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1. *Higher Mode Interference on Oceanic Love Waves Excited by Shallow Earthquakes as Inferred through Synthetic Waves.*

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Abstract

The interference of the fundamental and higher modes of Love waves is an important subject to be solved for the inversion problem of the velocity of surface waves. After a sufficient analysis of Love waves with respect to the contamination of the fundamental and higher modes, the inversion of the group and phase velocities of the waves seems to be meaningful for a searching examination of surface wave polarization anisotropy. For making clear the problem about the contamination of different modes the excitation of the fundamental and first higher modes of oceanic Love waves is investigated numerically by generating synthetic waves based on a couple of representative fault models. The wave form and velocity of the two modes which travelled about 7000 km, comparable to the length of the great circle path over the western Pacific, are investigated with special reference to the wave interference between the two modes. It is understood that the contamination of the first higher mode on the fundamental mode is severe for periods less than 100 sec, showing that the intensity of the amplitude spectra of the first higher mode is close to that of the fundamental mode near a period of 40 sec, and that the former is about one third of the latter at the period of 100 sec. The group velocities of Love waves contaminated by the two modes showed values lower and higher than those of the fundamental mode for periods shorter and longer than 70 sec, respectively. This group velocity dispersion curve has a trend approaching that of the first higher mode. The influence of the first, second, and third higher modes upon the fundamental mode is studied by calculating the theoretical amplitude spectral ratios of those higher modes to the fundamental mode.

1. Introduction

The long-period Love waves, whose dispersive nature is controlled by the physical property of the such crust and mantle materials as density and rigidity, is specially useful for elucidating the shear wave structure of the upper mantle, and several studies concerned with the observation and the inversion problem of Love waves in the Pacific have been reported (SANTO, 1961; SAITO and TAKEUCHI, 1966; ABE, 1972; YOSHIDA and SATÔ, 1975; FORSYTH, 1975; YOSHIDA, 1977; SCHLUE and KNOPOFF, 1977; YU and MITCHELL, 1979).

In early studies the polarization anisotropy of surface waves was found by AKI and KAMINUMA (1963) in Japan and MCEVILLY (1964) in the midcontinent of the United States, considering its mechanism from a viewpoint of petrological property in the upper mantle. On the other hand, the importance of the contamination of the fundamental mode Love wave by the first higher mode is pointed out by THATCHER and BRUNE (1969). They speculated that the interference of the first higher mode could explain the inconsistencies between Love and Rayleigh wave observations by a simple earth model with single plane isotropic layers.

BOORE (1969) investigated the effect of the higher mode contamination on phase velocities measured between two inline stations and suggested that the presence of the higher mode could produce significant scatter in the measured phase velocities, but that no consistent bias should result if a number of events were used.

In the observation and analysis of multi-mode Love waves, it seems to be very important that we have beforehand knowledge of both the excitation of the fundamental and higher modes and the velocity difference between the different modes. Without such knowledge results obtained in Love wave study are apt to be interpreted incorrectly when discussing the polarization anisotropy of surface waves, bringing about an unsatisfactory conclusion.

From Rayleigh wave analyses upper mantle models for the Pacific Ocean have been proposed by several investigators in connection with the heat flow, the gravity anomaly, the depth of the ocean bottom, and the ocean-floor age inferred from magnetic lineation (*e.g.*, LEEDS, KNOPOFF and KAUSEL, 1974; YOSHII, 1975; YOSHIDA, 1978).

In order to understand the relationship between the upper mantle structure and the dispersion characteristics of Love waves in the Pacific, we must make clear the interference phenomenon between multi-mode Love waves. Specially it is most important to know what relative amplitude ratio of the first higher mode to the fundamental mode is.

For the purpose of solving this problem, oceanic Love waves of the fundamental and first higher modes were synthesized on the basis of the earthquake dislocation theory (MARUYAMA, 1963), following the excitation formula of surface waves (SAITO, 1967). The seismograms were generated by using a reasonable upper mantle model in the western Pacific and the earthquake source was assumed to be located at the shallow part of the earth. The wave simulation was carried out for three seismic fault types; the strike-slip, the dip-slip along a vertical fault plane, and the thrust type with the fault plane having a dip angle of 45° vertically downwards. Since there is no proper regional models inferred from Love wave data, we tentatively employ the one constructed from Rayleigh waves.

2. Love wave dispersion and medium in the Pacific

In the Pacific, it is widely understood that, by the cooling mechanism of hot mantle materials such as dunites, the oceanic lithosphere is growing in proportion to the ocean floor age, starting from 0 m.y. in the East Pacific Rise up to 150 m.y. in the region of Izu Mariana Islands, the westernmost part of the Pacific. The oldest lithosphere is believed to have a thickness of 147.2 km (LEEDS, KNOPOFF, and KAUSEL, 1974), or 115.2 km (SCHLUE and KNOPOFF, 1977), or 85 km (YOSHIDA, 1978) from Rayleigh wave analysis.

These different estimations of thickness are caused mainly by the different initialization of the shear wave velocity in the low velocity zone (*LVZ*) and the depth range of the *LVZ* under the ocean bottom. The shear wave velocity in the *LVZ* and the deepest point of the *LVZ* in the starting model for the west Pacific are assumed, respectively, 4.10 km/sec and 180.2 km by Leeds et al., and Schlue and Knopoff, 4.30 km/sec and 220 km by the present author. For estimating the precise thickness of the oceanic lithosphere it is a key point to find out the shear wave velocity in the *LVZ* and the lower limit of the *LVZ* from other geophysical data independently.

It should be noticed that in the models proposed by Leeds et al., and Schlue and Knopoff, the *LVZ* shear wave velocity has the same value in the oldest and youngest regions, while a difference in the shear wave velocities exists in the model by the present author, indicating the low value 4.13 km/sec for the *LVZ* in the youngest region.

In Fig. 1 the dispersion curves of the fundamental and first higher models of Love waves, for the models *PC-MIN* (0 m.y.) and *PC-MAX* (150 m.y.) (YOSHIDA, 1978), are shown together with the curves for the model *8099-80*, which covers the western Pacific region with the ocean floor age of from 90 m.y. to 150 m.y. (YOSHII, 1975). The figure shows that in the Pacific the group velocities of the first two modes are crossing, in the period range from 10 to 100 sec, more tightly in the western Pacific than in the eastern Pacific, indicating the difficulty of the mode separation in the former region. In the phase velocities between the two modes, the curve-crossing periods do not appear in

