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31. *On the Force Equivalents of Dynamical Elastic Dislocations with Reference to the Earthquake Mechanism.**

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Introduction.

Methods of determining the nature of the motion at the foci of earthquakes have been developed by many seismologists (Hodgson 1959, 1961; Honda 1962). In every method one is led to correlate seismic observations with mathematical solutions to certain problems in elasticity theory. Recently the mathematical models on which these solutions are based have been examined from the viewpoint of dislocation theory.

A. V. Vvedenskaya (1956) found a system of forces which may be equivalent to a rupture accompanied by slipping in the theory of dislocations (Nabarro 1951). She has developed her method on the consideration that a rupture accompanied by slipping is the most probable form of movement in the earthquake foci under the conditions which occur in the earth's crust and in the upper part of the mantle, in which stresses may be supposed to have a considerable duration. It was interesting that the source in this case can be constructed by integration of the well-known force system type II, i. e. a pair of coplanar couples with moments of equal magnitude acting at right angles to one another, along the fault surface. However, Nabarro's formulae on which this method is based were obtained by replacing the system of static forces which is found in a static dislocation by the system of dynamic forces which has step function time dependence. A.V. Vvedenskaya (1959), after F. R. N. Nabarro, obtained formulae for the case of sudden formation of general Volterra dislocations (Volterra 1907), but she did not treat of the basis of the theory of general dynamic dislocations.

J. A. Steketee (1958 a, b) suggested independently that the theory of dislocations might be the proper tool for problems connected with faultplane studies of earthquakes and with fracture zones in the

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crust and mantle and states that as dislocation theory may be described as that part of the theory of elasticity dealing with surfaces across which the displacement components are discontinuous, the suggestion seems reasonable. He clarifies the fundamentals of the elasticity theory of static dislocations and considers a number of problems of static dislocations. He shows force equivalents of static dislocations with reference to the force models of earthquakes. However, his considerations were confined to static cases.

L. Knopoff and F. Gilbert (1959, 1960) applied dynamic dislocation theory to the consideration of the elastodonamic radiation resulting from the sudden occurence of an earthquake due to faulting. The fault plane is visualized as a geometrical discontinuity across which there exists a sudden discontinuity in either one component of the strain tensor or one component of the displacement vector. They developed the study on the basis of formulae obtained earlier by one of the authors (Knopoff 1956). They deal with the first motions (the high-frequency solution) from the impulsive excitation of the fault surface in detail. As to the force equivalents they show that the sudden formation of a fault results in the same first motions as that due to either double couples in the case of displacement dislocation faulting or to single forces in the case of strain dislocation faulting.

In this paper we shall present rigorous fundamental formulae in the case of general dynamic dislocations by deriving them from well-known relations in a straightforward fashion, and consider the force system equivalent to a dynamic dislocation without neglecting low-frequency terms. Mathematical notations here used are largely after J. A. Steketee (1958 b).

Theory.

We begin with the reciprocal theorem of Betti, which is a relation between two possible but different displacement fields, stress fields and body forces for a particular elastic body which occupies a region $D+S$ with S as its boundary. The mathematical statement is as follows:

$$\left.\begin{aligned} &\iiint_D \left(F_k^{(1)} - \frac{\partial^2 u_k^{(1)}}{\partial t^2}\right) u_k^{(2)} \rho dV + \iint_S \tau_{k\nu}^{(1)} u_k^{(2)} dS \\ =&\iiint_D \left(F_k^{(2)} - \frac{\partial^2 u_k^{(2)}}{\partial t^2}\right) u_k^{(1)} \rho dV + \iint_S \tau_{k\nu}^{(2)} u_k^{(1)} dS\ , \end{aligned}\right\} \tag{1}$$

where the superscripts inside parentheses refer to the two different sets of displacements, stresses and body forces; all quantities are referred to a rectangular cartesian coordinate system, and the summation convention applies. In equation (1), t denotes time, ρ density of the elastic body, $u_k^{(1)}$ and $u_k^{(2)}$ two sets of displacements, $\tau_{k\nu}^{(1)}$ and $\tau_{k\nu}^{(2)}$ the surface tractions, $F_k^{(1)}$ and $F_k^{(2)}$ the body forces ($k=1, 2, 3$). The surface traction $\tau_{k\nu}$ is a component of the force per unit area on a surface element as

$$\tau_{k\nu}=\tau_{kl}\nu_l \tag{2}$$

where ν_l's are the direction cosines of the outward normal to the surface element, τ_{kl} is a component of stress tensor ($k, l=1, 2, 3$).

Now we consider a homogeneous, isotropic, elastic body, which may be unstrained and at rest and occupies a region $D+S$ with S as its boundary. Then we imagine an open surface $\sum$ which may be situated entirely in the interior of the body, make a cut over $\sum$ and deform the two faces of the cut, which we denote as $\sum^+$ and $\sum^-$, in different ways by applying some force distibutions to them (Fig. 1). We consider two points P and Q in D, with cartesian coordinates (ξ_1, ξ_2, ξ_3) and (x_1, x_2, x_3) respectively, and define a vector $r(r_1, r_2, r_3)$ as fsllows,

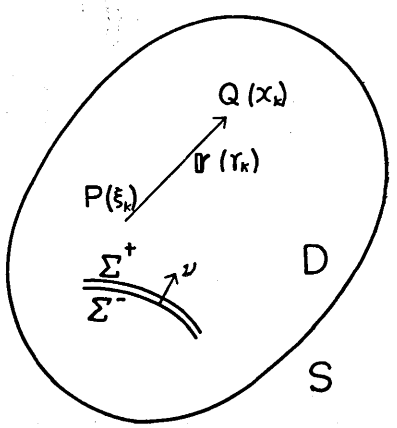

Fig. 1. Dislocation surface $\sum$.

$$\left.\begin{array}{l} r=\overrightarrow{PQ} \quad \text{or} \quad r_k=x_k-\xi_k \quad (k=1,\ 2,\ 3) \\ |r|=r=\sqrt{r_1^2+r_2^2+r_3^2}\ . \end{array}\right\} \tag{3}$$

Hereafter the expressions of differentiation with respect to ξ_k and x_k are simplified by the aid of subscripts and superscripts respectively as

$$\varphi_{,k}=\frac{\partial\varphi}{\partial\xi_k}\,, \qquad \varphi^{,k}=\frac{\partial\varphi}{\partial x_k}\,, \tag{4}$$

where φ denotes an arbitrary function.

If we apply proper tractions over S, $\sum^+$ and $\sum^-$, the body will be deformed as if it were a portion of an infinite elastic medium. When a force $e_m\delta(t-t_0)$ is applied in Q, where e_m denotes a unit vector in the positive x_m-direction and $\delta(t)$ Dirac's delta function, and when such surface tractions as will be generated by the force in an infinite medium are applied on S, $\sum^+$ and $\sum^-$, the displacement field and the stress field in D will be the same as in an infinite medium and they can be obtained from the well-known formulae (Love 1927). Therefore as the first set in the reciprocal theorem we take a body force $e_m\delta(t-t_0)(Q)$ in Q, displacement field $u_k^m(P, t)$ and stress field $\tau_{kl}^m(P, t)$ which are generated in P in an infinite elastic medium by the force. As the second set we may take an arbitrary possible displacement field $u_k(P, t)$ and the corresponding stress field $\tau_{kl}(P, t)$.

