BULLETIN OF THE EARTHQUAKE
RESEARCH INSTITUTE
Vol. 45 (1967) pp. 79-90

7. *The Wave Form of Tsunami on the Continental Shelf.*

By Tokutaro HATORI,

Earthquake Research Institute.

(Read Sept. 27, 1966.—Received Dec. 28, 1966.)

Abstract

From mareograms for some tsunamis observed at the head of Onagawa Bay, the wave form outside the bay is estimated, based on the theory of the multiple reflection of waves. The calculated waves are similar to a certain extent to the observed records at Enoshima which is located outside the bay. Besides, some records at Onahama and Ayukawa are analyzed by the same method.

For the Iturup, Alaska and other tsunamis, spectral analyses of the records at Miyako, Enoshima, Onagawa and Onahama are made, the coherences and phase differences for each station pair being calculated. Making use of these results, the existence of edge waves is examined with the aid of the theoretical dispersion curves of phase velocity.

1. Introduction

A method was proposed by R. Takahasi of obtaining a form of tsunami waves outside a bay from the motion of bay-water for a rectangular bay of uniform depth[1] and for a bay where both the breadth and depth vary[2]. K. Nakamura[3] discussed the response of bay-water due to the given variation of the water level at the mouth of a bay. Recently, making use of the theory of multiple reflection of elastic waves in a layer, K. Kanai et at[4] calculated the expected wave form underground from microtremors observed at the surface of ground the result showing good coincidence with the observed record underground.

---

1) R. TAKAHASI, "Motion of the Bay Water caused by Seismic Sea Waves," *Bull. Earthq. Res. Inst.*, **25** (1947), 1.

2) T. RIKITAKE, "The Form of Tsunami-waves outside a Bay as Inferred from the Motion of Bay-water," *Bull. Earthq. Res. Inst.*, **29** (1951), 277.

3) K. NAKAMURA, "Motion of Water due to Long Waves in a Rectangular Bay of Uniform Depth," *Sci. Rep. Tohoku Univ.*, Geophysics, [v], **12** (1961), 191.

4) K. KANAI, T. TANAKA and S. YOSHIZAWA, "On Microtremors, IX, Multiple Reflection Problem," *Bull. Earthq. Res. Inst.*, **43** (1965), 577.

In this paper, the wave forms for some tsunamis outside the bays of Onagawa, Ayukawa and Onahama are inferred from the tide-gauge records taken at the head of these bays on the principle of multiple reflection. In particular, the estimated waves calculated from the record obtained near the head of Onagawa Bay are compared with the record of tsunamis obtained at Enoshima which is located about 15 km east of Onagawa harbor.

For five tsunamis of 6 Nov. 1958 (Iturup), 13 Oct. 1963 (Iturup), 28 March 1964 (Alaska), 4 Nov. 1952 (Kamchatka), and 4 March 1952 (Tokachi-oki), the power spectrum was calculated at stations (Miyako, Enoshima, Onagawa and Onahama), coherence and phase difference being calculated for station pairs; Enoshima and Onahama, Enoshima and Miyako, and Onagawa and Onahama, and edge waves are examined on the basis of the theoretical dispersion curve of phase velocity.

2. Wave form at the mouth of a bay

If the bay is assumed rectangular, and constant depth, the wave form $\eta_0(t)$ at the mouth of the bay at time t can be obtained from the record of the water level variation $\eta(t)$ at the head of the bay as follows:

$$\eta_0(t)=1/2\{\eta(t-\tau)+\eta(t+\tau)\} \qquad (1)$$

where τ is the travel time of waves from the mouth to the head of the bay. To derive (1), bay head is considered a perfectly reflecting wall and the frictional effect in the bay is assumed negligible. If we approximate $\tau=T/4$ where T is the period of the fundamental seiche in the bay, we can estimate the wave form $\eta_0(t)$ at the mouth straightforwardly by using (1) from the knowledge of the water level variation $\eta(t)$ at the head of the bay and the seiche period T.

The application of the above method to some tsunami records obtained at various tide-gauge stations is given below. Fig. 1 shows the location of the tsunami observatory at Enoshima and tide-gauge stations at Miyako, Onagawa, Ayukawa and Onahama.

