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2. *Optimum Distribution of Seismic Observation Points. III.*

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1. Introduction

Arrival times at seismic observation points are used for the determination of earthquake parameters, consequently the distribution of stations has a large effect upon the errors of these parameters such as epicenter location, focal depth, origin time and wave velocity. In our previous papers[1),2)] the position of the epicenter was given in advance—cases applicable to volcanic earthquakes and similar phenomena—and such station distributions were discussed in which the errors were very small (or very large) by means of the Monte Carlo method.

In the present paper, however, a number of distributions are chosen, some of which were found to be good by the previous study, errors being calculated as functions of epicenter location. In this way we can determine the extent of errors, their distribution, expected minimum and maximum errors, existence of areas where results appear comparatively poor even for good station combinations.

2. Problem

At the stations $P_j(X_j, Y_j, Z_j)$ the equations for errors of five quantities (the location of focus, the origin time and the propagation velocity)

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1) Y. SATÔ, *Zisin*, **18** (1965), 9. (in Japanese).

2) Y. SATÔ and D. SKOKO, "Optimum Distribution of Seismic Observation Points. II", *Bull. Earthq. Res. Inst.*, **43** (1965), 451.

are given as follows:

$$(X_j-X_0)x+(Y_j-Y_0)y-Hh+V_0C_jt-D_jv/V_0=V_0C_je_j \quad (j=1, \cdots, 5) \tag{2.1}$$

where:

$$\left.\begin{array}{ll} x, y & \text{errors in epicenter location of } x \text{ and } y \text{ direction, respectively,} \\ h, t, v & \text{errors of focal depth, origin time and velocity,} \\ X_0, Y_0 & \text{coordinates of epicenter,} \\ D_j & =-[(X_j-X_0)^2+(Y_j-Y_0)^2+H^2], \\ C_j & =-(-D_j)^{1/2}, \\ H & =\text{focal depth}, \\ \text{and} & \\ V_0 & =\text{velocity of elastic waves.} \end{array}\right\} \tag{2.2}$$

For simplicity's sake the following assumptions are adopted as before, which will not spoil the generality of the theory.

i) smallness of errors $x, \cdots, v$,

ii) P_j, observational points, are on the surface ($Z_j=0$),

iii) velocity of the waves is constant,

iv) observational errors e_j of arrival times are $\pm\varepsilon$ or 0.

For fifteen distributions of stations, including satisfactory ones proved in our previous study[2], the errors $x, \cdots, v$ are calculated for all the possible combinations of e_j, the arrival time errors at the stations. These values are calculated as a function of epicenter location within an area around the stations. The standard deviations of the epicenter location (R), depth of the focus (Z), origin time (T) and velocity (V) are plotted and the contours with equal degrees of error are drawn in the figures.

3. Result

The nondimensional amount of the errors can be seen in Fig. 1, where the side of the square is taken as the unit of length and the time for a wave to travel that distance as the unit of time. The depth of the focus (H) is assumed to be 0.1 and the errors of arrival times $e_j=\pm 0.01$ or 0. ($\varepsilon=.01$).

If we assume

$$\left.\begin{array}{lll} & \text{unit of length (=side of the square)} & =100\,\text{km} \\ & \text{unit of velocity} & =10\,\text{km/sec}, \\ & \text{unit of time} & =10\,\text{sec}, \\ \text{and} & \text{observation error of time } (=\varepsilon) & =0.1\,\text{sec}. \end{array}\right\} \tag{3.1}$$

The 1st, 2nd, 3rd and 4th rows of the figure indicate the errors R, Z, T and V respectively, for 15 significant distributions. The black circles denote the position of the seismic observation points.

(1), (2) and (3) of Fig. 1 are examples of symmetrical distribution. The region of large errors at (1) is found around two perpendicular axes of symmetry and that of small errors towards the outer stations. Especially small errors at (2) and (3) are at the center of the square. The errors of the epicentral position at (1) vary from about 1 km (around the outer stations) to more than 10 km (within the central part of the square). The errors of the depth vary from about 7 km to more than a hundred, the errors of origin time from about 0.4 sec to more than 20 sec and that of velocity from about 0.4 km/sec to more than 20 km/sec*.

