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22. *Possibility of Detecting the Mantle Low-Velocity Layer by Geomagnetic Deep Sounding.*

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Summary

An estimate of electrical conductivity of the upper mantle indicates that the mantle low-velocity layer should have been detected by geomagnetic observation provided the temperature gradient required for the formation of the layer is assumed. The fact that no such layer has been reported from geomagnetic observation suggests that, even if some parts of the layer are highly conducting, they would not extend continuously on a large scale. Such a view is also supported by an estimate of bulk conductivity of a mantle having high-conducting inclusions.

A study is also made on size and depth of conductor embedded in a non-conducting mantle which may possibly be detected by geomagnetic observation. Even for a body of very high conductivity, it is concluded that the detectable depth is unexpectedly small. For example, a perfectly conducting sphere of radius 20 *km* can sensibly be detected when the depth of the top of the sphere amounts to only 14 *km* or less.

1. Introduction

B. Gutenberg[1),2),3),4),5),6),7),8)] made much effort to prove the existence of the "low-velocity layer" in the upper mantle by analysing time-distance curves for P and S waves. P. Caloi[9)] has pointed out special phases of P and S waves which seem to be guided by a low-velocity

1) B. GUTENBERG, *Bull. Seism. Soc. Amer.*, **38** (1948), 121.
2) B. GUTENBERG, *Bull. Seism. Soc. Amer.*, **43** (1953), 223.
3) B. GUTENBERG, *Bull. Geol. Soc. Amer.*, **65** (1954), 337.
4) B. GUTENBERG, *Geofis. pura e appl.*, **28** (1954), 1.
5) B. GUTENBERG, *Geol. Soc. Amer. Special Paper*, **62** (1955), 19.
6) B. GUTENBERG, *Trans. Amer. Geophys. Union*, **39** (1959), 486.
7) B. GUTENBERG, *Ann. di Geofis.*, **12** (1959), 439.
8) B. GUTENBERG, *Physics of the Earth's Interior*, Academic Press (1959).
9) P. CALOI, *Ann. di Geofis.*, **7** (1954), 491.

channel in the mantle at depths of between 50 and 250 *km*. The analyses by E. Vesanen and others[10] also favoured Gutenberg's view. Investigations[11),12),13),14] of dispersion of mantle Rayleigh waves have provided new evidence of the existence of the mantle low-velocity layer. Some more support of the low-velocity layer hypothesis has been found from the studies of G waves[15),16] and such like.

Recently it has been found by N. F. Ness and others[17] that even the oscillation periods of higher harmonic constituents of the earth's free oscillation could be successfully explained by the theory in which suitable account for the mantle low-velocity layer is taken.

Summarizing all the results in the above, it may be said that there is a sufficient number of different methods which support the mantle low-velocity layer hypothesis though the detailed structure of the layer has not yet been made clear. About the cause of the velocity minimum in the upper mantle nothing definite has been known. It is only surmised that the rate of decrease in seismic wave velocities caused by the increasing temperature would be somewhat larger than that of increase due to the pressure increase. P. E. Valle[18] reached, on the basis of solid state physics, a conclusion that the mantle low-velocity layer may be formed provided the temperature gradient exceeds $14°C/km$ for P waves and $11°C/km$ for S waves. G. J. F. MacDonald[19),20] has mentioned that an extrapolation of laboratory data favours a decrease in velocity provided a gradient of $6°C/km$ or thereabout is assumed.

