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# 34. *The Effects of a Bottle-neck on Tsunami.*

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## Introduction.

In the fore-going papers[1)-5)] the tsunamis in several kinds of canals have been studied. In this paper our concern is with the study of the tsunami in a semi-open canal with a bottle-neck, i.e., a narrow part (cf. Fig. 1).

## 1. General Theory.

Referring to Fig. 1, the origin of the coordinates is located at the rim of the canal. The directions of the $x$- and $y$-axes of the coordinates, the lengths $l_1$ and $l_2$ of the bottle-neck and the closed canal, and the widths $d_0$, $d_1$ and $d_2$ of the canals are defined as shown in Fig. 1. Then in order to use the method developed by the author,[4)] the buffer domains $D_{0,j}$ and $D_{2,j}$ ($j=1, 2$) are taken in the vicinity of the conjunction parts of the canals (Fig. 1). The widths $b_0$ and $b_1$ of the domains $D_{0,1}$ ($D_{0,2}$) and $D_{2,1}$ ($D_{2,2}$) are limited to the range

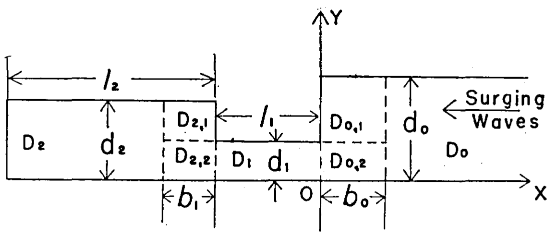


Fig. 1.

$$kb_j \ll 1 \quad (j=0, 1)\,, \tag{1}$$

where $k$ is the wave number of the surging waves. Other domains $D_0$ and $D_2$ are defined as shown in Fig. 1. The wave heights are denoted by $\zeta_j$ with the same suffices that are used in $D_j$ ($j=0;\ 1;\ 2;\ 0,1;\ 0,$

1) T. MOMOI, *Bull. Earthq. Res. Inst.*, **40** (1962), 719.
2) T. MOMOI, *ditto*, **40** (1962), 733.
3) T. MOMOI, *ditto*, **40** (1962), 747.
4) T. MOMOI, *ditto*, **41** (1963), 357.
5) T. MOMOI, *ditto*, **41** (1963), 375.

2; 2, 1; 2, 2).

Then the equations are

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\zeta_j=\frac{1}{c^2}\frac{\partial^2\zeta_j}{\partial t^2}\,, \tag{2}$$

where $c=\sqrt{gH}$ ($g$: the acceleration of gravity, $H$: the depth of water) and $t$ is the time variable.

When a periodic train of waves surges (Fig. 1), the solutions of (2), under the condition that there are no fluxes present at the rigid boundaries, are as given below:

in the domain $D_0$,

$$\zeta_0=\zeta e^{-ikx}+\sum_{m=0}^{\infty}\zeta_0^{(m)}\cos\frac{m\pi}{d_0}y\cdot e^{+ik_0^{(m)}x}\,; \tag{3}$$

in the domain $D_1$,

$$\zeta_1=\sum_{m=0}^{\infty}\{\zeta_1^{(m)}e^{+ik_1^{(m)}}+\bar{\zeta}_1^{(m)}e^{-ik_1^{(m)}x}\}\cos\frac{m\pi}{d_1}y\,; \tag{4}$$

in the domain $D_2$,

$$\zeta_2=\sum_{m=0}^{\infty}\zeta_2^{(m)}\cos\frac{m\pi}{d_2}y\cos k_2^{(m)}(x+l_1+l_2)\,; \tag{5}$$

in the domain $D_{0,1}$,

$$\zeta_{0,1}=\sum_{f_{0,1}}A_{0,1}\cos k_x^{(0,1)}x\cos k_y^{(0,1)}(y-d_0)\,; \tag{6}$$

in the domain $D_{0,2}$,

$$\zeta_{0,2}=\sum_{f_{0,2}}(A_{0,2}\cos k_x^{(0,2)}x+B_{0,2}\sin k_x^{(0,2)}x)\cos k_y^{(0,2)}y\,; \tag{7}$$