Three of the stations at distributions (4), (5) and (6) form a triangle, the 4th points being on one side, the 5th either on the side or within the triangle ((6) is a case of symmetrical distribution). The area of small errors exists around the stations, especially, near the center of the triangle. The minimum values of errors R, Z, T and V are about 1 km, 4 km, 0.4 sec and 0.4 km/sec respectively, while the maximum values are as large as 50 times of the minima.

(7), (8) and (9) are examples of irregular station distributions which prove to be very good. Again, small errors are generally at close proximity to the stations.

(11) (symmetrical), (12), (13), (14) and (15) are examples of flat distributions, the entire field being separated by narrow bands with large errors. Generally speaking, however, the errors are small if the epicenter is inside the polygon formed by stations.

4. Remarks

From the results obtained above it can be seen that the distribution of the seismological stations has a significant influence upon the accuracy of the determination of the earthquake parameters. Their positions have to be chosen carefully so that errors may remain small at the area of expected earthquake occurrence.

The above examples will give some clue to the choice of optimum distribution of stations, from which we can deduce the following few conclusions: —

* Smallness of errors is assumed in the calculation, therefore the amount of large errors is not accurate.

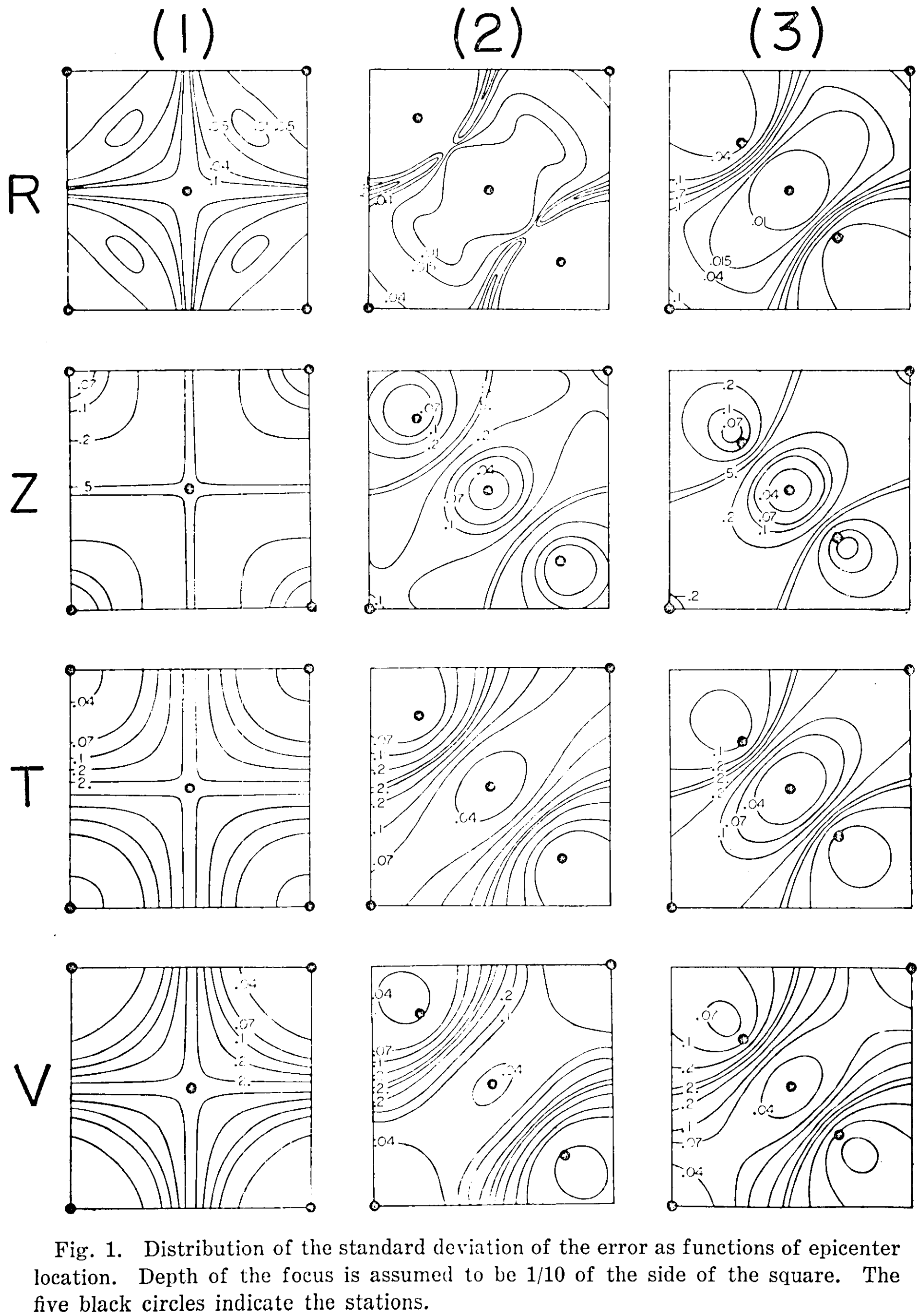

Fig. 1. Distribution of the standard deviation of the error as functions of epicenter location. Depth of the focus is assumed to be 1/10 of the side of the square. The five black circles indicate the stations.