According to seismic studies hitherto made, it seems highly likely that the depth of the mantle low-velocity layer is different for different regions. It might even be possible to think that the layer is not a continuous shell covering the whole earth but includes many portions in which a molten or partially molten state is attained. Such a view could be correlated to deep-seated magma reservoirs which are likely

10) E. VESANEN, M. NURMIA and M. T. PORKKA, *Geophysica*, **7** (1959), 1.
11) H. TAKEUCHI, F. PRESS and N. KOBAYASHI, *Bull. Seism. Soc. Amer.*, **49** (1959), 355.
12) H. TAKEUCHI, *Geophys. Journ.*, **4** (1961), 259.
13) N. JOBERT, *Geophys. Journ.*, **4** (1961), 242.
14) J. DORMAN, M. EWING and J. OLIVER, *Bull. Seism. Soc. Amer.*, **50** (1960), 87.
15) F. PRESS, *Journ. Geophys. Res.*, **64** (1959), 565.
16) M. LANDISMAN and Y. SATÔ, *Trans. Amer. Geophys. Union*, **39** (1958), 522.
17) N. F. NESS, J. C. HARRISON and L. B. SLICHTER, *Journ. Geophys. Res.*, **66** (1961) 621.
18) P. E. VALLE, *Ann. di Geofis.*, **9** (1956), 371.
19) G. J. F. MACDONALD and N. F. NESS, *Journ. Geophys. Res.*, **66** (1961), 1865.
20) G. J. F. MACDONALD, *Science*, **134** (1961), 1663.

to be in exsistence from place to place in the upper part of the mantle.

D. Shimozuru[21] has estimated the volume percentage of such a molten portion (magma pocket as it was called by him) in the mantle. In order to account for the decrease in shear modulus in the mantle low-velocity layer, the volume percentage of the molten portion has been calculated by him to be 11~15 percent.

If the mantle low-velocity layer is a shell-shaped one spread all over the earth, geomagnetic changes should be affected by the layer because the electrical conductivity in the layer is likely to be high. The intensity of magnetic field produced by the electric currents induced in the mantle by magnetic changes originating from outside the earth is largely controlled by the depth of the conducting region as has been studied by S. Chapman[22),23)], A. T. Price[23),24)], B. N. Lahiri[24] and T. Rikitake[25]. It would therefore be possible to check by geomagnetic method whether the mantle low-velocity layer is wide-spread or not. In Section 2, account is taken of this point.

In Section 3, the bulk conductivity of the low-velocity layer, which may be treated as a mixture of a non-conducting phase and a conducting one, is estimated in a way appropriate to an electromagnetic induction problem. Even if perfectly conducting inclusions are distributed at random in a continuons non-conductor, the increase in bulk conductivity would be small. Comparing the bluk conductivity with the apparent conductivity derived from analyses of geomagnetic changes, it may be possible to get some clue as to the volume of high conducting inclusion, that is the molten or partially molten portion of the mantle.

Discussion is also mede in Section 4 about size and depth of local high-conducting mass which might be detected by analysing geomagnetic changes like Sq, bay and so on. Since anomalies of geomagnetic variations have been reported from many observatories in recent years, this kind of study would be useful for getting some idea about the possible effect of mantle non-uniformity, though some of the anomalies[26] seem to be so large as well as complicated that electromagnetic induction in an isolated conductor cannot account for the phenomena.

21) D. SHIMOZURU, *Zisin*, **14** (1961), 227. (in Japanese)
22) S. CHAPMAN, *Phil. Trans. Roy. Soc. London A*, **218** (1919), 1.
23) S. CHAPMAN and A. T. PRICE, *Phil. Trans. Roy. Soc. London A*, **229** (1930), 427.
24) B. N. LAHIRI and A. T. PRICE, *Phil. Trans. Roy. Soc. London A*, **237** (1939), 509.
25) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **28** (1950), 45, 219, 263.
26) T. RIKITAKE, *Geophys. Journ.*, **2** (1959), 276.

2. Electrical conductivity in the upper mantle

It has been experimentally known[27),28),29),30)] that temperature dependence of electrical conductivity of rocks can be expressed as

$$\sigma=\sum_i e^{-A_i/T}\,, \tag{1}$$

where the summation should be made for all the conduction processes. T denotes absolute temperature.

