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38. *The Anomalous Behaviour of Geomagnetic Variations of Short Period in Japan and Its Relation to the Subterranean Structure. The 10th report.*

(*An analysis of the s.f.e. on Aug. 16, 1958.*)

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Summary

An *s.f.e.* that occurred on Aug. 16, 1958 is analysed in the hope of studying the underground structure beneath Japan. Since the inducing field prevails in a westerly direction in this case, it is expected to add something new to the previous studies because the analyses hitherto made have been mostly concerned with changes in a north-south direction. The distributions of magnetic potential for the *s.f.e.*, both external and internal origins, are obtained first. On the basis of the distributions of magnetic potential and vertical component in and around Japan it is concluded that the anomalous part of the induced field, that is not quite vanishing even for the present case, seems likely to be caused by the northerly change in the inducing field though it is much smaller than the westerly one. As far as we assume that the anomaly of geomagnetic variations of short period in Japan is caused by electric currents flowing in an underground circuit connected to the conducting part of the mantle, as has been discussed in the previous papers, the connecting points must be located on a line running approximately from east to west, so that very little current is excited in the circuit when the inducing field changes to an east-west direction.

The world's maps of magnetic potential enable us to discuss the earth's electrical state on a large scale. Applying a theory of electromagnetic induction by a time-dependent magnetic dipole in a spherical conductor, a conclusion which differs very little from the previous one for the average state of the earth is obtained.

1. Introdution

The anomaly of short-period geomagnetic variations in Japan has been studied by T. Rikitake and others[1)-8)] in fair detail. The possible cause of the anomaly has been discussed by Rikitake[5),6)]. The only way to account for the large amplitude of vertical component in the central area of Japan would be to suppose a highly conducting circuit, both end of which are connected to the conducting part of the mantle below a depth of a few hundred kilometers.

The shape of the underground circuit has been tentatively supposed to be roughly elliptical, about 1000 *km* in length and 200 *km* in width. If an intense electric current could somehow flow in the circuit at times of geomagnetic disturbance, the anomalous behaviour would be approximately explained.

The currents seem likely to come up from the deeper part of the mantle where electric currents are induced by changes in magnetic field orignating from outside the earth. Although the underground structure thus presumed beneath Japan is unusually complicated, there has been found no other way to account for the anomalous behaviour of geomagnetic variation in Japan.

The analyses related to the anomaly have been made mostly for *s. s. c.*'s, bays and polar magnetic storms. The changes in the inducing field for these variations are approximately directed to the north as far as the previous analyses are concerned. It has been pointed out[2)], however, that the vertical component becomes small at observatories in the central part of Japan whenever the magnetic field changes in the east-west direction. The apparent anisotropy thus found may have an important bearing on the underground structure.

It is the intention of this paper to analyse an *s. f. e.* of which the horizontal magnetic vector points roughly in a westerly direction in the

1) T. RIKITAKE, I. YOKOYAMA and Y. HISHIYAMA, *Bull. Earthq. Res. Inst.*, **30** (1952), 207; **31** (1953), 19, 89, 101 and 119.

2) T. RIKITAKE and I. YOKOYAMA, *Bull. Earthq. Res. Inst.*, **33** (1955), 297.

3) T. RIKITAKE, I. YOKOYAMA, S. UYEDA, T. YUKUTAKE and E. NAKAGAWA, *Bull. Earthq. Res. Inst.*, **36** (1958), 1.

4) T. RIKITAKE, S. UYEDA, T. YUKUTAKE, I. TANAOKA and E. NAKAGAWA, *Bull. Earthq. Res. Inst.*, **37** (1959), 1.

5) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **37** (1959), 545.

6) T. RIKITAKE, *Geophys. Journ.*, **2** (1959), 276.

7) T. RIKITAKE and I. YOKOYAMA, *Journ. Geomagn. Geoelectr.*, **5** (1953), 59.

8) T. RIKITAKE and I. YOKOYAMA, *Naturwissenschaften*, **41** (1954), 420.

hope of getting some clue for further investigation of the underground structure beneath Japan. An *s.f.e.* that occurred on Aug. 16, 1958 was chosen for the purpose because we could obtain magnetic data from a fair number of observatories.

In Section 2 the magnetic data collected are summarised. On the basis of these data, the characteristic distribution of the vertical component in and around Japan is studied in Section 3. Section 4 describes how to obtain the world's distribution of magnetic potential which is separated into external and internal parts in Section 5. In Section 5 is also described a special globe which is useful for the procedure of separation. The relation between the external and internal parts thus obtained is studied from two standpoints. Firstly in Section 6, the distribution of the electrical conductivity within the earth is discussed on the basis of the world's maps for magnetic potential of external and internal origin. Unlike the usual method based on a spherical harmonic analysis, a theory of electromagnetic induction by a time-dependent dipole is applied to the problem. Meanwhile, the potentials of both the parts around Japan are studied in Section 7. In the light of the investigations in these sections something new is added to the previous view on the possible cause of the anomaly of short-period variations in Japan as is dealt with in Section 8.

2. Data

Copies of magnetograms for an *s.f.e.* that occurred at $4h$ $34m$ *GMT* on Aug. 16, 1958 were collected by the writers from eight observatories in Japan as can be seen in Fig. 1. The locations of these observatories are listed in Table 1 together with the changes in the

Table 1. Magnetic changes for the *s.f.e.* on Aug. 16, 1958 at Japanese observatories.

IGY Number	Observatory	Geographic latitude	Geographic longitude	ΔH	ΔD	ΔZ	ΔX	ΔY
C 034	Memambetsu	43.9°N	144.2°E	9.9γ	−69.3γ	6.5γ	0.0γ	−70.0γ
C 117	Onagawa	38.4	141.5	13.7	−44.5	8.6	8.0	−45.8
—	Komoro	36.3	138.4	14.5	−48.0	−4.8	8.7	−49.4
C 147	Kakioka	36.2	140.2	11.2	−45.0	0.3	5.9	−46.0
—	Aburatsubo	35.2	139.6	20.6	−42.6	7.3	15.4	−44.7
C 214	Simosato	33.6	135.9	20.6	−42.8	14.0	16.2	−44.7
C 223	Aso	32.9	131.0	24.0	−39.8	−2.8	20.1	−41.9
C 245	Kanoya	31.4	130.9	31.4	−41.6	3.2	27.6	−44.1

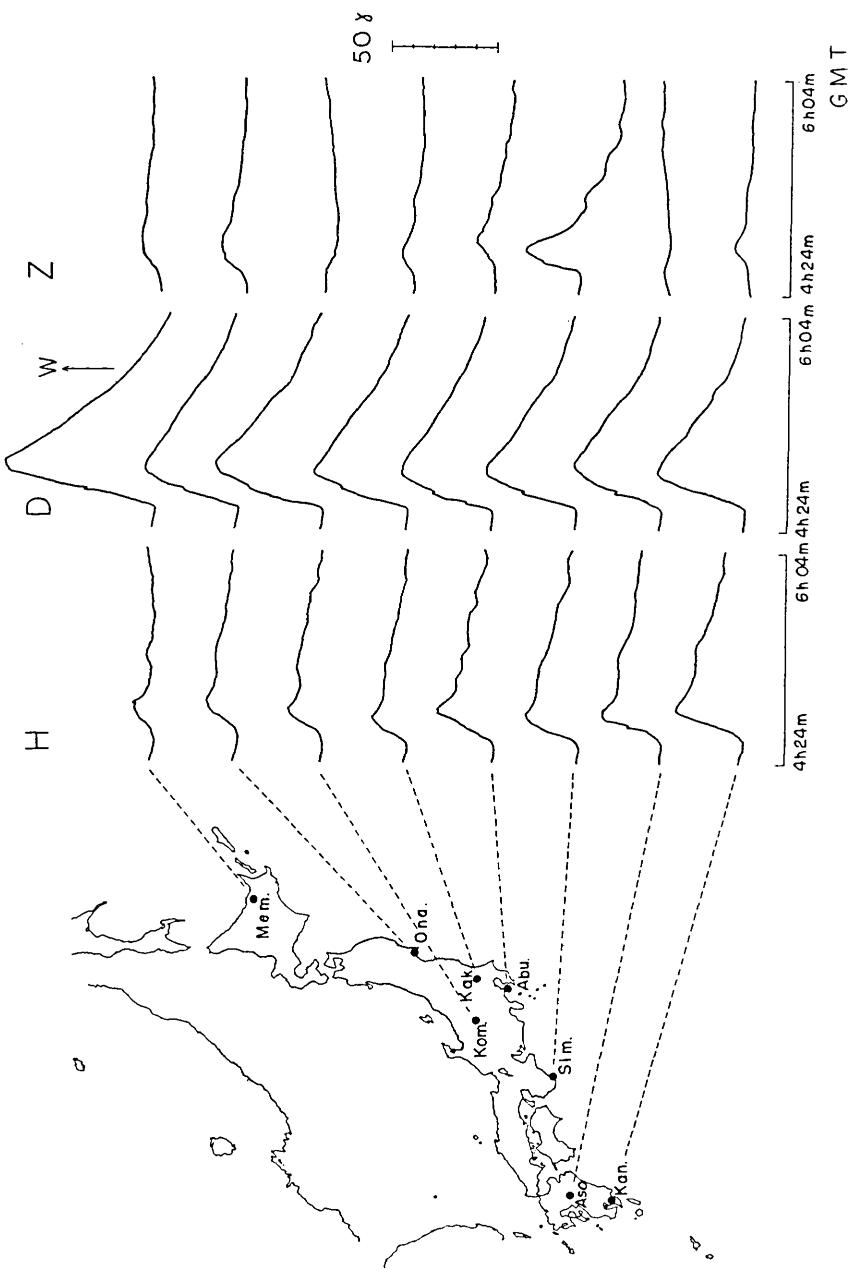

Fig. 1. The changes in the three geomagnetic components as observed at the eight Japanese observatories at the time of an *s.f.e.* on Aug. 16, 1958.