According to the well-known formula in Love (1927) the expression of $u_k^m(P, t)$ is as follows,

$$\begin{aligned} u_k^m(P, t) = \frac{1}{4\pi\rho}\Big\{\Big(\frac{1}{r}\Big)_{,mk}\int_{r/a}^{r/b}\tau\delta(t-t_0+\tau)d\tau \\ +\frac{1}{r}r_{,m}r_{,k}\Big[\frac{1}{a^2}\delta\Big(t-t_0+\frac{r}{a}\Big)-\frac{1}{b^2}\delta\Big(t-t_0+\frac{r}{b}\Big)\Big] \\ +\delta_{mk}\frac{1}{r}\Big[\frac{1}{b^2}\delta\Big(t-t_0+\frac{r}{b}\Big)\Big]\Big\}, \end{aligned} \tag{5}$$

where a and b are the velocities of P- and S-waves,

$$a^2=\frac{\lambda+2\mu}{\rho}, \qquad b^2=\frac{\mu}{\rho}, \tag{6}$$

and where λ and μ are Lamé constants. In equation (5) we take a converging wave as to $u_k^m(P, t)$. Stress components $\tau_{kl}^m(P, t)$ due to the displacement field (5) are computed from the general relation,

$$\tau_{kl}^m(P, t)=\lambda\delta_{kl}u_{n,n}^m+\mu(u_{k,l}^m+u_{l,k}^m), \tag{7}$$

where δ_{kl} is Kronecker delta.

The reciprocal theorem in our case may be written in the form,

$$\begin{aligned} \iiint_D\Big[\delta_{mk}\delta(t-t_0)(Q)-\frac{\partial^2u_k^m}{\partial t^2}\Big]u_k\rho dV+\iint_{\Sigma^++\Sigma^-+S}u_k\tau_{kl}^m\nu_l dS \\ =\iiint_D\Big[-\frac{\partial^2u_k}{\partial t^2}\Big]u_k^m\rho dV+\iint_{\Sigma^++\Sigma^-+S}u_k^m\tau_{kl}\nu_l dS, \end{aligned} \tag{8}$$

where integrations are taken over coordinates of variable P for fixed Q. The surfaces $\sum^+$ and $\sum^-$ being considered as two faces of the cut over an open surface $\sum$, $u_k^m(P, t)$ and $\tau_{kl}^m(P, t)$ as displacement and stress fields in an infinite elastic medium, $u_k^m(P, t)$ and $\tau_{kl}^m(P, t)$ will be continuous across $\sum$. Therefore following relations hold,

$$\nu_l^- = -\nu_l^+(=\nu_l) , \tag{9}$$

and

$$u_k^{m+} = u_k^{m-} , \qquad \tau_{kl}^{m+} = \tau_{kl}^{m-} , \tag{10}$$

where ν_l^+ and ν_l^- denote direction cosines of the normal to $\sum^+$ and $\sum^-$ respectively. We may use simply ν_l for ν_l^- by equation (9). The possible u_k's and τ_{kl}'s are here considered to be discontinuous generally across $\sum$; we define $\varDelta u_k$ and $\varDelta\tau_{kl}$ as follows,

$$\varDelta u_k = u_k^+ - u_k^- , \tag{11}$$

and

$$\varDelta\tau_{kl} = \tau_{kl}^+ - \tau_{kl}^- . \tag{12}$$

In equations (9), (10), (11) and (12), superscripts $+$ correspond to the values for a point P^+, originally being in P on $\sum$ but now situated on $\sum^+$, and superscripts $-$ for a point P^-, originally in P but now situated on $\sum^-$. Equation (8) may then be written in the form,

$$\iiint_D u_m(P, t)\delta(t-t_0)(Q)\rho dV = \iiint_D\left(u_k\frac{\partial^2 u_k^m}{\partial t^2} - u_k^m\frac{\partial^2 u_k}{\partial t^2}\right)\rho dV + \iint_\Sigma \varDelta u_k \tau_{kl}^m \nu_l d\Sigma$$
$$-\iint_\Sigma u_k^m \varDelta\tau_{kl}\nu_l d\Sigma - \iint_S u_k\tau_{kl}^m \nu_l dS + \iint_S u_k^m \tau_{kl}\nu_l dS . \tag{13}$$

The outer surface S is here assumed to be left free from forces, hence the last term of the right hand member of equation (13) vanishes.

Now we integrate equation (13) with respect to t from $-\infty$ to $+\infty$. The left hand side gives us $u_m(Q, t_0)$. Assuming the permissibility of the inversion of the order of integration and natural boundary conditions with respect to t, $(\partial/\partial t)u_k(\pm\infty) = u_k(\pm\infty) = 0$, we find the first term of the right hand member to vanish,

$$\left[u_k\frac{\partial u_k^m}{\partial t} - u_k^m\frac{\partial u_k}{\partial t}\right]_{-\infty}^{\infty} = 0 .$$

By virtue of the formula on δ-functions,

$$\int_{-\infty}^{\infty} f(t)\delta^{(n)}(t-\tau)dt = (-1)^n f^{(n)}(\tau) , \tag{14}$$

where n inside parenthese denotes n-th derivative, the integration of the second term of the right hand member of equation (13) gives us

$$\int_{-\infty}^{\infty} \varDelta u_k(t)\tau_{kl}^m(t)\nu_l dt$$

$$=\frac{1}{4\pi\rho}\Big\{6\mu\Big[-\delta_{kl}\frac{r_m}{r^5}-\delta_{mk}\frac{r_l}{r^5}-\delta_{lm}\frac{r_k}{r^5}+5\frac{r_k r_l r_m}{r^7}\Big]\int_{r/a}^{r/b}\tau\varDelta u_k(t_0-\tau)d\tau$$

$$+\Big[(\lambda-2\mu)\delta_{kl}\frac{r_m}{r^3}-2\mu\delta_{mk}\frac{r_l}{r^3}-2\mu\delta_{lm}\frac{r_k}{r^3}+12\mu\frac{r_k r_l r_m}{r^5}\Big]\Big[\frac{1}{a^2}\varDelta u_k\Big(t_0-\frac{r}{a}\Big)\Big]$$

$$+\Big[2\mu\delta_{kl}\frac{r_m}{r^3}+3\mu\delta_{mk}\frac{r_l}{r^3}+3\mu\delta_{lm}\frac{r_k}{r^3}-12\mu\frac{r_k r_l r_m}{r^5}\Big]\Big[\frac{1}{b^2}\varDelta u_k\Big(t_0-\frac{r}{b}\Big)\Big]$$

$$+\Big[\lambda\delta_{kl}\frac{r_m}{r^2}+2\mu\frac{r_k r_l r_m}{r^4}\Big]\Big[\frac{1}{a^3}\varDelta u_k'\Big(t_0-\frac{r}{a}\Big)\Big]$$

$$+\Big[\mu\delta_{mk}\frac{r_l}{r^2}+\mu\delta_{lm}\frac{r_k}{r^2}-2\mu\frac{r_k r_l r_m}{r^4}\Big]\Big[\frac{1}{b^3}\varDelta u_k'\Big(t_0-\frac{r}{b}\Big)\Big]\Big\}\nu_l\,. \tag{15}$$