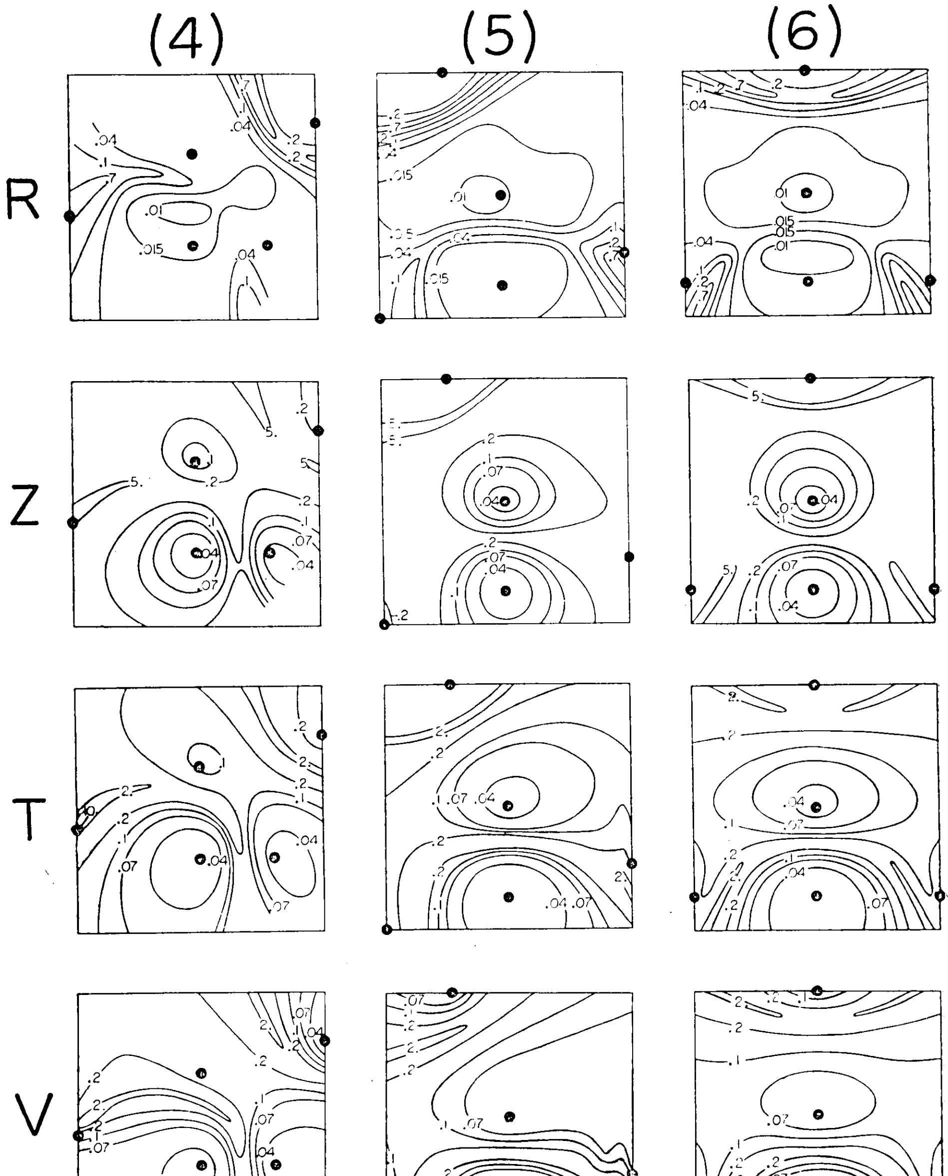

Fig. 1. (continued)

R: Error of the epicenter location (unit: length of the side of square)
Z: Error of the focal depth (unit: the same)
T: Error of the origin time (unit: time for the wave to travel the unit length)
V: Error of the propagation velocity.