three geomagnetic components at the maximum deviation stage of the *s.f.e.* at $4h$ $51m$. Assuming that the *s.f.e.* ended at $6h$ $04m$, deviations from the straight line connecting the point at $4h$ $34m$ to that at $6h$ $04m$ on magnetograms are taken as the changes, i. e. ΔH (horizontal intensity), ΔD (easterly declination) and ΔZ (vertical intensity).

Changes in the north (ΔX) and east (ΔY) directions that are calculated from ΔH and ΔD are also given in Table 1.

J. Veldkamp and D. Van Sabben[9] have made an extensive study of overhead electric current-systems of *s.f.e.*'s including the one concerned here. Dr. Veldkamp has kindly sent one of the writers (T. R.) a list of ΔH, ΔD and ΔZ at the maximum stage which is reproduced in Table 2 together with locations of observatories. In Fig. 2 are reproduced the current-systems at $4h$ $51m$ *GMT* on Aug. 16, 1958 drawn by Veldkamp and Van Sabben.

Table 2. Magnetic changes in units of gamma for the *s.f.e.* on Aug. 16, 1958 (After Veldkamp).

IGY Number	Observatory	Geographic latitude	Geographic longitude	ΔH	ΔD	ΔZ	ΔX	ΔY
A 050	Murmansk	69.0°*N*	33.1°*E*	0	+42	Very small	− 8	41
A 047	Tromsö	69.7	18.9 *E*	(−15)*)	(+25)	+15	−15	25
A 101	Reykjavik	64.2	21.7 *W*			No effect		
A 102	Big Delta	64.2	145.8 *W*	(0)	(+10)	0	− 5	8
A 065	Sodankylä	67.4	26.6 *E*	(−13)	(+36)	+12	−12	36
A 123	Dombas	62.1	9.1 *E*	0	+38	+17	3	38
A 124	Yakutsk	62.0	129.7 *E*	−21	−68	+14	−42	−57
A 134	Nurmijärvi	60.5	24.6 *E*	0	+43	+ 3	−27	43
B 009	Lovö	59.3	17.8 *E*	0	+42	0	0	42
A 149	Sitka	57.0	135.3 *W*	0	(+ 5)	0	− 2	4
B 019	Sverdlovsk	56.7	61.1 *E*	−22	+50	−18	−33	43
B 038	Eskdalemuir	55.3	3.2 *W*	(− 4)	+24	+ 6	0	24
B 035	Moscow	55.5	37.3 *E*	+14	+57	− 9	6	58
B 058	Wingst	53.7	9.1 *E*	(0)	+35	+ 7	2	35
C 362	Irkutsk	52.5	104.0 *E*	−52	−33	—	−53	−32
B 089	Swider	52.1	21.2 *E*	+ 5	+38	0	4	38
B 098	Valentia	51.9	10.2 *W*	0	(+14)	− 7	0	14
B 106	Göttingen	51.5	9.9 *E*	0	+40	+ 5	0	40
B 145	Lvov	49.9	23.7 *E*	+11	+27	+ 8	10	27

(to be contiuned)

9) J. VELDKAMP and D. VAN SABBEN, *Journ. Atmos. Terr. Phys.*, **18** (1960), 192.

Table 2. (continued)

IGY number	Observatary	Geographic latitude	Geographic longitude	ΔH	ΔD	ΔZ	ΔX	ΔY
B 191	Tihany	46.9°*N*	17.9°*E*	+ 6	+28	− 1	6	28
B 143	Pruhonice	50.0	14.5 *E*	0	+25	0	1	25
B 125	Kiev	50.5	30.5 *E*	+10	+35	− 9	7	36
B 172	Hurbanovo	47.9	18.2 *E*	+24	+ 6	− 3	6	24
C 016	Sakhalinsk	46.9	142.7 *E*	− 2	−53	+ 8	−10	−52
C 018	Odessa	46.8	30.9 *E*	+ 9	+36	− 6	8	36
C 051	Wladiwostok	43.8	132.0 *E*	−11	−65	(− 9)	−22	−62
B 267	Logrono	42.4	2.0 *W*	No effect				
C 364	Tiflis	41.7	44.8 *E*	+30	+45	− 2	29	47
C 076	Tashkent	41.4	69.2 *E*	+14	+50	+ 9	10	51
C 063	l'Aquila	42.4	13.3 *E*	+ 9	+19	− 2	10	19
—	Ksara	33.8	35.9 *E*	+13	+31	+11	12	32
C 251	Quetta	30.2	66.9 *E*	+13	+52	− 9	12	52
C 256	Helwan	29.9	31.3 *E*	+10	+19	(+ 5)	10	19
E 566	Kodaikanal	10.2	77.5 *E*	(+108)	(?)45	−27	—	—
C 277	Honolulu	21.3	158.1 *W*	No effect				
E 556	Guam	13.5	144.7 *E*	+25	−26	irregu-lar	26	−25
E 606	Koror	7.3	134.5 *E*	+96	−15	+52	96	−12
E 585	Fanning	3.9	159.4 *W*	0	− 4	− 2	0	− 4
E 578	Fuquene	5.5	73.7 *W*	No effect				
E 568	Addis Ababa	9.0	38.8 *E*	−28	+21	+ 4	−28	21
E 575	Paramaribo	5.8	55.2 *W*	No effect				
E 583	Bangui	4.6	18.6 *E*	+ 2	+45	+ 8	6	45
E 553	Mantinlupa	14.4	121.0 *E*	+50	−25	−30	50	−22
E 625	Hollandia	2.6 *S*	140.5 *E*	+38	0	−22	38	3
E 631	Binza	4.6	15.3 *E*	0	+48	−12	7	47
E 640	Luanda	8.8	13.2 *E*	(0)	(+30)	−20	6	29
E 644	Elisabethville	11.6	27.4 *E*	+19	+41	−10	25	38
E 634	Kuyper	6.0	106.7 *E*	+62	−31	−15	69	−17
E 646	Huancayo	12.1	75.3 *W*	No effect				
E 642	Port Moresby	9.4	147.2 *E*	+32	(+10)	+29	31	13
E 653	Apia	13.8	171.8 *W*	− 7	+11	+ 2	− 9	9
C 897	Darwin	12.3	131.0 *E*	+50	(+10)	(−15)	49	13
C 901	Tananarive	18.9	47.5 *E*	+18	+14	− 5	20	10
C 933	Gnangara	31.9	115.9 *E*	−13	−25	+14	−14	−24
C 957	Hermanus	34.4	19.2 *E*	(+ 1)	(+ 7)	(+10)	4	2
B 966	Toolangi	37.5	145.5 *E*	−27	+31	+ 8	−32	26
B 979	Amberley	43.1	172.7 *E*	−11	+13	+ 5	−15	8
A 961	Macquarie Is.	54.5	158.9 *E*	(− 7)	+ 8	+ 0	−10	4

*) Uncertain values are given in parentheses.

