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74. *Studies of the Thermal State of the Earth. The 20th Paper: Mountain Formation and Thermal Conduction.*

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Abstract

Time-variations of temperature within a mountain range are calculated by applying the relaxation technique to solving an equation of thermal conduction in the following two-dimensional mountain models. 1) A mountain range which is formed by a rapid erosion. 2) Mountain formation is caused by water erosion, the speed of which is very slow and constant. 3) A volcanic intrusion penetrated a flat plain. The results of the computations indicate that an L-type isotherm pattern would not be actually observed in geothermal measurements provided the geologically accepted age of mountains is true. It is possible to roughly estimate the age of a volcanic dome if it is younger than a few 10^4 years. Mt. Minakami-yama, a volcanic dome in the central area of the Matsushiro Earthquake Swarm, seems to be older than 10^4 years, the conclusion being consistent with the result obtained by a K-Ar method.

1. Introduction

Uyeda and Horai[1], who measured temperature distributions in metal mines in Japan, pointed out that, in some of the mines, the isotherms were running horizontally with no relation to the topographical features of ground surface and that, in some other mines, the isothermal surfaces were lying more or less parallel to the surface relief. They came to a conclusion that there are two types of isotherm distribution and called the former type "L-type" and the latter "D-type". Horai[2] speculated that such a distinction between the two types might be explained by taking the difference in the erosional history of the topography into

1) S. UYEDA and K. HORAI, *J. Geophys. Res.*, **69** (1964), 2121.
2) K. HORAI, *Bull. Earthq. Res. Inst.*, **42** (1964), 93.

account. Should Horai's speculation be true, there would be a possibility of roughly estimating the time required for reaching the present thermal state on the assumptions that the thermal process is utterly controlled by heat conduction and that the subsurface temperature measurements are made in fair detail.

What the author intends to advance in this paper is a theory of thermal process in a typical mountain with the purpose of applying the theory to a number of likely cases of mountain formation. The author will compute temperature variations with time by applying the relaxation technique, as will be outlined in Section 2, to the equation of thermal conduction in the three following two-dimensional mountain models. First of all, thermal process in a mountain range which is formed by a rapid erosion accompanied by landslide, fault, volcanic collapse and the like will be studied. It will be assumed in the next place that the speed of erosion is very slow and constant all the time. Such an erosion process could be caused by water and so may be called the water erosion. The third model will be an igneous intrusive mass which, penetrating the flat plain, formed a dome or a mountain range with a constant temperature at the initial state. All the results of computations for the above three models will be illustrated in Section 3.

In Section 4, the author will also examine the thermal state of some metal mines, such as Kamaishi, Ashio, Ikuno and Makimine, the temperature distributions of which were measured by Uyeda and Horai[3),4)] and Horai[5),6)] with special intention of comparing the actually observed temperature distributions to the theoretical ones as obtained in the previous sections. As a result of the investigation, a view that the existence of L-type isotherm distribution is implausible will be presented there.

Morimoto *et al.*[7)] estimated the age of Mt. Minakamiyama, a volcanic dome situated at the central area of the Matsushiro Earthquake Swarm. A few 10^5 years were suggested by them on the basis of topographic considerations as the age of the dome since its birth. A potassium-argon dating also indicated 3.5×10^5 years. The author would also like to extend the present theory in Section 5 in order to get some clue to a possible estimate of the time since the formation of the dome.

3) S. UYEDA and K. HORAI, *Bull. Earthq. Res. Inst.*, **41** (1963), 83.
4) S. UYEDA and K. HORAI, *Bull. Earthq. Res. Inst.*, **41** (1963), 109.
5) K. HORAI, *Bull. Earthq. Res. Inst.*, **41** (1963), 137.
6) K. HORAI, *Bull. Earthq. Res. Inst.*, **41** (1963), 149.
7) R. MORIMOTO *et al.*, *Bull. Earthq. Res. Inst.*, **44** (1966), 423.

2. Relaxation method and stability of solutions

The differential equation of thermal conduction is written as

$$\frac{\partial\theta}{\partial t}=\kappa\nabla^2\theta \ , \tag{1}$$

where θ, κ and $\nabla^2\theta$ denote temperature, thermal diffusivity and Laplacian θ, *i.e.*

$$\nabla^2\theta=\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2} \ . \tag{2}$$

In a two-dimensional case, $\nabla^2\theta$ can be approximated as follows;

$$\begin{aligned}\nabla^2\theta=[&\theta(x-h,\, y,\, t)+\theta(x+h,\, y,\, t)+\theta(x,\, y-h,\, t)\\ &+\theta(x,\, y+h,\, t)-4\theta(x,\, y,\, t)]/h^2 \ ,\end{aligned} \tag{3}$$

where h is a spacing between two grid points forming square relaxation patterns. The left-hand member of equation (1) being approximately written as

$$\frac{\partial\theta}{\partial t}=\frac{\theta(x,\, y,\, t+\Delta t)-\theta(x,\, y,\, t)}{\Delta t} \ , \tag{4}$$

the relaxation form of (1) can be obtained by substituting (4) to (1), *i.e.*

$$\theta(x,\, y,\, t+\Delta t)=\theta(x,\, y,\, t)+\kappa\Delta t\nabla^2\theta \ . \tag{5}$$

The right-hand members of this equation are functions at t, while the left-hand one at $t+\Delta t$. If the three-dimensional temperature distribution at t is given at all grid points of the pattern, we can readily compute the temperature distribution at $t+\Delta t$. Starting from an initial temperature distribution, therefore, it is possible to calculate decreases in temperature at all grid points in conjunction with proper boundary conditions.