Onagawa

This is the only place where the estimated wave form at the bay head can be compared with the observed wave records, since Enoshima is located near the mouth of the bay. Figs. 2, 3 and 4 show the com-

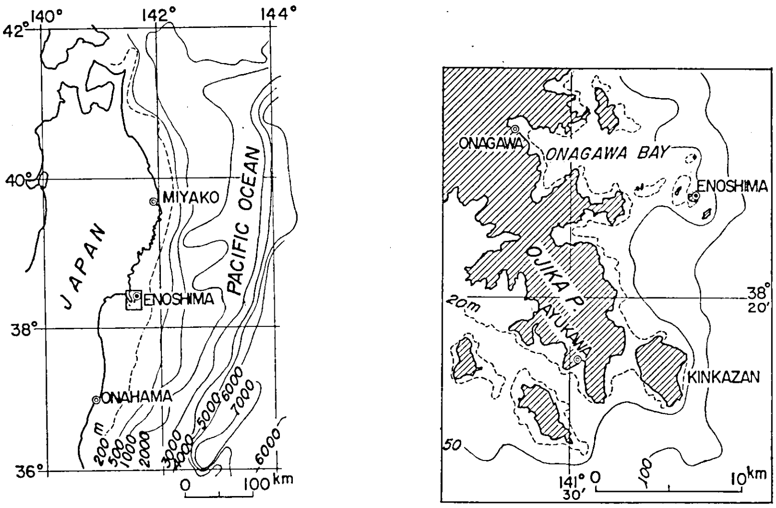

Fig. 1. Geographical distribution of the tsunami observatory (Enoshima) and the tide-gauge stations.

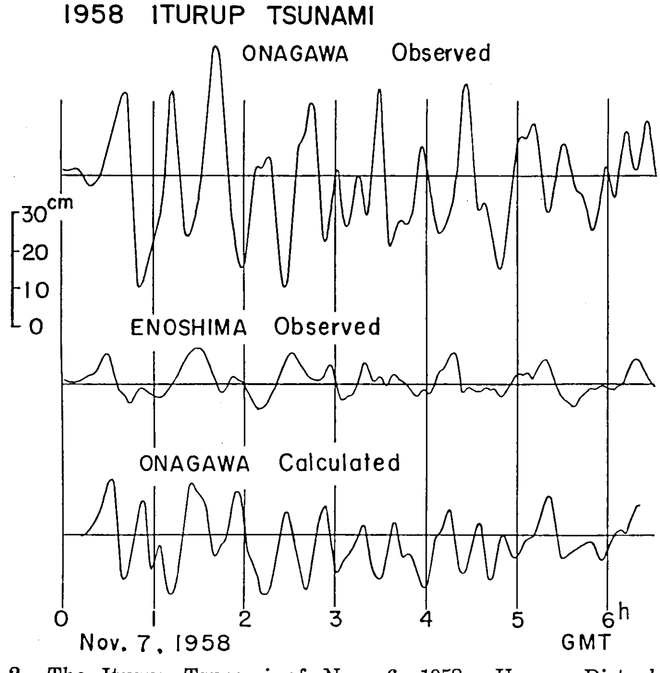

Fig. 2. The Iturup Tsunami of Nov. 6, 1958. Upper: Disturbance at Onagawa Bay from which the tide is eliminated. Middle: Observed record at Enoshima. Lower: Wave form at the mouth of the bay estimated from the record at the head of the bay.

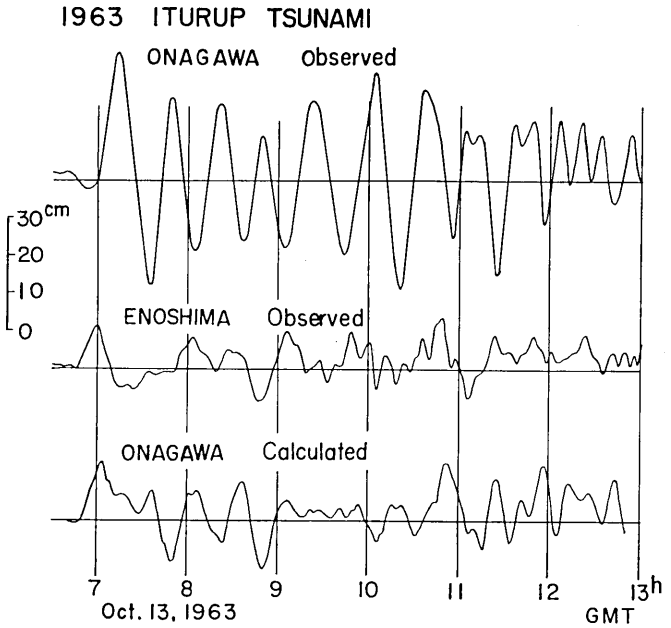

Fig. 3. The Iturup Tsunami of Oct. 13, 1963. Upper: Disturbance at Onagawa Bay from which the tide is eliminated. Middle: Observed record at Enoshima. Lower: Estimated wave form at the mouth of the bay.