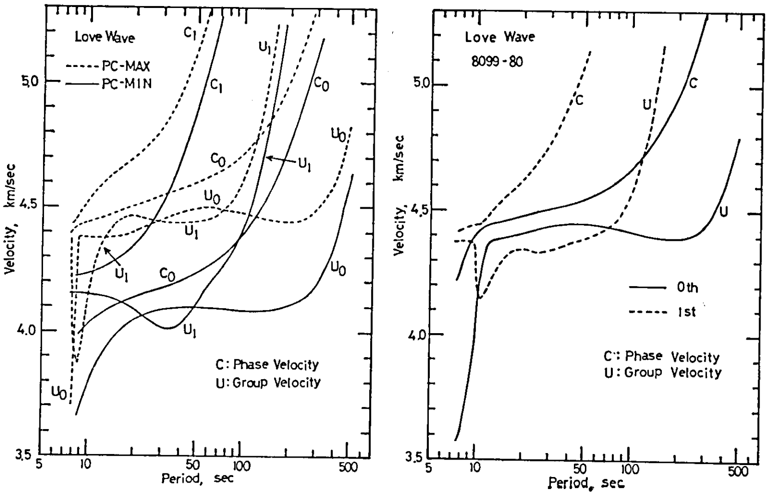


Fig. 1. Phase and group velocities calculated for the models *PC-MAX*, *PC-MIN* and *8099-80*. *PC-MAX* and *PC-MIN* correspond to the regions of the highest- and lowest-velocity-areas in the Pacific, locating in regions where the ocean-floor age is 150 m.y. or more and 0 m.y. respectively (YOSHIDA, 1978). *8099-80* corresponds to the region where the ocean-floor age is from 90 m.y. or more in the western Pacific (YOSHII, 1975).

the Pacific. However it can be said that the contamination of the fundamental mode by the first higher mode is unavoidable when we look at the mode interference from the viewpoint of the group velocity characteristics.

The excitation of these two modes is another important problem since the interference never occurs when one mode is negligible when compared to the other. The model *8099-80* was employed for this purpose because it is the model proposed to represent the average upper mantle structure in the western Pacific, so the synthesized waves based on this model can be used for comparison with actual observed waves propagating the ocean.

According to the dispersion curves calculated for the model *8099-80* (Fig. 1), the group velocity of the fundamental mode is about 0.1 km/sec more than that of the first higher mode in the period ranging from 10 to 50 sec, the velocity indicating uniformly 4.4 km/sec in the period ranging from 10 to 300 sec. It can be inferred that this flat portion of the group velocity in the wide period range contributes to the generation of the isolated wave; the so called "*G* wave" travelling the typical oceanic region (SATÔ, 1958). Wave forms similar to the *G* wave are often observed on the horizontal component seismograms, which traverse a large part of the Pacific.

3. Method

For the analysis of Love waves having the epicentral distance $5000\sim9000$ km, with the wave being assumed to be propagated in the western Pacific, the curvature of the earth's layering cannot be neglected, particularly in the group velocity calculation of long period surface waves over 40 sec.

For the generation of long-period surface waves the earth flattening technique is exploited (*e.g.*, BISWAS and KNOPOFF, 1970; SCHWAB and KNOPOFF, 1973; KAUSEL and SCHWAB, 1973; SCHWAB and KAUSEL, 1976; CALCAGNILE, PANZA, SCHWAB and KAUSEL, 1976), in which the transformation of the coordinate system from the spherical layered earth to the flat layered earth is needed, while, in early times, the direct calculation of free oscillations of the earth was employed for making the synthetic waves (USAMI, SATÔ and LANDISMAN, 1964, 1965; SATÔ, USAMI and LANDISMAN, 1968). The calculation of torsional oscillation is employed for the present simulation of the oceanic Love waves.

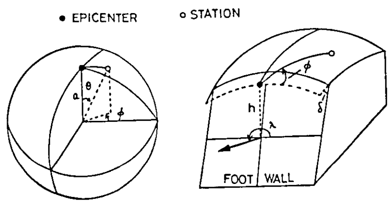


Fig. 2. Coordinate system and source geometry. a, θ and ϕ are the radius of the earth, the colatitude, and the azimuth measured from the fault strike respectively. h is the focal depth. λ and δ are the slip-angle and the dip-angles.

The geometry between the epicenter and the station, and the coordinate system is shown in Fig. 2. According to SAITO (1967) and TAKEUCHI and SAITO (1972), the displacement of the transverse component of torsional oscillation is written as follows.

$$
{}_iu_n(t)=M_0\times\frac{y_1(r)}{{}_i\omega_n^2 I_1}\cdot\frac{2n+1}{4\pi n(n+1)}\cdot\left\{\frac{y_2(r_s)}{\mu_s}\cdot\frac{\partial P_n^1(\cos\theta)}{\partial\theta}\right.
$$
$$
\times(d_1\sin\varphi+d_2\cos\varphi)+\frac{y_1(r_s)}{r_s}\cdot\frac{\partial P_n^2(\cos\theta)}{\partial\theta}
$$
$$
\left.\times(d_3\sin 2\varphi+d_4\cos 2\varphi\right\}\times\exp\left(-\frac{{}_i\omega_n\cdot t}{2{}_iQ_n}\right)\times\cos{}_i\omega_n t \qquad (1)
$$

Here the seismic source is assumed to be a double-couple point source without moment, and the source time function to be a step function. Such a force system is the replacement of the earthquake slip dislocation (MARUYAMA, 1963; BURRIDGE and KNOPOFF, 1964). M_0 is the seismic moment and μ_s is the rigidity of the source layer $r_s=a-h$. $P_n^m(\cos\theta)$ is the associated Legendre function. ${}_i\omega_n$ is the eigenfrequency of the torsional oscillation for the mode i and the order number n, and ${}_iQ_n$ is the dimensionless attenuation factor. I_1 is the energy integral expressed as

$$I_1=\int_0^a \rho r^2 y_1^2(r)dr \tag{2}$$

y_1 and y_2 are the radial factors of the displacement and stress. The radiation coefficients d_1, d_2, d_3 and d_4 depending on the dip angle δ and the slip angle λ, are given by

$$\left.\begin{aligned} d_1&=\cos\delta\cdot\cos\lambda \\ d_2&=-\cos(2\delta)\cdot\sin\lambda \\ d_3&=1/2\cdot\sin(2\delta)\cdot\sin\lambda \\ d_4&=\sin\delta\cdot\cos\lambda \end{aligned}\right\} \tag{3}$$

The attenuation factor ${}_iQ_n$ of Love waves is known to change slightly with the wave period, from the observational data (*e.g.* SATÔ, 1958; BEN-MENAHEM, 1965; MITCHELL, LEITE, YU and HERRMANN, 1976) and also from a theoretical viewpoint (YAMAKAWA and SATÔ, 1964; ANDERSON, BEN-MENAHEM, and ARCHAMBEAU, 1965; SATO, 1967).

However the Q values of oceanic Love waves of both the fundamental and first higher modes are very close to 100 in the period ranging from 20 to 150 sec (FUKAO and ABE, 1971). Hence in the forthcoming wave synthesis the constant value 100 is assumed for ${}_iQ_n$. The calculation was made for 34 points of n ranging from 20 to 1150, corresponding 356 to 8 sec respectively for the fundamental mode. For the first higher mode it was made for 37 points of n ranging from 13 to 1150, covering the periods from 320 to 8 sec. For the intermediate n the interpolation was performed by using the spline function.