For peridotite, that is thought to compose the upper mantle, the conduction process can be approximately divided into three stages. The conduction process that predominates at relatively lower temperatures is the impurity conduction, while the intrinsic electronic conductivity becomes important for intermediate temperatures. At much higher temperatures, the conduction is mainly ionic.

The influence of pressure on these conduction processes can be approximately estimated on the basis of solid state physics. Rikitake[31)] has applied the theory of ionic crystal to the estimation, while H. Hughes[29)] measured the pressure-effect for a magnesium-rich olivine. Assuming the compressibility of olivine at zero pressure and temperature as 0.8×10^{-12} *c. g. s.*, K. Noritomi[30)] has obtained expressions for the pressure-effect as follows.

For the temperatures between 600 and 1100°C, the conductivity can be expressed as

$$\sigma=\sigma_0{}^*\exp\left(\frac{-E_0}{2kT}K\right), \tag{2}$$

where E_0 and k are the activation energy and Boltzmann's constant, while

$$\sigma_0{}^*=\sigma_0\exp\left(\frac{-E_0}{2kT}\right). \tag{3}$$

According to Noritomi's experiments, it is obtained that

$$\sigma_0=10^{-9}\,e.\,m.\,u. \quad E_0=2.8\,eV\,. \tag{4}$$

27) T. NAGATA, *Bull. Earthq. Res. Inst.*, **15** (1937), 663.
28) H. P. COSTER, *M. N. R. A. S. Geophys. Suppl.*, **5** (1948), 193.
29) H. HUGHES, *Journ. Geophys. Res.*, **60** (1955), 187.
30) K. NORITOMI, *Journ. Min. Coll. Akita Univ. Ser. A*, **1** (1961), 27.
31) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **30** (1952), 13.

K is a quantity derived from a theory of pressure-effect like Rikitake's and is given as

$$K=\{\exp(0.4)\}\{\exp[-0.4\exp(-0.4\times10^{-12}p)]-\exp(0.4)\} \quad (5)$$

in which p is the pressure in units of dyn/cm^2.

Meanwhile, he has also obtained

$$\left.\begin{array}{ll} \sigma=\sigma_0{}^*\exp\left(\frac{-E_0}{kT}K\right), & \sigma_0{}^*=\sigma_0\exp\left(\frac{-E_0}{kT}\right) \\ \sigma_0=10\ e.m.u., & E_0=4.0\ eV \end{array}\right\} \quad (6)$$

for a temperature-range 1100∼1150°C and

$$\left.\begin{array}{ll} \sigma=\sigma_0{}^*\exp\left(\frac{-E_0}{kT}K\right), & \sigma_0{}^*=\sigma_0\exp\left(\frac{-E_0}{kT}\right) \\ \sigma_0=10^{-8}\ e.m.u., & E_0=0.8\ eV \end{array}\right\} \quad (7)$$

for temperatures higher than 1150°C.

Denoting the above conduction mechanisms respectively by A, B and C, the conductivity values expected for various temperatures at a depth of 150 km, where the minimum of seismic velocities is likely to occur, are calculated as shown in Table 1 for each mechanism. The pressure value at that depth is taken as $0.048\times10^{12}\ dyn/cm^2$ after K. E. Bullen[32].

Table 1. Estimated conductivities in $e.m.u.$ for the three conduction mechanisms at the pressure corresponding to 150 km in depth.

Temperature	A	B	C
1200°K	5.2×10^{-16}	9.7×10^{-18}	4.6×10^{-12}
1400	3.9×10^{-15}	3.6×10^{-15}	8.2×10^{-12}
1600	2.1×10^{-14}	3.9×10^{-13}	2.0×10^{-11}
1800	6.2×10^{-14}	9.7×10^{-12}	4.0×10^{-11}

On the basis of the change in Poission's ratio in the upper mantle, Shimozuru[21] estimated the temperature at the depth of 150 km as about 1500°C which is much higher than that obtained by Rikitake[31]. As long as the studies based on solid state physics relevant to seismic wave velocities are concerned[18),19], a temperature not greatly different

32) K. E. BULLEN, *Bull. Seism. Soc. Amer.*, **30** (1940), 246.

from 1500°C seems to be required to cause a remarkable derease in velocity at that depth. If 1500°C is taken for granted at the depth of 150 km, a value of the order of 10^{-13} $e.m.u.$ or larger is reached by any conduction mechanism except A as can be seen in Table 1.