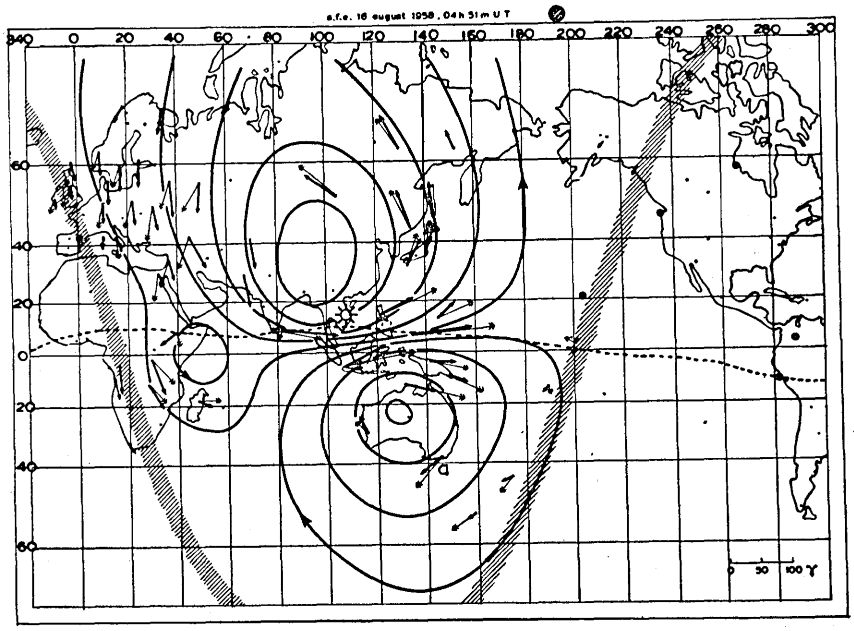

Fig. 2. Map of horizontal electric currents in the ionosphere, necessary to produce the horizontal components of the *s.f.e.* currents at 4 *h* 51 *m GMT* on Aug. 16, 1958. Heavy arrows with single head represent the *s.f.e.* currents, light arrows with double head the currents of the normal daily variation at the same time. (After Veldkamp and Van Sabben.)

3. Changes in and around Japan and their characteristics

On the basis of the Japanese and Russian data in Tables 1 and 2, the overhead current-arrows for the maximum stage of the *s.f.e.* at observatories in and around Japan are shown in Fig. 3. We see that these current-arrows fit in with the current-systems drawn by Veldkamp and Van Sabben. Nothing extraordinary can be seen in the distribution of horizontal vectors of the *s.f.e.*

Going back to Fig. 1, however, we observe that the magnetograms for the vertical component take different shapes from observatory to observatory. The amplitude of ΔZ at Simosato is especially large. Although the ΔZ anomaly of magnetic change for the *s.f.e.* is not so remarkable as that for a bay, it is apparent that We still observe some anomaly, which presumably reflects the unusual underground condition beneath Japan, even in this case.

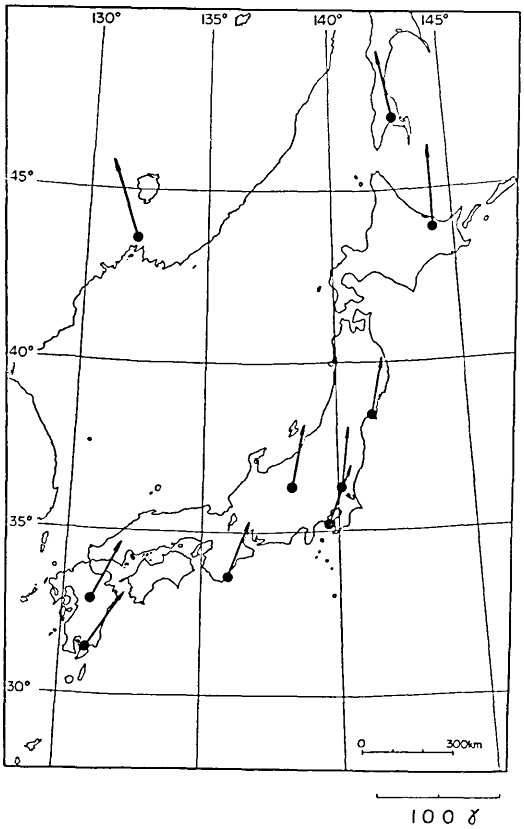

Fig. 3. The current-arrows over Japan for the *s.f.e.* at $4h$ $51m$ *GMT* on Aug. 16, 1958.

In the cases of bay, it has long been recognised (1) that ΔZ is very large and (2) that the parallelism between ΔH and ΔZ is nearly perfect at observatories in the central part of Japan. Such a tendency does not hold good for an *s.f.e.* The difference between bay and *s.f.e.* is well demonstrate in Figs. 4 and 5.

The changes at the eight Japanese observatories during the *s.f.e.* are divided into six parts of equal interval and then the growth and decay of magnetic vectors in the horizontal plane are shown on the left in Fig. 4 while ΔZ's are shown on the right as functions of time. We see that the westerly component is predominating at all the observatories.

Meanwhile, similar diagrams for a positive bay that occurred on April 18, 1958 are shown in Fig. 5. In this case, it is seen that the northerly component is predominating. Looking at the ΔZ curves for both cases, we observe that there exists a surprisingly large difference in the behaviour of ΔZ between the *s.f.e.* and the bay. In the latter case, ΔZ is extremely large in the central area of Japan amounting to the same amplitude as that of ΔH at Simosato. On the contrary, ΔZ is generally small in the former case and its sign is not necessarily the same throughout the observatories.

A tendency of this kind has been well noticed[2] at a few observatories. At Aburatsubo, for instance, ΔZ is very small when the magnetic vector is directed to the east-west direction in the horizontal plane. As far as we assume electromagnetic induction within the earth, we may understand the above characteristics as an apparent anisotropy in the response

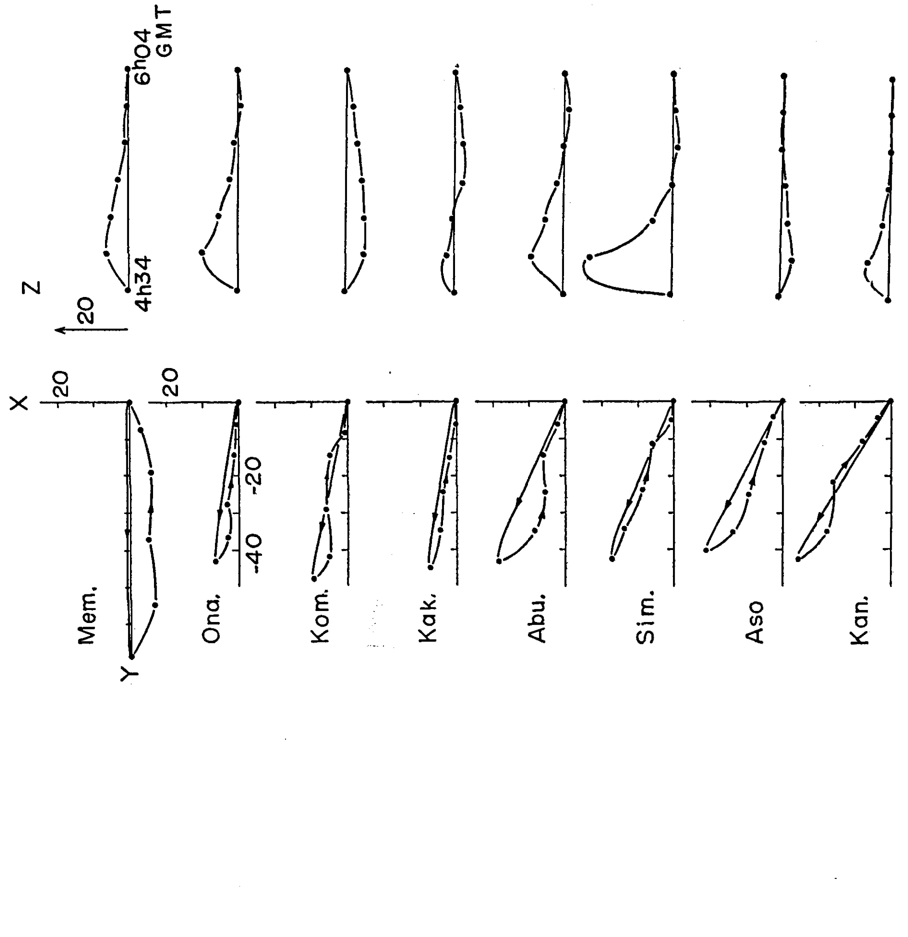

Fig. 4. The vector diagrams in the horizontal plane and the changes in vertical component as observed at the eight Japanese observatories at the time of the *s.f.e.* on Aug. 16, 1958.