in the domain $D_{2,1}$,

$$\zeta_{2,1}=\sum_{f_{2,1}}A_{2,1}\cos k_x^{(2,1)}(x+l_1)\cdot\cos k_y^{(2,1)}(y-d_2)\,; \tag{8}$$

in the domain $D_{2,2}$,

$$\zeta_{2,2}=\sum_{f_{2,2}}\{A_{2,2}\cos k_x^{(2,2)}(x+l_1)+B_{2,2}\sin k_x^{(2,2)}(x+l_1)\}\cos k_y^{(2,2)}y\,; \tag{9}$$

where the time factor $\exp(-i\omega t)$ is omitted as usual; $\zeta$ is the amplitude of the surging wave; $\zeta^{(m)}$, $\bar{\zeta}^{(m)}$ ($m=0, 1, 2, \cdots$), $A$ and $B$ with suffices relevant to the domains denote arbitrary constants in each domain; $k_j^{(m)}=\sqrt{k^2-(m\pi/d_j)^2}$ $(j=0,\ 1,\ 2)$; $(k_x^{(j)})^2+(k_y^{(j)})^2=k^2$ $(j=0,\ 1;\ 0,\ 2;\ 2,\ 1;\ 2,\ 2)$; $\sum_{f_j}$ $(j=0,\ 1;\ 0,\ 2;\ 2,\ 1;\ 2,\ 2)$ is the integration under the condition $(k_x^{(j)})^2+(k_y^{(j)})^2=k^2$.

In order to determine the arbitrary constants in the above expression (3)-(9), there are the following conditions available[6]:

$$\left.\begin{array}{lll} & \zeta_i=\zeta_j & \text{(the continuitiy of the wave height)} \\ \text{and} & \dfrac{\partial\zeta_i}{\partial n}=\dfrac{\partial\zeta_j}{\partial n} & \text{(the continuity of the flux)}\,, \end{array}\right\} \tag{10}$$

at the boundaries between the neighbouring domains $D_i$ and $D_j$, where $\partial/\partial n$ is the normal derivative at the boundary.

Connecting the expressions (3)-(9) by use of the condition (10) and using Momoi's method, we have the following equations for the first modes of the waves:

$$d_0e^{+ikb_0}\zeta_0^{(0)}-(+ikb_0d_0+d_1)\zeta_1^{(0)}-(+ikb_0d_0-d_1)\bar{\zeta}_1^{(0)}=d_0e^{-ikb_0}\zeta\,, \tag{11}$$

$$-d_0e^{+ikb_0}\zeta_0^{(0)}+(+ikb_0d_1+d_0)\zeta_1^{(0)}+(-ikb_0d_1+d_0)\bar{\zeta}_1^{(0)}=d_0e^{-ikb_0}\zeta\,, \tag{12}$$

$$(-ikb_1d_2+d_1)e^{-ikl_1}\zeta_1^{(0)}-(+ikb_1d_2+d_1)e^{+ikl_1}\bar{\zeta}_1^{(0)}=\zeta_2^{(0)}(+id_2)\sin k(l_2-b_1)\,, \tag{13}$$

$$(-ikb_1d_1+d_2)e^{-ikl_1}\zeta_1^{(0)}+(+ikb_1d_1+d_2)e^{+ikl_1}\bar{\zeta}_1^{(0)}=\zeta_2^{(0)}d_2\cos k(l_2-b_1)\,. \tag{14}$$

The derivation of the above equations follows exactly the same line as in the preceding paper,[7] being obtained to the first order of $kd$ ($d$: the dimension of the width of the canal).