It is very important for solving the relaxation equation (5) to choose appropriate values of h and Δt. An inappropriate choice leads to unstable solutions in which the temperature distribution is sometimes oscillating. The double Fourier transform of $\theta(x, y, t)$ with respect to x and y is given as

$$\Theta(\omega_1,\, \omega_2,\, t)=\iint_{-\infty}^{\infty}\theta(x,\, y,\, t)\exp\{-j(\omega_1 x+\omega_2 y)\}dxdy \ , \tag{6}$$

where ω_1 and ω_2 are frequencies in the directions of x and y axes, respectively, and $j=\sqrt{-1}$. Similar transformation in regard to (5) leads to

$$\Theta(\omega_1, \omega_2, t+\Delta t)=\left\{1-\frac{4\kappa\Delta t}{h^2}\left(\sin^2\frac{\omega_1 h}{2}+\sin^2\frac{\omega_2 h}{2}\right)\right\}\Theta(\omega_1, \omega_2, t)$$
$$\geqq\left(1-\frac{8\kappa\Delta t}{h^2}\right)\Theta(\omega_1, \omega_2, t) ,$$

so that we see that the following relation should hold good;

$$\left|\frac{\Theta(\omega_1, \omega_2, t+\Delta t)}{\Theta(\omega_1, \omega_2, t)}\right|\geqq 1-\frac{8\kappa\Delta t}{h^2}\geqq 0 . \tag{7}$$

The condition for which the solutions are stable is,

$$\frac{\kappa\Delta t}{h^2}\leqq\frac{1}{8} . \tag{8}$$

in a two-dimensional case.

3. Changes in temperature in two-dimensional mountain models

3-1. Mountain range formed by a rapid erosion

Suppose that the topography is originally a flat plain in which isotherms are running horizontally and that the ground is subjected to an erosion which is so rapid that the flat plain is converted into a mountain range within a short period of time. In that case, it is supposed that the isotherms would have remained almost unchanged for some time (L-type) and their shapes would gradually be modified later in such a fashion as to run more or less parallel to the surface relief (D-type). Since the speed of erosion is very slow, it is most unlikely that a mountain should be formed actually in a short period by water erosion only. It would be necessary to imagine that some landslide, fault-slip and such like would accelerate erosion which develops the slope. Geological evidence that a valley is apt to develop along a fault line may in fact support the model considered in actual mountain formation.

The slope-angle is taken as 45° and the altitude of right-angled top above river floor at the foot is assumed as ten in units of h throughout the calculation in this section. $z=0$ indicates the surface of the flat plain before erosion, and z-axis is taken downwards. $\theta(x, y, 0)=\beta z$ is adopted as the initial condition, where β is the original geothermal gradient. The boundary conditions are $\theta=0$ on the surface and $\theta=\theta_0$ at a plane defined by $z=h\theta_0/\beta$, the plane being deep enough not to be

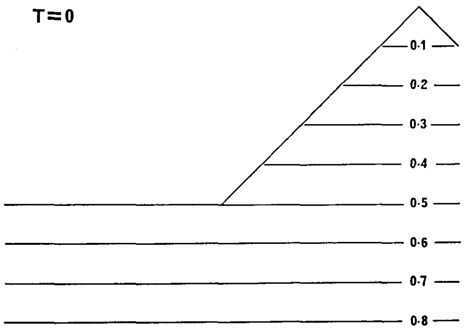

Fig. 1a. Isotherm distribution at $t=0$ (a rapid erosion).

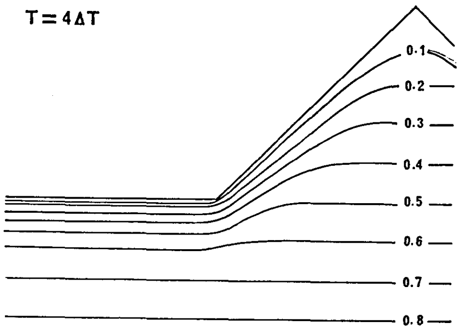

Fig. 1b. Isotherm distribution at $t=4\,\Delta t$ (a rapid erosion).

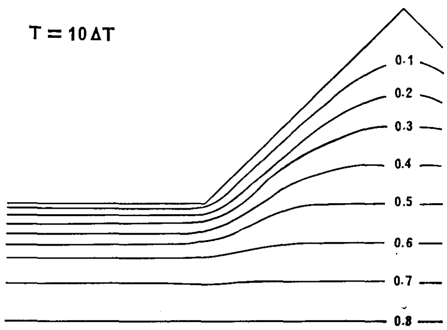

Fig. 1c. Isotherm distribution at $t=10\,\Delta t$ (a rapid erosion).