4. Fault type and radiation pattern of the first two modes at a period of 50 sec

The amplitude radiation of surface waves at the source depends chiefly on the magnitude of the earthquake, or the seismic moment. However, the fault type, the focal depth and the rock property near the source also affect the radiation. Generally the higher modes are excited strongly as the focal depth increases while the opposite is true for the fundamental mode (JOBERT, 1964).

In deep earthquakes the radiation of the higher mode of continental Love waves is strong and interference between the fundamental and higher modes is likely to occur, so the shallow earthquakes are primarily used for the velocity determination on the continent. In the continent having no well-developed *LVZ*, the higher mode Love waves from shallow earthquakes are not dominant. However the excitation of the higher modes of oceanic Love waves cannot be overlooked even if the earthquake is shallow. The estimation of the higher mode excitation, especially for the first higher mode, is an indispensable subject for the treatment of oceanic Love waves.

Among shallow earthquakes those of the strike-slip type along the vertical fault plane strongly excite Love waves. These earthquakes occur chiefly in the inlands. The thrust fault or the normal fault often occurs near the subduction zone of the oceanic plate around the circum-Pacific seismic zone. For the three fault types Love waves of the fundamental and first higher modes were synthesized.

4.1 Fault type

It is well known that the period range in which the contamination of multi-mode Love waves occurs is below the wave period 100 sec. For the inversion problem we usually use waves longer than 30 or 40 sec. In Fig. 3 the azimuthal dependence of the amplitude radiation patterns together with the space phases of Love waves of the fundamental and first higher modes, calculated for the model *8099-80*, are shown only for the period 50 sec, since this period seems to be adequate in making clear the difference of the two modes in the radiation amplitude. The calculation was made for the source shallower than 100 km, by employing the method carried out in YOSHIDA (1982b). Fig. 3 shows that, for the sources of the left lateral strike-slip type ($\delta=90°$, $\lambda=0°$) and the thrust fault type ($\delta=45°$, $\lambda=90°$), the radiations of the first higher mode are nearly equal to, but slightly less than those of the fundamental mode, for focal depths 10, 30 and 45 km.

However, the radiation pattern of the two modes greatly varies for the pure dip-slip source ($\delta=90°$, $\lambda=90°$). Except for the focal depth 10 km the first higher mode is vastly higher than the fundamental mode. It is understood that among the three fault types the amplitude radiation of the two modes is highest for the pure strike-slip, then the thrust fault with the dip angle 45°, and lowest for the pure dip-slip.

For reference, the radiation pattern and the space phase of Rayleigh waves calculated for the model PC-MAX, for the fault type of the pure dip-slip motion, are shown in Fig. 4. It is interesting that, even for the dip-slip, the Rayleigh wave radiation of the fundamental mode exceeds that of the first higher mode for the earthquakes shallower than 80 km. For oceanic Love waves, the amplitude ratio of the first higher mode to the fundamental mode becomes highest for

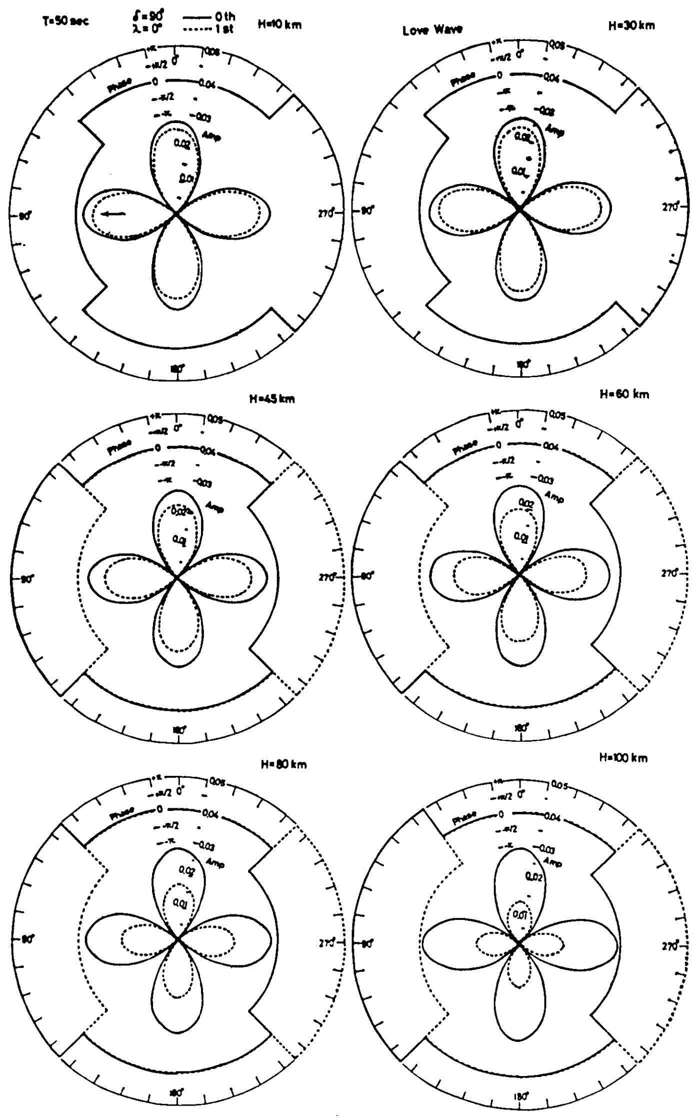


Fig. 3a.

Fig. 3. Azimuthal dependence of amplitude radiation patterns and space phases of the fundamental and first higher modes of Love waves at a period of 50 sec. The calculation is done for shallow earthquakes of three fault types (a) the strike-slip, (b) the thrust fault, and (c) the dip-slip. Arrows mean the direction at which synthetic seismograms are generated.