Solving the above equations (11)-(14), we obtain:
for the first mode of waves in the domain $D_2$,

$$\zeta_2^{(0)}=\frac{8d_0d_1}{F}\cdot\zeta\,, \tag{15}$$

where

$$\begin{aligned} F=&(d_0-d_1)(d_1+d_2)e^{+ik(l_2+l_1)}+(d_0-d_1)(d_1-d_2)e^{-ik(l_2-l_1)} \\ &+(d_0+d_1)(d_1+d_2)e^{-ik(l_2+l_1)}+(d_0+d_1)(d_1-d_2)e^{+ik(l_2-l_1)}\,: \end{aligned} \tag{16}$$

$$\bar{\zeta}_1^{(0)}=\frac{\zeta_2^{(0)}}{4d_1}\cdot e^{-ikl_1}\cdot\{(d_1+d_2)e^{-ikl_2}+(d_1-d_2)e^{+ikl_2}\}\,, \tag{17}$$

$$\zeta_1^{(0)}=\frac{\zeta_2^{(0)}}{4d_1}\cdot e^{+ikl_1}\cdot\{(d_1+d_2)e^{+ikl_2}+(d_1-d_2)e^{-ikl_2}\}\,, \tag{18}$$

6) T. Momoi, *loc. cit.*, 1)-5).
7) T. Momoi, *loc. cit.*, 5).

(17) and (18) being the expressions for the progressive and the retrogressive waves respectively and $\zeta_2^{(0)}$ in the above expressions being given by (15): for the first mode of retrogressive waves in the domain $D_0$,

$$\zeta_0^{(0)}=\frac{G}{F}\cdot\zeta\ , \tag{19}$$

where

$$G=(d_0-d_1)(d_1+d_2)e^{-ik(l_2+l_1)}+(d_0-d_1)(d_1-d_2)e^{+ik(l_2-l_1)}$$
$$+(d_0+d_1)(d_1+d_2)e^{+ik(l_2+l_1)}+(d_0+d_1)(d_1-d_2)e^{-ik(l_2-l_1)}\ . \tag{20}$$

As far as the higher modes of waves are concerned, following the same procedures as in the previous paper[8] and the foregoing analysis for the first mode, we have:

in the domain $D_2$,

$$\zeta_2^{(m)}=\frac{16kb_1d_0d_2\sin\left(m\pi\dfrac{d_1}{d_2}\right)}{F\cdot m\pi\cdot\cosh\left(\dfrac{m\pi}{d_1}\right)(l_2-b_1)}\cdot\sin kl_2\cdot\zeta\ ; \tag{21}$$

in the domain $D_0$,

$$\zeta_0^{(m)}=-\frac{8kb_0d_0}{F\cdot m\pi}\cdot e^{(m\pi/d_0)b_0}\cdot\sin\left(m\pi\frac{d_1}{d_0}\right)\cdot$$
$$\cdot\{(d_1+d_2)\sin k(l_1+l_2)+(d_1-d_2)\sin k(l_1-l_2)\}\cdot\zeta\ ; \tag{22}$$

where the above two expressions are obtained to the approximation of the first order of $kd$, and the expression $F$ in the denominators is described in (16); in the domain $D_1$, the higher modes are null when consideration is limited to the first order of $kd$. (These modes are of the second order. Refer to the preceding paper.[9])

Taking the absolute values of the first modes, the expressions (15)–(20) become;

$$|\zeta_2^{(0)}|=\frac{8d_0d_1}{|F|}\cdot\zeta\ , \tag{15'}$$

where

8) T. MOMOI, *loc. cit.*, 5).
9) T. MOMOI, *loc. cit.*, 5).

$$|F|^2=4(d_0^2+d_1^2)(d_1^2+d_2^2)+4(d_0^2+d_1^2)(d_1^2-d_2^2)\cos 2kl_2+4(d_0^2-d_1^2)(d_1^2-d_2^2)\cos 2kl_1$$
$$+2(d_0^2-d_1^2)(d_1+d_2)^2\cos\{2k(l_2+l_1)\}+2(d_0^2-d_1^2)(d_1-d_2)^2\cos\{2k(l_2-l_1)\}\ ; \tag{16'}$$

$$|\bar{\zeta}_1^{(0)}|\equiv|\zeta_1^{(0)}|=2\sqrt{2}\,d_0\cdot\frac{|I|}{|F|}\cdot\zeta\ , \tag{17'}$$

where

$$|I|^2=(d_1^2+d_2^2)+(d_1^2-d_2^2)\cos 2kl_2 \tag{18'}$$

and $|F|^2$ has already been given in (16');

$$|\zeta_0^{(0)}|\equiv\zeta\ . \tag{19'}$$

Although some ambiguity is present in the determination of the buffer domain, qualitative arguments on the higher modes of waves would be permissible (reference should be made to the preceding paper[10]).