In the case of Rikitake's model[31] of the upper mantle in which an ionic conduction with $\sigma_0=10^{-4}$ $e.m.u.$ and $E_0=2.3\,eV$ has been presumed at zero pressure, a conductivity value of the order of 10^{-11} $e.m.u.$ is obtained at the depth of 150 km provided the temperature is higher than 1200°C.

If the conductivity exceeds 10^{-13} $e.m.u.$ in a layer spread so widely that the whole earth is surrounded by it, the amplitude of the internal part of a geomagnetic change, bay or such like, must be approximately $(n/n+1)q^{2n+1}$ times as large as the amplitude of the external part so long as the duration-time of the change is one hour or shorter, wherein n is the degree of spherical harmonic function by which the distribution of the change is described and q is the ratio of the radius of the upper surface of the layer to the earth's radius. Taking a depth of 150 km, the amplitude ratio is estimated as 0.47 for $n=1$. The analyses of actual data, however, indicate a value 0.42 or so, the difference between the former and the latter being significantly noticeable even if we take the accuracy of the analyses into account. It is therefore concluded that the mantle low-velocity layer hypothesis, which assumes a molten or partly molten layer on a large scale, cannot be acceptable from the geomagnetic point of view. If the layer is composed of peridotite-like material in which inclusions of molten or partly molten states are scattered, the bulk conductivity might be low as has been inferred from analyses of geomagnetic variations. On this point, discussion is made in the following section.

3. Bulk conductivity in the upper mantle with high-conducting inclusions

Let us suppose that a spherical conductor (radius: a) embedded in a non-conducting medium is placed in a uniform magnetic field varying with a time-factor $e^{i\alpha t}$. Dropping the time-factor, the magnetic potential of the field, which is directed to the $\theta=0$ direction of spherical coordinates r, θ and ϕ, can be written as

$$W_e = rH\cos\theta \qquad (8)$$

where H is the intensity of the field, while the origin of the coordinate system is taken at the centre of the sphere.

The magnetic potential of the field which is produced by the electric currents induced in the sphere by the uniform field is easily obtained as[33)]

$$W_i = a^3 r^{-2} I(i\alpha) H \cos\theta \tag{9}$$

where

$$I(i\alpha) = \frac{1}{2}\left(1 - \frac{F_1(ka)}{F_0(ka)}\right), \quad k^2 = 4\pi\sigma i\alpha \tag{10}$$

in which σ denotes the electrical conductivity of the sphere and the magnetic permeability is assumed as unity in electromagetic unit. Functions F_0 and F_1 are defined by

$$F_1(x) = \frac{\sinh x}{x}, \quad F_1(x) = \frac{3}{x^2}\left(\cosh x - \frac{\sinh x}{x}\right). \tag{11}$$

If there are n spheres and if the mutual interaction between these spheres can be neglected, the induced magnetic potential for a large r may be written as

$$W_i = n a^3 r^{-2} I(i\alpha) H \cos\theta\,. \tag{12}$$

Let us then take a sphere of radius b which contains n conducting spheres. If the bulk conductivity of this sphere is denoted by σ', induced magnetic potential W_i' can be obtained as

$$W_i' = b^3 r^{-2} I'(i\alpha) H \cos\theta \tag{13}$$

where

$$I'(i\alpha) = \frac{1}{2}\left(1 - \frac{F_1(k'a)}{F_0(k'a)}\right), \quad k'^2 = 4\pi\sigma' i\alpha\,. \tag{14}$$

Putting $|W_i| = |W_i'|$, we get

$$\mathrm{mod.}\, I'(i\alpha) = s\, \mathrm{mod.}\, I(i\alpha)\,, \tag{15}$$

where s is the ratio of the volume occupied by the inclusions to that of the large sphere as defined by

$$s = na^3/b^3\,. \tag{16}$$

33) S. CHAPMAN and J. BARTELS, *Geomagnetism* Ch. 12 (1940).