The integration of the third term gives us

$$-\int_{-\infty}^{\infty} u_k^m(t)\varDelta\tau_{kl}(t)\nu_l dt=-\frac{1}{4\pi\rho}\Big\{\Big[-\delta_{mk}\frac{1}{r^3}+3\frac{r_m r_k}{r^5}\Big]\int_{r/a}^{r/b}\tau\varDelta\tau_{kl}(t_0-\tau)d\tau$$

$$+\Big[\frac{r_m r_k}{r^3}\Big]\Big[\frac{1}{a^2}\varDelta\tau_{kl}\Big(t_0-\frac{r}{a}\Big)\Big]$$

$$+\Big[\delta_{mk}\frac{1}{r}-\frac{r_m r_k}{r^3}\Big]\Big[\frac{1}{b^2}\varDelta\tau_{kl}\Big(t_0-\frac{r}{b}\Big)\Big]\Big\}\nu_l\,. \tag{16}$$

To obtain a formula for an infinite medium, we assume that S recedes to infinity requiring at the same time that Q is not at infinity and $\sum$ has no points at infinity either. Then in the fourth term of the right hand member of equation (13) we have $dS \fallingdotseq r^2 d\Omega$, where $d\Omega$ denotes the solid angle subtended by dS at Q, and integration gives following terms:

$$\left.\begin{aligned}
&\int_{r/a}^{r/b}\tau u(t_0-\tau)d\tau\cdot O(r^{-2})\,,\\
&u\Big(t_0-\frac{r}{c}\Big)\cdot O(r^0)\,,\\
&u'\Big(t_0-\frac{r}{c}\Big)\cdot O(r)\,,
\end{aligned}\right\} \tag{17}$$

where c represents a or b. If we make r sufficiently large, contributions from these terms can be neglected; we may assume some small dissipation.

Thus we can compute displacement components at an arbitrary point Q in an infinite medium as a function of t, if the possible discontinuities in displacement components and in stress components are given on the dislocation surface $\sum$ as functions of t.

For the sake of simplicity we define $\psi_k(t)$ and $\psi_{kl}(t)$ as follows,

$$\left.\begin{aligned} \psi_k(t)&=\int_0^t dt'\int_0^{t'} \Delta u_k(t'')dt'' \\ \psi_{kl}(t)&=\int_0^t dt'\int_0^{t'} \Delta\tau_{kl}(t'')dt'' \ . \end{aligned}\right\} \tag{18}$$

This type of integral is used after V. I. Keylis-Borok (1950). The expression of $u_m(Q, t)$ may then be written

$$u_m(Q,\ t)=\iint T^m d\sum-\iint U^m d\sum\ , \tag{19}$$

where we write t for t_0 in equations (15) and (16),

$$\begin{aligned} T^m=\frac{1}{4\pi\rho}\Big\{&6\mu\Big[-\delta_{kl}\frac{r_m}{r^5}-\delta_{mk}\frac{r_l}{r^5}-\delta_{lm}\frac{r_k}{r^5}+5\frac{r_kr_lr_m}{r^7}\Big]\Big[\psi_k\Big(t-\frac{r}{a}\Big)-\psi_k\Big(t-\frac{r}{b}\Big)\Big] \\ &+6\mu\Big[-\delta_{kl}\frac{r_m}{r^4}-\delta_{mk}\frac{r_l}{r^4}-\delta_{lm}\frac{r_k}{r^4}+5\frac{r_kr_lr_m}{r^6}\Big]\Big[\frac{1}{a}\psi'_k\Big(t-\frac{r}{a}\Big)-\frac{1}{b}\psi'_k\Big(t-\frac{r}{b}\Big)\Big] \\ &+\Big[(\lambda-2\mu)\delta_{kl}\frac{r_m}{r^3}-2\mu\delta_{mk}\frac{r_l}{r^3}-2\mu\delta_{lm}\frac{r_k}{r^3}+12\mu\frac{r_kr_lr_m}{r^5}\Big]\Big[\frac{1}{a^2}\psi''_k\Big(t-\frac{r}{a}\Big)\Big] \\ &+\Big[2\mu\delta_{kl}\frac{r_m}{r^3}+3\mu\delta_{mk}\frac{r_l}{r^3}+3\mu\delta_{lm}\frac{r_k}{r^3}-12\mu\frac{r_kr_lr_m}{r^5}\Big]\Big[\frac{1}{b^2}\psi''_k\Big(t-\frac{r}{b}\Big)\Big] \\ &+\Big[\lambda\delta_{kl}\frac{r_m}{r^2}+2\mu\frac{r_kr_lr_m}{r^4}\Big]\Big[\frac{1}{a^3}\psi'''_k\Big(t-\frac{r}{a}\Big)\Big] \\ &+\Big[\mu\delta_{mk}\frac{r_l}{r^2}+\mu\delta_{lm}\frac{r_k}{r^2}-2\mu\frac{r_kr_lr_m}{r^4}\Big]\Big[\frac{1}{b^3}\psi'''_k\Big(t-\frac{r}{b}\Big)\Big]\Big\}\nu_l\ , \end{aligned} \tag{20}$$

and

$$\begin{aligned} U^m=\frac{1}{4\pi\rho}\Big\{&\Big[-\delta_{mk}\frac{1}{r^3}+3\frac{r_mr_k}{r^5}\Big]\Big[\psi_{kl}\Big(t-\frac{r}{a}\Big)-\psi_{kl}\Big(t-\frac{r}{b}\Big)\Big] \\ &+\Big[-\delta_{mk}\frac{1}{r^2}+3\frac{r_mr_k}{r^4}\Big]\Big[\frac{1}{a}\psi'_{kl}\Big(t-\frac{r}{a}\Big)-\frac{1}{b}\psi'_{kl}\Big(t-\frac{r}{b}\Big)\Big] \\ &+\Big[\frac{r_mr_k}{r^3}\Big]\Big[\frac{1}{a^2}\psi''_{kl}\Big(t-\frac{r}{a}\Big)\Big] \\ &+\Big[\delta_{mk}\frac{1}{r}-\frac{r_mr_k}{r^3}\Big]\Big[\frac{1}{b^2}\psi''_{kl}\Big(t-\frac{r}{b}\Big)\Big]\Big\}\nu_l\ . \end{aligned} \tag{21}$$

