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35. *A Surface Integral Representation of the Tectonomagnetic Field Based on the Linear Piezomagnetic Effect.*

By Yoichi SASAI,

Earthquake Research Institute.

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Abstract

A representation theorem of the linear piezomagnetic field is formulated for a homogeneous and isotropic magneto-elastic material. The tectonomagnetic field is given by surface integrals of the displacement and its normal derivatives over the strained body. This is a corrected version of previous results (SASAI 1980). Applying the theorem to a medium including a dislocation surface within it, we find that the dislocation surface behaves as a magnetic sheet. For a special type of dislocations where all the stress components are continuous across the dislocation surface, the magnetic sheet is simply a double layer, of which moment is given by the inner product of the displacement discontinuity and the magnetization vector. The seismomagnetic moment thus defined is useful to intuitively presume coseismic magnetic changes, which is demonstrated for the seismomagnetic effect accompanying the 1946 Nankaido Earthquake of M 8.1. With the aid of potential theory, the tectonomagnetic field at the Earth's surface is found to contain some information of the strain field at the observation site. This gives a measure of sensitivity of the magnetic measurement as a strain sensor, which amounts to roughly 10 μ-strain per nT in a strongly magnetized region. The use of representation theorem greatly reduces efforts of tectonomagnetic calculations in comparison with the traditional dipole force law. It is exemplified by actually applying the theorem to the Mogi model.

1. Introduction

SASAI (1980) developed a Green's function method for calculating the piezomagnetic anomaly field associated with an arbitrary dislocation model. In the last section of that paper, another method for tectonomagnetic modelling was suggested: the piezomagnetic potential can be expressed by an integral of the model displacement field and its derivatives over the entire surface of the strained body. The latter approach has merit in that we can deal with tectonic models including some differently magnetized blocks, in contrast to the limitation that a Green function derived by SASAI (1980) is available only to a uniformly magnetized single layer of horizontally infinite extent. An error was found, however, in the prescribed surface integral formula. Corrections are made here on some of the previous results. A surface integral approach for tectonomagnetic modelling is reformulated properly. We will hereafter refer to the previous work (SASAI 1980) as paper I.

A new theoretical approach to tectonomagnetic modelling was devised by BONAFEDE and SABADINI (1980). From thermodynamical considerations, they derived constitutive relations of aeolotropic magneto-elastic materials. Combining those with the equation of motion and the Ampère-Maxwell equation, they presented a system of equations in which the coupling between the displacement and magnetic field explicitly appears. On the assumption that the Green function exists for the above equations a representation theorem was formulated for the piezomagnetic vector potentials. Applying the theorem to a magneto-elastic body including a dislocation surface within it, they showed that the magnetic field due to a dislocation event such as faulting is equivalent to that of a distribution of magnetic dipoles on the fault surface, of which intensities are proportional to the seismic moment tensor density through a set of piezomagnetic coefficients.

This is a clearcut physical image on the tectonomagnetic field caused by dislocation events. A similar feature was noticed by the present writer for the basic characteristics of the elementary piezomagnetic potentials, which are the Green functions associated with point dislocations within a semi-infinite medium (§ 4 in paper I). He found that elementary potentials consist of dipoles and some higher-order multipoles placed at the dislocation point and its image points with respect to the Currie depth, so that a dislocation surface can be regarded as a magnetic sheet. Its entity was obscured by some mathematical details, partly by taking into account the Currie point isotherms, which was entirely ignored in Bonafede and Sabadini's results. Although Bonafede and Sabadini dealt with the isothermal process, the theory was extended to include a more general thermo-elasto-magnetic coupling by BONAFEDE and BOSCHI (1980).

Since the Green function for a general aeolotropic medium is assumed *a priori* to exist but its definite form is not presented, we cannot actually calculate the tectonomagnetic field for such a complicated state of matter.

In fact, Bonafede and Boschi's seismomagnetic calculation of the Teal Creek fault (i. e. faulting induced by the Cannikin explosion) was made for a uniformly magnetized Earth. Very little has been known about the anisotropy of the piezomagnetic parameters. Hence we are obliged to regard the Earth's crust, at most, as an assemblage of differently magnetized blocks : each segment consists of a homogeneous and isotropic magneto-elastic material. A surface integral representation for a homogeneous and isotropic piezomagnetic body is still of some practical use.

The present formulation follows traditional lines of tectonomagnetic modelling as initiated by STACEY (1964). Some points are discriminated from Bonafede and Sabadini's method.

1) The source of the magnetic field is the magnetization in contrast to the conduction current in Bonafede and Sabadini's formalism. This enables us to describe the magnetic field in terms of the scalar potential rather than the vector potential.

2) The basic equation of the magnetic field is the Gauss law for the magnetic induction instead of the Ampère-Maxwell equation.

3) An important simplification is that we neglect the coupling terms, namely the magnetostrictive stresses in the stress equation and the secondary induced magnetization due to the incremental piezomagnetic field in the magnetic induction equation. The problem is thus reduced to an uncoupled one, in which we may solve a Poisson equation with definite magnetic sources expressed by the known displacement field.

4) The constitutive law of the linear piezomagnetism is based on the uniaxial compression tests and its extension in three dimensions by superposition (e. g. NAGATA 1970, STACEY and JOHNSTON 1972). Apparently it has the same tensor form as Bonafede and Sabadini's piezomagnetic parameters. The formula applies to stress-induced changes in the remanent magnetization as well as the susceptibility change, while Bonafede and Sabadini's results are derived only for the remanence change.

5) We are concerned here with the static problem. The elasto-dynamic magnetic change is a future subject.

2. Fundamental Equation and Representation Theorem

The basic equation for the scalar potential of the piezomagnetic field is given by

$$\nabla^2 W_k = 4\pi \operatorname{div} \Delta \boldsymbol{M}_k \qquad (k=x,\ y,\ z) \tag{2.1}$$

$\Delta \boldsymbol{M}_k$ indicates the stress-induced magnetization vector. The above equation can be derived from the Gauss law:

$$\mathrm{div}\,\boldsymbol{B}=0 \tag{2.2}$$

and interrelations among the scalar potential W, the magnetic field $\boldsymbol{H}$, the magnetization $\boldsymbol{M}$ and the magnetic induction $\boldsymbol{B}$ as

$$\left.\begin{aligned}\boldsymbol{H}&=-\mathrm{grad}\,W\\ \boldsymbol{B}&=\boldsymbol{H}+4\pi\boldsymbol{M}\end{aligned}\right\} \tag{2.3}$$

The displacement field of the elastic material bearing the magnetization $\Delta\boldsymbol{M}_k$ satisfies the equation of static equilibrium under the body force F_k:

$$(\lambda+\mu)\frac{\partial}{\partial x_k}\mathrm{div}\,\boldsymbol{u}+\mu\nabla^2 u_k+F_k=0 \tag{2.4}$$

where λ and μ are Lamè's constants. We consider a simple case where the magnetic and elastic properties of the strained material are homogeneous and isotropic. We also assume that $u_k(\boldsymbol{r})$ is already known by solving the equation (2.4) under appropriate boundary conditions.