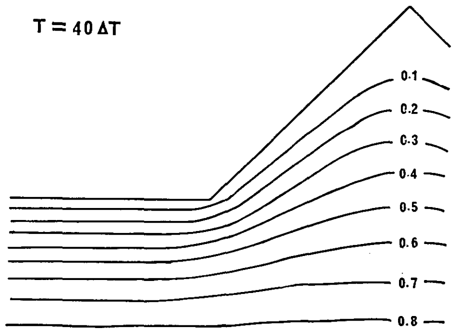

Fig. 1d. Isotherm distribution at $t=40\,\Delta t$ (a rapid erosion).

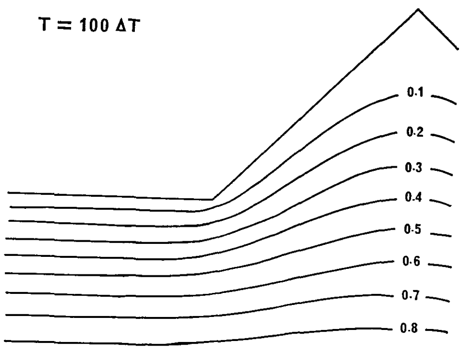

Fig. 1e. Isotherm distribution at $t=100\,\Delta t$ (a rapid erosion).

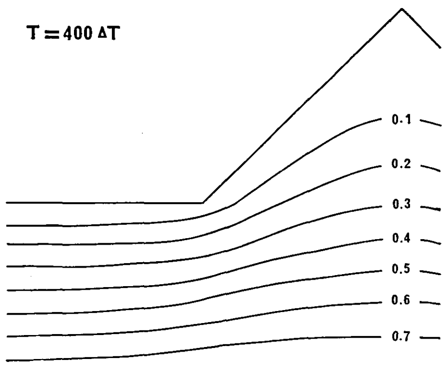

Fig. 1f. Isotherm distribution at $t=400\,\Delta t$ (a rapid erosion).

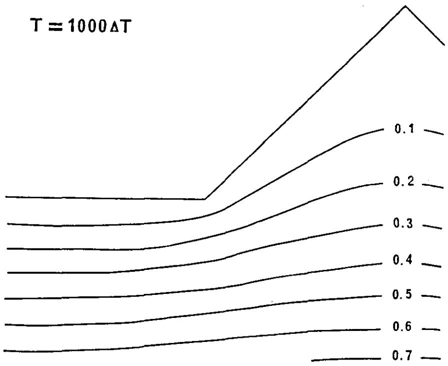

Fig. 1g. Isotherm distribution at $t=1000\,\Delta t$ (a rapid erosion).

influenced by the cooling. The unit of time Δt is taken as $h^2/8\kappa$ by taking the limit of stability for two-dimensional relaxation method (see (9)) into account. Temperatures being measured in units of θ_0, Fig. 1a shows the initial temperature distribution in the case of $\beta=0.05$ in units of θ_0/h. Figs. 1b, 1c, $\cdots$, and 1g show temperature distributions at times of $4\Delta t$, $10\Delta t$, $\cdots$, and $1000\Delta t$, respectively. Although the temperature distribution tends to become the D-type near the surface for small values of t, the interior distribution keeps the initial L-type isotherm pattern (see Figs. 1b and 1c). The L-type isotherm distribution disappears after $t=40\Delta t$ as shown in Fig. 1d, all isotherms being identified as the D-type. Assuming that $h=100$ m and $\kappa=10^{-2}$ cm^2/sec, $40\Delta t$ is estimated as about 1600 years, a very short period of time for a mountain problem. As far as the present model goes, therefore, it may be said that almost all the mountain ranges should exhibit the D-type isotherm distributions.

3-2. Mountain range formed by a gradual erosion

In contrast to the rapid formation of mountain body, as dealt with in the last sub-section, the author would here like to take up a mountain formation which is so slow that the lowering of flat plane takes place with a constant speed while the mountain body is constantly subjected to cooling. Such a model may well be applied to an erosion by water flowing over the mountain slope. Although it has been known that the general geometry of a slope eroded by water is an exponential curve according to Strahler[8], the author assumes that the slope-angle is always kept at 45° for simplification and the water hits the originally flat surface with a constant mass-carrying velocity. The initial isotherms

8) A. STRAHLER, *Bull. Geol. Soc. Amer.*, **63** (1952), 923.

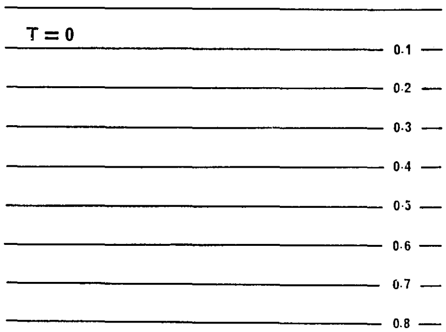

Fig. 2a. Isotherm distribution at $t=0$ (a gradual erosion).