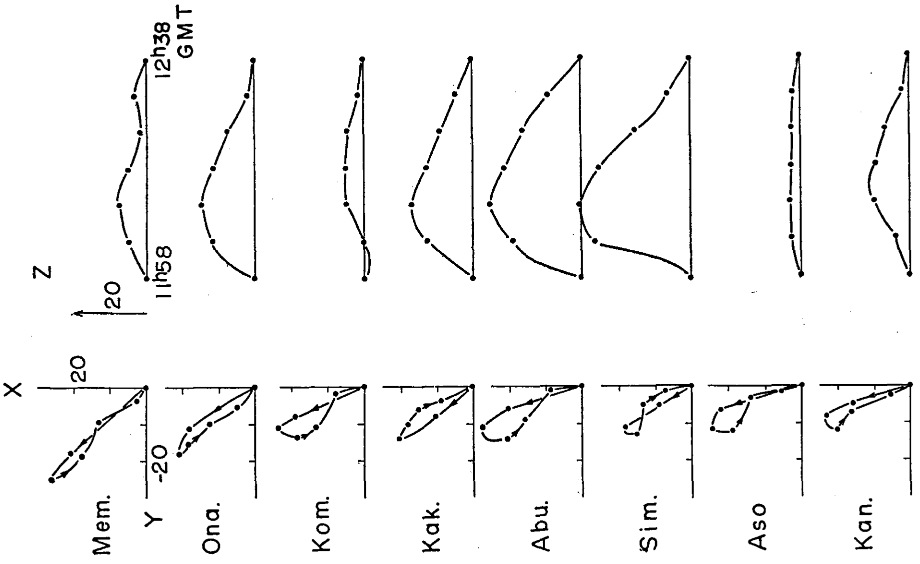

Fig. 5. The vector diagrams in the horizontal plane and the changes in vertical component as observed at the eight Japanese observatories at the time of the bay on April 18, 1958.

of the earth's interior underneath Japan when the direction of inducing field changes.

Since the very large ΔZ which, according to previous studies is originating within the earth, is observed when the inducing field is directed to the north or south, we may naturally think that the field induced by a geomagnetic change in that direction is anomalous. The values of $\Delta Z/\Delta H$ at these observatories have been statistically obtained for an inducing field changing almost to a north-south direction. Making use of the proportional constants hitherto known, we may then estimate approximate ΔZ's which can be expected at the observatories as a result of the north-south change in the magnetic field even in the case of the present *s.f.e.* Let us denote this by ΔZ^* which is estimated at the maximum stage of the *s.f.e.* as given in Table 3. $\Delta Z-\Delta Z^*$ is also shown in Table 3.

Table 3.

Observatory	ΔZ	ΔZ^*	$\Delta Z-\Delta Z^*$
Sakhalinsk	8.0γ	− 0.2γ	8.2γ
Memambetsu	6.5	0.7	5.8
Onagawa	8.6	4.1	4.5
Komoro	− 4.8	1.5	−6.3
Kakioka	0.3	6.3	−6.0
Aburatsubo	7.3	10.5	−5.2
Simosato	14.0	20.6	−6.6
Aso	− 2.8	3.2	−6.0
Kanoya	3.2	12.1	−8.9

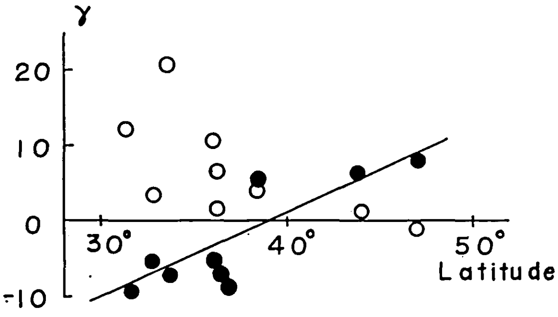

Fig. 6. The changes in ΔZ (hollow circle) and $\Delta Z-\Delta Z^*$ (full circle) with the latitude.

$\Delta Z-\Delta Z^*$ thus obtained may be taken as the approximate ΔZ which should have been observed provided the inducing field had changed only in the east-west direction. The distribution of $\Delta Z-\Delta Z^*$ is fairly regular as can be seen in Fig. 6 in which $\Delta Z-\Delta Z^*$ and ΔZ are plotted taking the latitudes of observatories as the abscissa. In contrast to the large scattering of ΔZ, $\Delta Z-\Delta Z^*$ is distributed fairly smoothly along a straight line. It is not difficult to understand the gradual chage from negative values to positive ones as the latitude increases on the basis of the overhead current-system shown in Fig. 2. Roughly speaking, therefore, it may be said that nothing anomalous takes place in association with induction by a field changing to the east-west direction over Japan. It seems to the writers that the apparent anisotropy discussed above has an important bearing on the underground structure beneath Japan. Any theory that accounts for the anomaly of geomagnetic variations in Japan should also be able to account for the anisotropy.

4. World distribution of the magnetic potential for the *s.f.e.*

In order to make a more detailed study on the *s.f.e.*, it is planned to separate the part of the magnetic field originating from inside the earth from that originating from outside the earth. For that purpose, it is required to obtain the distribution of magnetic potential.

On the basis of ΔX and ΔY given in Tables 1 and 2, the contours of equal variation are first drawn as shown in Figs. 7 and 8. Irregularities near the magnetic equator probably due to the intense electric current right above there are ignored. Making use of readings for ΔX and ΔY at mesh points for meridians and parallel circles of 10-degree interval, magnetic potential ΔW is graphically calculated after the

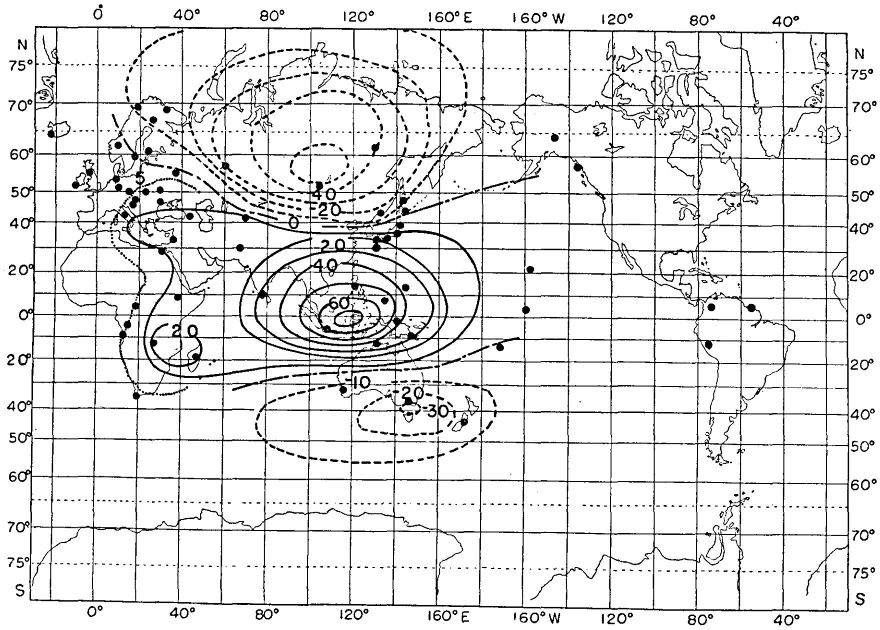

Fig. 7. The world distribution of ΔX in units of gamma.

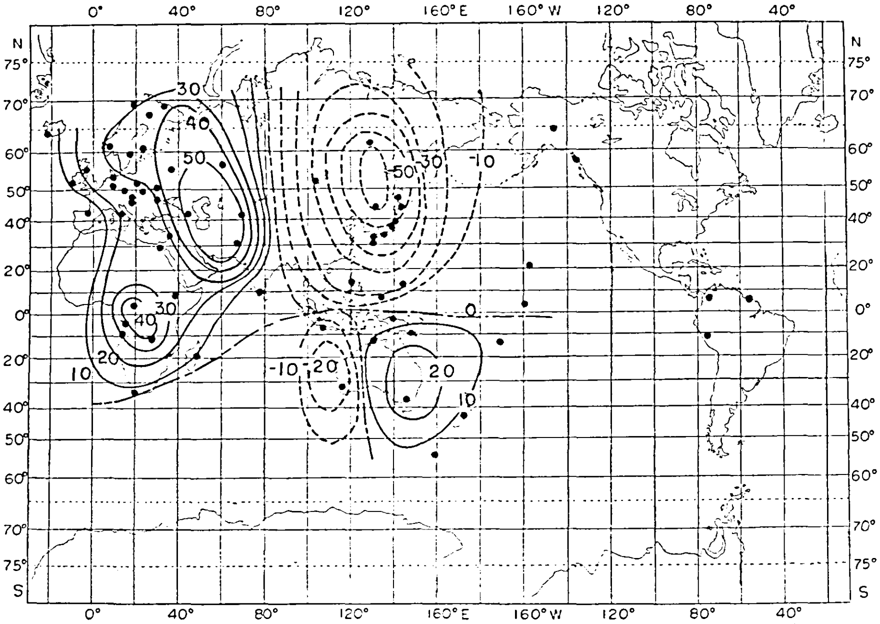

Fig. 8. The world distribution of ΔY in units of gamma.