In the following section, some discussions and analyses for particular cases are given.

## 2. Discussion of Particular Cases.

Firstly, our concern is with the higher modes of waves which are described in (21) and (22).

(i) when $\qquad kl_2=m\pi\ (m=1,\ 2,\ 3,\cdots)$;

$\zeta_2^{(m)}$ (the lateral modes of waves in the domain $D_2$) vanishes.

On partially differentiating (16') with respect to $l_2$, we have

$$\frac{\partial|F|^2}{\partial l_2}=-4k[2(d_0^2+d_1^2)(d_1^2-d_2^2)\sin 2kl_2+(d_0^2-d_1^2)(d_1+d_2)^2\sin\{2k(l_2+l_1)\}$$
$$+(d_0^2-d_1^2)(d_1-d_2)^2\sin\{2k(l_2-l_1)\}]\ . \tag{23}$$

From the above expression, it turns out that when $kl_2=m\pi$ the first mode of the waves in the domain $D_2$ (15') does not necessarily take the extreme value. But for approximate values of $l_1$, (23) vanishes. Hence we use the term *semi-resonance* in this case.

$$\left.\begin{array}{lll}\text{(ii) when} & k(l_1+l_2)=m\pi & (m=1,\ 2,\ 3,\cdots)\\ \text{and} & k(l_1-l_2)=n\pi & (n=0,\ \pm1,\ \pm2,\cdots)\end{array}\right\}\ ; \tag{24}$$

10) T. MOMOI, *loc. cit.*, 5).

$\zeta_0^{(m)}$ (the lateral modes of waves in the domain $D_0$) disappers.

Solving (24),

$$\left.\begin{aligned} 2kl_1&=(m+n)\pi\ ,\\ 2kl_2&=(m-n)\pi\ .\end{aligned}\right\} \tag{25}$$

But $\zeta_2^{(m)}$ does not necessarily vanish.

Differentiating (16′) with respect to $l_1$, we have

$$\frac{\partial|F|^2}{\partial l_1}=-4k[2(d_0^2-d_1^2)(d_1^2-d_2^2)\sin 2kl_1+(d_0^2-d_1^2)(d_1+d_2)^2\sin\{2k(l_2+l_1)\}$$
$$-(d_0^2-d_1^2)(d_1-d_2)^2\sin\{2k(l_2-l_1)\}]\ . \tag{26}$$

Putting (24) or (25) for (23) and (26), both derivatives are found to vanish for the relations (24) or (25). Then the first mode of waves in the domain $D_2$ takes the extreme values for the present case. These extreme values are classified in the following way:

when both $m$ and $n$ are even (or odd),

$$|\zeta_2^{(0)}|=2\zeta \quad \text{(maximum value)};$$

when $m$ and $n$ are even (or odd) and odd (or even) respectively,

$$|\zeta_2^{(0)}|=2\cdot\frac{d_1}{d_2}\cdot\zeta \quad \text{(maximum value)}\ ,$$

where

$$d_1<d_2\ .$$

The more particular case where $l_1\to 0$ is treated in the later small section (iv).

(iii) when

and

$$\left.\begin{aligned} &kl_2=m\pi \quad (m=1,\ 2,\ 3,\cdots)\\ &k(l_1+l_2)=n\pi \quad (n=1,\ 2,\ 3,\cdots)\\ &k(l_1-l_2)=q\pi \quad (q=0,\ \pm1,\ \pm2,\cdots)\end{aligned}\right\};$$

$\zeta_0^{(m)}$ and $\zeta_2^{(m)}$ both vanish.

After simple considerations, it follows that $m$ and $n$ are both even or odd. Therefore, this case turns out to be a particular example of the foregoing small section. This case is completely resonant.