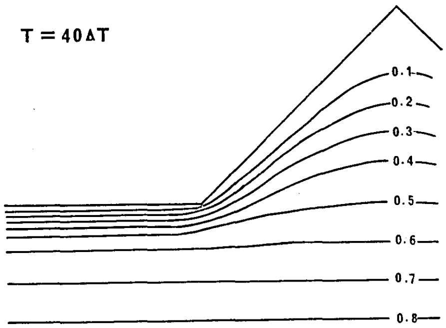

Fig. 2b. Isotherm distribution at $t=40\,\Delta t$ (a gradual erosion).

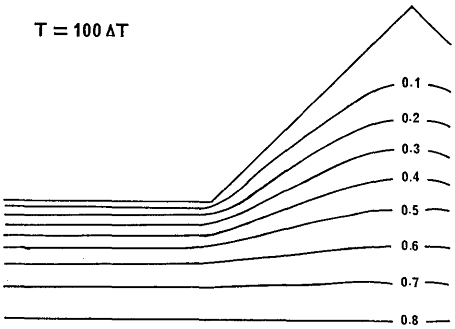

Fig. 2c. Isotherm distribution at $t=100\,\Delta t$ (a gradual erosion).

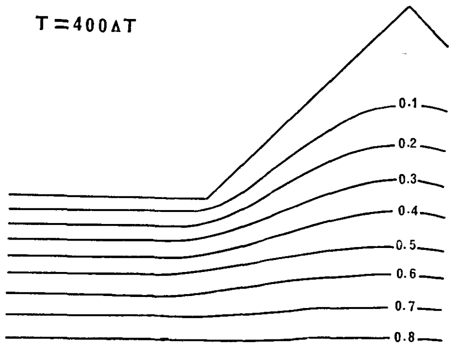

Fig. 2d. Isotherm distribution at $t=400\,\Delta t$ (a gradual erosion).

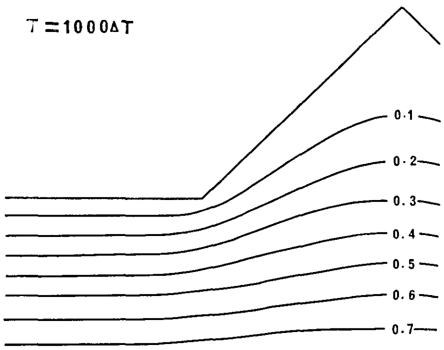

Fig. 2e. Isotherm distribution at $t=1000\,\Delta t$ (a gradual erosion)

run horizontally under the surface as shown in Fig. 2a. The initial and boundary conditions are the same as those mentioned in the last subsection except the boundary varying with time.

Figs. 2b, 2c, $\cdots$, and 2e show temperature distributions at times of $40\Delta t$, $100\Delta t$, $\cdots$, and $1000\Delta t$, respectively. The main difference between the rapid erosion and the gradual one is the rate of decrease in temperature especially under the level of the river floor. The isotherm distribution varies greatly with time in the case of rapid erosion, while the isotherms are almost stationary in the case of slow erosion. The L-type isotherm pattern cannot be found after $t=40\Delta t$ as shown in Fig. 2b. It is most unlikely that any L-type isotherm distribution would be found in an actual mountain which is older than a few tens of thousand years old.

3-3. Mountain range formed by a rock intrusion

The volcanic activity realizes this model. Let us suppose that a large-scale intrusive mass of high temperature lifts up penetrating the flat surface, under which the initial isotherms run horizontally. The heat flowing from the hot mass greatly disturbs the initial equilibrium of temperature, while the mass of the lava is gradually cooling from the surface. The initial geothermal gradient within the flat plain is assumed as 0.03°C/m and a uniform initial temperature of the intrusive rock mass is assumed as 1000°C. The mass is assumed to extend to an infinite depth. The boundary conditions are such that the temperature keeps 0°C on the surface relief all the time and the vertical heat flow is continuous at $z=30h$.

Figs. 3a, 3b, $\cdots$, 3e and 3f show the geothermal states at times of $10\Delta t$, $40\Delta t$, $\cdots$, and $2000\Delta t$, respectively, where time-step Δt amounts to about 10 years provided $h=50$ m and $\kappa=10^{-2}$ cm^2/sec. The temperature at the central part of the intrusive mass does not appreciably differ from the initial temperature until some 400 years have elapsed. The interior of the volcano is still hotter than 300°C after 2×10^4 years. It seems therefore likely that gases emanate and hot springs flow out from a volcanic mass even at a period a few 10^4 years after its intrusion. A bottomless hot mass is assumed in the present calculation. It is natural, however, to think that the decrease in temperature would be more rapid in the case of a lava mass flowing over a sedimentary layer. As an example of such a case, cooling of a mountain which may represent Mt. Minakamiyama in the Matsushiro area will be studied in Section 5.