The two basic equations (2.1) and (2.4) can be combined to yield the fundamental equation of the piezomagnetic potential through constitutive laws of linear piezomagnetism and elasticity. SASAI (1980) presented a linear relationship between the magnetization change and stress components as follows:

$$\begin{pmatrix}\Delta J_x\\ \Delta J_y\\ \Delta J_z\end{pmatrix}=\beta\begin{pmatrix}\tau_{xx}-\dfrac{\tau_{yy}+\tau_{zz}}{2}, & \dfrac{3}{2}\tau_{xy}, & \dfrac{3}{2}\tau_{xz}\\ \dfrac{3}{2}\tau_{yx}, & \tau_{yy}-\dfrac{\tau_{zz}+\tau_{xx}}{2}, & \dfrac{3}{2}\tau_{yz}\\ \dfrac{3}{2}\tau_{zx}, & \dfrac{3}{2}\tau_{zy}, & \tau_{zz}-\dfrac{\tau_{xx}+\tau_{yy}}{2}\end{pmatrix}\begin{pmatrix}J_x\\ J_y\\ J_z\end{pmatrix} \tag{2.5}$$

where β is the stress sensitivity. Note that the equation (2.5) is opposite in sign to the previous one (eq. (2.18) in paper I). This is because we now follow the standard sign convention in elasticity (i. e. compression is negative) in the definition of β. The new convention has been introduced by HAO *et al.* (1982) in order to avoid confusion in the modelling process. It is noteworthy that eq. (2.5) can be written as

$$\Delta\boldsymbol{J}=\frac{3}{2}\beta\boldsymbol{T}'\boldsymbol{J} \tag{2.5$'$}$$

where $\boldsymbol{T}'$ is the deviatoric stress tensor, whose component is given by $\tau'_{mn}=\tau_{mn}-1/3\,\delta_{mn}\tau_{ll}$. On the other hand, the constitutive relation of linear elasticity is Hooke's law:

$$\tau_{mn}=\delta_{mn}\lambda \operatorname{div} \boldsymbol{u}+\mu\left(\frac{\partial u_m}{\partial x_n}+\frac{\partial u_n}{\partial x_m}\right) \tag{2.6}$$

Let us consider the piezomagnetic field associated with each Cartesian component of the magnetization separately. The stress-induced magnetization vector $\boldsymbol{\Delta M}_k$ is obtained by substituting $\boldsymbol{J}=J_k\boldsymbol{e}_k$ into (2.5): $\boldsymbol{e}_k$ is the unit vector in the k-th direction. With the aid of (2.6), we have the following expression:

$$\Delta M_{kl}=\beta J_k\mu\left\{\frac{3}{2}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right)-\delta_{kl} \operatorname{div} \boldsymbol{u}\right\} \tag{2.7}$$

By making use of (2.7) and (2.4), we arrive at the fundamental equation of the piezomagnetic potential:

$$\nabla^2 W_k=4\pi \operatorname{div} \boldsymbol{\Delta M}_k=4\pi C_k\left(\nabla^2 u_k-\frac{1}{3\lambda+2\mu}F_k\right) \tag{2.8}$$

where

$$C_k=\frac{1}{2}\beta J_k\mu\frac{3\lambda+2\mu}{\lambda+\mu} \tag{2.9}$$

The boundary conditions at the surface of the magnetic body are the continuity of the potential itself as well as of the normal component of $\boldsymbol{B}$. These are expressed as follows:

$$W_{k+}=W_{k-} \tag{2.10a}$$

$$\left[\frac{\partial W_k}{\partial n}\right]_-^+=-4\pi\boldsymbol{\Delta M}_k\cdot\boldsymbol{n} \tag{2.10b}$$

where $\boldsymbol{n}$ is the outward normal to the boundary surface S surrounding a volume V. Thus the problem is reduced to solving the equation (2.8) under the conditions (2.10).

The above institution of the problem involves some approximations as summarized in the following:

1) The magnetization of ferromagnetic materials consists of induced and remanent magnetizations: $\boldsymbol{M}=\chi\boldsymbol{H}+\boldsymbol{J}_R$. Precisely speaking, $\boldsymbol{\Delta M}_k$ on the righthand side of eq. (2.1) should include the secondary induced magnetization caused by piezomagnetic field changes. This effect is, however, by two orders of magnitude smaller than the primary part. Thus we may regard $\boldsymbol{\Delta M}_k$ as a function of stress alone and not that of W.

2) In the stress equation (2.6), we neglect magnetostriction, which is too small to compare with mechanical strain.

The solution of eq. (2.1) satisfying the boundary conditions (2.10) is well established (e. g. STRATTON 1941):

$$W_k(\boldsymbol{r}) = -\iiint_V \frac{\operatorname{div} \Delta \boldsymbol{M}_k}{\rho} dV + \iint_S \frac{\Delta \boldsymbol{M}_k \cdot \boldsymbol{n}}{\rho} \tag{2.11}$$

This is identical to the dipole law of force:

$$W_k(\boldsymbol{r}) = \iiint_V \Delta \boldsymbol{M}_k \cdot \nabla \left(\frac{1}{\rho} \right) dV \tag{2.12}$$

which has been utilized in ordinary tectonomagnetic modelling. The function $(4\pi\rho)^{-1}$ is the fundamental solution of the Laplace-Poisson type equations, which satisfies

$$\left. \begin{aligned} &\nabla^2 \left(\frac{1}{4\pi\rho} \right) = -\delta(\boldsymbol{r} - \boldsymbol{r}') \\ &\rho = |\boldsymbol{r} - \boldsymbol{r}'| \end{aligned} \right\} \tag{2.13}$$

Substituting the third identity of (2.8) into (2.11) and applying Green's theorem together with (2.13), we obtain

$$\begin{aligned} W_k(\boldsymbol{r}) = {} & 4\pi C_k u_k(\boldsymbol{r}) \theta(\boldsymbol{r} \in V) + \frac{C_k}{3\lambda + 2\mu} \iiint_V F_k \frac{1}{\rho} dV \\ & + \iint_S \left[\left\{ -C_k \frac{\partial u_k(\boldsymbol{r}')}{\partial n'} + \Delta \boldsymbol{M}_k \cdot \boldsymbol{n}' \right\} \frac{1}{\rho} + \{ C_k u_k(\boldsymbol{r}') \} \frac{\partial}{\partial n'} \left(\frac{1}{\rho} \right) \right] dS' \end{aligned} \tag{2.14}$$

where

$$\theta(\boldsymbol{r} \in V) = \begin{cases} 1 & (\boldsymbol{r} \in V) \\ 0 & (\boldsymbol{r} \notin V) \end{cases} \tag{2.15}$$

The equation (2.14) is a representation theorem of the linear piezomagnetic field. The previous result (eq. (6.11) in paper I) lacks the term $\Delta \boldsymbol{M}_k \cdot \boldsymbol{n}$ in the single layer potential, so that it cannot fulfill the boundary condition (2.10b). The continuity of the potential on S is guaranteed by a jump in the potential value across the double layer, which has already been discussed in paper I.

Since we are mainly concerned with the piezomagnetic change in the free space, we will present here a formula for the potential outside the magneto-elastic body:

$$\begin{aligned} W_k(\boldsymbol{r}) = {} & C_k \iint_S \left[\left\{ -\frac{\partial u_k(\boldsymbol{r}')}{\partial n'} + \frac{2(\lambda + \mu)}{3\lambda + 2\mu} \Delta \boldsymbol{m}_k \cdot \boldsymbol{n}' \right\} \frac{1}{\rho} + \{ u_k(\boldsymbol{r}') \} \frac{\partial}{\partial n'} \left(\frac{1}{\rho} \right) \right] dS' \\ & + \frac{C_k}{3\lambda + 2\mu} \iiint_V F_k \frac{1}{\rho} dV, \end{aligned} \tag{2.16}$$

where

$$\Delta m_{kl} = \frac{3}{2} \left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right) - \delta_{kl} \operatorname{div} \boldsymbol{u}\,. \tag{2.17}$$

3. Magnetic Source Equivalents and the Seismomagnetic Moment

We have derived eq. (2.16) with the aid of Green's theorem. This implies that the displacement field $\boldsymbol{u}(\boldsymbol{r})$ and its first order derivatives must be continuous in $V+S$, while its second order derivatives be piece-wise continuous within V. When the magnetoelastic body V involves an internal dislocation surface Σ, eq. (2.16) no longer holds as it is. In this case, we divide V into two parts with a surface S' including Σ, as shown in Fig. 1. Then we apply the formula (2.16) to each part separately and add each other. The unit normal is assigned to point outward, and we define

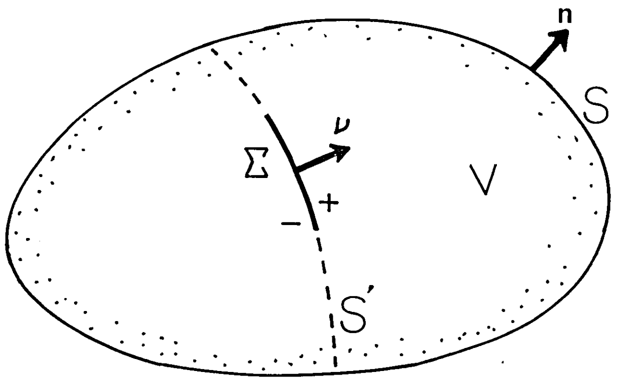


Fig. 1. A magneto-elastic body V with its surface S and an internal dislocation surface Σ. **n** and $\boldsymbol{\nu}$ are unit normal vectors. S′ is a subdivisional surface including Σ.