In order to clear up the nature of displacement field $u_m(Q, t)$ we turn back to the formula of Love (1927). When a body force in the positive x_k-direction $e_k\chi(t)$ with time dependence in the form $\chi(t)$ is applied in P in an infinite medium, the m-component of the displacement field in Q, $U_k^m[\psi]$, may be expressed as follows,

$$
\begin{aligned}
U_k^m[\psi] &= \frac{1}{4\pi\rho}\Big\{\Big(\frac{1}{r}\Big)^{\cdot mk}\int_{r/a}^{r/b}\tau\chi(t-\tau)d\tau \\
&\quad + \frac{1}{r}r^{\cdot m}r^{\cdot k}\Big[\frac{1}{a^2}\chi\Big(t-\frac{r}{a}\Big)-\frac{1}{b^2}\chi\Big(t-\frac{r}{b}\Big)\Big]+\delta_{mk}\frac{1}{r}\Big[\frac{1}{b^2}\chi\Big(t-\frac{r}{b}\Big)\Big]\Big\} \\
&= \frac{1}{4\pi\rho}\Big\{\Big[-\delta_{mk}\frac{1}{r^3}+3\frac{r_m r_k}{r^5}\Big]\Big[\psi\Big(t-\frac{r}{a}\Big)-\psi\Big(t-\frac{r}{b}\Big)\Big] \\
&\quad + \Big[-\delta_{mk}\frac{1}{r^2}+3\frac{r_m r_k}{r^4}\Big]\Big[\frac{1}{a}\psi'\Big(t-\frac{r}{a}\Big)-\frac{1}{b}\psi'\Big(t-\frac{r}{b}\Big)\Big] \\
&\quad + \Big[\frac{r_m r_k}{r^3}\Big]\Big[\frac{1}{a^2}\psi''\Big(t-\frac{r}{a}\Big)\Big]+\Big[\delta_{mk}\frac{1}{r}-\frac{r_m r_k}{r^3}\Big]\Big[\frac{1}{b^2}\psi''\Big(t-\frac{r}{b}\Big)\Big]\Big\} \\
&= U_m^k[\psi]\,,
\end{aligned}
\tag{22}
$$

where $\psi(t)$ is defined as

$$
\psi(t)=\int_0^t dt'\int_0^{t'}\chi(t'')dt''\,. \tag{23}
$$

In equation (22) we take a diverging wave as in ordinary circumstances.

From $U_k^m[\psi]$ we can compute a displacement field $T_{kl}^m[\psi]$ defined by such combinations of derivatives of $U_m^k[\psi]$ with respect to x_k's, the coordinates of Q, as follows,

$$
T_{kl}^m[\psi]=-\lambda\delta_{kl}(U_m^n[\psi])^{\cdot n}-\mu(U_m^k[\psi])^{\cdot l}-\mu(U_m^l[\psi])^{\cdot k}=T_{lk}^m[\psi]\,. \tag{24}
$$

We obtain

$$
\begin{aligned}
&T_{kl}^m[\psi] \\
&= \frac{1}{4\pi\rho}\Big\{6\mu\Big[-\delta_{kl}\frac{r_m}{r^5}-\delta_{mk}\frac{r_l}{r^5}-\delta_{lm}\frac{r_k}{r^5}+5\frac{r_k r_l r_m}{r^7}\Big]\Big[\psi\Big(t-\frac{r}{a}\Big)-\psi\Big(t-\frac{r}{b}\Big)\Big] \\
&\quad + 6\mu\Big[-\delta_{kl}\frac{r_m}{r^4}-\delta_{mk}\frac{r_l}{r^4}-\delta_{lm}\frac{r_k}{r^4}+5\frac{r_k r_l r_m}{r^6}\Big]\Big[\frac{1}{a}\psi'\Big(t-\frac{r}{a}\Big)-\frac{1}{b}\psi'\Big(t-\frac{r}{b}\Big)\Big] \\
&\quad + \Big[(\lambda-2\mu)\delta_{kl}\frac{r_m}{r^3}-2\mu\delta_{mk}\frac{r_l}{r^3}-2\mu\delta_{lm}\frac{r_k}{r^3}+12\mu\frac{r_k r_l r_m}{r^5}\Big]\Big[\frac{1}{a^2}\psi''\Big(t-\frac{r}{a}\Big)\Big]
\end{aligned}
$$

$$+\left[2\mu\delta_{kl}\frac{r_m}{r^3}+3\mu\delta_{mk}\frac{r_l}{r^3}+3\mu\delta_{lm}\frac{r_k}{r^3}-12\mu\frac{r_kr_lr_m}{r^5}\right]\left[\frac{1}{b^2}\psi''\left(t-\frac{r}{b}\right)\right]$$

$$+\left[\lambda\delta_{kl}\frac{r_m}{r^2}+2\mu\frac{r_kr_lr_m}{r^4}\right]\left[\frac{1}{a^3}\psi'''\left(t-\frac{r}{a}\right)\right]$$

$$+\left[\mu\delta_{mk}\frac{r_l}{r^2}+\mu\delta_{lm}\frac{r_k}{r^2}-2\mu\frac{r_kr_lr_m}{r^4}\right]\left[\frac{1}{b^3}\psi'''\left(t-\frac{r}{b}\right)\right]\Bigg\}\ . \tag{25}$$

The displacement fields $U_m^{k,l}$ are usually referred to as nuclei of strain; for $k=l$ we have a double force without moment, for $k\neq l$ it is a double force with moment; for example from the definition

$$U_m^{1,1}=\lim_{\Delta x_1\to 0}\left\{\frac{1}{\Delta x_1}U_m^1(x_1+\Delta x_1,\cdots)-\frac{1}{\Delta x_1}U_m^1(x_1,\cdots)\right\}$$

we observe that $U_m^{1,1}$ is considered as the m-component of the displacement field in Q due to a double force without moment in P (Fig. 2). It follows that $T_{kl}^m[\psi]$ may be considered as the displacement field due to some combination of nuclei of strain; in the case $k=l$ that is a center of dilatation which is a combination of three equal, mutually perpendicular double forces without moment and an additional double force without moment in the k-direction, A-nucleus after Steketee; in the case $k\neq l$ it is a combination of two coplanar mutually perpendicular double forces with moment, B-nucleus after Steketee. If we denote a center of dilatation by a sphere after Steketee, the two combinations may be schematically represented as in Fig. 3.

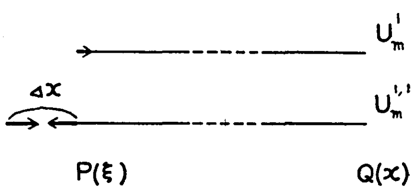

Fig. 2. U_m^1 is the displacement field in Q due to a body force in the x_1-direction in P. $U_m^{1,1}$ is the displacement field in Q due to a double force in P.