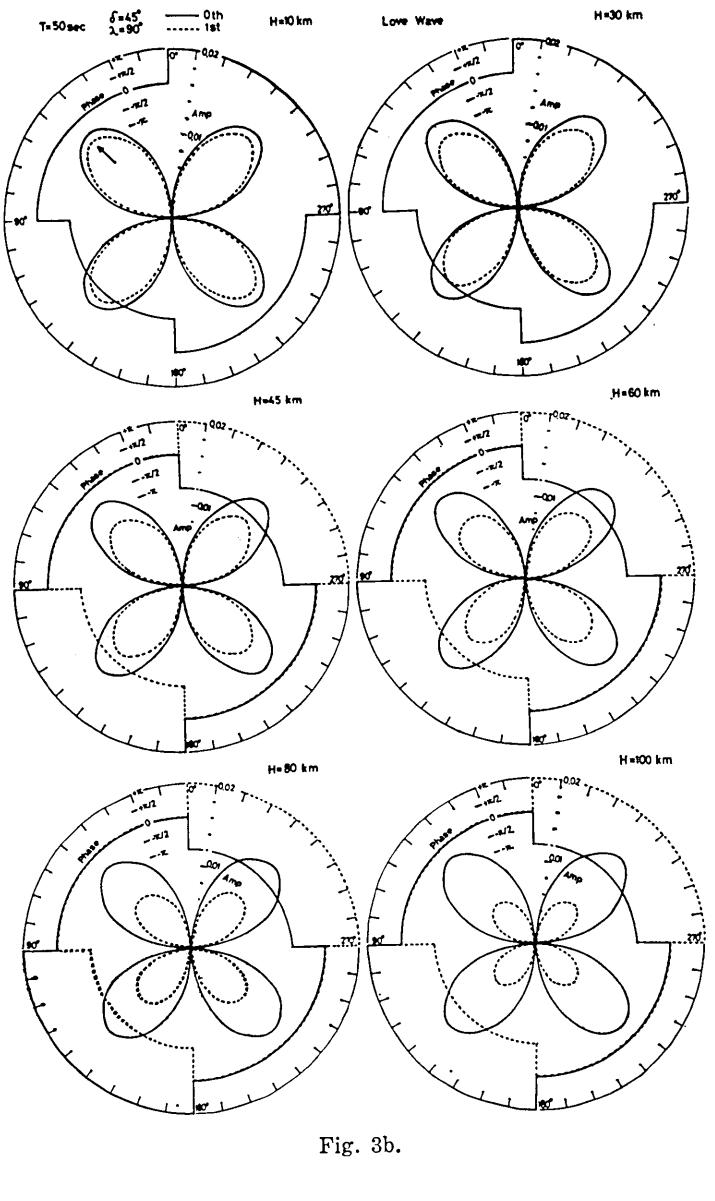

T=50sec
δ=45°
λ=90°
0th
1st
H=10 km
Love Wave
H=30 km
H=45 km
H=60 km
H=80 km
H=100 km
Phase
Amp
0.02
0.01
0°
90°
180°
270°

Fig. 3b.

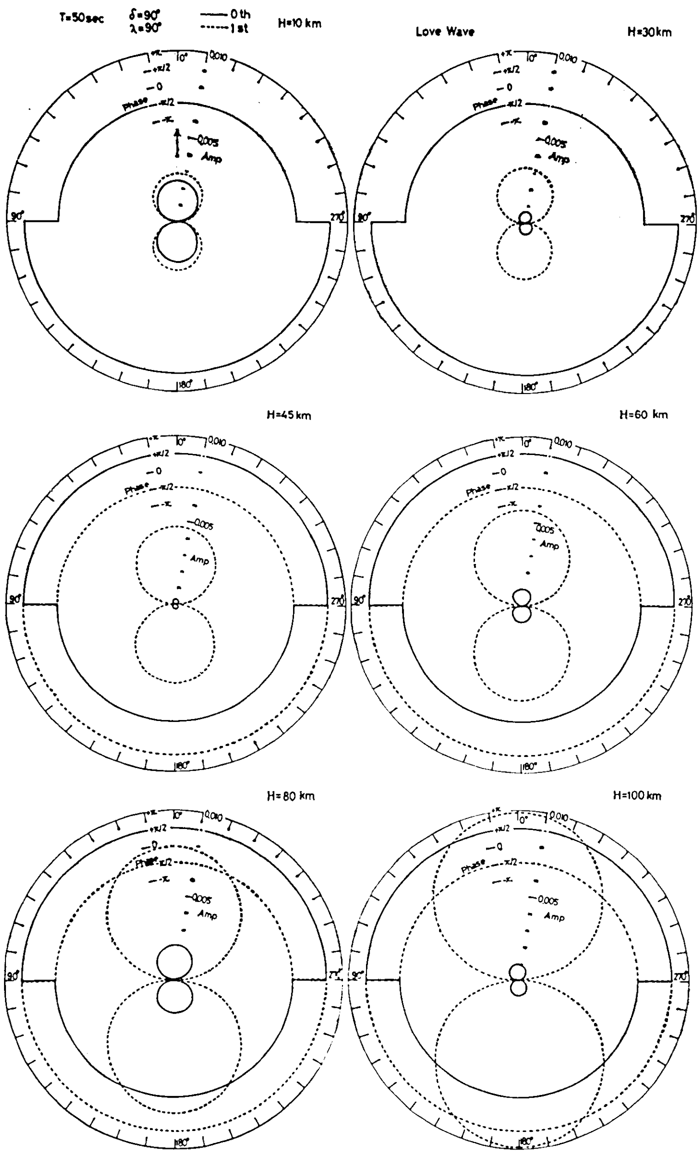

T=50sec
δ=90°
λ=90°
0 th
1 st
H=10 km
Love Wave
H=30km
H=45 km
H=60 km
H=80 km
H=100 km
Phase
Amp
0.005
0.010
10°
90°
270°
180°

Fig. 3c.

the dip-slip source.

4.2 Depth dependence

As the depth increases the decrease of the radiation of the first higher mode of Love waves is seen for both the strike-slip and thrust faults, while the fundamental mode keeps a constant radiation in the depth range from 10 to 100 km (Fig. 3). However for the dip-slip source the first higher mode increases the radiation in proportion to the focal depth.

It can be also observed from Fig. 3 that the azimuthal dependence of radiation patterns of the amplitude and the phase do not vary even if the depth differs, though the absolute amplitude varies with the depth. We should pay attention to the fact that, for earthquakes shallower than 60 km, the radiation of the first higher mode is comparable to or larger than the fundamental mode at the period 50 sec, suggesting that near that period the radiations of the first higher mode of Love waves, excited by earthquakes with various fault types around the western Pacific, can be never neglected for the treatment of Love waves.

4.3 Space phase

For phase velocity determination by means of the single-station method, the initial phase of surface waves must be calculated. The space phase, depending on the fault type, the medium property near the source, the wave period, and the focal depth, is one of three phase factors of the initial phase. We notice from Fig. 3 that for the sources of the strike-slip and the thrust fault there are two phase characteristics between the fundamental and first higher modes. The two space phases have the same phase angle or there is a difference of 2π between them. The former phase characteristic appears in the space phases for the focal depth of 10 and 30 km, while the latter for the depths over 45 km.

For the dip-slip source (Fig. 3), however, a phase difference of π exists between the two modes in all the azimuths for focal depths over 45 km. This discrepancy is physically significant. In the space phase of Rayleigh waves (Fig. 4), a difference of π is also observed between the two modes for deep sources.

5. Synthesized Love waves

Following the formulation (1) synthetic Love waves of the fundamental and first higher modes were calculated using the model *8099-80* and are shown in Fig. 5. The waves were generated at the maximum amplitude azimuth indicated by arrows in Fig. 3.