$$\left.\begin{aligned} &\boldsymbol{\nu}=\boldsymbol{\nu}_-=-\boldsymbol{\nu}_+ && \text{on } \Sigma \\ &\boldsymbol{n}_{S'+}=-\boldsymbol{n}_{S'-} && \text{on } S' \text{ besides } \Sigma \end{aligned}\right\} \tag{3.1}$$

The contribution of the surface integral from S' except for Σ becomes zero owing to (3.1). Hence we have

$$W_k=W_k^{(\Sigma)}+W_k^{(S)}+W_k^{(F)} \tag{3.2}$$

where

$$W_k^{(\Sigma)}=C_k\iint_\Sigma\left\{\left[\text{grad } u_k(\boldsymbol{r}')-\frac{2(\lambda+\mu)}{3\lambda+2\mu}\Delta\boldsymbol{m}_k\right]_-^+\cdot\boldsymbol{\nu}\left(\frac{1}{\rho}\right)\right.$$
$$\left.-[u_k(\boldsymbol{r}')]_-^+\frac{\partial}{\partial\nu'}\left(\frac{1}{\rho}\right)\right\}d\Sigma \tag{3.3}$$

$$W_k^{(S)}=C_k\iint_S\left[\left\{-\text{grad } u_k(\boldsymbol{r}')+\frac{2(\lambda+\mu)}{3\lambda+2\mu}\Delta\boldsymbol{m}_k\right\}\cdot\boldsymbol{n}\left(\frac{1}{\rho}\right)+\{u_k(\boldsymbol{r}')\}\frac{\partial}{\partial n'}\left(\frac{1}{\rho}\right)\right]dS \tag{3.4}$$

and

$$W_k^{(F)}=\frac{C_k}{3\lambda+2\mu}\iiint_V\frac{F_k}{\rho}dV \tag{3.5}$$

The symbol $[\ ]^{\pm}$ represents the discontinuity of the quantity within the bracket.

Let us investigate the characteristics of the dislocation-related potential $W_k^{(\Sigma)}$. In case of natural dislocation events within the Earth such as seismic faulting, dyke formation by intrusive magmas and so on, no external force acts upon the dislocation surface. Hence the traction across Σ must be continuous. Then we obtain

$$W_k^{(\Sigma)}=C_k\iint_\Sigma\left\{\left[\text{grad } u_k(\boldsymbol{r}')-\frac{(\lambda+\mu)(\lambda+2\mu)}{\mu(3\lambda+2\mu)}(\text{div } \boldsymbol{u})e_k\right]_-^+\cdot\boldsymbol{\nu}\left(\frac{1}{\rho}\right)-[u_k(\boldsymbol{r}')]^{\pm}\frac{\partial}{\partial\nu}\left(\frac{1}{\rho}\right)\right\}d\Sigma \tag{3.6}$$

Further, we often prefer a simple model, in which all the stress components be continuous across the dislocation surface, as is the case with the Volterra dislocations. In this special case, all the spatial derivatives of the displacement become continuous, and the single layer term in (3.6) vanishes. Eq. (3.6) reduces to

$$W_k^{(\Sigma)}=-C_k\iint_\Sigma[u_k(\boldsymbol{r}')]^{\pm}\frac{\partial}{\partial\nu}\left(\frac{1}{\rho}\right)d\Sigma \tag{3.7}$$

Summing up for $k=x,\ y,\ z$, we have a resultant potential $W^{(\Sigma)}$ as follows:

$$W^{(\Sigma)}=\iint_\Sigma m\frac{\partial}{\partial\nu}\left(\frac{1}{\rho}\right)d\Sigma \tag{3.8}$$

where

$$m=-C_0\boldsymbol{e}_J\cdot\Delta\boldsymbol{u} \tag{3.9}$$

$$C_0=\frac{1}{2}\beta J\mu\frac{3\lambda+2\mu}{\lambda+\mu} \tag{3.10}$$

$\boldsymbol{e}_J$ is the unit vector in the magnetized direction, $\Delta\boldsymbol{u}$ the vector representation of the displacement discontinuity and J the intensity of the total magnetization. Thus the piezomagnetic field accompanying a dislocation surface is equivalent to that of a double layer Σ, whose magnetic moment is given by (3.9).

At far-field distances, where we may regard the earthquake fault as a point source, we have an expression:

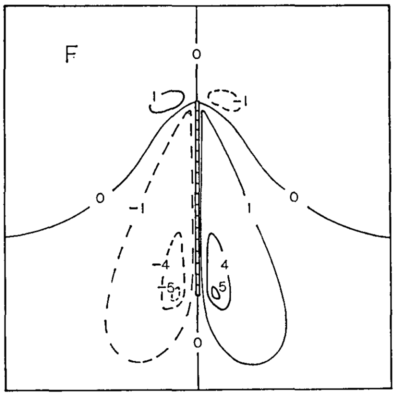
$$W^{(\Sigma)}=M\frac{\partial}{\partial\nu}\left(\frac{1}{\rho}\right) \tag{3.11}$$

where

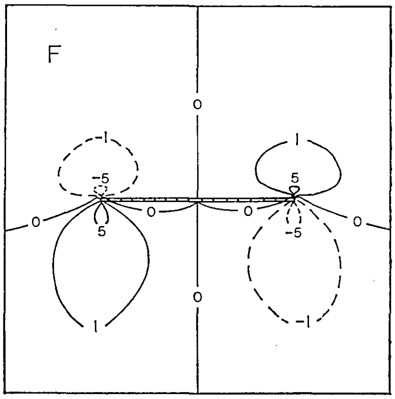
$$\left.\begin{aligned} M&=-\frac{1}{2}\beta\frac{3\lambda+2\mu}{\lambda+\mu}JM_0\cos\varphi \\ \varphi&=(\boldsymbol{J},\ \boldsymbol{\Delta u}) \end{aligned}\right\} \tag{3.12}$$

M_0 is the seismic moment (i.e. $M_0=\mu\Delta uA$; A is the fault area. See text: e.g. AKI and RICHARDS 1980). φ indicates the angle between the magnetization and the slip vector. The magnetic moment given by (3.9) or (3.12) is suitable for measuring the seismomagnetic effect. For an earthquake fault, we may call M the total seismomagnetic moment and m the seismomagnetic moment density.

The important result in this section is that the dislocation surface within a magneto-elastic body is nothing but the sources and sinks of the magnetic lines of force. The surface magnetic source distribution in eq. (3.6) corresponds to the magnetic source equivalents as named by BONAFEDE and SABADINI (1980). In general Somigliana dislocations, we have not only the double layer but also the single layer. The total intensity of these monopoles summed over the dislocation surface should vanish, or else their counterpart should appear in the surface integral term $W_k^{(S)}$.



(a)



(b)

Fig. 2a. The total field magnetic change associated with a vertical rectangular strike-slip fault with the magnetic N-S orientation (reproduced from Fig. 8(d) in paper I).

Fig. 2b. The total field magnetic change of the same fault but with the magnetic E-W orientation (reproduced from Fig. 10(d) in paper I).

The concept of the seismomagnetic moment is useful for intuitively understanding the dislocation-related magnetic field near the dislocation sources. Note that the type of dislocations is not limited to the shearing offset but is allowed for the tension crack. Moreover, eq. (3.9) tells us that the intensity of the magnetic source equivalent or seismomagnetic moment depends on the angle between the magnetization and the dislocation vector. In some instances the seismomagnetic moment becomes null, e.g. an $E-W$ striking fault. In fact the magnetic field significantly diminishes in the case of the $E-W$ striking vertical transcurrent fault as compared with the $N-S$ fault, which is demonstrated in Fig. 2a and 2b as reproduced from paper I. However, the magnetic field does not completely disappear, owing to additional terms $W_k^{(S)}$ and $W_k^{(F)}$ in the total potential (3.2).