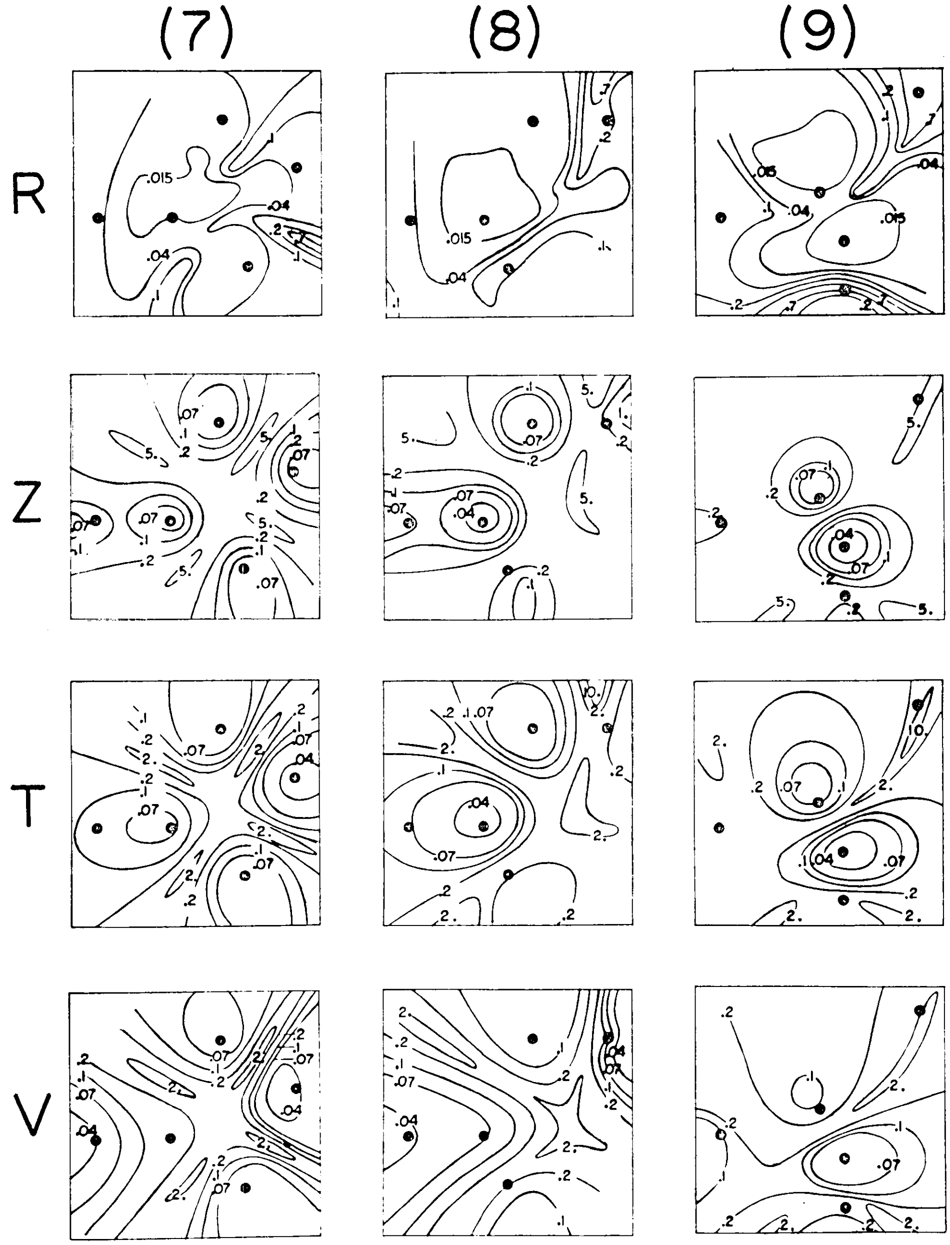

Fig. 1. (continued)