Here we show that nucleus B is equivalent to a combination of two mutually perpendicular double forces without moment (Fig. 4). If we denote by σ and ν two directions perpendicular to one another, and by α the direction which makes an angle $\pi/4$ with σ and ν, by β the direction which makes an angle $3\pi/4$ with σ and an angle $\pi/4$ with ν, either of which is in the plane made by σ and ν (Fig. 4), we obtain the following equation which means that the two nuclei shown in Fig. 4 are the same.

$$-(U_m^{\sigma,\nu}+U_m^{\nu,\sigma})=-U_m^{\alpha,\alpha}+U_m^{\beta,\beta}\ , \tag{26}$$

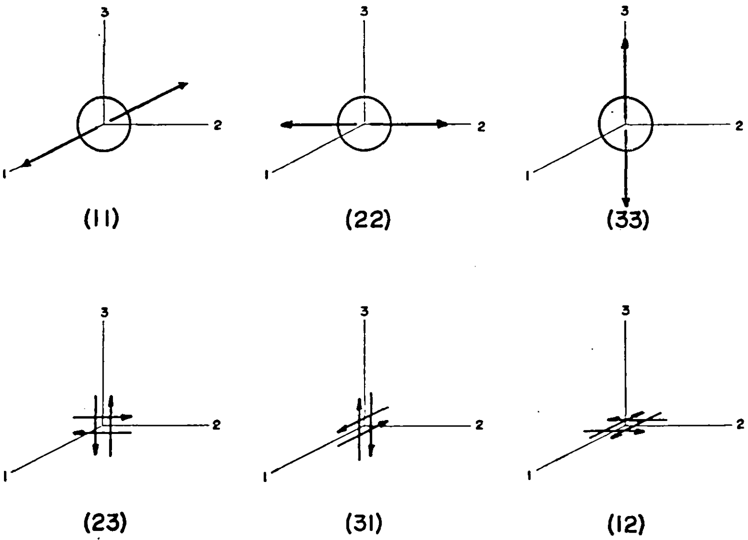

Fig. 3. (kl) is a combination of double forces in P which generates the displacement field T^m_{kl} in Q. (11), (22) and (33) are A-nuclei; (23), (31) and (12) are B-unclei.

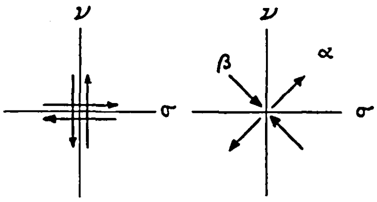

Fig. 4. Equivalent two nuclei.

where U^α_m, for example, represents as before the m-component of the displacement field in Q, when a body force $\chi(t)$ is applied in the α-direction in P; $U^{\alpha,\alpha}_m$ reprensents the directional derivative of U^α_m in Q along the α-direction, that is,

$$U^{\alpha,\alpha}_m=\frac{1}{\sqrt{2}}(U^{\alpha,\sigma}_m+U^{\alpha,\nu}_m)=\frac{1}{2}(U^{\sigma,\sigma}_m+U^{\nu,\nu}_m+U^{\sigma,\nu}_m+U^{\nu,\sigma}_m)\ .$$

From equations (19), (20), (21), (22) and (25) we observe that $u_m(Q, t)$ in (19) may also be written in the form,

$$u_m(Q,\ t)=\iint T^m_{kl}[\psi_k]\nu_l d\Sigma+\iint U^m_k[-\psi_{kl}]\nu_l d\Sigma\ . \tag{27}$$

It follows that the displacement components $u_m(Q, t)$ in equation (19) may be considered as the resultant effect of a distribution of strain nuclei of which strengthes change with time in just the same manner as Δu_k over Σ and a distribution of single forces of which strengthes change with time in just the same manner as $-\Delta\tau_{kl}$ over $\sum$. It is natural that the effect of discontinuity in stress can be described by a single force. With the aid of equations (9) and (12) we have

$$-\Delta\tau_{kl}\nu_l d\sum=(\tau^+_{kl}\nu^+_l+\tau^-_{kl}\nu^-_l)d\sum\ ,$$

where the right hand side can be considered as the k-component of the vector sum of the forces exerted across $d\sum^+$ and $d\sum^-$ from the outside of the body (or by the inner portion surrounded by $\sum^+$ and $\sum^-$). Hence as we can see from (18) and the equation

$$U^m_k[-\psi_{kl}]\nu_l d\sum=U^m_k[-\psi_{kl}\nu_l d\sum]\ ,$$

the second term of the right hand member of (27) is interpreted as a displacement component due to a distribution of single forces exerted across $\sum^+$ and $\sum^-$ from the outside of the body (or by the inner portion surrounded by $\sum^+$ and $\sum^-$).

In order to clarify the meaning of equation (27) further, we consider a surface element $d\sum$ of $\sum$ with the normal in the x_3-direction (Fig. 5). Its contribution to the displacement in Q is

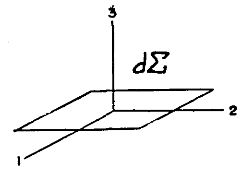

Fig. 5. Surface element $d\sum$ in P with the normal in the x_3-direction.

$$\begin{aligned} du_m(Q,\ t)=&\{T^m_{13}[\psi_1]+T^m_{23}[\psi_2]+T^m_{33}[\psi_3]\}d\sum \\ &+\{U^m_1[-\psi_{13}]+U^m_2[-\psi_{23}]+U^m_3[-\psi_{33}]\}d\sum\ . \end{aligned} \tag{28}$$

The double forces and single forces equivalent to Δu_k and $-\Delta\tau_{k3}$ in P on $d\sum$ are shown in Fig. 6. The contribution of discontinuty in displacement in P $\Delta u_k(P,\ t)$ to the displacement field in Q is the same as that of a strain nucleus in P, ($k3$), which varies its strength with time in the same manner as $\Delta u_k(P,\ t)$. The contribution of discontinuity in stress in P $\Delta\tau_{k3}(P,\ t)$ to the displacement field in Q is the same as that of a single force in P, (k), which varies its strength with time in the same manner as $-\Delta\tau_{k3}(P,\ t)$. It is clear that Δu_1 and Δu_2, which are in directions perpendicular to the normal, may be called the slip, while Δu_3 represents the discontinuity in displacement of the two faces $d\sum^+$ and $d\sum^-$ in the direction of their normal. The effect of the slip can be described by nuclei of type B while the effect of normal discontinuity is described by a nucleus A.

For some purposes it is convenient to express the time factor of

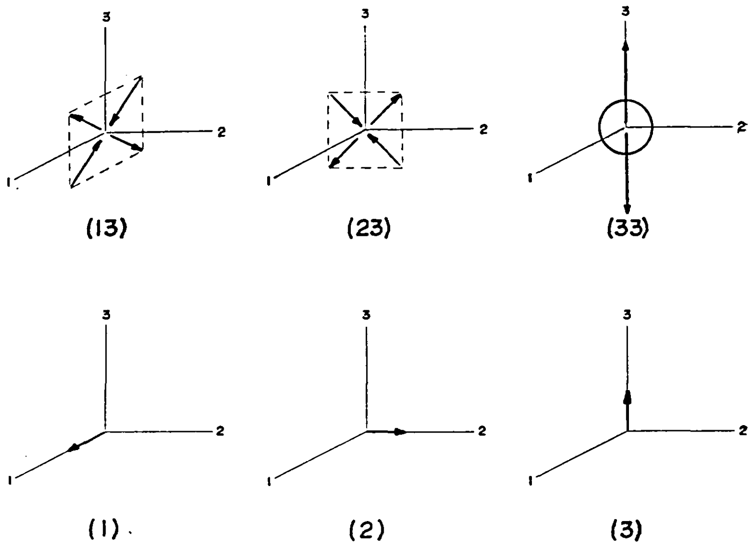

Fig. 6. Nucleus ($k3$) in P varies its strength with time in just the same manner as $\Delta u_k(P, t)$. Single force (k) in P varies its strength with time in just the same manner as $-\Delta\tau_{k3}(P, t)$.