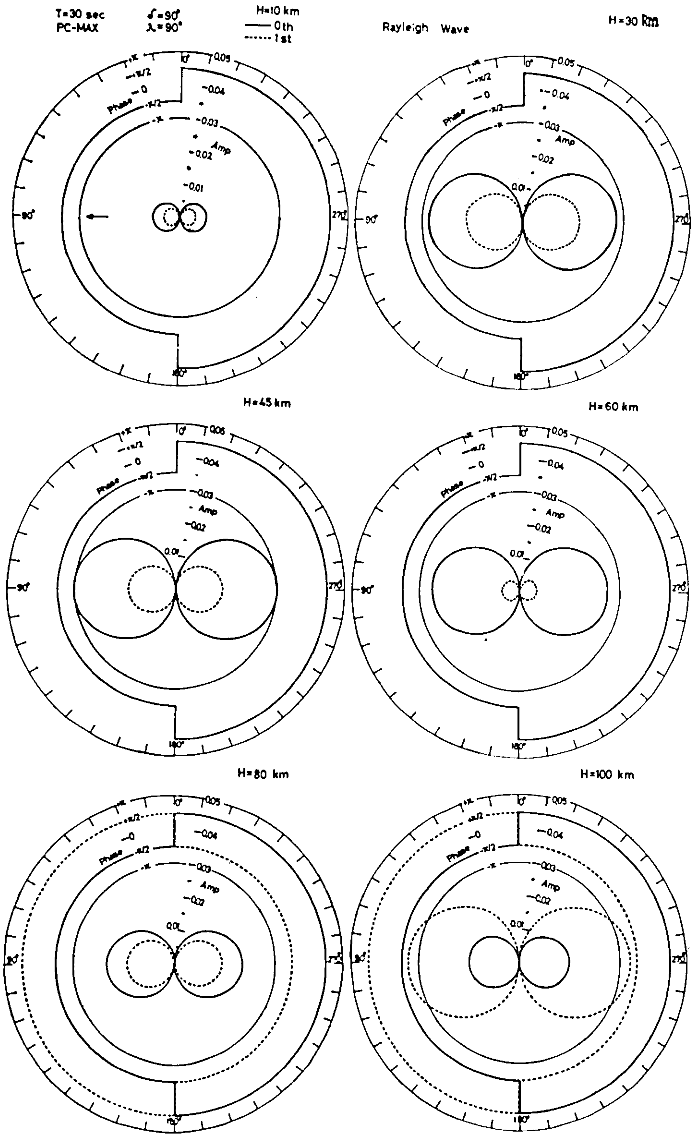
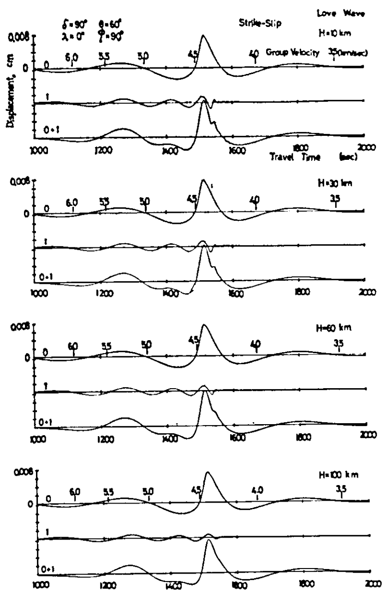
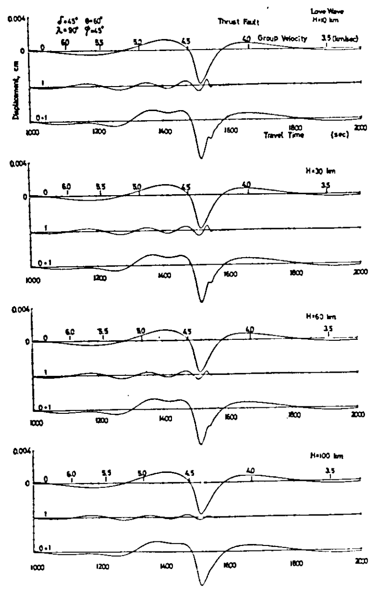


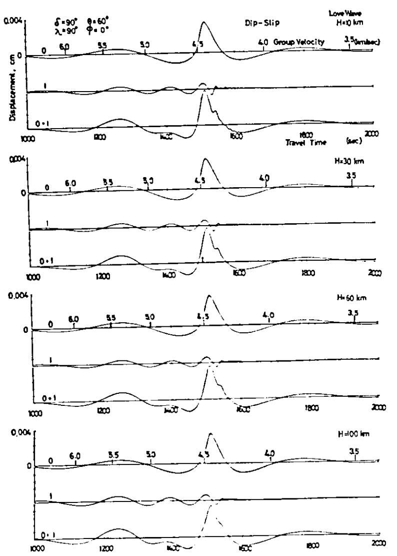
Fig. 4. Amplitude radiation patterns and space phases of the fundamental and first higher modes of Rayleigh waves at a period 30 sec. The calculation is done for shallow earthquakes of the dip-slip type. The figures are prepared for comparison with Love waves. For the arrow see the caption in Fig. 3.



(a)



(b)



(c)

Fig. 5. Synthetic Love waves generated for various focal depths H ranging from 10 to 100 km. The synthesis is done for earthquakes of (a) the strike-slip, (b) the thrust fault, and (c) the dip-slip. The epicentral distance is assumed to be $\Delta=60°$. The numerals in the abscissa are the travel time in sec, and the group velocity in km/sec. 0: Fundamental mode, 1: First higher mode, 0+1: Combination of the fundamental and first higher modes.

5.1. Wave form

We see from Fig. 5 that, for all fault types, the wave forms of the two modes hardly vary with the focal depth, each mode keeping a similar form in the depth range from 10 to 100 km. In the fundamental mode long period waves composed of several hundreds of seconds seem to be dominant, while in the first higher mode the wave components having shorter periods than the fundamental mode are clearly excited.

It is a significant feature that the fundamental mode's maximum amplitude is limited to the group velocity range from 4.5 km/sec to 4.3 km/sec, but the first higher mode has no such strong peak and the wavelets arrive faster than the group velocity 4.2 km.

As we look on the waves produced by the interference of the fundamental and first higher modes, the waves being expressed by the sum of the two modes in Fig. 5, it can be said that the contaminated waves keep the original wave form of the fundamental mode. The wave forms of the fundamental mode showing no clear dispersion may be closely related with the well-developed LVZ expanding to a depth of about 200 km under the western Pacific.

It is an interesting phenomenon that the wave forms of the fundamental and first higher modes generated by the strike-slip source at azimuth 90° (Fig. 5a), are extremely similar to those generated by the dip-slip source at azimuth 0° (Fig. 5c). This phenomenon must result in the common geometrical circumstance between the direction of the motion of the fault plane and the location of the station where the observation point is located to the azimuth normal to the direction of the motion of the hanging wall (see also Fig. 2). The difference of the waves caused by the two sources is found in the amplitude level, namely, the amplitude for the strike-slip source is approximately twice that of the dip-slip source for both fundamental and first higher modes.

According to (3), for the strike-slip source $d_1=d_2=d_3=0$ and $d_4=1$, and for the dip-slip source $d_1=d_3=d_4=0$, and $d_2=-1$. Therefore the expression (1) shows that the difference of the amplitude level for two sources comes from different excitation functions, namely the function $y_1(r_s)/r_s \cdot \partial P_n^2(\cos\theta)/\partial\theta$ contributes to the wave radiation for the strike-slip source,while $y_2(r_s)/\mu_s \cdot \partial P_n^1(\cos\theta)/\partial\theta$ for the dip-slip source.