If exact account for phase relation between the inducing and induced fields is to be taken, it is not possible to assume that $|W_i|=|W_i'|$. But for a crude estimate of the bulk conductivity aimed at in this section, such a simple theory would work to some extent. It has also been customary to estimate the apparent conductivity of a medium with inclusions of different conductivity by applying a uniform electric field from outside the system. The writer thinks, however, that such an estimate would not lead to an accurate result for an electromagnetic induction problem considered here.

Assuming that the mean radius of high-conducting inclusion is 20 *km*, for example, the bulk conductivity of a sphere of 200 *km* in radius is calculated for various periods of geomagnetic change as can be seen in Table 2. It is also assumed that s is 0.1 as has been suggested by Shimozuru[21] for the explanation of the mantle low-velocity layer.

Table 2. Bulk conductivities for geomagnetic changes of various periods.

σ \ Period	1 *min.*	1 *hour*	6 *hours*	24 *hours*
10^{-12} *e. m. u.*	5.3×10^{-16}	9.7×10^{-16}	9.9×10^{-16}	1.6×10^{-15}
10^{-10}	2.6×10^{-15}	6.0×10^{-14}	9.5×10^{-14}	1.5×10^{-13}
∞	3.0×10^{-15}	1.8×10^{-13}	1.1×10^{-12}	4.3×10^{-12}

We therefore see that, on condition that conductivity of inclusions amounts to $10^{-11}\sim10^{-10}$ *e. m. u.*, the calculated bulk conductivity is too small to be detected by a geomagnetic analysis. Although the bulk conductivity seems to increase as the period increases, the induced magnetic field becomes markedly small for a large-period change, so that geomagnetic changes are hardly affected in this case.

A simple estimate of bulk conductivity in the upper mantle, 10 percent of which is composed of high-conducting inclusions, reveals the impossibility of finding geomagnetic effect due to the mantle low-velocity layer. In Section 2 it is concluded that the layer would not be a wide-spread one composed of high-conducting material. Even if the layer is formed as a result of local high-temperature inclusions, it is concluded in this section that it would affect geomagnetic changes very little.

4. Size and depth of a local high-conducting mass possibly detected by geomagnetic method

It is intended in this section to examine the size of a conductor, embedded in an insulating mantle, which can possibly be detected by a geomagnetic change. In the first place, electromagnetic induction in a sphere (radius: a) having various values of conductivity (σ) by a uniform time-varying magnetic field applied from outside is studied. The induced magetic moments for a number of combinations of coductivity and period have been calculated by Rikitake[34] as reproduced in Table 3.

Table 3. The magnetic moment in electromagnetic unit induced by a uniform magnetic field of unit strength.

Table 3-1. $a=2\,km$

Period	σ(*e. m. u.*)		
	10^{-12}	10^{-10}	∞
1 *min.*	1.4×10^{13}	1.4×10^{15}	4.0×10^{15}
1 *hour*	2.3×10^{11}	2.3×10^{13}	〃
2	1.2×10^{11}	1.2×10^{13}	〃
6	3.8×10^{10}	3.8×10^{12}	〃
12	1.9×10^{10}	1.9×10^{12}	〃
24	9.6×10^{9}	9.6×10^{11}	〃