the displacement field in the form $exp(i\omega t)$. Let the Fourier transform of $\chi(t)$ be denoted as $\bar{\chi}(\omega)$, then we have by definition

$$\left.\begin{aligned}\bar{\chi}(\omega)&=\int_{-\infty}^{\infty}\chi(t)e^{-i\omega t}dt\,,\\ \chi(t)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{\chi}(\omega)e^{i\omega t}d\omega\,.\end{aligned}\right\}\tag{29}$$

The factor which appears in the form of integral in equation (22), by the aid of (29), can be written as

$$\begin{aligned}\int_{r/a}^{r/b}\tau\chi(t-\tau)d\tau=&\frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{\chi}(\omega)d\omega\left\{-\frac{1}{\omega^2}[e^{i\omega(t-r/a)}-e^{i\omega(t-r/b)}]\right\}\\ &+\frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{\chi}(\omega)d\omega\left\{\frac{1}{i\omega}\left[\frac{r}{a}e^{i\omega(t-r/a)}-\frac{r}{b}e^{i\omega(t-r/b)}\right]\right\}\,,\end{aligned}$$

so that as to $\psi(t)$ in (23) we may take the following integral

$$\psi(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{-\frac{\bar{\chi}(\omega)}{\omega^2}\right\}e^{i\omega t}d\omega\,,$$

it follows that

$$\psi^{(n)}(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}(i\omega)^{n-2}\,\bar{\chi}(\omega)e^{i\omega t}d\omega\,, \tag{30}$$

where $\psi^{(n)}(t)$ denotes n-th derivative of $\psi(t)$ with respect to t ($n=0$, 1, 2, $\cdots$). In order to write our expressions in compact forms we introduce spherical Hankel function of the second kind $h_n^{(2)}(\rho)$ defined as

$$h_n^{(2)}(\rho)=\sqrt{\frac{\pi}{2\rho}}H_{n+1/2}^{(2)}(\rho)\,, \tag{31}$$

where $H_{n+1/2}^{(2)}$ is Hankel function of the second kind. For a few integral values of the order n we have

$$\left.\begin{aligned}
h_0^{(2)}(\rho)&=\left(\frac{i}{\rho}\right)e^{-i\rho}\\
h_1^{(2)}(\rho)&=\left(-\frac{1}{\rho}+\frac{i}{\rho^2}\right)e^{-i\rho}\\
h_2^{(2)}(\rho)&=\left(-\frac{i}{\rho}-3\frac{1}{\rho^2}+3\frac{i}{\rho^3}\right)e^{-i\rho}\\
h_3^{(2)}(\rho)&=\left(\frac{1}{\rho}-6\frac{i}{\rho^2}-15\frac{1}{\rho^3}+15\frac{i}{\rho^4}\right)e^{-i\rho}\,.
\end{aligned}\right\} \tag{32}$$

Let the Fourier transform of $\Delta u_k(P, t)$ be denoted as $\Delta\bar{u}_k(P, \omega)$ or simply $\Delta\bar{u}_k(\omega)$, and the Fourier transform of $\Delta\tau_{kl}(P, t)$ as $\Delta\bar{\tau}_{kl}(P, \omega)$ or simply $\Delta\bar{\tau}_{kl}(\omega)$, that is,

$$\left.\begin{aligned}
\Delta\bar{u}_k(\omega)&=\int_{-\infty}^{\infty}\Delta u_k(t)e^{-i\omega t}dt\,, &\quad \Delta u_k(t)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Delta\bar{u}_k(\omega)e^{i\omega t}d\omega\,,\\
\Delta\bar{\tau}_{kl}(\omega)&=\int_{-\infty}^{\infty}\Delta\tau_{kl}(t)e^{-i\omega t}dt\,, &\quad \Delta\tau_{kl}(t)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Delta\bar{\tau}_{kl}(\omega)e^{i\omega t}d\omega\,.
\end{aligned}\right\} \tag{33}$$

Replacing $\bar{\chi}(\omega)$ in equation (30) by $\Delta\bar{u}_k(\omega)$ or by $\Delta\bar{\tau}_{kl}(\omega)$ and using (32) and (33), equation (19) becomes

$$\begin{aligned}
u_m(Q, t)=&\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\omega t}d\omega\iint\Delta\bar{u}_kT_{kl}^{m}(\omega)\nu_l d\Sigma\\
&-\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\omega t}d\omega\iint U_k^m(\omega)\Delta\bar{\tau}_{kl}\nu_l d\Sigma\,,
\end{aligned} \tag{34}$$

where

$$T^m_{kl}(\omega)=\frac{i}{4\pi}\left\{\alpha^2 h_1^{(2)}(\alpha r)\left[-\left(\frac{\lambda+(2/5)\mu}{\lambda+2\mu}\right)\left(\delta_{kl}\frac{r_m}{r}\right)-\left(\frac{(2/5)\mu}{\lambda+2\mu}\right)\left(\delta_{mk}\frac{r_l}{r}+\delta_{lm}\frac{r_k}{r}\right)\right]\right.$$

$$+\alpha^2 h_3^{(2)}(\alpha r)\left[-\left(\frac{(2/5)\mu}{\lambda+2\mu}\right)\left(\delta_{kl}\frac{r_m}{r}+\delta_{mk}\frac{r_l}{r}+\delta_{lm}\frac{r_k}{r}\right)+\left(\frac{2\mu}{\lambda+2\mu}\right)\frac{r_k r_l r_m}{r^3}\right]$$

$$+\beta^2 h_1^{(2)}(\beta r)\left[\frac{2}{5}\left(\delta_{kl}\frac{r_m}{r}\right)-\frac{3}{5}\left(\delta_{mk}\frac{r_l}{r}+\delta_{lm}\frac{r_k}{r}\right)\right]$$

$$\left.+\beta^2 h_3^{(2)}(\beta r)\left[\frac{2}{5}\left(\delta_{kl}\frac{r_m}{r}+\delta_{mk}\frac{r_l}{r}+\delta_{lm}\frac{r_k}{r}\right)-2\frac{r_k r_l r_m}{r^3}\right]\right\} \tag{35}$$

and

$$U^m_k(\omega)=\frac{i}{4\pi}\left\{\frac{\alpha}{\lambda+2\mu}\left[\left(-\frac{1}{3}\delta_{mk}\right)h_0^{(2)}(\alpha r)+\left(-\frac{1}{3}\delta_{mk}+\frac{r_m r_k}{r^2}\right)h_2^{(2)}(\alpha r)\right]\right.$$