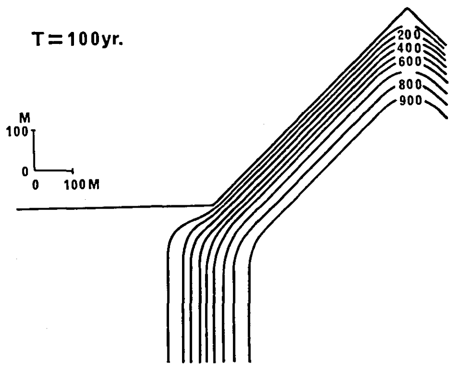

Fig. 3a. Isotherm distribution at t=100 yr. (a rock intrusion).

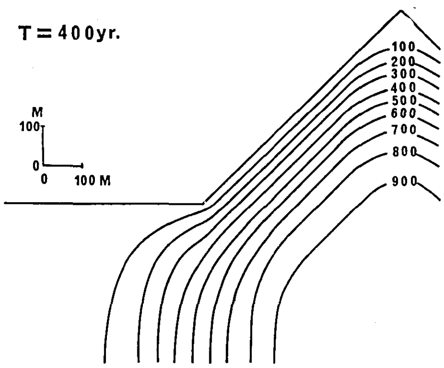

Fig. 3b. Isotherm distribution at t=400 yr. (a rock intrusion).

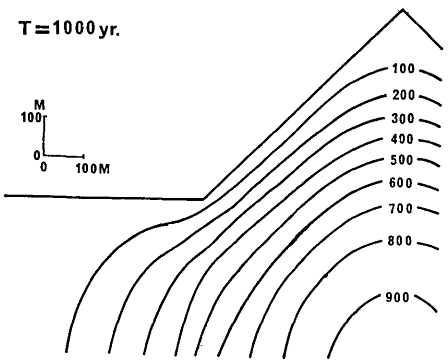

Fig. 3c. Isotherm distribution at t=1000 yr. (a rock intrusion).

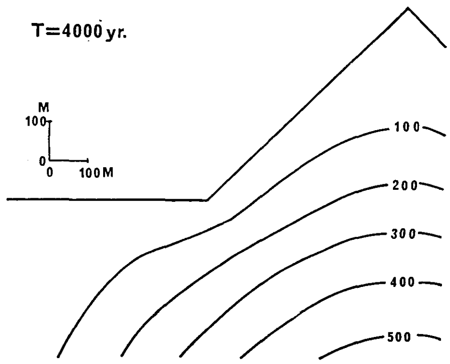

Fig. 3d. Isotherm distribution at t=4000 yr. (a rock intrusion).

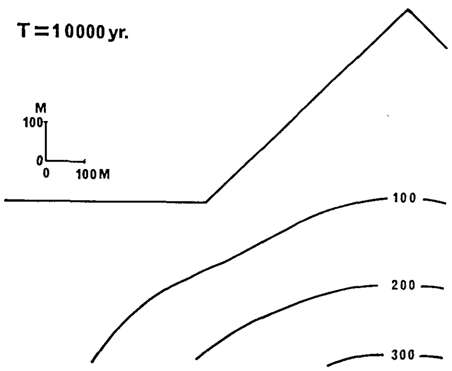

Fig. 3e. Isotherm distribution at t=10000 yr. (a rock intrusion).

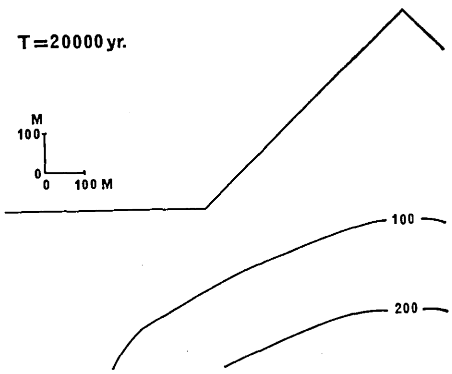

Fig. 3f. Isotherm distribution at t=20000 yr. (a rock intrusion).

4. Discussion on the temperature distributions in some Japanese metal mines

Uyeda and Horai measured temperature distributions in twenty metal mines. Of these, 13 mines are classified as the L-type and the remaining the D-type. The author selected four of these mines as typical examples by the reason that there were many sites of measurements. Ashio and Makimine belong to the L-type, while Kamaishi and Ikuno to the D-type. The model isothermal sections of these four mines are shown in Figs. 4a, 4b, 4c and 4d, respectively. It seems in the figures that all the isotherms are running parallel to the slope except in Ashio. As the sites of measurement are arranged almost along a vertical line right under the top of the mountain in the case of Ashio Mine, the author is of the opinion that we cannot decide whether the isothermal type is L or not. Judging from the results obtained in the last section, an L-type isotherm distribution can be seen only at a very young age since the beginning of erosion. Since most of the topographical features in Japan are presumed to have been formed during the latest glacial ages in Pleistocenes, which is too old to keep L-type isotherms, it is highly unlikely that an L-type isotherm is actually observed at the present time. Ore bodies of Kamaishi, Makimine, Ashio and Ikuno are considered