The Seismomagnetic Effect of the 1946 Nankaido Earthquake

Now we are to interpret the coseismic magnetic change associated with the 1946 great Nankaido Earthquake of M 8.1, Japan (KATO and UTASHIRO 1949: Fig. 3). This is one of the most notable seismomagnetic effect detected with a reliable observation technique, as reviewed by NAGATA (1969). Fig. 4 shows the fault model of the Nankaido earthquake determined by IWASAKI and MATSU'URA (1981). Their model consists of two thrust-type fault planes. Notice that Katsu-ura observatory is located

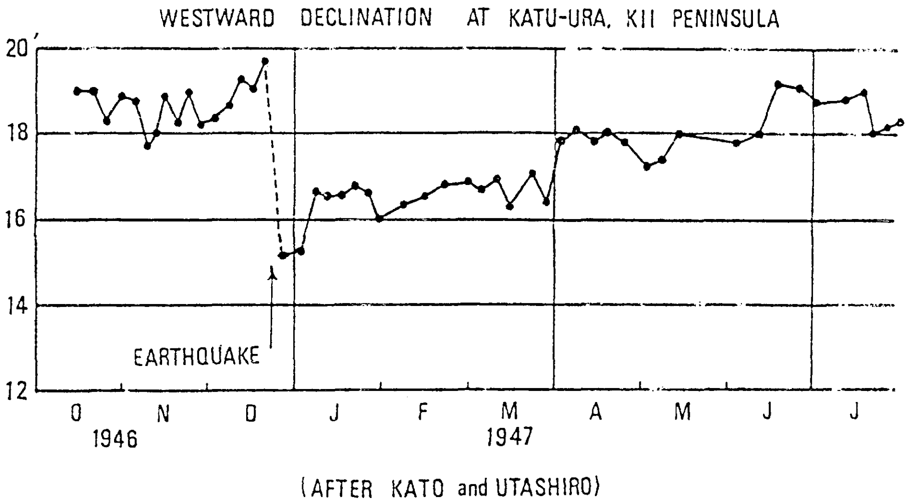


Fig. 3. Changes in the declination at Katsu-ura Hydrographic Observatory associated with the Great Nankaido Earthquake of 1946. Unit in minutes of arc. Westward positive. Referenced to Kakioka Magnetic Observatory. (Original: KATO and UTASHIRO 1949. Reproduced from NAGATA 1969).

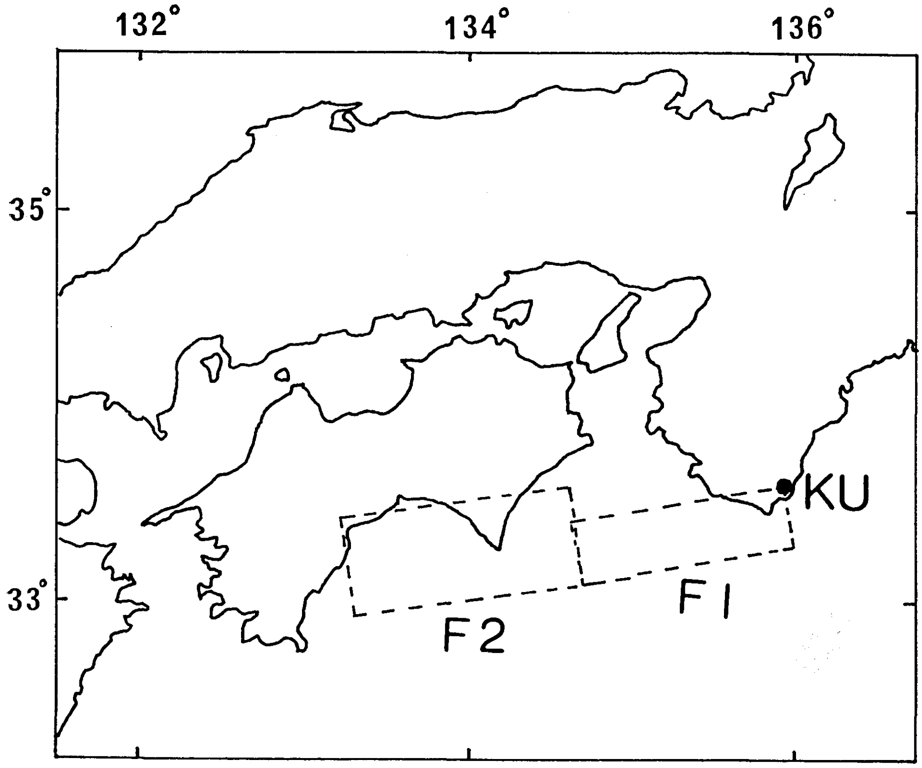


Fig. 4. A fault model of the 1946 Nankaido Earthquake, consisting of two thrust-type faults F1 and F2. The black circle at the eastern edge of F1 indicates Katsu-ura observatory. (After IWASAKI and MATSU'URA 1981).

Table 1. Fault parameters of F-1 and the field direction (After IWASAKI and MATSU'URA 1981)

fault length	122 km
fault width	38 km
depth of burial	2.3 km
dislocation (thrust)	3.0 m
dip angle	158 deg.
slip angle	−112 deg.
mag. inclination	47 deg.
mag. declination (West)	6 deg.

just on the eastern edge of the fault F-1. Fault parameters and the direction of the ambient magnetic field are listed in Table 1.

The slip vector is roughly anti-parallel to the geomagnetic field, so that the seismomagnetic moment of this earthquake is expected to be large with its magnetized direction being upward. A uniform slip over the whole fault surface produces a uniformly magnetized plate magnet, whose magnetic field is equivalent to that of a line current along the perimeters of the fault. The current circulates anti-clockwise as viewed from above. Katsu-ura is just above a line current flowing northward along the eastern periphery of the fault, which results in the *horizontally east* magnetic field. Thus an idea of the seismomagnetic moment successfully explains the eastward coseismic change in the declination as depicted in Fig. 3.

Another remarkable phenomenon is the gradual recovery of the coseismic change of up to $30-40\,nT$ after a five months' period. HAMANO (1979) argued that the coseismic change may be ascribable to the production of PRM, and the postquake recovery to its disappearance because of the magnetic instability. In this paper, we do not deal with the PRM and the irreversible piezomagnetic effect, which are not in linear relation to stresses (NAGATA 1970). It is not clear at the present stage that the PRM effect could interpret the coseismic change. Another possibility was also suggested by Hamano that any gradual crustal movement might have caused the postseismic changes in the D component. If this were the case, it would be natural to presume an eastward extension of aseismic faulting. The equivalent line current beneath Katsu-ura recedes eastward: the horizontally eastward magnetic field diminishes while the upward field prevails instead. Thus we might expect the decrease in the inclination at Katsu-ura during the postquake period. Unfortunately, observations of the geomagnetic dip were not conducted in those days at the Katsu-ura hydrographic observatory (UTASHIRO 1982: personal communication).

4. Tectonic Strain and the Corresponding Magnetic Field

In paper I, we discussed the general relationship between the crustal strain and tectonomagnetic changes as deduced from the representation theorem. Some portions of the previous argument should be altered according to the revised theorem (2.14).

In an infinite magneto-elastic medium, the correspondence between the piezomagnetic potential and the displacement field is most precise. Disregarding the source terms $W_k^{(\Sigma)}$ and $W_k^{(F)}$, the potential is simply proportional to the displacement:

$$W_k(\boldsymbol{r})=4\pi C_k u_k(\boldsymbol{r}) \tag{4.1}$$

The total field change in the medium is given by

$$\Delta F = -4\pi C_0 \frac{\partial u_f}{\partial f} \tag{4.2}$$

where

$$u_f = u_x \cos I_0 + u_z \sin I_0 \tag{4.3}$$

$$\frac{\partial}{\partial f} = \boldsymbol{e}_f \cdot \boldsymbol{\nabla} = \cos I_0 \frac{\partial}{\partial x} + \sin I_0 \frac{\partial}{\partial z} \tag{4.4}$$

C_0 is already defined by (3.10). The unit vector $\boldsymbol{e}_f$ indicates the direction of the ambient field which is assumed to coincide with that of the uniform magnetization $\boldsymbol{e}_J$. For a model earth with material constants $\beta = 1.0 \times 10^{-4}$ bar^{-1}, $J = 1.0 \times 10^{-3}$ emu/cc and $\lambda = \mu = 3.5 \times 10^{11}$ cgs, $1\,nT$ change in the total field corresponds to a strain change of 1.8×10^{-5}. This gives a measure of the sensitivity of a magnetometer as a strain gauge. Since we follow the standard sign convention of the stress field, the simple extension along $\boldsymbol{e}_f$ axis brings about the decrease in the total field.