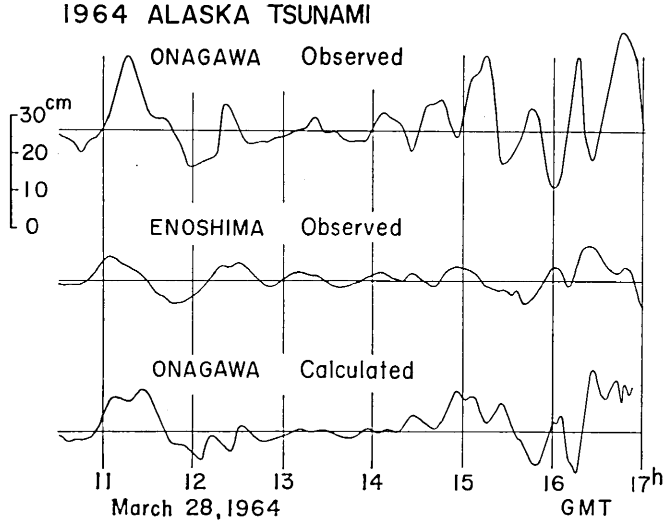

Fig. 4. The Alaska Tsunami of March 28, 1964. Upper: Disturbance at Onagawa Bay from which the tide is eliminated. Middle: Observed record at Enoshima. Lower: Estimated wave form at the mouth of the bay.

parison between the observed records and the calculated wave forms in the cases of the Iturup Tsunamis of 1958, 1963 and Alaska Tsunami of 1964. For the calculation, the records are read at every 2.5 minutes interval and the seiche period in Onagawa Bay is assumed to be 40 minutes inferred from the data of daily observation.

In these figures, the upper one shows the tide-gauge record at Onagawa taken by the Magnetic Observatory, Tohoku Univ., from which the ordinary tide is eliminated, the middle one is the record at Enoshima Tsunami Station observed by the tsunami recorder, and the lower one is the wave form estimated at the mouth of the bay from the record at Onagawa. Comparing the estimated wave form with the observed record at Enoshima, it may be said that a fairly good agreement is obtained and that this method would therefore be useful to estimate the wave form at a bay mouth. The coincidence of the estimated and the observed arrival time of the first crest shows that the travel time τ is correctly estimated.

Onahama

The coastal line in the vicinity of Onahama is relatively simple. The tide-gauge is installed within a harbor, in which the proper period of oscillation is 15 minutes. Fig. 5 shows the records of the tsunamis of 1958, 1963 (Iturup) and 1964 (Alaska), from which the ordinary tide was eliminated, and the estimated waves outside the harbor.

Although the calculated waves still contain short period components, the wave forms became simpler than the original and we assume these as tsunami waves on the continental shelf off Onahama.

Ayukawa

The tide-gauge is installed at the head of the bay. The mouth of the bay faces a channel formed by the islands as shown in Fig. 1. The seiche period of 8 minutes is predominant in the bay. The upper figure in Fig. 6 shows the record of the 1958 Iturup Tsunami from which the ordinary tide was eliminated.

The middle figures in Fig. 6 (symbol A) show the estimated wave forms at the mouth of the bay for the Iturup Tsunami of 1958. T. Rikitake[2)] obtained the wave form at the mouth of the bay for the Urup Tsunami of 1918 (lower figure of A) by means of a different method. It is clear that the 1918 tsunami was larger than the 1958 tsunami in amplitude, but the wave forms look very similar and contain