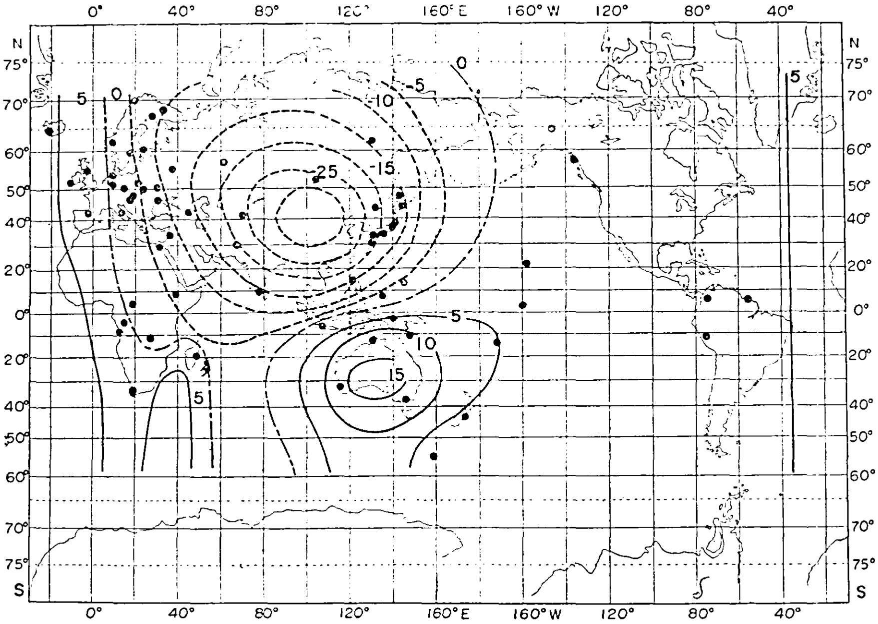

Fig. 9. The world distribution of ΔW in units of gamma divided by the earth's radius.

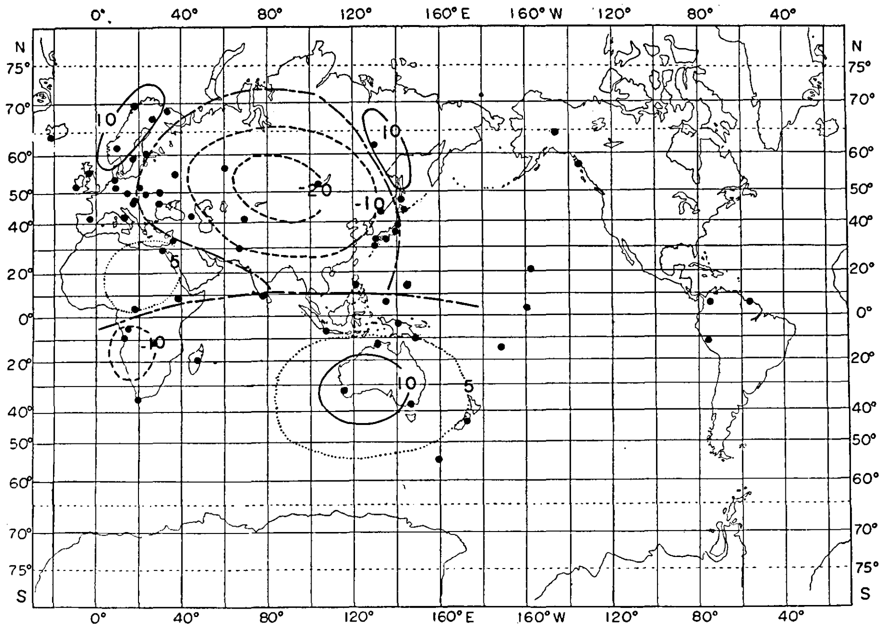

Fig. 10. The world distribution of ΔZ.

method[1)] previously utilised. The distribution of ΔW is illustrated in Fig. 9, the general aspect of ΔW being naturally compatible with the current-system in Fig. 2.

It is also necessary to have the distribution of ΔZ over the earth. Although ΔZ is affected by irregularities more strongly than ΔX or ΔY, it is not impossible to draw a contour map like Fig. 10 which may show the general distribution of ΔZ. It is not intended in Fig. 10 to visualise local anomalous distributions such as those found in Japan. In applying the procedure stated in the following section to an observatory in Japan, the local distribution around the observatory is taken into account, though we may use the distribution shown in Fig. 10 as a basis in estimating the effect arising from a distant part of the earth.

5. Separation of the potential into parts of external and internal origin

Combining ΔW and ΔZ obtained in the last section, it is possible to separate ΔW into parts arising from outside (ΔW_e) and inside (ΔW_i) the earth. For that purpose, a method of surface integral due to E. H. Vestine[10] may be used. No detailed theory of the method is described here.

According to Vestine, $\Delta W_e - \Delta W_i$ at a certain point on the earth (radius: a) is given by

$$\Delta W_e - \Delta W_i = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi/2}(\Delta W + 2a\Delta Z)\cos\psi d\psi d\phi\ , \qquad (1)$$

where the pole of polar coordinate-system $(a,\ \theta,\ \phi)$ is taken at the point at which the separation is required. ψ is written in place of $\theta/2$. If we know the potential apart from a constant U, (1) becomes

$$\Delta W_e - \Delta W_i = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi/2}(\Delta W - U + 2a\Delta Z)\cos\psi d\psi d\phi + U\ . \qquad (2)$$

In order to make separation at many points, it is necessary to perform many numerical integrations. A special globe is prepared by the writers for the purpose. The surface of a globe of 32 cm in diameter,

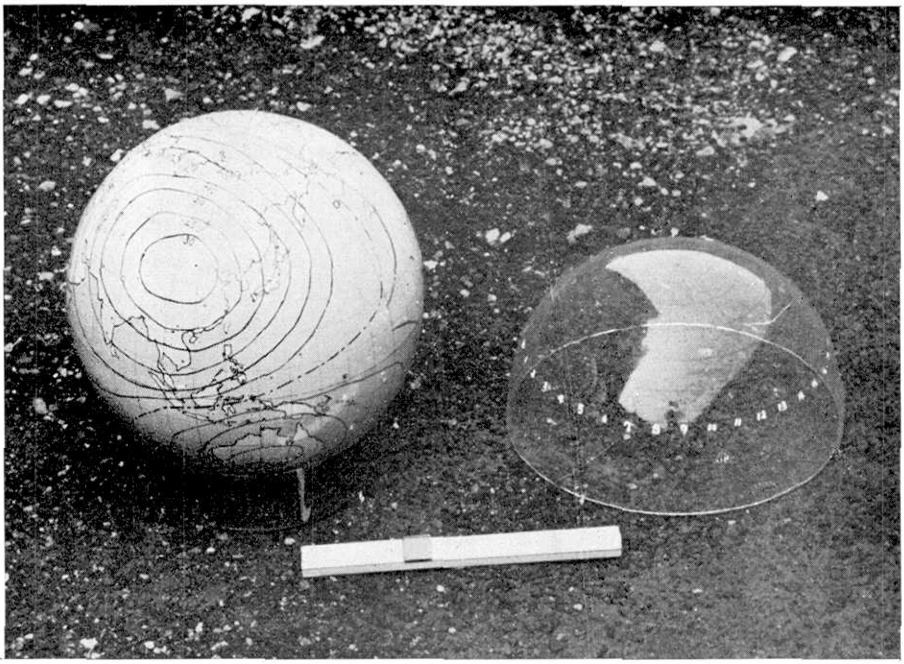

Fig. 11. The globe specially made for separation of magnetic potential and the transparent hemi-spherical sheet.

10) E. H. VESTINE, *Terr. Mag.*, **46** (1941), 27.

Table 4. External and internal parts of the magnetic potential in units of gamma divided by the earth's radius for observatories outside Japan.