Rayleigh waves of the fundamental and first higher modes calculated for the model PC-MAX, which are shown in Fig. 6 for comparison, suggest that for focal depths shallower than 80 km the higher mode is excited much less than the fundamental mode. We easily understand that the mode interference rarely occurs in oceanic Rayleigh waves.

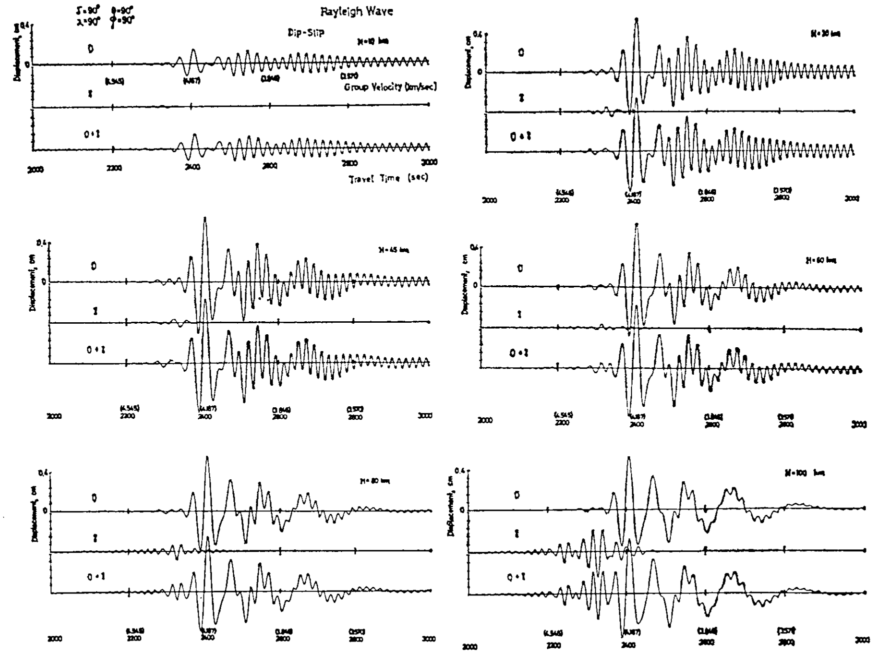


Fig. 6. Synthetic Rayleigh waves generated for the dip-slip source. Seismograms made for various shallow focal depths are prepared for comparison with Love waves. The epicentral distance is assumed to be $\Delta=90°$. 0: Fundamental mode, 1: First higher mode, 0+1: Combination of the two modes.

5.2 First higher mode and Q

In the problem of the mode interference of oceanic Love waves, the Q value of the first higher mode is an important factor. However, the observation of Q of Love wave is virtually limited to the fundamental mode, so in the synthesis of the first higher mode we must employ the value theoretically determined as a function of period.

According to the calculation of Q by FUKAO and ABE (1971), the first higher mode (the fundamental mode) has values of about 100(400), 130(100), 200(100), and 300(100) at periods of 10, 20-100, 200, and 300 sec respectively. In this calculation the oceanic model *CIT11A* (KOVACH and ANDERSON, 1964) for the shear velocity and the model *MM8* (ANDERSON, BEN-MENAHEM and ARCHAMBEAU, 1965) for the shear wave Q were used.

In the present analysis the first higher mode was generated by assuming a constant Q value of 100. This assumed Q is slightly less than that calculated by FUKAO and ABE (1971). To understand the effect of different Q values on the wave forms, synthetic waves of the first higher mode were made for focal depths of 10, 45 and 100 km by assuming Q values higher than 100 and are shown in Fig. 7.

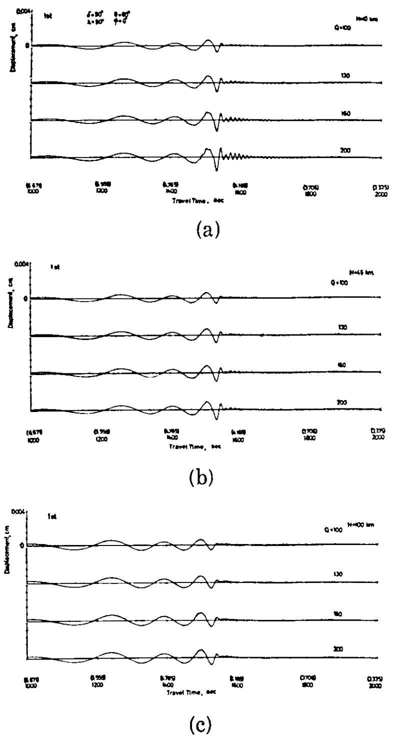


Fig. 7. Synthetic Love waves of the first higher mode generated for the dip-slip source by assuming different Q values. The focal depth H is assumed to be (a) 10 km, (b) 45 km, and (c) 100 km.

The seismic source is assumed to be the dip-slip motion. We notice from the figure that the short period components apparently increase as the Q value increases. This trend is sensitive for shallow sources, and the effect of high Q values is extremely weak for a focal depth of 100 km.

For the study of the higher mode interference on the fundamental mode of long-period Love waves, the wave amplitude in the period range from 40 to 100 sec is particularly important. In this range the Q value of the first higher mode may be slightly higher than 100. The observed Q values of the fundamental mode are close to 100 in the period range from 50 to 300 sec (BEN-MENAHEM, 1965).

We can conclude from Fig. 7 that the wave forms of the first higher modes for $Q=100$ and values higher than 100 are not much different for both focal depths of 10 and 45 km. There is a possibility that the first higher modes displayed in Fig. 5 have slightly longer period components than what they have since the calculated Q values of that mode are high compared to the assumed value 100 at the period range from 200 to 300 sec.

5.3 Spectra

The fundamental and first higher modes of Love waves, generated for the two seismic sources of the strike-slip and the dip-slip motions with a focal depth of 10 km, were Fourier analyzed. The amplitude spectra of the waves having a time length of 1000 sec from the travel time 1000 to 2000 sec, traced in Fig. 5, were calculated and are shown in Fig. 8. We see from the figure that the spectral shapes are very similar, although amplitudes for the strike-slip source are about double the values for the dip-slip source. It should be noted that the amplitudes of the fundamental and first higher modes intersect at the wave period 40 sec, beyond this period the former dominates the latter.

Considering the high Q values of the fundamental mode around the period 20 sec, observed by TSAI and AKI (1969), the amplitude spectra of that mode vary towards a higher level than displayed in Fig. 8 and are not accurate. So this spectral picture can be useful for estimating the amplitudes of the two modes for the periods longer than 30 or 40 sec.

The amplitude spectra clearly demonstrate that the first higher mode interferes with the fundamental mode strongly for periods shorter than 40 sec, and weakly for periods longer than 100 sec. The spectra of the interfered waves, which are the sum of the fundamental and first higher modes in the time domain, are nearly equal to or larger than the fundamental mode in most of the period range considered here, except for around 25 sec. The interference by the first higher mode apparently seems to amplify the fundamental mode in amplitude.