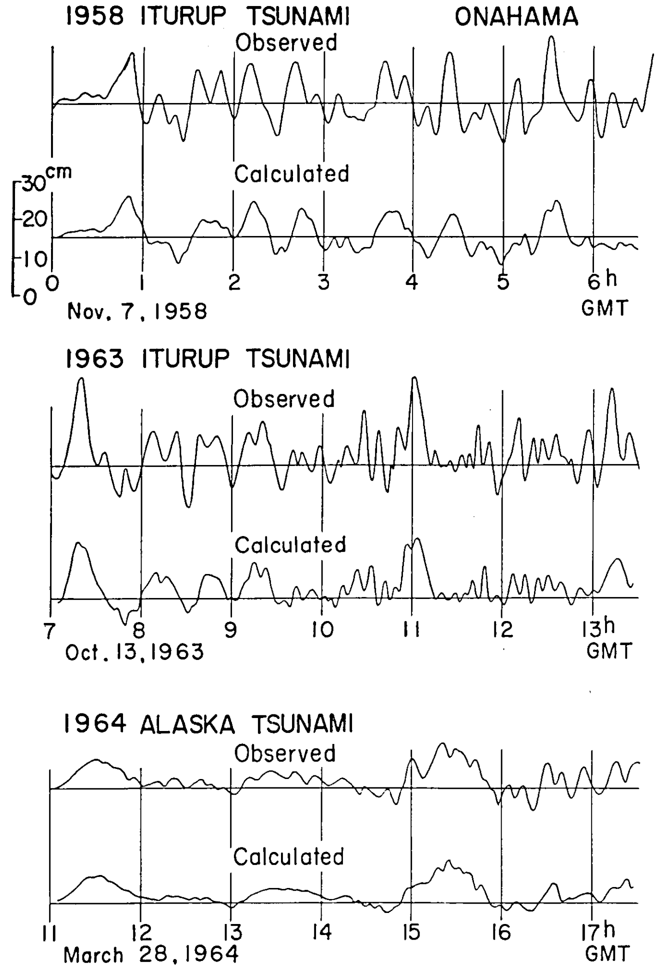

Fig. 5. Different tsunamis at Onahama. Disturbance of bay-water from which the tide is eliminated and the estimated wave form on the continental shelf by means of the present method.

components with the proper period of about 20 minutes. The spectral analysis[5)] of the records at Ayukawa for the tsunamis of 1952 (Tokachi-oki and Kamchatka) also indicated dominant components with the proper period of 20 minutes. When the mean depth and length of the channel are assumed to be 30 m and 11 km respectively, the seiche period of

5) R. TAKAHASI and I. AIDA, "Spectra of Several Tsunamis observed on the Coast of Japan," *Bull. Earthq. Res. Inst.*, **41** (1963), 299, (in Japanese).

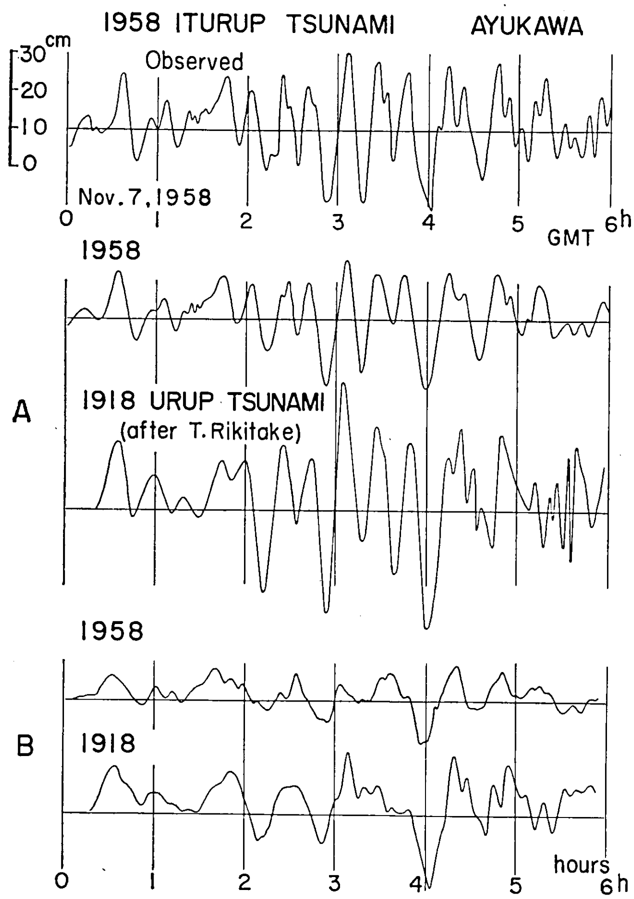

Fig. 6. Tsunamis of the Iturup (1958) and Urup (Sept. 7, 1918) at Ayukawa. Upper: Disturbance of bay-water from which the tide is eliminated. Middle (A): Wave forms estimated at the mouth of the bay. Lower (B): Wave forms estimated on the continental shelf.

about 20 minutes is obtained. Therefore, it seems that the seiche of the channel is often excited by tsunamis.