No.	Observatory	ΔW_e	ΔW_i	No.	Observatory	ΔW_e	ΔW_i
A 050	Murmansk	—	—	C 063	l'Aquila	− 1.9	−0.2
A 047	Tromsö	−3.7	−1.3	—	Ksara	− 6.1	−1.9
A 101	Reykjavik	—	—	C 251	Quetta	−15.4	−5.7
A 102	Big Delta	1.6	1.4	C 256	Helwan	− 5.0	−2.0
A 065	Sodankylä	− 5.2	−2.6	E 566	Kodaikanal	−10.6	−5.0
A 123	Dombas	− 1.3	−0.4	C 277	Honolulu	1.9	1.1
A 124	Yakutsk	−10.4	−4.6	E 556	Guam	− 3.2	−1.7
A 134	Nurmijärvi	− 6.0	−2.5	E 606	Koror	− 2.5	−1.5
B 009	Lovö	− 3.6	−1.4	E 585	Fanning	2.2	0.8
A 149	Sitka	0.9	2.2	E 578	Fuquene	—	—
B 019	Sverdlovsk	−14.4	−4.6	E 568	Addis Ababa	− 4.4	−1.6
B 038	Eskdalemuir	0.9	1.2	E 575	Paramaribo	—	—
B 035	Moscow	− 9.0	−3.0	E 583	Bangui	− 0.7	0.2
B 058	Wingst	− 1.2	0.2	E 553	Mantinlupa	−10.5	−6.5
C 362	Irkutsk	—	—	E 625	Hollandia	3.1	1.0
B 089	Swider	− 4.3	−0.8	E 631	Binza	− 0.6	1.6
B 098	Valentia	1.5	1.6	E 640	Luanda	− 0.5	2.0
B 106	Göttingen	− 1.2	0.2	E 644	Elisabethville	− 2.0	1.0
B 145	Lvov	− 4.9	−1.2	E 634	Kuyper	0.7	0.3
B 191	Tihany	− 3.4	−0.6	E 646	Huancayo	—	—
B 143	Pruhonice	− 2.5	−0.1	E 642	Port Moresby	6.8	3.2
B 125	Kiev	− 7.0	−2.0	E 653	Apia	3.8	1.2
B 172	Hurbanovo	− 3.4	−0.7	C 897	Darwin	8.1	4.0
C 016	Sakhalinsk	−10.9	−5.7	C 901	Tananarive	0.7	2.4
C 018	Odessa	− 6.2	−1.8	C 933	Gnangara	10.0	4.0
C 051	Wladiwostok	−15.0	−7.1	C 957	Hermanus	0.4	1.6
B 267	Logrone	—	—	B 966	Toolangi	9.6	3.4
C 364	Tiflis	− 9.9	−3.1	B 979	Amberley	3.7	0.3
C 076	Tashkent	−16.6	−6.3	A 961	Macquarie Is.	4.0	0.1

Table 5. External and internal parts of the magnetic potential in units of gamma divided by the earth's radius for Japanese observatories.

No.	Observatory	ΔW_e	ΔW_i	No.	Observatory	ΔW_e	ΔW_i
C 034	Memambetsu	−10.5	−5.5	—	Aburatsubo	−11.3	−6.3
C 117	Onagawa	−11.1	−5.9	C 214	Simosato	−12.5	−7.1
—	Komoro	−11.8	−6.3	C 223	Aso	−14.1	−8.0
C 147	Kakioka	−11.6	−6.5	C 245	Kanoya	−14.2	−7.9

on which meridians, parallel circles, shapes of land and locations of magnetic observatories are drawn, is covered with transparent plastic paint, so that contours can be drawn on the surface with ink. If we wipe the surface with a wet cloth, the drawings are easily erased. Meanwhile a transparent hemi-spherical sheet of 2 *mm* in thickness is made to be placed in close contact with the surface of the globe. Meridians and parallel circles are printed on the hemi-spherical sheet at 10-degree intervals. In Fig. 11 are seen the globe and the sheet.

In actual use the pole of the hemi-spherical sheet is put right on the point where the separation is required. Values of ΔW and ΔZ at each mesh point are then read off. Numerical integrations are performed with these values. Values of ΔW and ΔZ along parallel circles $\theta=10°$, $20°$, $30°, \cdots$, $170°$ are read off at mesh points of 10-degree intervals in longitude. Averaged values $\Delta\overline{W}$ and $\Delta\overline{Z}$ with regard to longitude are then calculated. The numerical integration is then performed by the modified formula of (2) such as

$$\Delta W_e - \Delta W_i = \int_0^{\pi/2} (\Delta\overline{W} - U + 2a\Delta\overline{Z}) \cos\psi d\psi + U . \qquad (3)$$

In order to bring out the local anomaly found in Japan, numerical integration from the pole to parallel circle $\theta=10°$ is performed with $\Delta\overline{W}$ and $\Delta\overline{Z}$ read every 1-degree in longitude.

With $\Delta W_e - \Delta W_i$ thus obtained and $\Delta W_e + \Delta W_i (=\Delta W)$, we can obtain ΔW_e and ΔW_i. It has been shown that we actually obtain $\Delta W_e + U$ instead of ΔW_e, so that, if ΔW is indeterminate by a constant U, we see that ΔW_e is also indeterminate by an amount U. Meanwhile, ΔW_i is free from a constant.

The separation is made at all the observatories. ΔW_e's and ΔW_i's are given in Tables 4 and 5 respectively for foreign and domestic observatories.

6. Large-scale relationship between ΔW_e and ΔW_i

The ΔW_e's and ΔW_i's given in Tables 4 and 5 enable us to provide world equipotential maps as can be seen Fig. 12 for the external part and in Fig. 13 for the internal part. It is interesting to note that $\Delta W_i / \Delta W_e$ amounts to 1/2 or a little less for the parts of the earth's surface where ΔW_e and ΔW_i take fairly large values. The fact supports

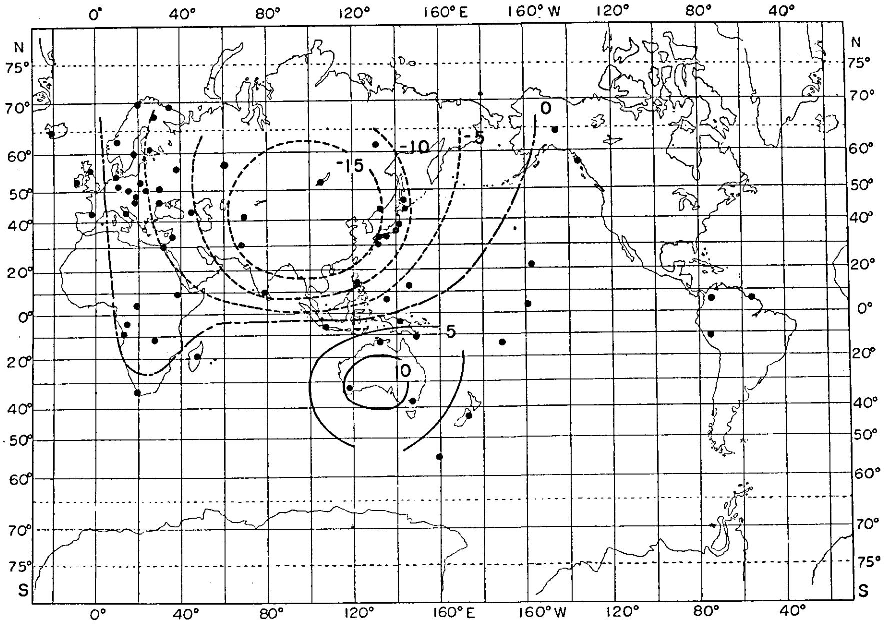

Fig. 12. The world distribution of ΔW_e in units of gamma divided by the earth's radius.

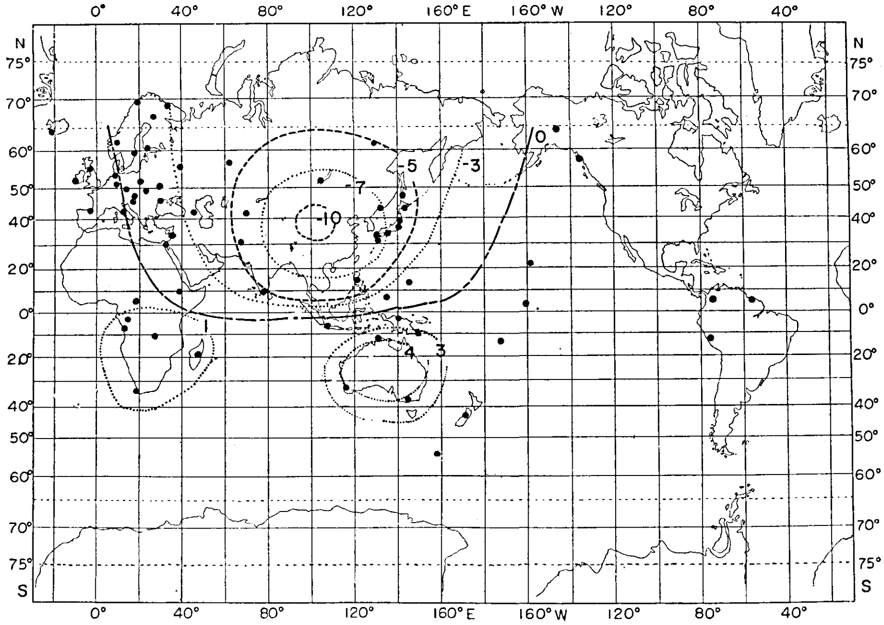

Fig. 13. The world distribution of ΔW_i in units of gamma divided by the earth's radius.

the theory that $\varDelta W_i$ is produced by the electric currents induced by the change in $\varDelta W_e$ within the earth.