Table 3-2. $a=20\,km$

Period	σ(*e. m. u.*)		
	10^{-12}	10^{-10}	∞
1 *min.*	1.4×10^{18}	3.6×10^{18}	4.0×10^{18}
1 *hour*	2.3×10^{16}	1.7×10^{18}	〃
2	1.2×10^{16}	1.1×10^{18}	〃
6	3.8×10^{15}	3.7×10^{17}	〃
12	1.9×10^{15}	1.5×10^{17}	〃
24	9.6×10^{14}	1.5×10^{17}	〃

Table 3-3. $a=100\,km$

Period	σ(*e. m. u.*)		
	10^{-12}	10^{-10}	∞
1 *min.*	4.2×10^{20}	4.9×10^{20}	5.0×10^{20}
1 *hour*	7.4×10^{19}	4.3×10^{20}	〃
2	3.9×10^{19}	4.1×10^{20}	〃
6	1.8×10^{19}	3.5×10^{20}	〃
12	5.0×10^{18}	3.0×10^{20}	〃
23	3.2×10^{18}	2.3×10^{20}	〃

Table 3-4. $a=200\,km$

Period	σ(*e. m. u.*)		
	10^{-12}	10^{-10}	∞
1 *min.*	3.6×10^{21}	4.0×10^{21}	4.0×10^{21}
1 *hour*	1.7×10^{21}	3.7×10^{21}	〃
2	1.1×10^{21}	3.6×10^{21}	〃
6	3.7×10^{20}	3.4×10^{21}	〃
12	1.5×10^{20}	3.1×10^{21}	〃
24	1.5×10^{20}	2.8×10^{21}	〃

Supposing that a sphere is buried in an earth which is regarded as a non-conductor and that an external field is applied perpendicularly to the ground surface, depths from the ground surface to the top of the sphere, the electric currents induced in which produce a field amounting to one tenth of the inducing one at the ground surface, are

34) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **37** (1959), 545.

estimated for various periods. The results are shown in Figs. 1, 2, 3 and 4 respectively for $a=2$, 20, 100 and 200 *km*.

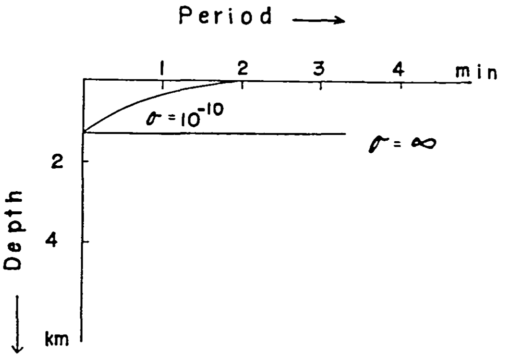

Fig. 1. Curves showing periods and depths of the top of the conducting sphere which give an induced field amounting to 10 percent of the inducing one at the ground surface. $a=2\,km$ is assumed.

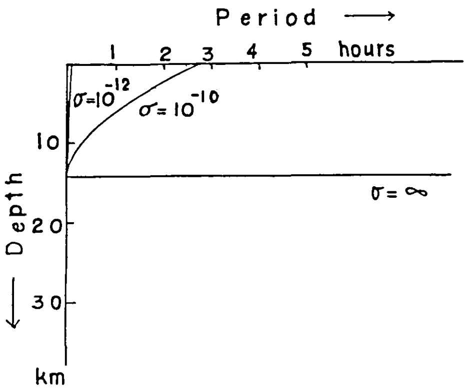

Fig. 2. $a=20\,km$.

Looking at these figures, we see that the depth of a conductor that can be detected by geomagnetic observation is not very large.

Supposing a sphere of $2\,km$ in radius, for instance, an intensity of the induced field larger than the 10 percent of the inducing one can be expected only for depths of the top of sphere shallower than $1.3\,km$ even for the perfectly conducting case.