Actually, magnetic measurements are made in the free space, where the analogy between the strain and the piezomagnetic field fails. The surface magnetic field, however, does contain some information of the crustal strain just at the observation site. Applying the formula (2.16) to a magnetic medium bounded by the free surface $z=0$ and the Currie point isotherm $z=H$, we obtain the potential in the free space $(z<0)$:

$$W_k(\boldsymbol{r}) = W_k^{(0)} + W_k^{(H)} + W_k^{(\Sigma)} + W_k^{(F)} \tag{4.5}$$

where

$$W_k^{(0)} = C_k \iint_{-\infty}^{\infty} \left[\left\{ \frac{\partial u_k}{\partial z'} - \Delta m_{kz} \right\} \frac{1}{\rho} - \{u_k(\boldsymbol{r}')\} \frac{\partial}{\partial z'} \left(\frac{1}{\rho} \right) \right]_{z'=+0} dx'dy' \tag{4.6}$$

$$W_k^{(H)} = C_k \iint_{-\infty}^{\infty} \left[\left\{ -\frac{\partial u_k}{\partial z'} + \Delta m_{kz} \right\} \frac{1}{\rho} + \{u_k(\boldsymbol{r}')\} \frac{\partial}{\partial z'} \left(\frac{1}{\rho} \right) \right]_{z'=H} dx'dy' \tag{4.7}$$

We take the x axis in the magnetic north direction and the z axis positive downward. Taking into account the traction free boundary condition at $z'=0$, we obtain

$$\left.\begin{aligned} W_x^{(0)} &= C_x \iint_{-\infty}^{\infty} \left[\left\{ \frac{\partial u_x}{\partial z'} \right\} \frac{1}{\rho} - \{u_x(\boldsymbol{r}')\} \frac{\partial}{\partial z'} \left(\frac{1}{\rho} \right) \right]_{z'=+0} dx'dy' \\ W_z^{(0)} &= C_z \iint_{-\infty}^{\infty} \left[-\frac{\lambda+2\mu}{\lambda} \left\{ \frac{\partial u_z}{\partial z'} \right\} \frac{1}{\rho} - \{u_z(\boldsymbol{r}')\} \frac{\partial}{\partial z'} \left(\frac{1}{\rho} \right) \right]_{z'=+0} dx'dy' \end{aligned}\right\} \tag{4.8}$$

Now we apply two theorems on the derivatives of single and double layer potentials when a point approaches the source layer. Proper conditions should be assumed for the surface curvature and the smoothness of the surface density distribution (KELLOGG 1929). The normal derivatives of a single layer potential $U^{(s)}$ with a surface density $\sigma(\boldsymbol{p})$ on the positive and negative side of the layer are given by

$$\left.\begin{aligned}\left(\frac{\partial U^{(s)}}{\partial n}\right)_+ &= -2\pi\sigma(\boldsymbol{p})+\iint_S \sigma(\boldsymbol{p}')\frac{\partial}{\partial n'}\left(\frac{1}{\rho}\right)dS' \\ \left(\frac{\partial U^{(s)}}{\partial n}\right)_- &= 2\pi\sigma(\boldsymbol{p})+\iint_S \sigma(\boldsymbol{p}')\frac{\partial}{\partial n'}\left(\frac{1}{\rho}\right)dS'\end{aligned}\right\} \tag{4.9}$$

On the other hand, tangential derivatives of a double layer potential $U^{(d)}$ with a surface moment density $\mu(\boldsymbol{p})$ have a similar property:

$$\left.\begin{aligned}\left(\frac{\partial U^{(d)}}{\partial t}\right)_+ &= 2\pi\frac{\partial \mu(\boldsymbol{p})}{\partial t}+\iint_S \mu(\boldsymbol{p}')\frac{\partial^2}{\partial t\partial n'}\left(\frac{1}{\rho}\right)dS' \\ \left(\frac{\partial U^{(d)}}{\partial t}\right)_- &= -2\pi\frac{\partial \mu(\boldsymbol{p})}{\partial t}+\iint_S \mu(\boldsymbol{p}')\frac{\partial^2}{\partial t\partial n'}\left(\frac{1}{\rho}\right)dS'\end{aligned}\right\} \tag{4.10}$$

A proof of this theorem is found in COURANT and HILBERT (1937). (The sign of the first terms on the righthand side should be exchanged in eq. (6.17) in paper I, which was a misprint.) Since the source layer is simply a plane, integral terms in eqs. (4.9) and (4.10) vanish.

Applying these formulas to derivatives of $W_x^{(0)}$ and $W_z^{(0)}$, we obtain the magnetic field arising from the free surface potential $W_k^{(0)}$:

$$\Delta X^{(0)} = -2\pi C_0\frac{\partial u_f}{\partial x} - C_0\frac{\partial U_p^{(0)}}{\partial x} \tag{4.11a}$$

$$\Delta Y^{(0)} = -2\pi C_0\frac{\partial u_f}{\partial y} - C_0\frac{\partial U_p^{(0)}}{\partial y} \tag{4.11b}$$

$$\Delta Z^{(0)} = -2\pi C_0\frac{\partial u_p}{\partial z} - C_0\frac{\partial V_f^{(0)}}{\partial z} \tag{4.11c}$$

where

$$u_p = u_f - \frac{2(\lambda+\mu)}{\lambda}u_z \sin I_0 \tag{4.12}$$

$$U_p^{(0)} = \iint_{-\infty}^{\infty}\left[\frac{\partial u_p}{\partial z'}\frac{1}{\rho}\right]_{z'=+0}dx'dy' \tag{4.13}$$

$$V_f^{(0)} = \iint_{-\infty}^{\infty}\left[u_f\frac{\partial}{\partial z'}\left(\frac{1}{\rho}\right)\right]_{z'=+0}dx'dy' \tag{4.14}$$

The contribution to the total field is given by

$$\Delta F^{(0)} = -2\pi C_0\left(\frac{\partial u_f}{\partial f} - \frac{2(\lambda+\mu)}{\lambda}\frac{\partial u_z}{\partial z}\sin I_0\right)$$

$$-C_0\left(\frac{\partial U_p^{(0)}}{\partial x}\cos I_0+\frac{\partial V_f^{(0)}}{\partial z}\sin I_0\right) \tag{4.15}$$

The first term of eq. (4.15) represents a linear combination of the surface strain components just at the observation site. In paper I, the corresponding term was simply $-2\pi C_0\frac{\partial u_f}{\partial f}$(i. e. eq. (6.22) in paper I: the sign is opposite because of new stress convention for β), which was incorrect owing to overlooking the boundary condition (2.10b).

If the first term of $\Delta F^{(0)}$ in eq. (4.15) is a representative of the resultant total field as estimated from the total piezomagnetic potential (4.5), the tectonomagnetic observation is nothing but a kind of strain measurement. However, things are different from such simple circumstances. In the next section, we will apply the representation theorem to obtain the piezomagnetic field associated with the Mogi model, in which we will find each term in eq. (4.5) having the same order of magnitude: some cancel and other augment each other. Moreover, the latter integral terms in eq. (4.15) are comparable with the first one. In the special case of the Mogi model, $\Delta F^{(0)}$ is actually zero: the first and the latter terms completely cancel each other.

Thus we cannot say anything definite about the strain change just at the observation site by the magnetic measurement. Although the direct correspondence between the piezomagnetic field and the pointwise strain fails in the free space, we may say that a few nT changes are expected around the strain field of 10^{-5} with a magnetization of 10^{-3} emu/cc.

5. Piezomagnetic Field Associated with the Mogi Model—Application of the Theory

The Mogi model was introduced to interpret surface deformations accompanying volcanic eruptions (MOGI 1958). Mechanically it is a center of dilatation within a semi-infinite elastic solid. The piezomagnetic change associated with the Mogi model was computed numerically by DAVIS (1976). SASAI (1980) analytically solved the problem for the point pressure source. The latter approach was essentially based on the dipole force law (2.12), in which the volumetric integration was achieved by the double Fourier transform and its inverse. By applying the representation theorem (2.16) we can easily get at the same result.