It has been customary in studying $\varDelta W_e$ and $\varDelta W_i$ to make spherical harmonic analyses and to discuss the relationship between the coefficients of the external and internal parts of magnetic potential. Looking at Figs. 12 and 13 in which there are practically no variations on the night-time hemi-sphere, it is naturally foreseen that a fairly large number of spherical harmonics are needed for accurate analyses. In order to avoid laborious procedures of spherical harmonic analysis, an attempt for comparison between $\varDelta W_e$ and $\varDelta W_i$ based on quite a different stand-point is made in the following.

The concentric equi-potentials in Figs. 12 and 13 suggest that the distribution of potential could well be approximated if we take a pair of radially polarized magnetic dipoles at points suitably located outside the earth for Fig. 12 and inside the earth for Fig. 13.

Let us suppose a radial dipole placed at a point on the $\theta=0$ axis. The distance between the dipole and the centre of the earth is z_e. As can be seen in Fig. 14, the magnetic potential at the earth's surface due to the dipole is given as

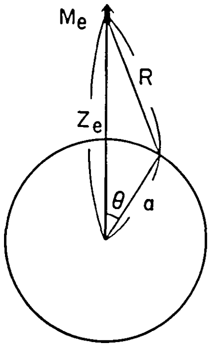

Fig. 14. A magnetic dipole placed outside the earth.

$$w_e = M_e \frac{z_e - a\cos\theta}{R^3}\,, \tag{4}$$

where

$$R^2 = z_e^2 + a^2 - 2az_e\cos\theta\,, \tag{5}$$

and M_e denotes the magnetic moment of the dipole.

Putting

$$\nu = a/z_e\,, \tag{6}$$

(4) can be rewritten as

$$w_e = \frac{M_e\nu^2}{a^2}\frac{1-\nu\cos\theta}{(1+\nu^2-2\nu\cos\theta)^{3/2}}\,, \tag{7}$$

which is further written as

$$w_e/w_{e,\theta=0} = \frac{1-\nu\cos\theta}{(1+\nu^2-2\nu\cos\theta)^{3/2}}\,, \tag{8}$$

where

$$w_{e,\theta=0}=\frac{M_e}{a^2}\frac{\nu^2}{(1-\nu)^{1/2}}\,. \qquad (9)$$

We therefore see that $w_e/w_{e,\theta=0}$ can be expressed as a function of θ taking ν as parameter. In Fig. 15 are shown $W_e/W_{e,\theta=0}$ for $\nu=0.500$ ($z_e=2.0a$) and 0.556 ($z_e=1.8a$).

On the other hand, the distribution of ΔW_e is again drawn on the globe which was used for the separation in the last section. On the assumption that the distribution of ΔW_e is concentric around its centre, the averaged readings along the parallel circles of 10-degree intervals in latitude can be plotted in Fig. 15, ΔW_e being measured by taking its maximum value as the unit. In this case only the distribution for the vortex in the northern hemisphere is dealt with because it is much more intense than the one in the southern hemi-sphere. Since ΔW_e takes a non-zero value, 1.5 *gammas* or so, even at observatories in the dark hemi-sphere, we assume that the potential is not free from a constant, so that the points in Fig. 15 are corrected for this constant part.

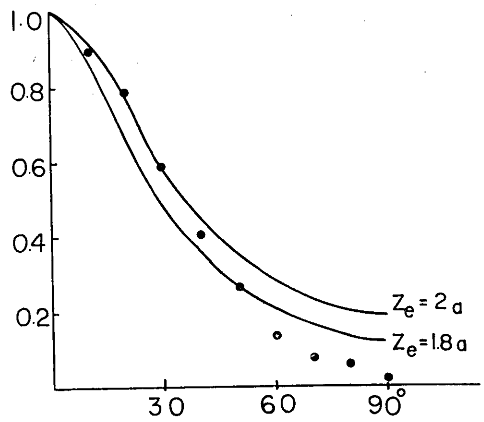

Fig. 15. $w_e/w_{e,\theta=0}$ for $z_e=2.0a$ and $1.8a$. The dots are derived from the distribution of ΔW_e.

Comparing the points for actual ΔW_e with the theoretical curves, we may presume that the curve for $z_e=1.9a$ fits into the actual distribution so long as the distribution is assumed to be presented by that of a dipole. The magnetic moment of the dipole is calculated from (9) to be $M_e=1.0\times10^{23}$ *e. m. u.*

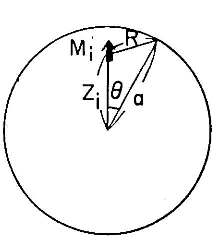

Fig. 16. A magnetic dipole placed inside the earth.

When a dipole is assumed within the earth, as schematically shown in Fig. 16, the potential at the earth's surface can be expressed as

$$w_i/w_{i,\theta=0}=\frac{\nu\cos\theta-\kappa}{(\kappa^2+\nu^2-2\kappa\nu\cos\theta)^{3/2}}\,, \qquad (10)$$

where

$$w_{i,\theta=0}=\frac{M_i}{a^2}\frac{\nu^2}{(\nu-\kappa)^{1/2}}, \tag{11}$$

and

$$\kappa=\frac{z_i}{z_e}, \tag{12}$$

while ν is already defined by (6).

Comparison between the distribution of $\varDelta W_i$ in the northern hemisphere and theoretical curves for $z_i=0.47a$ and $0.43a$ is demonstrated in Fig. 17. Owing to the scattering of the points derived from $\varDelta W_i$, any z_i between the above two values may be chosen. The magnetic moment of the dipole is estimated as 0.10 and 0.087×10^{23} $e.m.u.$ respectively for $z_i=0.47a$ and $0.43a$.

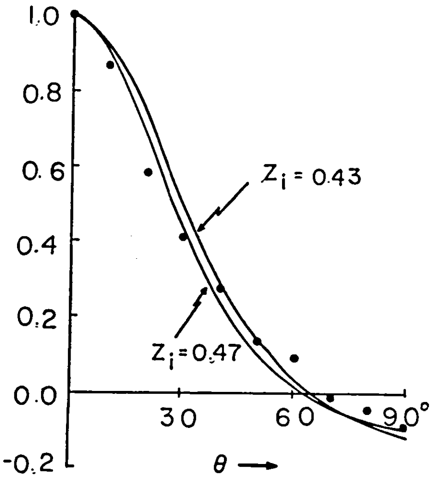

Fig. 17. $w_i/w_{i,\theta=0}$ for $z_i=0.47a$ and $0.43a$. The dots are derived from the distribution of $\varDelta W_i$.

We are now in a position to examine how well the relation between M_e and M_i fits into electrical property in the earth as hitherto known or to determine the electrical conductivity of the earth's interior from the relation if possible. For that purpose, a theory of electromagnetic induction in a spherical conductor by a time-dependent dipole is developed in the following.

The righthand-side of (4) can be expanded as

$$\left.\begin{aligned} w_e&=M_e\frac{1}{r}\sum_{n=0}^{\infty}n\frac{z_e^{n-1}}{r}P_n(\cos\theta) \quad \text{for } r>z_e\,, \\ &=-M_e\frac{1}{z_e^2}\sum_{n=0}^{\infty}(n+1)\left(\frac{r}{z_e}\right)^n P_n(\cos\theta) \quad \text{for } r<z_e\,, \end{aligned}\right\} \tag{13}$$

where r denotes the radial distance from the centre of the earth. $P_n(\cos\theta)$ is the Legendre function of degree n. If we write the potential in a form

$$W_e=a\sum_n e_n\left(\frac{r}{a}\right)^n P_n(\cos\theta), \tag{14}$$

we have

$$e_n = -\frac{M_e}{z_e^3}(n+1)\nu^{n-1} . \tag{15}$$

In the cases of a rapid geomagnetic change like *s. s. c.*, *s. f. e.* and bay, it has been proved[11] that coefficient i_n arising from the electric currents induced in the conducting part of the earth is approximately given as

$$i_n = \frac{n}{n+1} q^{2n+1} e_n , \tag{16}$$

where q denotes the radius of the conducting part measured in units of the earth's radius. The phase difference between e_n and i_n is negligibly small for such a rapid change. In that case the induced potential becomes

$$w_i = a \sum_n i_n \left(\frac{a}{r}\right)^{n+1} P_n(\cos\theta) , \tag{17}$$

which, with the aid of (15) and (16), can be written as

$$w_i = -k^3 M_e \frac{1}{r} \sum_n n \frac{(kqa)^{n-1}}{r^n} P_n(\cos\theta) , \tag{18}$$

where

$$k = qa/z_e = q\nu . \tag{19}$$

We therefore see that the induced potential is represented by that of a dipole of which the magnetic moment is given by

$$M_i = k^3 M_e = (q\nu)^3 M_e , \tag{20}$$

and its position is defined by

$$z_i = kqa = q^2 \nu a . \tag{21}$$

If an inducing dipole is placed at $z_e = 1.9a$ as obtained before, M_i/M_e and q are calculated for $z_i = 0.47a$ and $0.43a$ as given in Table 6.