$$\left.+\frac{\beta}{\mu}\left[\left(-\frac{2}{3}\delta_{mk}\right)h_0^{(2)}(\beta r)+\left(\frac{1}{3}\delta_{mk}-\frac{r_m r_k}{r^2}\right)h_2^{(2)}(\beta r)\right]\right\}, \tag{36}$$

and where

$$\alpha=\frac{\omega}{a}, \qquad \beta=\frac{\omega}{b}. \tag{37}$$

The meaning of $T^m_{kl}(\omega)$ and $U^m_k(\omega)$ here defined are comprehensible owing to (22), (23) and (24). By putting

$$\chi(t)=e^{i\omega t}$$

in (22), (23) and (24), we have

$$T^m_{kl}[\chi]=T^m_{kl}(\omega)e^{i\omega t}$$

and

$$U^m_k[\chi]=U^m_k(\omega)e^{i\omega t}.$$

Hence $T^m_{kl}(\omega)exp(i\omega t)$ is the m-component of the displacement field in Q due to such dynamic double forces as shown in Fig. 3 the time dependence of which is $exp(i\omega t)$ in P, while $U^m_k(\omega)exp(i\omega t)$ is the m-component of the displacement field in Q due to a single dynamic force in the k-direction the time dependence of which is $exp(i\omega t)$ in P.

The formulae for static cases will be easily obtained from (19), (20) and (21). Here, for form's sake, by tending ω in (35) and (36) to zero we find formulae for static cases. In view of the Laurent expansions of the spherical Hankel functions,

$$h_0^{(2)}(\rho)=i\rho^{-1}+1-\frac{1}{2}i\rho+\cdots$$

$$h_1^{(2)}(\rho)=i\rho^{-2}+0+\frac{1}{2}i+\frac{1}{3}\rho+\cdots$$

$$h_2^{(2)}(\rho)=3i\rho^{-3}+0+\frac{1}{2}i\rho^{-1}+0+\frac{1}{8}i\rho+\cdots$$

$$h_3^{(2)}(\rho)=15i\rho^{-4}+0+\frac{3}{2}i\rho^{-2}+0+\frac{1}{8}i+0+\cdots$$

we obtain, as $\omega\rightarrow 0$,

$$T_{kl}^{m}(0)=\frac{1}{4\pi}\left\{\left(\frac{\mu}{\lambda+2\mu}\right)\left(-\delta_{kl}\frac{r_m}{r^3}+\delta_{mk}\frac{r_l}{r^3}+\delta_{lm}\frac{r_k}{r^3}\right)+3\left(\frac{\lambda+\mu}{\lambda+2\mu}\right)\frac{r_k r_l r_m}{r^5}\right\} \tag{38}$$

and

$$U_k^m(0)=\frac{1}{8\pi\mu}\left\{\left(\frac{\lambda+3\mu}{\lambda+2\mu}\right)\delta_{mk}\frac{1}{r}+\left(\frac{\lambda+\mu}{\lambda+2\mu}\right)\frac{r_m r_k}{r^3}\right\}. \tag{39}$$

Interchanging the order of integration in (34), for static cases we can write in the form

$$u_m(Q)=\iint\Delta u_k T_{kl}^{m}\nu_l d\sum-\iint U_k^m \Delta\tau_{kl}\nu_l d\sum\ , \tag{40}$$

where T_{kl}^m is $T_{kl}^m(0)$ in (38) and U_k^m is $U_k^m(0)$ in (39). In equation (40) T_{kl}^m represents the m-component of the displacement field in Q due to such combinations of static double forces as shown in Fig. 3 in P, while U_k^m represents the m-component of the displacement field in Q due to a single static force in the k-direction in P. The first term in the right hand side of equation (40) is the same as shown in Steketee (1958 a, b).

Remarks.

We have seen that the displacement field due to a dynamic dislocation is exactly equivalent to that due to a distribution of some combinations of dynamic double forces in the absence of the dislocation surface, if we define a dislocation as discontinuities in displacement components or stress components across a dislocation surface. In this way for a moving dislocation we have a train of the above-stated elementary force systems distributed on the whole surface, each of which varies its strength with time in the form of an approximate step function of which outset shifts from point to point in accordance with the velocity and the direction of propagation of the dislocation.

As for the discontinuities in components of the stress tensor, there are some occasions when we are justified in neglecting them. We imagine that it is under uniform pressure or uniform tension between $\sum^+$ and $\sum^-$; the former is realized by inserting some fluid between them. In these cases the normal stress in P^+ and the normal stress in P^- are of equal magnitude and of opposite direction and the tangential stresses are zero, therefore we have $\Delta\tau_{kl}=0$, since the coordinate system of reference (concerning independent variables in the surface integrations) is taken naturally as the body-fixed coordinate system which may be fixed in the body in the initial moment. Besides we can imagine the following occasion, that is, surfaces $\sum^+$ and $\sum^-$ are deformed as they are in contact with each other. Then an element of $\sum^+$ exerts a force on the contacting element of $\sum^-$ and vice versa, where two forces should be of equal magnitude and of opposite direction. Although the point P^+ is situated in the point P^- at time $t=0$, they are in different positions at arbitrary time t; $d\sum^+$ and $d\sum^-$ are not always in contact with each other. However, if the relative displacement is not large, we have the relation

$$\tau_{kl}^{+}\nu_l^{+}d\sum \fallingdotseq -\tau_{kl}^{-}\nu_l^{-}d\sum \quad \text{or} \quad \Delta\tau_{kl}\fallingdotseq 0 . \tag{41}$$

Thus, if $\sum^+$ and $\sum^-$ are deformed in any way, in some situations we can reasonably assume the relation (41) chiefly on the basis of the law of action and reaction, and the second terms of the right hand side of equations (19), (27), (28), (34) and (40) will vanish.

So far as the theory of ideal elasticity is concerned, the displacement field associated with a dislocation over $\sum$ is independent of the initial state of stress and strain owing to the linearity of the equations of motion. Therefore if an earthquake is the result of the vanishing of accumulated strains along certain surfaces, since our equations will be applicable to the discontinuities in displacement across the surfaces which will be caused under those circumstances, our equations might be important in the study of earthquakes.

The formulae (19) and (34) are equations applicable to the most general dislocations in which $\Delta u_k(P, t)$ may vary from point to point along $\sum$ and may vary with time t. V. I. Keylis-Borok (1961) pointed out that in a model obtained by analogy with dislocation theory there is a peculiar distribution of pre-faulting stress which can not correspond to a real earthquake source. However, this difficulty will be overcome by taking Δu_k as a function of coordinates, which corresponds to a Somigliana dislocation in static cases (Steketee 1958b), and by taking Δu_k sufficiently small in the neighborhood of circumference of $\sum$, while Keylis-Borok calculated the stress due to a simple type of discontinuity in u_1, Δu_1=const. along $\sum$, which corresponds to a special case of Volterra dislocation in static cases (Steketee 1958b).

According to dislocation theory it is possible to suppose the nature of the focus of an earthquake, if we can accout for seismic observations on the surface of the earth either by an *A*-nucleus or a *B*-nucleus in an infinite elastic medium with due regards to the effects which actual seismic waves will suffer within and on the earth's crust or mantle.