The displacement field due to a center of dilatation at a point $A(0, 0, D)$ within a semi-infinite elastic medium is given by (MINDLIN and CHENG 1950, YAMAKAWA 1955)

$$\frac{2\mu}{C}u_x=\frac{x}{R_1^3}+\frac{\lambda+3\mu}{\lambda+\mu}\frac{x}{R_2^3}-\frac{6xz(z+D)}{R_2^5} \tag{5.1a}$$

$$\frac{2\mu}{C}u_z=\frac{z-D}{R_1^3}+\frac{(\lambda-\mu)z-(\lambda+3\mu)D}{(\lambda+\mu)R_2^3}-\frac{6z(z+D)^2}{R_2^5} \tag{5.1b}$$

where

$$\left.\begin{aligned} R_1&=\{x^2+y^2+(z-D)^2\}^{1/2}\\ R_2&=\{x^2+y^2+(z+D)^2\}^{1/2}\end{aligned}\right\} \tag{5.2}$$

The moment of a strain nucleus C may be given by

$$C=-\frac{1}{2}a^3\Delta P \tag{5.3}$$

provided that a small sphere of radius a centered at A is pumped by hydrostatic pressure ΔP. The upper crustal layer bounded by $z=0$ and $z=H$ is assumed uniformly magnetized. When the Currie point isotherm is deeper than the pressure source, the singular point A is included in the integrated area. We then subdivide the magnetic layer into two parts: the upper layer is bounded by $z=0$ and $z=D-\varepsilon_1$ ($\varepsilon_1\to0$), while the lower is bounded by $z=D+\varepsilon_2$ ($\varepsilon_2\to0$) and $z=H$. According to the surface integral formula (2.16), the total piezomagnetic potential consists of contributions from these four sheets.

The contribution from the free surface is represented by

$$\frac{2\mu}{C}W_x^{(0)}=-\frac{2(\lambda+2\mu)}{\lambda+\mu}C_x\iint_{-\infty}^{\infty}\left[\frac{3x'D}{R_0^5}\frac{1}{\rho_0}+\frac{x'}{R_0^3}\frac{z}{\rho_0^3}\right]dx'dy' \tag{5.4a}$$

$$\frac{2\mu}{C}W_z^{(0)}=\frac{2(\lambda+2\mu)}{\lambda+\mu}C_z\iint_{-\infty}^{\infty}\left[-\left(\frac{1}{R_0^3}-\frac{3D^2}{R_0^5}\right)\frac{1}{\rho_0}+\frac{D}{R_0^3}\frac{z}{\rho_0^3}\right]dx'dy' \tag{5.4b}$$

where

$$\left.\begin{aligned} R_0&=(x'^2+y'^2+D^2)^{1/2}\\ \rho_0&=\{(x-x')^2+(y-y')^2+z^2\}^{1/2}\end{aligned}\right\} \tag{5.5}$$

By taking the Fourier transform, the convolution integrals in (5.4) are converted into the products of each transform in the wave number domain. All the required Fourier transforms and their inverses are tabulated in SASAI (1979). The Fourier transforms of $W_k^{(0)}$'s are given as follows:

$$\begin{aligned}\frac{2\mu}{C}\overline{W}_x^{(0)}&=4\pi\frac{\lambda+2\mu}{\lambda+\mu}C_x\left\{\frac{i\xi}{\alpha}e^{-\alpha(D-z)}-\frac{i\xi}{\alpha}e^{-\alpha(D-z)}\right\}\\ &=0\end{aligned} \tag{5.6a}$$

$$\frac{2\mu}{C}\overline{W}_z^{(0)}=4\pi\frac{\lambda+2\mu}{\lambda+\mu}C_z\left\{e^{-\alpha(D+|z|)}+\frac{z}{|z|}e^{-\alpha(D+|z|)}\right\}$$

$$=0 \qquad (\because\ |z|=-z,\ z<0) \tag{5.6b}$$

Thus we have

$$W_x^{(0)}=W_z^{(0)}=0 \tag{5.7}$$

Although the integrands in eq. (5.4) are not pointwise zero, the single and double layer terms are equivalent to dipoles placed at A and they completely cancel each other. Such a circumstance is not generally seen because the vertical strike-slip fault, for example, has a non-zero $W_k^{(0)}$ component.

The contribution from the Currie point isotherm is expressed as follows:

$$\frac{2\mu}{C}W_x^{(H)}=C_x\iint_{-\infty}^{\infty}\Bigg[3\Bigg\{-\frac{3\lambda+4\mu}{3\lambda+2\mu}\frac{x'(H-D)}{R_1^5}+\Bigg(-\Bigg(\frac{\lambda-\mu}{\lambda+\mu}+\frac{6\mu}{3\lambda+2\mu}\Bigg)H$$

$$+\Bigg(\frac{\lambda+3\mu}{\lambda+\mu}-\frac{2\mu}{3\lambda+2\mu}\Bigg)D\Bigg)\frac{x'}{R_2^5}+\frac{10(3\lambda+4\mu)}{3\lambda+2\mu}\frac{H(H+D)^2x'}{R_2^7}\Bigg\}\frac{1}{\rho_H}$$

$$-\Bigg\{-\frac{x'}{R_1^3}-\frac{\lambda+3\mu}{\lambda+\mu}\frac{x'}{R_2^3}+\frac{6x'H(H+D)}{R_2^5}\Bigg\}\frac{z-H}{\rho_H^3}\Bigg]dx'dy' \tag{5.8a}$$

$$\frac{2\mu}{C}W_z^{(H)}=C_z\iint_{-\infty}^{\infty}\Bigg[\Bigg\{-\frac{3\lambda+4\mu}{3\lambda+2\mu}\Bigg[-\frac{1}{R_1^3}+\frac{3(H-D)^2}{R_1^5}-\frac{\lambda-\mu}{\lambda+\mu}\frac{1}{R_2^3}$$

$$+3\Bigg(\frac{7\lambda+5\mu}{\lambda+\mu}H+\frac{\lambda-\mu}{\lambda+\mu}D\Bigg)\frac{H+D}{R_2^5}-\frac{30H(H+D)^3}{R_2^7}\Bigg]$$

$$+\frac{8\mu}{3\lambda+2\mu}\Bigg[\frac{1}{R_2^3}-\frac{3(H+D)^2}{R_2^5}\Bigg]\Bigg\}\frac{1}{\rho_H}-\Bigg\{-\frac{H-D}{R_1^3}$$

$$+\Bigg(-\frac{\lambda-\mu}{\lambda+\mu}H+\frac{\lambda+3\mu}{\lambda+\mu}D\Bigg)\frac{1}{R_2^3}+\frac{6H(H+D)^2}{R_2^5}\Bigg\}\frac{z-H}{\rho_H^3}\Bigg]dx'dy' \tag{5.8b}$$

where

$$\left.\begin{aligned}R_1&=\{x'^2+y'^2+(H-D)^2\}^{1/2}\\ R_2&=\{x'^2+y'^2+(H+D)^2\}^{1/2}\\ \rho_H&=\{(x-x')^2+(y-y')^2+(z-H)^2\}^{1/2}\end{aligned}\right\} \tag{5.9}$$

Taking the Fourier transforms of (5.8a) and (5.8b) and inverting them, we obtain

$$\frac{2\mu}{C}W_x^{(H)}=4\pi C_x\Big[-\frac{\mu}{3\lambda+2\mu}\frac{x}{\rho_3^3}+\frac{6(\lambda+\mu)}{3\lambda+2\mu}H\frac{3xD_3}{\rho_3^5}$$

$$+\begin{cases}-\dfrac{3(\lambda+\mu)}{3\lambda+2\mu}\dfrac{x}{\rho_2^3}\Big] & (H>D)\\[2ex] \dfrac{\mu}{3\lambda+2\mu}\dfrac{x}{\rho_1^3}\Big] & (H<D)\end{cases} \tag{5.10a}$$

$$\frac{2\mu}{C}W_z^{(H)}=4\pi C_z\Big[\frac{\mu}{3\lambda+2\mu}\frac{D_3}{\rho_3^3}+\frac{6(\lambda+\mu)}{3\lambda+2\mu}H\left(-\frac{1}{\rho_3^3}+\frac{3D_3^2}{\rho_3^5}\right)$$

$$+\begin{cases}-\dfrac{3(\lambda+\mu)}{3\lambda+2\mu}\dfrac{D_2}{\rho_2^3}\Big] & (H>D)\\[2ex] -\dfrac{\mu}{3\lambda+2\mu}\dfrac{D_1}{\rho_1^3}\Big] & (H<D)\end{cases} \tag{5.10b}$$

where

$$\left.\begin{aligned}&\rho_i=(x^2+y^2+D_i^2)^{1/2}\\ &D_1=D-z,\ D_2=2H-D-z,\ D_3=2H+D-z\end{aligned}\right\} \tag{5.11}$$