Table 6.

z_i	M_i/M_e	q
0.47 a	0.087	0.94
0.43 a	0.11	0.90

It is therefore seen that, if we assume $q = 0.90$, M_i/M_e thus calculated approximately agrees with M_i/M_e obtained on the basis of

11) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **28** (1950), 45 and 219.

the observational data. The relationship between ΔW_e and ΔW_i of the present *s.f.e.* may be then accounted for by taking an earth-model of which the upper 600 *km* is non-conducting while the conductivity is fairly high (exceeding 10^{-12} *e.m.u.* or so) in the lower depth. This conclusion is a little different from the electrical state of the earth previously obtained by one of the writers (T.R.) who, by analysing variations of various kinds, has concluded that the earth is on the average nearly non-conducting down to a depth of 400 *km*. Whether or not the difference is significant is not known because the present analysis is made only for one *s.f.e.* If we can repeat the same procedure for many variations, it would be possible to discuss errors of determination and to investigate even the regional difference of the size of the conducting part of the earth.

7. ΔW_e and ΔW_i in and around Japan

On the basis of ΔW_e's and ΔW_i's at the eight Japanese and two Russian observatories, it is possible to examine the distribution of magnetic potential, both external and internal, in and around Japan though more observatories are needed for a detailed study.

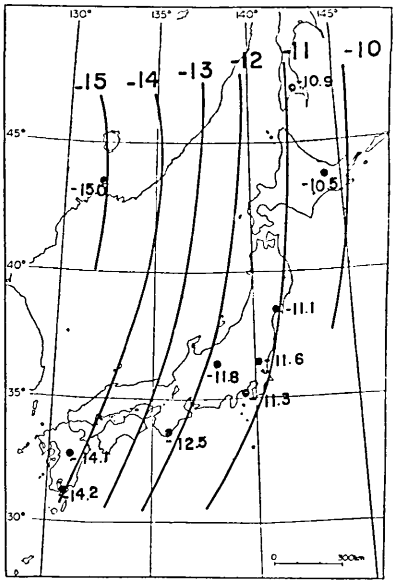

Fig. 18. The distribution of ΔW_e in units of gamma divided by a in and around Japan.

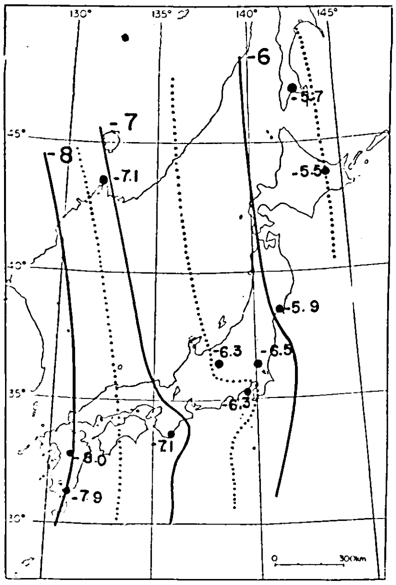

Fig. 19. The distribution of ΔW_i in units of gamma divided by a in and around Japan.

Fig. 18 shows the equipotentials for the external part, the distribution being fairly regular. We may say that there is nothing particular in the distribution of ΔW_e in and around Japan. Meanwhile the equipotentials for the internal part indicate slight irregularities in the central part of Japan as can be seen in Fig. 19.

In one of the previous papers[1], it has been shown that the distribution of the induced potential exhibits an anomalous behaviour in the case of a polar magnetic storm. The present irregularity is of the same character as that for a polar magnetic storm or a bay though the extent of anomaly seems to be a little smaller. In Section 3 it has been emphasised that the anomaly of geomagnetic variation in Japan appears whenever the magnetic field changes in the north-south direction while the anomaly is not observed very clearly in the case of a change in the east-west direction. Since the magnetic vector of the present *s.f.e.* is mainly directed to the west, it is natural that the anomaly is observed less clearly than that for a polar magnetic storm or a bay of which the magnetic field vectors necessarily point to the north-south direction when the anomaly of ΔZ becomes maximum.

The conclusion made in Section 3 that the anomaly in association with the present *s.f.e.* is likely to be caused by the north-south change of the inducing field is also supported by the distribution of ΔW_i shown in Fig. 19.

8. Discussion and conclusion

The detailed analysis of an *s.f.e.* that occured on Aug. 16, 1958 confirms that the relation between the fields arising from outside and inside the earth can be accounted for by the theory of electromagnetic induction. Although the depth of the non-conducting layer is here estimated as some 600 *km* under the Asian Continent, the significance of difference between this value and the hitherto surmised value, 400 *km* say, as the average for the whole earth is not known. Until many more investigations are conducted, it would be difficult to conclude whether or not there are any significant differences in the distribution of the electrical conductivity from region to region in the earth's interior.

It is of interest and importance that the anomaly of geomagnetic variation observed in Japan is mainly caused by the induction by a magnetic field changing in the north-south direction. It is demonstrated in Sections 3 and 7 that the magnetic vector directed to the west has very little to do with the conspicuous anomaly which is usually observed in the vertical component in the cases of bays and polar magnetic storms. It seems to the writers that the fact has an important bearing on the physical interpretaion of the anomaly.

The cause of the anomaly has been speculated by one of the writers (T. R.) to be an underground ring-shaped circuit in which an intense electric current flow whenever we observe a magnetic disturbance, bay, polar magnetic storm or such like. The circuit is supposed to be connected to the conducting part of the mantle. Even a non-conducting wedge penetrating in the deeper part of the mantle has to be assumed to exist between the connecting points, otherwise no intense current which is enough for producing the anomalous magnetic field can be expected. Such a model has been supposed mostly from studies on magnetic variations in the north-south direction. In order to account for the apparent anisotropy brought to light in this paper, the conneting points, where the currents induced in the conducting mantle flow in and come out of the hypothetical circuit, must lie on a line running from east to west. If it is so, no potential difference would appear in the case of a field changing in the east-west direction. On the contrary, the potential difference would become the highest when the field changes in the north-south direction.

The reason why we have such a complicated structure beneath Japan is not clear. It might be possible to suppose that the branch circuit is connected to the conducting mantle at both sides of the wedge-shaped penetration which is the seat of deep-focus earthquakes. But such a view is still a matter of speculation.

In conclusion the writers thank Dr. Veldkamp who furnished his collection of magnetic data at the request of one of the writers. The writers are also indebted to the members of the Japanese observatories who kindly took the trouble of sending them the copies of magnetograms. Part of the present study was made with the financial aid of the Research Grant from the Ministry of Education. The writers are grateful for the aid given by the Ministry.

38. 日本における地磁気短周期変化の異常と地下構造（第10報）

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1958年8月16日の地磁気 *s.f.e.* 変化を解析した．日本における地磁気短周期変化の異常の研究は，従来主として地球磁場の変化が南北方向に卓越する場合について実施されてきたが，今回の *s.f.e.* の場合には，変化ベクトルが西を向いているので，従来の結果にさらに新事実をつけ加えるものと期待された．

日本の8観測所および多数の各国観測所の資料にもとづいて，変化ポテンシャルの分布図をつくり，特製の地球儀を使用して，ポテンシャルを地球外および内に原因を有する部分に分離した．この両部分を比較することにより，日本付近の地磁気変化異常は地球内部に原因することが再確認された．

この異常は，磁場の変化ベクトルが東西を向く場合には，南北を向く場合にくらべてきわめて小さくなる．鉛直分力について，今回の *s.f.e.* の南北成分の影響を経験的法則により除去すると，異常はほとんどなくなつてしまうことがわかつた．したがつて，第一近似として，東西方向の変化のみによつては，異常磁場は発生しないことが結論される．この結果を従来考えられてきたモデルに適用すると，地下100～200 *km* の深さにあると推定されるリング状回路の両端はほぼ東西の線上において，電気伝導度の大きいマントルの部分に連結しているものと推測される．

なお，汎世界的磁気ポテンシャルの分布にもとづいて，アジア大陸下の電気的性質を調べることができたが，その結果は従来地球の平均状態として得られていたものと大きな差はない．