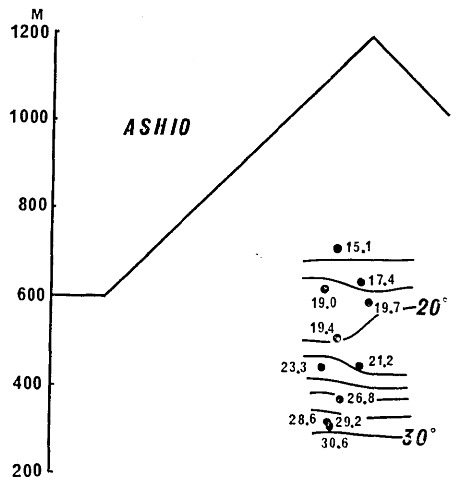

Fig. 4a. Isothermal section of Ashio Mine (after Uyeda and Horai).

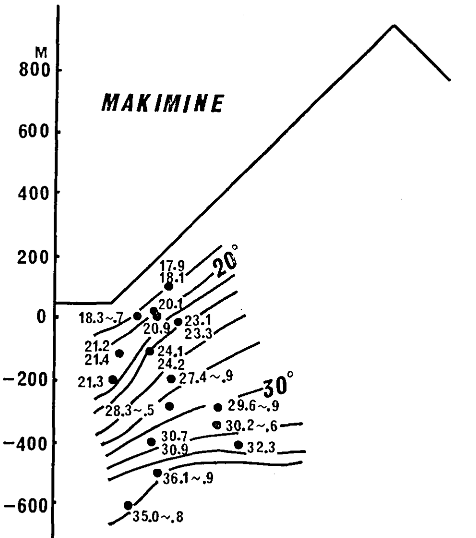

Fig. 4b. Isothermal section of Makimine Mine (after Horai).

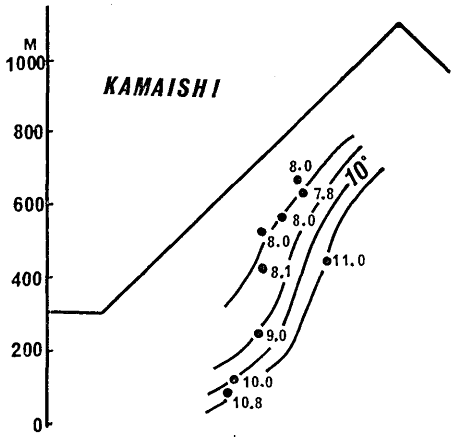

Fig. 4c. Isothermal section of Kamaishi Mine (after Horai).

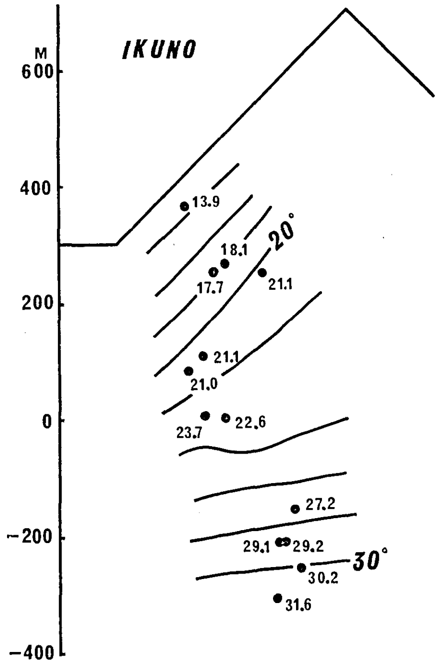

Fig. 4d. Isothermal section of Ikuno Mine (after Uyeda and Horai).

to deposit in the late Paleozoic (2×10^8 yr.), the late Mesozoic (1×10^8 yr.), the Quaternary and Tertiary (10^7 yr.), respectively. The deposition times in these mines are too old to influence today's isotherm distributions.

5. Mt. Minakamiyama—a volcanic dome at the central area of the Matsushiro Earthquake Swarm

Mt. Minakamiyama is a lava dome of pyroxine andesite penetrating the valley system formed by Quaternary alluvium and diluvium. The volcanic activity that resulted in the formation of the dome is estimated as 3.5×10^5 years ago by a K-Ar dating. Epicenters of the Matsushiro

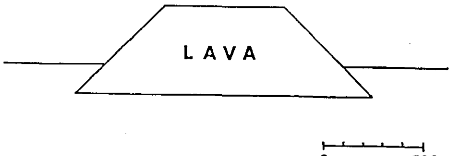

Fig. 5. Model Geometry of Mt. Minakamiyama.

Earthquake Swarm are distributed in a roughly circular area around Mt. Minakamiyama about 10 km in diameter, while depths of the foci range are from one km to ten beneath the mountain.

The shape of Mt. Minakamiyama is a circular cone 600 m in diameter at the top-plane, the slope-angle being about 45°. A geological column

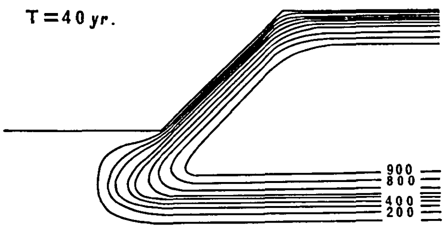

Fig. 6a. Isotherm distribution at t=40 yr. (Mt. Minakamiyama).