Finally, the contribution from the pressure source $W_k^{(p)}$ is considered as the sum of $W_k^{(D-\varepsilon_1)}$ and $W_k^{(D+\varepsilon_2)}$ passing to a limit by diminishing ε_1 and ε_2 infinitesimally:

$$W_k^{(p)}=\lim_{\substack{\varepsilon_1\to 0\\ \varepsilon_2\to 0}}\{W_k^{(D-\varepsilon_1)}+W_k^{(D+\varepsilon_2)}\} \tag{5.12}$$

$W_k^{(D-\varepsilon_1)}$ is obtained simply by replacing H with $D-\varepsilon_1$ in eqs. (5.8), while $W_k^{(D+\varepsilon_2)}$ is equal to the negative of eqs. (5.8) replacing H with $D+\varepsilon_2$ because the outward normal vector points upward. Correspondingly we have solutions for $W_k^{(D-\varepsilon_1)}$ as the $H<D$ case of eqs. (5.10) in place of H with $D-\varepsilon_1$ and for $W_k^{(D+\varepsilon_2)}$ as the negative of the $H>D$ case of eqs. (5.10) in place of H with $D+\varepsilon_2$. Thus we have

$$\frac{2\mu}{C}W_x^{(p)}=4\pi C_x\frac{3\lambda+4\mu}{3\lambda+2\mu}\frac{x}{\rho_1^3} \tag{5.13a}$$

$$\frac{2\mu}{C}W_z^{(p)}=4\pi C_z\frac{D_1}{\rho_1^3} \tag{5.13b}$$

These terms appear only when the pressure source is buried in the magnetic layer.

Summing up (5.7), (5.10) and (5.13), we arrive at the piezomagnetic potential associated with the Mogi model:

$$\frac{2\mu}{C}W_x=4\pi C_x\left[\frac{\mu}{3\lambda+2\mu}\left(\frac{x}{\rho_1^3}-\frac{x}{\rho_3^3}\right)+\frac{6(\lambda+\mu)}{3\lambda+2\mu}H\frac{3xD_3}{\rho_3^5}\right.$$

$$+\begin{cases}\left.\dfrac{3(\lambda+\mu)}{3\lambda+2\mu}\left(\dfrac{x}{\rho_1^3}-\dfrac{x}{\rho_2^3}\right)\right] & (H>D)\\[2ex] \left.0\qquad\qquad\right] & (H<D)\end{cases}\tag{4.14a}$$

$$\frac{2\mu}{C}W_z=4\pi C_z\left[-\frac{\mu}{3\lambda+2\mu}\left(\frac{D_1}{\rho_1^3}-\frac{D_3}{\rho_3^3}\right)+\frac{6(\lambda+\mu)}{3\lambda+2\mu}H\left(-\frac{1}{\rho_3^3}+\frac{3D_3^2}{\rho_3^5}\right)\right.$$

$$+\begin{cases}\left.\dfrac{3(\lambda+\mu)}{3\lambda+2\mu}\left(\dfrac{D_1}{\rho_1^3}-\dfrac{D_2}{\rho_2^3}\right)\right] & (H>D)\\[2ex] \left.0\qquad\qquad\right] & (H<D)\end{cases}\tag{5.14b}$$

These are quite identical with previously obtained solutions by SASAI (1979), in which more complicated and lengthy manipulations are required.

The same procedure as exhibited in this section is applicable to derive elementary piezomagnetic potentials. It would diminish much of the effort of volumetric integrations as conducted in paper I. Applying the representation theorem to dislocation sources within a semi-infinite elastic medium, we can discriminate each term of elementary potentials as originating from the Earth's surface, the Currie point isotherm and the dislocation source respectively.

6. Discussion

Since the magnetic layer occupies only a limited portion of the crust, we must be careful to apply the concept of the seismomagnetic moment to the actual Earth. How can we imagine the seismomagnetic effect if the earthquake fault intersects the Currie point isotherm or the whole fault lies beneath the Currie depth? To answer the question, we must know the displacement field caused by dislocations in a semi-infinite elastic medium (i. e. MARUYAMA 1964). In paper I, we have obtained the piezomagnetic potentials associated with elementary dislocations on the basis of Maruyama's solutions. The elementary piezomagnetic potentials consist of dipoles and multipoles at the dislocation source and its image points. This is true even if the dislocation point is located beneath the magnetized layer.

Fig. 5 illustrates a typical example: i. e. the equivalent magnetic sources of the seismomagnetic effect associated with a two-dimensional

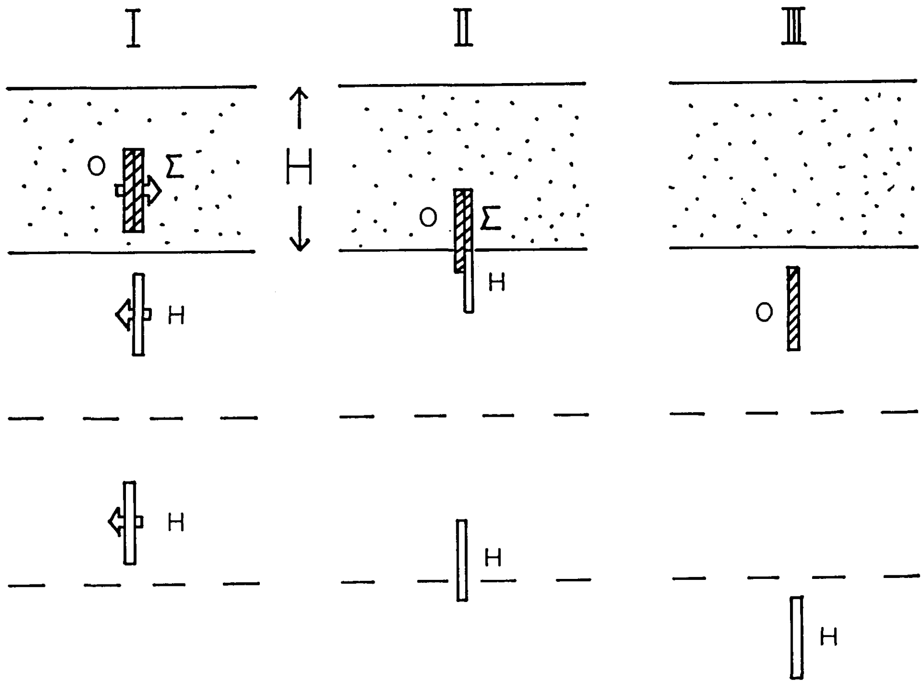


Fig. 5. Equivalent plate magnets showing sources of the seismomagnetic effect associated with an infinitely long vertical strike-slip fault in the N-S orientation. H is the Currie depth. 0, H and Σ indicate that the magnets correspond to the surface integral terms $W_x^{(0)}$, $W_x^{(H)}$ and $W_x^{(\Sigma)}$ respectively. (Rewritten from Fig. 7 in paper I).

vertical strike-slip fault. Now we can interpret these magnets as those corresponding to three surface integral terms $W_x^{(0)}$, $W_x^{(H)}$ and $W_x^{(\Sigma)}$ in eq. (4.5), which are specified in the figure. Clearly the seismomagnetic moment alone is insufficient to totally represent the coseismic magnetic field. When the fault lies within the magnetic crust (case I), the overall feature of the coseismic change can be described by the seismomagnetic moment, but its intensity should be doubled owing to the term $W_x^{(0)}$. When faulting occurs beneath the Currie depth (case III), the dislocation-related term $W_x^{(\Sigma)}$ does not exist. Nonetheless the term $W_x^{(0)}$ produces a bar magnet just at the fault position, its intensity being the same as the seismomagnetic moment which might be defined in a semi-infinitely magnetized Earth. Case II is rather complicated, in which the Currie depth term $W_x^{(H)}$ has a certain influence. In any case we may qualitatively presume the coseismic magnetic field by a plate magnet at the fault position, whose magnetization is given by the seismomagnetic moment. The rigorous solution should be obtainable by integrating elementary potentials over a dislocation surface. Seismomagnetic calculations of a rectangular fault with an arbitrary orientation, dip angle and slip vector are now in progress and will be published elsewhere (OHSHIMAN and SASAI, in preparation).