The lower figures in Fig. 6 (symbol B) show the estimated wave forms of 1918 and 1958 tsunamis outside the channel by taking the channel seiches of about 20 minutes into account. It can be seen that the estimated wave pattern of the 1918 tsunami is similar to that of the 1958 tsunami to a certain extent. For these two cases the tsunami origins were near to each other.

As shown in Figs. 2, 3 and 5, the records of the 1958 and 1963 Iturup Tsunamis are very similar in shape, especially in several of the initial waves. Taking the similarity of the 1918 and 1958 tsunamis into account, it seems that these tsunamis (1918, 1958 and 1963) were generated and propagated in a similar manner, namely the source position, source mechamism and the travel path were similar.

3. Spectral analysis

In cases of five tsunamis, if the estimated wave forms were not dominantly influenced by the topographic effect in the vicinity of the respective bay mouth, the coherence and phase difference are calculated by the Tukey's method for each of the station pairs (Enoshima and Onahama, Enoshima and Miyako, and Onagawa and Onahama) beside the power spectrum. The conditions of analysis are as follows: Time length of the record is 6.5 hours including the initial wave for which the records were sampled at every 2.5 minutes interval, so that the total number of data points is 157 and the lag is taken as 72. The records at Enoshima are corrected with respect to sensitivity and oscillatory char-

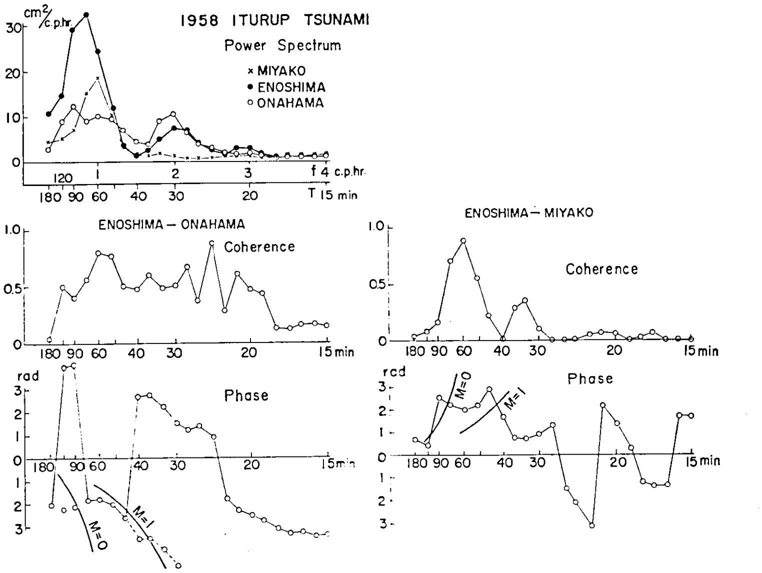

Fig. 7. Power spectrum of the 1958 Iturup Tsunami at Miyako, Enoshima and at Onahama (upper). Coherences and phase differences between the simultaneous records at Enoshima and Onahama, and at Enoshima and Miyako (lower).

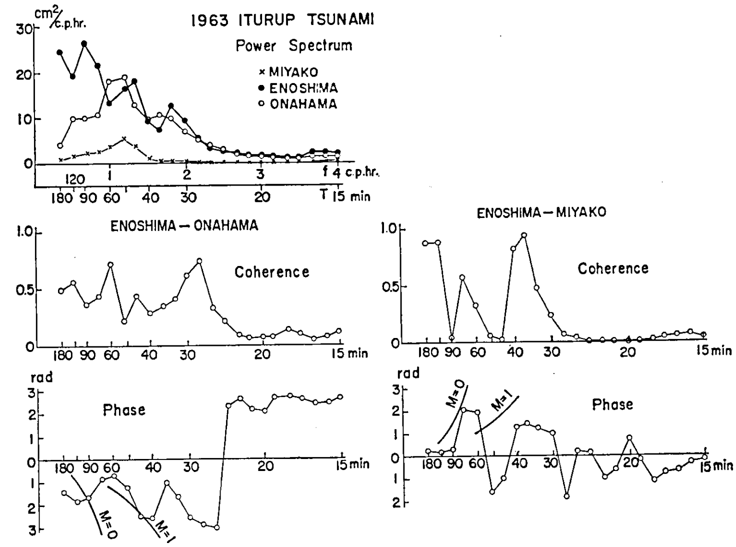

Fig. 8. Power spectrum of the 1963 Iturup Tsunami at Miyako, Enoshima and at Onahama (upper). Coherences and phase differences between the simultaneous records at Enoshima and Onahama, and at Enoshima and Miyako (lower).