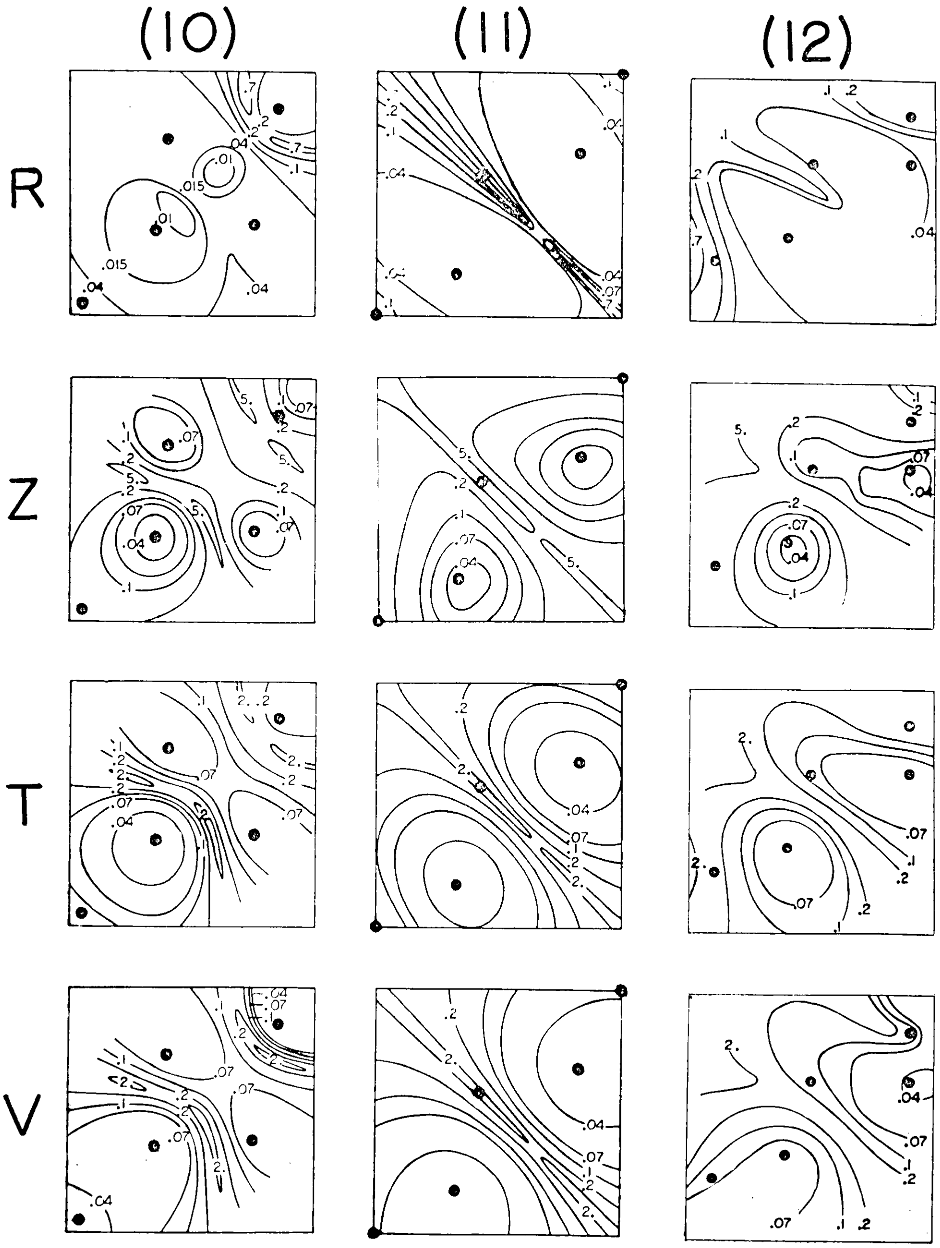

Fig. 1. (continued)

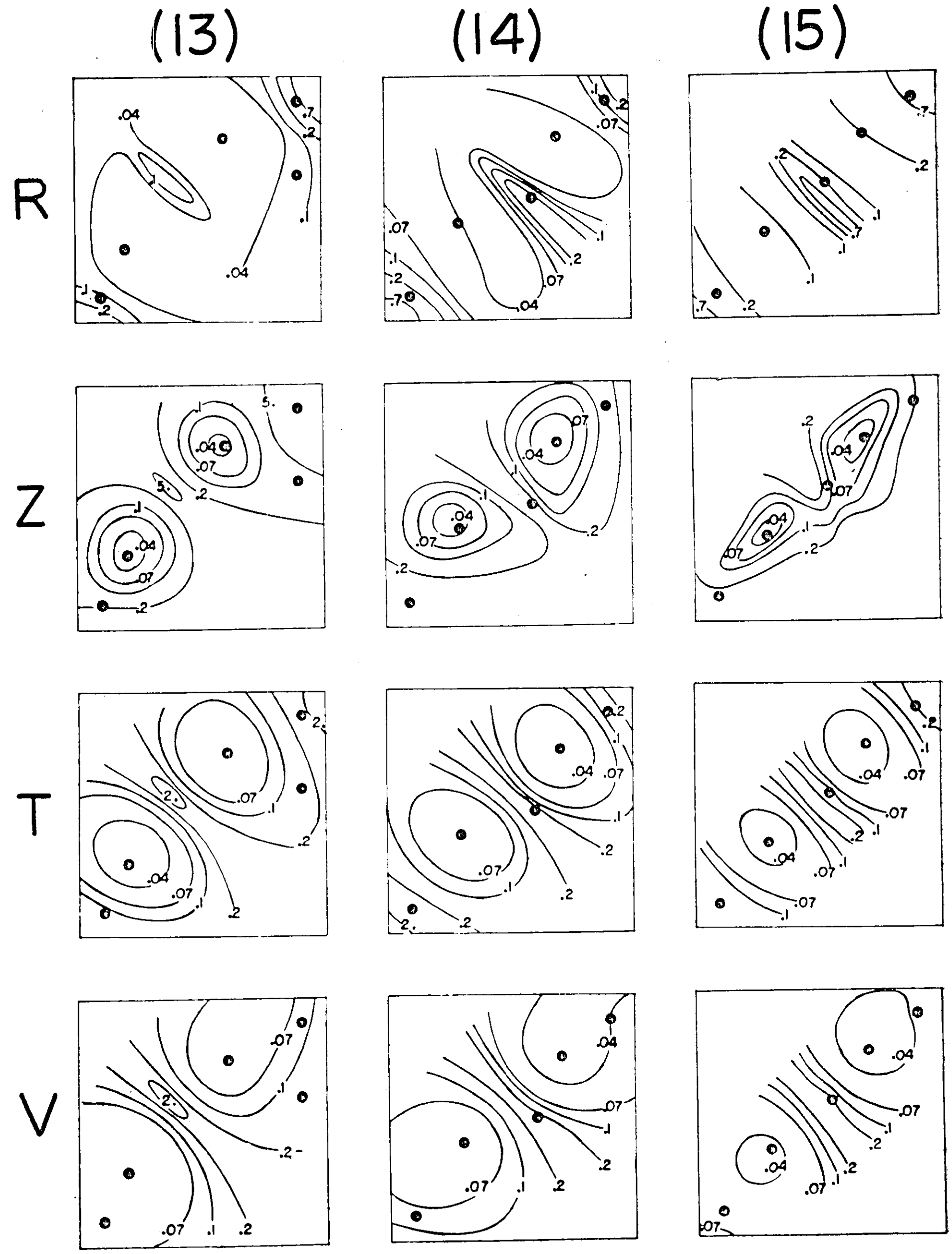

Fig. 1. (continued)

The area around a station generally consists of a small degree of error.