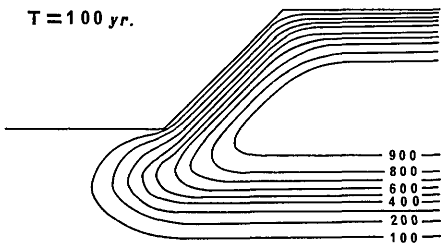

Fig. 6b. Isotherm distribution at t=100 yr. (Mt. Minakamiyama).

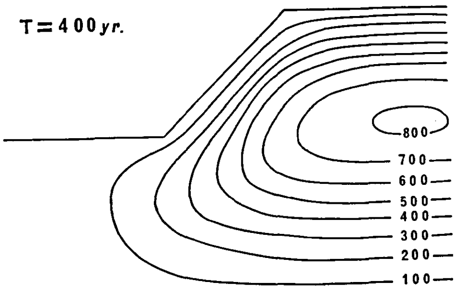

Fig. 6c. Isotherm distribution at t=400 yr. (Mt. Minakamiyama).

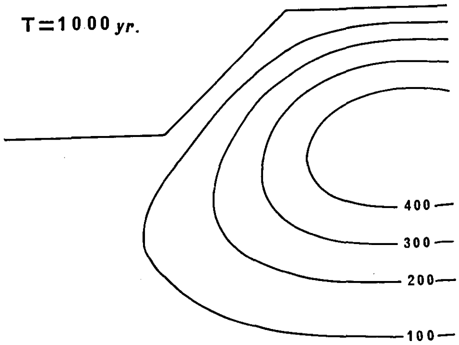

Fig. 6d. Isotherm distribution at t=1000 yr. (Mt. Minakamiyama).

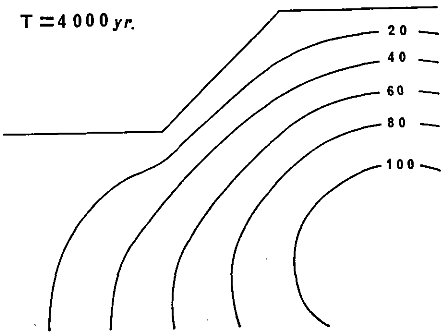

Fig. 6e. Isotherm distribution at t=4000 yr. (Mt. Minakamiyama).

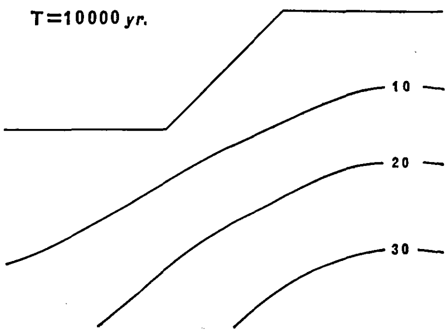

Fig. 6f. Isotherm distribution at t=10000 yr. (Mt. Minakamiyama).

by diamond-drilling on the foot of the dome pierced through andesite lava at a depth of about 150 m beneath the surface. Accordingly, it is not unreasonable to assume the geometry of Mt. Minakamiyama as shown in Fig. 5.

It is assumed that the initial geothermal gradient had been 0.03°C/m before the dome was formed. On the basis of the melting point of wet andesite, the initial temperature is assumed as 1000°C uniformly in all the mass of the lava dome. The temperature at the boundary contacting with the atmosphere is always taken as 0°C, and vertical heat flow is continuous at $z=30h$. Figs. 6a, 6b, $\cdots$, and 6f show isotherm distributions at about 40, 100, $\cdots$, and 10000 years, respectively, Δt is estimated as approximately 10 years by assuming that the thermal diffusivity of andesite is $10^{-2}\,\mathrm{cm^2/sec}$ and $h=50$ m. As seen in Figs. 6b and 6c, the central part of the dome keeps the initial temperature during a few ten years after the formation of the dome.

There are a number of drifts inside the lava of Mt. Minakamiyama which were drilled there as shelters for air raids during World War II. As it is said that nobody felt hot in the drifts, it is evident that the dome has entirely cooled down. As illustrated here in a series of figures, the temperature gets lower than 20°C at the central part of the dome only after 10000 years. Therefore, one can estimate that the age of Mt. Minakamiyama should be older than 10000 years at least. The result obtained here is consistent with the age determined by a K-Ar method.

The above calculation being made on a two-dimensional model, however, the cooling would be faster if a three-dimensional treatment is properly made. The cooling rate of a sphere is about 1.7 times greater than that of a cylinder, the radius of its section being equal to the radius of the sphere (see Appendix). Taking this fact into consideration, the age of Mt. Minakamiyama may be estimated to be older than 6000 years at least.