B-nucleus after Steketee is the well-known force system type II, and as we have seen above, it corresponds to a slip fault.

The normal discontinuty in displacement to which a nucleus of type *A* after Steketee corresponds might appear in the actual earth, if the voluminal change of material along a certain surface, which might be caused by the phase change of the focal substance, should occur indeed in the earthquake focus. Or, as Steketee (1958 b) suggests, if the magma suddenly forces itself into cracks and crevasses in the wall of a cavity in the earth, the source might be essentially described by dynamic *A*-nuclei. However, if the voluminal change might not be the major feature of the actual earthquake, a single *A*-nucleus will not represent the focus.

For the earthquake mechanism type I, no simple dislocation model is fit and Knopoff and Gilbert (1960) have suggested a configuration of two dislocation surfaces parallel to one another, the dislocations on which are of opposite sign. If one sticks to a single surface of elastic dislocation in an infinite medium, type I must be interpreted by two forces of opposite direction located at some distance on the surface.

Voluminal models of the earthquake focus have been studied usually by making use of the solution of equation of motion in terms of spherical harmonics. However, dislocation theory also provides some plain means to voluminal models. The extention of a voluminal source in one direction may be considered as a positive normal discontinuity in dislocation theory which corresponds to an *A*-nucleus in the same direction, while the contraction in another direction may be considered as a negative normal discontinuity which corresponds to an *A*-nucleus of opposite sign in the latter direction. Therefore in some cases we shall be able to get an equivalent representation of a voluminal model by taking more than one dislocation surfaces and by superposing the *A*-nuclei in different directions, the strengthes of which may vary from point to point and with time.

Here we consider an example of the force system corresponding to a focus of which volume suffers little change. If a small voluminal source suffers an extention in one direction, e. g. in the x_3-direction, and contractions in two directions at right angles to the former, e. g. in the x_1- and x_2-directions, and if the latter two mutually perpendicular contractions are of equal magnitude, the corresponding equivalent force system can be obtained by superposing a positive nucleus as (33) in Fig. 3 and two negative nuclei of equal strength in the x_1- and x_2-directions. The resultant force system is a combination of a center of compression and a double force without moment. If we denote by a sphere of broken line a center of compression, the combination may be schematically represented as in Fig. 7. This force system makes the conical type distribution of initial motions which has been detected and discussed by Japanese seismologists since 1931 (e. g. Ishimoto (1932), Kawasumi (1933, 1934), a brief review in Mikumo (1959)). Ishimoto (1932) considered this model as the evidence of his "magma intrusion theory" or "plutonic earthquake theory" of the earthquake occurrence, and it is the Model A of Kawasumi (1933, 1934) (Kawasumi's Model B agrees with *B*-nucleus after Steketee).

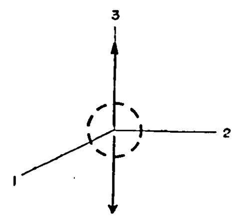

Fig. 7. Nucleus corresponding to Ishimoto's model.

In the same way *B*-nucleus in dislocation theory can be considered from the point of a voluminal model. Since the superposition of a center of dilatation and a center of compression of equal strength has no effect, it is clear that a nucleus of type *B* corresponds to a small source which suffers no voluminal change but suffers an extention in one direction and a contraction of equal magnitude in another direction perpendicular to the former.

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31. 弾性的転位に等価な力の模型と発震機構の力の模型

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地震の震源の特性を定めるために様々の方法が考えられている．これらの方法はいずれも，観測と弾性論のある種の問題に対する数学的な解とを関連させることに外ならない．この数学的な解をみちびく数学的なモデルそのものの検討を，弾性転位論の立場から行なつているのは，A. V. Vvedenskaya, L. Knopoff と F. Gilbert, J. A. Steketee 等である．Vvedenskaya, Knopoff と Gilbert は，地震の震源に近似できると思われる無限弾性体の中の特定の転位から発生する波動の初動について，立ちいつた考察を行なつた．Steketee は無限弾性体の中の静的な転位にともなう変位の場を，転位に等価な力の模型が極めて見やすい形に表現し，従来の発震機構の力の模型に関連づけようとした．

ここでは，Steketee の表現を，無限弾性体の中の時間的に変化する一般の転位に拡張して，それらに完全に等価な力の模型を調べ，同時にそれらから発生する波動の省略なしの表現を，時間的変化が一般の任意の函数の場合，$\exp(i\omega t)$ の場合，無変化すなわち静的な場合のおのおのについて示した．

我々は転位という概念を，次のような仮想的な過程から得ている．すなわち弾性体の中に一つの開いた曲面 Σ を考え，これに沿つて切口を入れ，その切口の両側の面に力を加えてこれらを相対的に変位させるとき，式 (11), (12) によつて定義される変位の不連続，応力の不連続を転位と名づけているのである．本論において示したことは，このようにして定義された転位が時間的に任意に変化するとき，無限弾性体の中に作られる変位の場は，その転位と同じ場所に同じ時間的変化を行なうある force system を置いた場合に作られる変位の場とまつたく同じであるということである．その force system としては，変位の不連続に対しては Fig. 3 に示される型の double force の組合わせであり，応力の不連続に対しては single force である．さらにくわしくは，転位面の微小面分における法線方向の変位の不連続に対しては A 型の force system が，接線方向の変位の不連続に対しては B 型の force system が対応する．(ここにいう A 型，B 型は静的な場合に対する Steketee の命名に従う.) 転位の全体に対しては，これらの force system の面積分が対応する．もし転位が有限の速さで進行するならば，これに対しては，変位の不連続のベクトルと法線のベクトルとによつてそれぞれの点において定まる force system を関与した面全体に並べ，それらの時間的変化を，各々は段階函数に近いものであつて，その立ち上がりが転位の進行にともなつて時間的に次々に遅れて行くようにとることになる．

一般の議論においては，形式を整えるために応力の不連続をも考慮してきたが，物質の割れ目に流体が入りこむ場合や，その二つの面が互に接触したまま相互に滑る場合等においては，作用反作用の法則によつて応力の不連続は 0 ととるべきであることが示される．

線型の弾性論の範囲では，一つの転位にともなう変位の場は初期状態に無関係であるから，以上の結果をそのまま初期応力の存在する場合に用いることができる．したがつてもし地震がある面の近傍に蓄えられた歪エネルギーの解放であるならば，その時生ずる転位面における変位の不連続に対してこれらの式を用いることが許されるから，これらの式は地震学において有用であるかも知れない．

特に slip fault に対しては，たとえそれが進行する fault であつても，あるいはさらに他の任意の時間的変化を仮定するとしても，B 型 (すなわち type II) の force system が，線型の弾性論から帰結するもつとも自然な force system であることがわかる．ただしそれらの force system の積分の結果として，合成の force system が多少変形されることはあるかも知れない．

最後に，震源として転位面という面を想定するこのような転位のモデルと，従来の球その他の容積の変形を考える震源モデルとのつながりについて述べた．