal to the arc of circle traced while the plate had been at rest, the construction for in stroke fig. 2 threw S inside *b′a′c′*; but decreasing the radius of *bac* by only 2 p.c. caused both constructions fig. 1 and 2 to be real and gave a gradually decreasing vel. for each semi-oscillation. This shews the necessity of having an actual measurement of the distance between centres of plate and of pointer. When the plate ran quickly it was clearly seen that the zigzags ran alternately on different sides of the centre.

I examined also the path traced by an elastic pointer placed sensibly at right angles to a radius, and made to vibrate upon the plate. I found it impossible to find one circle *cab* such, that S would fall upon different sides of it for alternate strokes, shewing that the elastic arc traced by the pointer cannot be assumed to be circular.

And lastly I examined the registers of different large earthquakes, made by the bracket machine with its pointer sensibly at right angles to the radius point of contact when at rest. The construction for the first few in strokes either by fig. 3 on the assumption of the right angle or by fig. 2 gave an impossible result, as S always fell inside *b′a′c′*.—Impossible, I mean, upon the hypothesis that the plate was rotating constantly and the pointer oscillating with a S.H. motion. That the plate was rotating constantly in one direction, and with an almost constant rate, was shewn beyond all doubt on the margin by hatches made with a pendulum. Hence the inference pointed at is, that the tracing point was not simply oscillating in an arc of a circle with respect to the earth or that the distance between centre of plate and hinge of pointer was varying periodically. I consider, that the earthquake record resembles more closely the path traced by the vibrating elastic pointer than that traced by an oscillating pointer and infer, that the pointer of the bracket instrument traces, relative to the earth, some elastic curve during an earthquake rather than a circular arc.

I submit—That time records taken by inspection from the path traced upon a circular plate by the bracket machine, after the fashion of a clock face, whether by drawing radii or arcs with a constant circular template, are deceptive as they may involve the fallacy that a point can move in two circles at a time—That results accurately developed upon the cinematical principles of figs. 1, and 2, but upon the assumption that the pointer was set at right angles to a radius in this instrument, which has no fine adjustments, and could not be expected to remain in adjustment, are worthless for the in stroke, even when we know that the pointer was constrained to oscillate in an arc of a circle—That the development by figs. 1 and 2 for the path traced by a point constrained to oscillate in an arc of circle, even when the measured lengths are given, is a very delicate exercise in draughting—And that the large in strokes on the earthquake records, which were given to me by Mr. Milne, lead to an impossible result on the assumption that the tracing point was oscillating in a circular arc about the hinge of pointer while the distance between centre of plate and hinge remained constant.