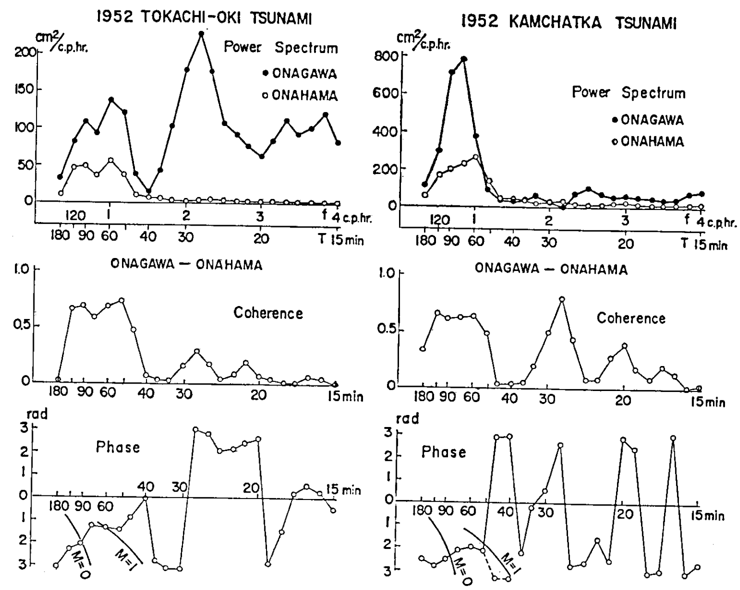

Fig. 9. Power spectrum of the 1952 Tokachi-oki Tsunami and the 1952 Kamchatka Tsunami at Onagawa and at Onahama (upper). Coherences and phase differences between the simultaneous records at Onagawa and Onahama (lower).

acteristics of the instrument. At Onahama, Onagawa and Miyako, the calculated waves at the mouth of the bay are used. Examples of the waves to be used are shown in Figs. 2 to 5.

Figs. 7 to 10 show the power spectrum (upper figure), coherence (middle figure), and phase difference (lower figure) for each tsunami. From the result of the spectral analysis, the peaks of the spectra for each tsunami show in Table 1. From Table 1, it seems that the position of the predominant peak of the tsunami spectrum move to the low frequency part as the earthquake magnitude or the travel time increases.

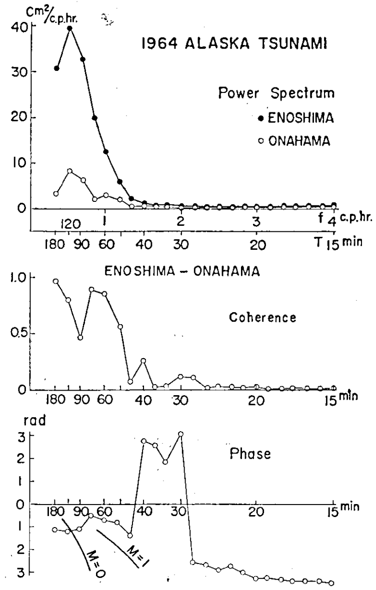

Fig. 10. Power spectrum of the 1964 Alaska Tsunami at Enoshima and at Onahama (upper). Coherence and phase difference between the simultaneous records at Enoshima and Onahama (lower).