Flat distributions show two distinctive regions with small errors separated by a band of large errors, which are, however, fairly narrow and small.

Distributions not symmetrical have the advantage over those which are symmetrical. Latter ones often show very large errors though the area may not be large.

If we remember existence of such unfavorable areas and pay attention to the treatment of the results, all the examples given here can be recommended.

In these examples, when the numerical values given in (3.1) are employed, minimum errors arrived at are:—

Epicenter location	1 km
Depth of the focus	5 km
Origin time	0.5 sec
Velocity	0.5 km/sec

5. Acknowledgments

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2. 地震観測点の最良分布 III

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1. 地震波の到達時刻を用ゐて震源の位置，発震時，速度等を定める時，地震観測点の分布が，上記諸量の決定精度に大きな影響を持つことは明らかである．さきには，震源の位置があらかじめほぼ定まつてゐるとして，モンテカルロ法によつて問題をといた.[1),2)] この解は火山地震等に適用しうるであらうが，一般の場合としては，むしろ観測点を固定し，震央の位置をいろいろに変へて考へるのが望ましい．

2. それで今回は，さきの計算でよいとわかつたものを含む 15 の代表的な観測点分布を選び，前と同様の計算を行なつた．

すなはち，

1) 誤差は小さく，その 2 乗を無視しうる．
2) 5 個の観測点は一平面上にある．
3) 波の伝播速度は一定である．
4) 観測誤差は $\pm\varepsilon$ または 0 の 3 種にかぎる．

を仮定し，5 観測点に 3 種の誤差をわりあてるあらゆる可能な組合せ 3^5 個について，震央位置，震源の深さ，発震時，伝播速度の誤差を求めて，その標準偏差 (順に R, Z, T, V) を計算した．その際，震源の深さは一定 (正方形 1 辺の 1/10) としたが，震央は順次移動させて，上記 4 個の量を震央位置の関数として求めた．これを図上に書き入れて，値の等しい点をむすんだのが第 1 図である．図中に書き込まれた数は，正方形の一辺を長さの単位に，この長さを波が伝播するのに要する時間を時間の単位にとつた無次元の量である．誤差の大きさとしては，$\varepsilon=0.01$ を採用してある．もし，

正方形の一辺の長さ	100 km
伝播速度	10 km/sec

と取れば，

時間の単位	10 sec
観測誤差 (ε)	0.1 sec

となる．

3. 得られた結果を要約すれば次の通りである．

1) 観測点の分布は震源位置，発震時，伝播速度等の計算に大きな影響をもつ．従つて，その設置に際しては十分の考慮を払ふべきである．

2) 第 1 図に挙げた例はいづれも，分布としては良いものであり，ことに観測点附近に地震が起つた時には精度よく定まる．

3) 比較的平たい観測点分布は，それの作る多角形の中に震央があれば，大体良い結果を与へるが，誤差の大きな区域がバンド状に表はれて，精度の良い地域を二分する傾向がある．

4) 不規則な分布の方が，対称を保つた規則的な分布よりも，どちらかと言へば望ましい．

5) 誤差の平均値 R, Z, T, V の最小値は，図にも見られる通り，上記の数値を採用する時，

震央位置	1 km
震源の深さ	5 km
発震時	0.5 sec
速度	0.5 km/sec

の程度である．従つて，多くの観測点を設け，最小二乗法によるのでない限り，この位の誤差は覚悟しなくてはならない．

6) 以上の点を考慮して扱ふならば，ここに挙げた 15 例，またはこれに類似のものは，時に応じて採用してよいものであらう．