In case the magneto-elastic body is too complicatedly shaped to obtain analytic expressions for the displacement field, we need numerical values of the displacement and its derivatives at discrete points on the surface. The boundary element method (BEM) seems suitable for such a purpose (see text: e.g. BREBBIA and WALKER 1980). The surface integral method developed in this paper is similar to the BEM. Actually the basic idea of reducing the volume integral to a surface one is common in both methods. Only the boundary values of the displacement field are required to get at the tectonomagnetic field. They would be obtained by numerically solving the boundary integral equation for the static equilibrium problem (2.4).

The representation theorem tells us that surfaces of an isolated magnetic body are sources and sinks of the magnetic lines of force. Hence we may expect the enhancement of tectonomagnetic signals near the edge of differently magnetized blocks even if these boundaries are not mechanically singular points. In computing the tectonomagnetic field in such cases, the surface integral representation (2.16) will provide us with a powerful means for more accurate and less laborious modelling than the ordinary volumetric integrations.

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References

AKI, K. and P.G. RICHARDS, 1980, *Quantitative Seismology, Theory and Methods*, Vol. 1, Chapter 3, W.H. Freeman and Co., San Francisco.

BONAFEDE, M. and E. BOSCHI, 1980, A physical model of magnetic anomalies related to tectonic sources, in Dziewonskii, A.M. and E. Boschi (Ed.): *Proceedings of the International School of Physics≪Enrico Fermi≫, Cource LXXVIII, Physics of the Earth's Interior*, North-Holland Pub. Co., Amsterdam, 650-668.

BONAFEDE, M. and R. SABADINI, 1980, A theoretical approach to the seismo-magnetic effect, *Boll. Geof. Teor. Appl.*, 22, 105-116.

BREBBIA, C.A. and S. WALKER, 1980, *Boundary Element Techniques in Engineering*, Butterworth & Co., London.

COURANT, R. and D. HILBERT, 1937, *Methoden der Mathematischen Physik*, Band 2, Kapitel 4, Springer, Berlin.

DAVIS, P.M., 1976, The computed piezomagnetic anomaly field for Kilauea Volcano, Hawaii, *J. Geomag. Geoelectr.*, 28, 113-122.

HAMANO, Y., 1979, Pressure demagnetization of NRM of rocks by uniaxial compression (in Japanese), *Soc. Ter. Mag. Electr. Japan Program and Abstracts, 1979 Spring Meeting*, p. 170.

HAO, J.Q., L.M. HASTIE and F.D. STACEY, 1982, Theory of the seismomagnetic effect: a reassessment, *Phys. Earth Planet. Int.*, 28, 129-140.

IWASAKI, T. and M. MATSU'URA, 1981, An optimal fault model of the 1946 Nankaido Earthquake by the inversion analysis of geodetic data (in Japanese), *Seism. Soc. Japan Program and Abstracts, 1981 Spring Meeting*, p. 135.

KATO, Y. and S. UTASHIRO, 1949, On the changes of the terrestrial magnetic field accompanying the Great Nankaido Earthquake of 1946, *Sci. Rep. Tohoku Univ., Sendai, Japan*, Ser. 5, **1**, 40-41.

KELLOGG, O.D., 1929, *Foundation of Potential Theory*, Chapter VI, Springer, Berlin.

MARUYAMA, T., 1964, Statical elastic dislocations within an infinite and semi-infinite medium, *Bull. Earthq. Res. Inst.*, 42, 289-368.

MINDLIN, R.D. and D.H. CHENG, 1950, Nuclei of strain in the semiinfinite solid, *J. Applied Phys.*, 21, 926-930.

MOGI, K., 1958, Relations between the eruptions of various volcanoes and the deformations of the ground surfaces around them, *Bull. Earthq. Res. Inst.*, 36, 99-134.

NAGATA, T., 1969, Tectonomagnetism, *I.A.G.A. Bull.*, 27, 12-43.

NAGATA, T., 1970, Basic magnetic properties of rocks under mechanical stresses, *Tectonophysics*, **9**, 167-195.

OHSHIMAN, N. and Y. SASAI, 1983, The seismomagnetic effect of inclined rectangular faults, *in preparation.*

SASAI, Y., 1979, The piezomagnetic field associated with the Mogi model, *Bull. Earthq. Res. Inst.*, 54, 1-29.

SASAI, Y., 1980, Application of the elasticity theory of dislocations to tectonomagnetic modelling, *Bull. Earthq. Res. Inst.*, 55, 387-447.

STACEY, F.D., 1964, The seismomagnetic effect, *Pure Appl. Geophys.*, 58, 5-22.

STACEY, F.D. and M.J.S. JOHNSTON, 1972, Theory of the piezomagnetic effect in titanomagnetite-bearing rocks, *Pure Appl. Geophys.*, **97**, 146-155.

STRATTON, J.A., 1941, *Electromagnetic Theory*, Chapter III, McGraw-Hill, New York.

YAMAKAWA, N., 1955, On the strain produced in a semi-infinite elastic body by an interior source of stress (in Japanese), *J. Seism. Soc. Japan* (ii), 8, 84-98.

35. 線形ピエゾ磁気変化の面積分による表現

地震研究所　笹　井　洋　一

筆者（1980）は一様均質な磁気弾性体が歪んでいる時，ピエゾ磁気効果で生ずる磁場を，その物体の表面変位と垂直微分の面積分で表現する式を得た．この定式化の一部に誤りがあったので訂正し，あわせて前報より多少ふみこんだ考察を行なった．筆者とは独立に，同じような定式化が BONAFEDE and SABADINI (1980) によって行なわれ，一般の異方性磁気弾性体の作る磁場のベクトル・ポテンシャルについての表現定理が得られている．筆者の方法では，磁化を磁場の源泉とし，スカラー・ポテンシャルで場を記述する．STACEY (1964) 以来の仮定に従って，磁歪と二次的誘導磁化を無視し，帯磁率と硬い残留磁化の応力変化を同等に扱った．磁場についてのガウスの法則と弾性体の静的つりあいの式とを，磁気弾性体の構成法則（線形ピエゾ磁気公式とフックの法則）で結合して，基礎方程式を導く．これは変位場で表わされた源泉項を持つポアソン方程式になる．体積分項を含むその解をグリーンの公式を用いて変形して，上記の表現定理を得た．この式をくい違い面を含む物体に適用すると，くい違い面の位置にも磁気的一重層と二重層が現われる．特にくい違い面上で全ての応力成分が連続な場合（ヴォルテラ型くい違い等はこれに当る），二重層のみが残り，その磁気モーメントはくい違い量と磁化ベクトルの内積で与えられる．この量は地震地磁気効果の大きさを表わすのに最適なので，これを地震地磁気モーメントと呼ぶことにする．地震地磁気モーメントの考えを用いると，地震に伴う地磁気変化の様相を定性的に理解するのが容易になる．一例として有名な南海道地震に伴った地磁気変化 (KATO and UTASHIRO, 1949) を考察し，観測された通り，偏角の東偏が期待されることを示した．表現定理の形から．磁気ポテンシャルと変位，磁場と歪の対応関係が予想される．ポテンシャル論の定理を用いて，地表での磁場変化がその点の歪変化のある成分に比例した項を含むことを示した．大体，10 μ-strain 程度の歪に対して，1 nT の地磁気変化が対応する．しかし実際の磁場変化にはそれ以外の項の寄与が大きく，磁場観測が直ちに観測点の歪だけを測っているとはいえない．表現定理を茂木モデル (MOGI 1958) に適用して，これに伴う地磁気変化を求めた．結果は従来得られていた体積分による解 (SASAI 1979) と全く一致するが，本稿の方法による方が著しく計算量を節約できる．実際の地球では地殻の上部しか帯磁していないため，地震地磁気モーメントの考えをそのまま適用できない場合も多い．しかし簡単な2次元断層モデルでは，非磁性の断層面上にも，見掛上地震地磁気モーメントと等しい磁気二重層が現われる．最後に本稿の方法が境界要素法と原理的に同じ考えに基づいていること，更に複雑な形状の磁気弾性体による磁場を数値的に求めるのに，境界要素法が有効である点にも触れた．