4. Edge wave

The empirical dispersion curves of group velocity for the Iturup (1958, 1963)[6] and Alaska Tsunami (1964)[7] calculated from the records observed at Enoshima coincided approximately with the theoretical dispersion curve calculated by K. Nakamura for a simple shelf geometry. Recently I. Aida[8] calculated dispersion curves of phase

Table 1. Peaks of the spectra for the tsunami waves

Date	Location	Earthquake magnitude	Peaks of the spectra (min)
1952 March 4	Tokachi-oki	8.1	60, 28
1952 Nov. 4	Kamchatka	8.25	90
1958 Nov. 6	Iturup	8.2	60, 30
1963 Oct. 13	Iturup	8.1	50, 33
1964 March 28	Alaska	8.5	120

6) T. HATORI and R. TAKAHASI, "On the Iturup Tsunami of Oct. 13, 1963, as Observed along the Coast of Japan," *Bull. Earthq. Res. Inst.*, **42** (1964), 543.
7) T. HATORI, "On the Alaska Tsunami of March 28, 1964, as Observed along the Coast of Japan," *Bull. Earthq. Res. Inst.*, **43** (1965), 399.
8) I. AIDA, "Water Level Oscillations on the Continental shelf in the Vicinity of Miyagi-Enoshima," *Bull. Earthq. Res. Inst.*, **45** (1967), 61.

velcity of edge waves which are propagated along the continental shelf in the vicinity of Enoshima. Based on this dispersion curve, the phase differences of the fundamental and the second mode edge waves for station pairs ; Enoshima and Onahama, Enoshima and Miyako, and Onagawa and Onahama, are calculated and shown with a solid line in the lower figure in Figs. 7 to 10. The distances D of propagation from Enoshima to Onahama and to Miyako are taken as 160 km and 140 km respectively.

As shown in the figures, the theoretical curve of the second mode seems to fit roughly in these observed values, at station pairs of Enoshima and Onahama for the Iturup Tsunami, in which the coherence is 0.7 or more. However, for other tsunamis, edge waves are not found clearly.

The phase velocity of a tsunami along the coast is expressed as $D/t=2\pi D/T\alpha$, where T is the period of tsunami and α, the phase difference. For the Iturup Tsunami of 1958, it is taken as $T\doteqdot 60$ min and $\alpha=2$ rad from Fig. 7. The calculated velocity between Enoshima and Onahama is 140 m/sec, which corresponds to the velocity of tsunami propagated in the mean depth of 2000 m, if the wave front is perpendicular to the coast line.

5. Conclusion

Based on the theory of multiple reflection of waves in a bay, wave form of a tsunami at the mouth of a bay is inferred from the observed record at the head of a bay.

Spectral peaks at Enoshima and Onahama for the distant tsunami fall in the bands of periods from 50 to 120 minutes. There seems to be a tendency that the lower frequency part in the tsunami spectrum is predominant for a larger earthquake.

The existence of edge waves was examined by comparing the phase differences for station pairs computed from the cross spectral analysis and those expected theoretically. However, this expectation was not so well defined in the present result.

The author wishes to express his sincere thanks to Prof. K. Kajiura for his guidance. Thanks are also due to Dr. I. Aida for his consent in making the computer program available.

7. 陸棚における津波の波形

地震研究所 羽鳥徳太郎

1958年，1963年のエトロフ津波および1964年のアラスカ津波について，女川，小名浜，鮎川の各湾奥で観測された検潮記録から，湾口における津波の波形を求めた．解析方法は波の重複反射の理論にもとずいて，次の様に行なう．記録された波から，平常の潮汐成分をとり除き，この波に湾の静振周期の半周期ずらして加算を行なっている．とくに女川湾外には江ノ島の津波計による同時観測が行なわれているので，江ノ島の記録と湾口の推定波形とを対比すると，波形，振幅がかなりよく合致する結果を得た．したがって，湾奥で観測した津波記録から，湾口における波形を推定するのに，この方法は有用であろう．

次に数個の遠地津波について，江ノ島の記録および宮古，女川，小名浜の湾外における推定波形から，それぞれのパワースペクトルを求めた．これからアラスカ津波では120分，カムチャツカ津波では90分，1958年のエトロフ津波では60分と30分，1963年のエトロフ津波では50分と33分，十勝沖津波では60分と28分にスペクトルのピークがあった．この解析結果によって，マグニチュードの大きい地震ほど津波スペクトルのピークの位置は長周期部分へ移動するという傾向を示している．

エトロフ，アラスカ津波について，江ノ島の記録から群速度の分散曲線を求めたところ，陸棚の単純なモデルから得た計算値と近似する結果を得ている．今回これらの遠地津波について，宮古，江ノ島，小名浜などの6.5時間分の記録から，江ノ島—小名浜，江ノ島—宮古など二点間の，波の相関および位相差を計算した．この結果を位相速度の理論的分散曲線と対比すると，今回の解析結果に対しEdge waveの検出は十分にできなかった．