In the spectra of the first higher modes in Fig. 8, the amplitudes in the period range from 100 to 300 sec are expected to approach those of the fundamental mode, since it is probable that the Q values of the first higher mode for those periods are larger than the assumed value 100. The spectral peak which is located near the period 200 seems to keep its position even if high Q values are assumed in the

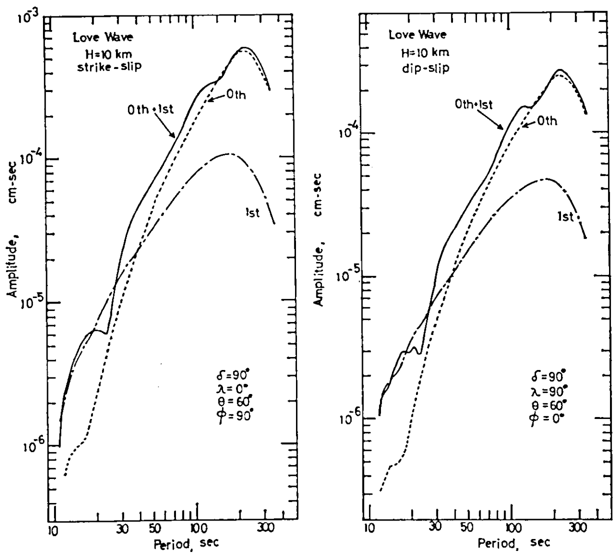


Fig. 8. Fourier amplitude spectra of Love waves of the fundamental (0th), first higher (1st) modes, and their combination, generated by the strike-slip and dip-slip sources at a focal depth of 10 km.

synthetic waves. We also notice that the fundamental mode has a peak at the period slightly longer than the first higher mode.

5.4 Group velocity

The mode separation of observed oceanic Lowe waves which are contaminated by higher modes is generally not easy. The difficulty mainly comes from the similar group velocities of the fundamental and higher modes for periods less than 100 sec.

The group velocities of the fundamental and first higher modes, which were synthesized for the dip-slip source and whose spectral characteristics were examined in the previous section, were determined by means of the multiple filtering technique (DZIEWONSKI, BLOCH and LANDISMAN, 1969) and are shown in Fig. 9. The filtering parameters for oceanic surface waves are described in YOSHIDA (1982a). The group velocity curves were obtained by calculating the wavetrain of each mode separately.

If we interpret that the two curves represented in the figure are very close to the dispersion curves of Love waves travelling the western Pacific, it will be a key for the solution of the higher mode contamination to know how the constant group velocity 4.44 km/sec is scattered by the interference of the higher modes. The feature of the group velocity of the first higher mode in the western Pacific will be summarized as follows; The velocity for periods less than 70 sec is lower than the fundamental mode, while it is higher for periods over 70 sec (see also Fig. 1).

According to Fig. 9, the group velocity of the interfered waves

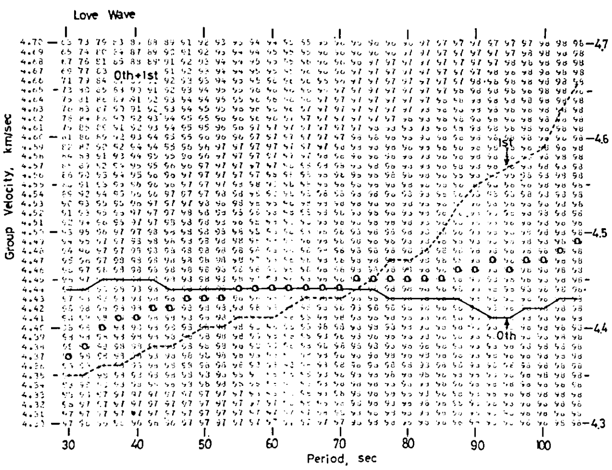


Fig. 9. Group velocities (open circles) of the synthetic waves of the combination of the fundamental and first higher modes shown in Fig. 5 (c) for a focal depth of 10 km, together with the group velocities of the fundamental and first higher modes.

shows a characteristic that the velocity of the interfered waves has the general trend that the group velocity curve is as a whole close to that of the fundamental mode. The velocity of the interfered wave is shifted to that of the first higher mode at periods less than 40 sec or more than 90 sec. These biases can be interpreted as the effect of the contamination of the first higher mode. However this effect of the mode contamination on the velocity is a very simple case. BOORE (1969) showed oscillating dispersion curves of the phase velocity in the period range from 10 to 100 sec, the velocity being calculated for synthetic waves of oceanic Love waves of the fundamental mode interfered by the first higher mode. They demonstrated that complex phase velocity curves result in the effect of the interference between different modes.

As seen from Fig. 9 the fundamental mode does not show clear dispersion, but the first higher mode shows a normal dispersion. Hence the wave form of the latter mode vary with the epicentral distance, and the interfered waves are also expected to vary with the distance. We understand that the velocity bias of the interfered wave appearing in Fig. 9 are simply caused at the point having an epicentral distance of $\Delta=60°$ which was assumed in the present analysis.

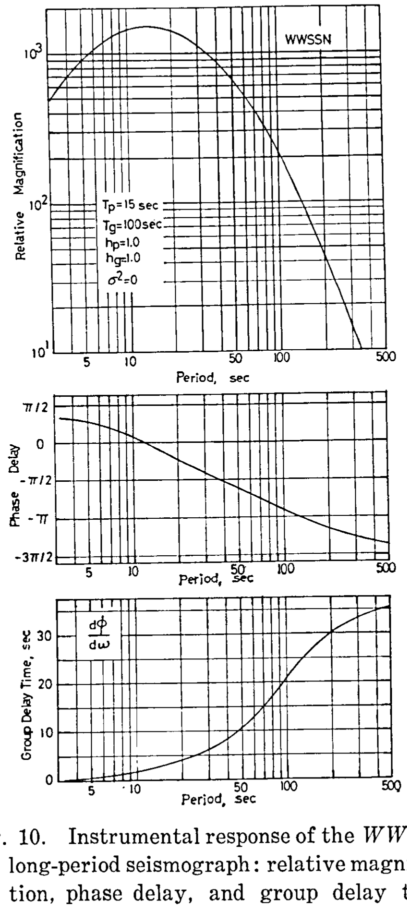


Fig. 10. Instrumental response of the *WWSSN* long-period seismograph: relative magnification, phase delay, and group delay time. T_p and h_p are the natural period and damping constant of the pendulum, respectively. T_g and h_g are those of the galvanometer. σ is the coupling factor between the seismometer and the galvanometer.

6. Phase velocity

Phase velocity of surface waves is determined either by means of the single-station method or by the two station method. If the observed waves of the fundamental mode are contaminated by higher modes, the calculated phase angle is scattered and involves errors, which lowers the accuracy of the phase velocity. Oceanic Love waves experiencing mode interference may have anomalously high apparent phase velocity (THATCHER and BRUNE, 1969), or show positive or negative perturbation of the velocity (BOORE, 1969). We notice that the velocity of oceanic Love waves interfered by higher modes can be determined with high accuracy by taking into consideration the excitation of both fundamental and higher modes, though it is not easy to simulate multi-mode Love waves only for the purpose of the velocity determination.