6. Conclusions

The author studied in this paper the thermal history of a mountain on the basis of three conceivable ways of mountain forming. The first is the case in which a mountain is formed by a rapid erosion accompanied with a fault or a collapse. The second one bases on erosion by water, the speed of which is very slow, and the last one is caused by an igneous intrusion. On the basis of the results of change in temperature in these three cases, the author concluded that it is not realistic that an L-type isotherm pattern would actually be obtained by field observations. It seems likely that the isotherm distribution identified as an L-type by Uyeda and Horai is only a part of a D-type pattern.

The following conclusions are obtained with respect to possible geothermal dating. It is difficult to estimate the time when a mountain was formed by erosion on the basis of the sub-surface temperature distribution observed. On the contrary, it seems possible to estimate roughly the age of a volcanic dome provided it is younger than a few 10^4 years. The author concluded that Mt. Minakamiyama should have been older than 10000 years of age. This is consistent with the result obtained by a K-Ar dating method.

7. Acknowledgment

The author wishes to express his thanks to Professor T. Rikitake under whose constant encouragement the present work has been carried out.

APPENDIX: Cooling Rate of Sphere and Cylinder

Temperature at the center of a sphere of which the radius is taken as unit length is written as

$$\theta=2\sum_{n=1}^{\infty}(-1)^{n+1}\exp(-n^2\pi^2\kappa t)\ .$$

at time t. The initial temperature is taken as unity, the boundary temperature being kept at 0°C on the surface of the sphere. Temperature on the central axis of a cylinder of infinite length, having the same radius is given by

$$\theta=2\sum_{n=1}^{\infty}\frac{\exp(-\lambda_n^2\kappa t)}{\lambda_n J_1(\lambda_n)}\ .$$

where λ_n is one of the roots of $J_0(\lambda_n)=0$ where J_0 is Bessel function of degree zero. The initial and boundary conditions are the same as those in the case of the sphere. Fig. 7 shows the cooling of the sphere and cylinder as calculated from the two expressions. The cooling rates are estimated as being about 5.7 and 9.9 respectively for the case of the sphere and the cylinder, provided tangents of the linear parts of these two curves are adopted. It is then obtained that the cooling speed of the sphere is about 1.7 times faster than that of the cylinder.

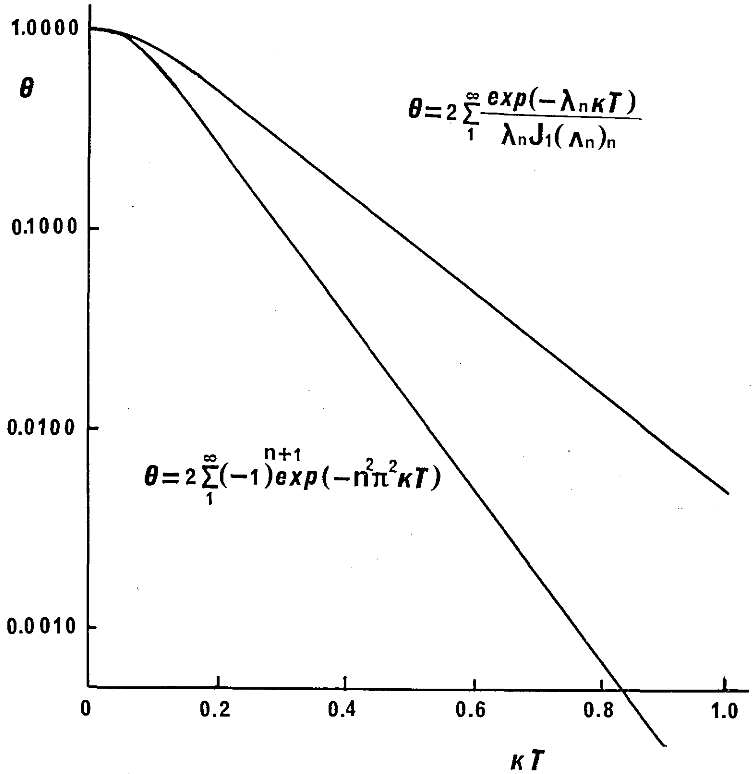

Fig. 7. Cooling of a sphere and a cylinder.

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74. 地 球 熱 学 第20報 山岳形成と熱伝導

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山岳が浸蝕や火山活動によつて形成された後，温度分布が時間の経過とともに，どのように変化して行くかを，熱伝導方程式によつて計算した．山岳形成モデルとして

1) 断層・地すべりの類によつて急に形成された場合．
2) ゆるやかな浸蝕で形成されて来た場合．
3) 火山活動によつて形成された場合．

を採用した．

その結果として次のことがわかつた．

1) 上田・宝来によれば，山の等温面が地形に関係なく高度に関係するもの（L型）と地形地表面に沿つて平行に分布するもの（D型）とがあるが，計算の結果 L 型は山の形成後極めて短期間を除いては存在しないことがわかつた．
2) 松代地震地域の皆神山ドームの年代は地形の浸蝕の度合からして数万年から数十万年，K-Ar 年代決定法で 35 万年と推定された．このドームの冷却を計算したところ 1 万年以上の年代を有するとの結論に達し，これらの結果と矛盾しない．