(iv) when $\qquad l_1\to 0$ ;

the expressions of the wave heights become as follows:

from (15′) and (16′),

$$|\zeta_2^{(0)}| = 2\sqrt{2}\, d_0 \cdot \frac{1}{|K|} \cdot \zeta \ , \tag{15''}$$

where

$$|K|^2 = (d_0^2 + d_2^2) + (d_0^2 - d_2^2) \cos 2kl_2 \ ; \tag{16''}$$

from (21),

$$\zeta_2^{(m)} = \frac{4kb_1 d_0 d_2 \sin\left(m\pi \dfrac{d_1}{d_2}\right)}{L \cdot d_1 m\pi \cdot \cosh\left(\dfrac{m\pi}{d_1}\right)(l_2 - b_1)} \cdot \sin kl_2 \cdot \zeta \ , \tag{21'}$$

where

$$L = d_0 \cos kl_2 - i d_2 \sin kl_2 \ ;$$

from (22),

$$\zeta_0^{(m)} = -\frac{4kb_0 d_0 d_2}{L \cdot d_1 \cdot m\pi} \cdot e^{(m\pi/d_0) b_0} \cdot \sin\left(m\pi \frac{d_1}{d_0}\right) \cdot \sin kl_2 \cdot \zeta \ . \tag{22'}$$

Also the first and the second derivatives of $|K|^2$ in terms of $l_2$ become as follows:

$$\frac{\partial |K|^2}{\partial l_2} = 2k(d_2^2 - d_0^2) \sin 2kl_2 \ , \tag{27}$$

and

$$\frac{\partial |K|^2}{\partial l_2^2} = 4k^2(d_2^2 - d_0^2) \cos 2kl_2 \ . \tag{28}$$

When $kl_2 = m\pi$ $(m = 1, 2, 3, \cdots)$

$$\left.\begin{aligned} &\frac{\partial |K|^2}{\partial l_2} = 0 \quad \text{(from (27))} \\ \text{and} \quad & \\ &\frac{\partial |K|^2}{\partial l_2} = 4k^2(d_2^2 - d_0^2) \quad \text{(from (28))} \ . \end{aligned}\right\} \tag{29}$$

From (29) it turns out that:

$$\left.\begin{array}{l}\text{when } \left.\begin{array}{l}kl_2=m\pi\\ d_2>d_0\end{array}\right\},\ |\zeta_2^{(0)}| \text{ takes a maximum value } 2\zeta \text{ (Fig. 2)};\\ \text{when } \left.\begin{array}{l}kl_2=m\pi\\ d_2<d_0\end{array}\right\},\ |\zeta_2^{(0)}| \text{ takes a minimum value } 2\zeta \text{ (Fig. 3)}.\end{array}\right\} \quad (30)$$

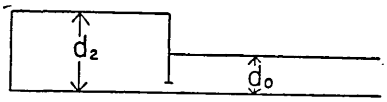


Fig. 2.

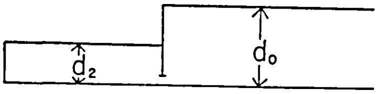


Fig. 3.

Then the lateral modes of waves (21′) and (22′) vanish by the existence of the factor $\sin kl_2$.

After some reductions, we find that:

for the former cases of (30), the minimum value of $|\zeta_2^{(0)}|$ is $2\dfrac{d_0}{d_2}\zeta$ $(d_2>d_0)$;

for the latter case of (30), the maximum value of $|\zeta_2^{(0)}|$ is $2\dfrac{d_0}{d_2}\zeta$ $(d_2<d_0)$.

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## 34. 津波に対する狭水路（ボトルネック）の影響

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本論説において，筆者はボトルネックが津波に対してどのように働くかを調べて見た．得た結論の中で最も重要な点は，

進入波の波長とボトルネックの長さおよび湾の長さとの間に特別な関係があるとき，湾口の内外の横モードが消え去ること．

これらのことは **Discusion of Particular Cases** の節の中で述べてある．