We usually analyze surface waves recorded by the *WWSSN* long-period seismograph which indicates the maximum amplitude magnification at the wave period 15 sec and has a specific phase delay as shown in Fig. 10. So the recorded surface waves show no true ground displacements as shown in Fig. 5. The synthetic waves were corrected for the instrumental response of the *WWSSN* seismograph, the calculation based on the formula by HAGIWARA (1958). The instrumental corrections were done on the synthetic seismograms shown in Fig. 5 by applying the method of TANAKA, YOSHIZAWA and OSAWA (1975).

In that method the correction of the amplitude magnification and phase delay of the seismograph was performed in the frequency domain by the use of FFT algorithm, and the accuracy is known to be high. The corrected waves are plotted in Fig. 11.

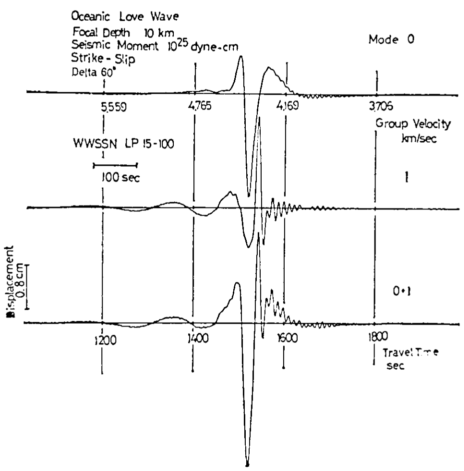


Fig. 11. Synthetic Love waves of the fundamental and first higher modes and their combination, corrected for the instrumental response of the *WWSSN* long-period seismograph. Group velocity and travel time are given in the abscisa.

The waves in Fig. 11 correspond to those for the focal depth 10 km in Fig. 5a. Comparing the two sets of seismograms we easily see that the waves of the first higher mode convolved with the *WWSSN* seismograph characteristics are emphasized in amplitude, disturbing the phase of the fundamental mode, specially of the short period component. We clearly understand from Fig. 11 that the phase angle of the fundamental mode cannot be calculated accurately from the higher mode contaminated waves, and so the phase velocity, as well as the group velocity, should be determined by estimating the excitation of the higher modes of oceanic Love waves.

7. Influence of the second and third higher modes upon the fundamental Love waves

So far we have investigated the relative amplitude ratios between the fundamental and first higher modes. However, it will be necessary to study approximately the influence of the second and third higher modes upon the fundamental mode of oceanic Love waves.

To make clear the relative amplitude ratios between the first four modes, synthetic waves excited by a shallow earthquake were generated using the model *8099-80* and a test model (*PC-L43*) for the West Pacific constructed by the present author, with the assumption of the constant Q values of 100 for the periods from 40 to 200 sec for the four modes and an epicentral distance of about 5000 km, and they were Fourier analyzed. The spectral ratios of the first, second and third higher modes to the fundamental mode obtained for the test model and *8099-80* are shown in Fig. 12 by solid and dotted lines respectively.

If we set up a criterion level of the amplitude ratios of the higher modes to the fundamental mode at 0.3, in which the higher modes cannot be neglected, Fig. 12 indicates that the first higher mode significantly influences the fundamental mode for the periods from 40 to about 130 sec, the second higher mode up to 70 sec, and the third higher mode up to 50 sec. If we set up the criterion level

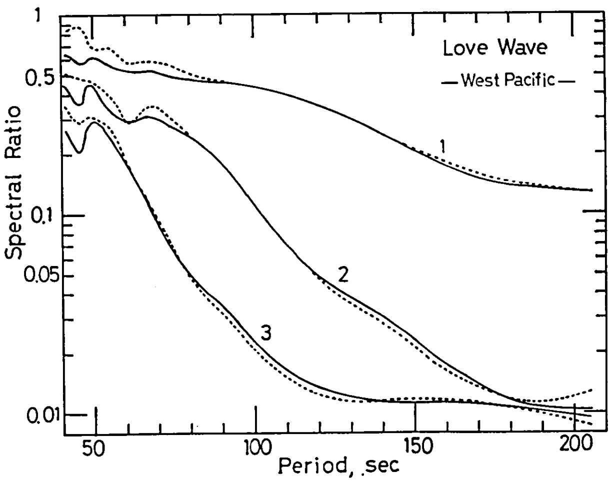


Fig. 12. Amplitude spectral ratios of the first (1), second (2), and third (3) higher modes to the fundamental mode.

at 0.5, the first higher mode cannot be neglected up to a period of 80 sec and other higher modes can be neglected for the periods longer than 40 sec.

In the present analysis synthetic waves were used for solving the interference phenomenon of the fundamental and higher modes of oceanic Love waves. The test model used for the calculation of the spectral ratios of the higher modes to the fundamental mode was also constructed by the simulation method of Love waves. Simulations of the waves will be a powerful means for the investigation of the relationship between the regional underground structures and the surface wave dispersion.

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1. 浅発地震によって励起される海洋性ラブ波における高次モードの干渉
――合成波による推定――

地震研究所　吉田　満

ラブ波基本モードと高次モードの干渉は，表面波速度の逆問題にとって，解かれるべき一つの重要な課題である．基本モードと高次モードの混合に関するラブ波の十分な解析の後に，表面波群速度と位相速度のインバージョンは表面波 Polarization anisotropy の厳重な検討に対して意味をもつと思われる．異種モードの混合に関する問題を明らかにする為に，海洋性ラブ波の基本及び高次モードの励起が，二，三の代表的な断層モデルに基づいて合成波をつくる事により数値的に見積られた．西太平洋上の大円弧の長さに匹敵する約 7000 km 伝播した二つのモードの波形と速度を，特に二種モードの干渉に関連して調べた．基本モードに対する一次高次モードの混合は周期 100 秒以内で激烈であり，40 秒付近では，一次高次モードの振幅スペクトルの強度は基本モードのそれに近く，100 秒では前者は後者のおよそ3分の1である事が理解出来た．二種モードが干渉した波の群速度は基本モードのそれよりも 70 秒以下では低く，70 秒以上では高くなる事を示している．この群速度分散曲線は一次高次モードのそれに近づく傾向がみられる．

二次，三次高次モードが基本モードに及ぼす影響の度合を高次モードの基本モードに対する振幅スペクトル比から推定した．高次モードが無視出来ない周期範囲はその振幅スペクトル比の基準を仮に 30% とすると，一次高次モードが周期 40 秒から 130 秒まで，二次高次モードが 70 秒まで，三次高次モードが 50 秒までと推定される．基準レベルを 50% とすると一次高次モードは周期 40 秒から 80 秒までは無視出来ず，二次，三次高次モードは周期 40 秒以上のどの周期でも無視出来る．