If the conductivity is taken as 10^{-10} *e. m. u.*, a likely value for molten lava, no changes having periods larger than 2 *minutes* can give rise to the 10 percent field intensity. For a 1 *minute* period change, the detectable depth becomes as small as $0.4\,km$ or so. According to Rikitake[35], who repeated magnetic surveys over Volcano Mihara at the time of the 1950 eruption, the temperature in a volume having a radius of $2\,km$ seemed likely to exceed the Curie-point, the top surface of the volume being at the depth of $3\,km$ from the surface. From what we have been dealing with in the above, it is impossible to detect the hot mass of such size and depth by observing anomalous changes in the geomagnetic field even though there is good reason to believe that a high conductivity exists in the interior of the volcano. It should also be pointed out that the 10 percent anomaly is not very large in actual observation of geomagnetic

35) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **29** (1951), 161.

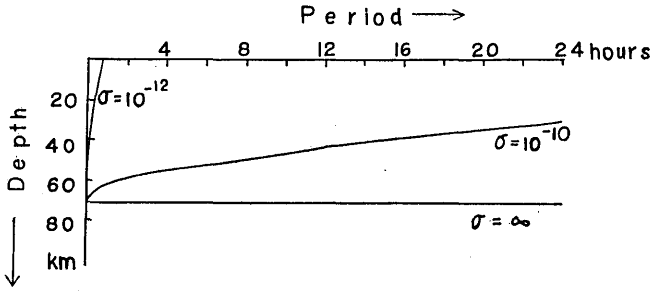

Fig. 3. $a=100\ km$.

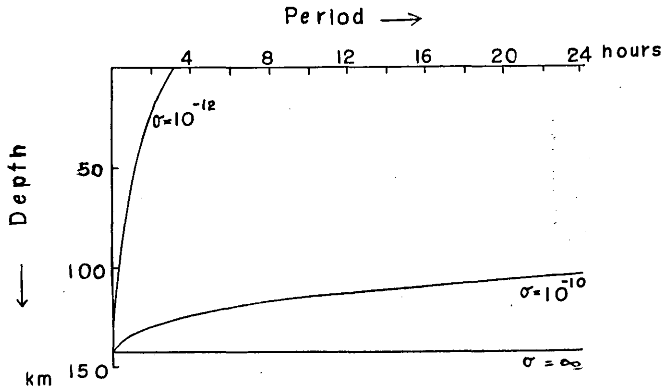

Fig. 4. $a=200\ km$.

variation, so that the depth that causes an anomalous field of noticeable extent would become much smaller. Turning to much larger conductors, the conclusion that the detectable depth is not very large holds good again as can be seen in Figs. 2, 3 and 4. We therefore see that there are no geomagnetic means to detect individually a high-conducting inclusion at the level of the mantle low-velocity layer.

Next, the writer would here like to estimate the influence of a flat conductor on geomagnetic changes in contrast to spherical ones in the above. For that purpose, electromagnetic induction in a thin circular disk placed in a uniform magnetic field is here considered. A. A. Ashour[36], who solved the problem by making use of integral equation, has shown that the induced current intensity is given by

$$J\left(\frac{r}{a}\right)=\lambda H\left(\frac{r}{a}\right)^{-1/2}\sum_{n=0}^{\infty}\phi_n\left(\frac{r}{a}\right)\lambda^n\,, \tag{17}$$

36) A. A. Ashour, *Quart. Journ. Mech. Applied Math.*, **3** (1950), 119.

where H, r and a denote the external magnetic field, radial distance in the disk plane from the centre of the disk and the radius of the disk. λ is defined by

$$\lambda=-i\frac{\pi a\sigma D}{T} \tag{18}$$

in which σ, D and T represent the conductivity, thickness of the disk and the period of magnetic change. Ashour has calculated $x^{-1/2}\phi_n(x)$ for various combinations of n and x, so that his result which has been given in the form of a table can be utilized in the present calculation. Since the current distribution, that is necessarily circular, is obtained in this way, the magnetic field produced by the currents directly above the disk can be easily calculated by integration.

It is difficult, however, to apply Ashour's method to a highly conducting disk because the series in (17) does not converge. The electromagnetic induction in a perfectly conducting disk may be solved by considering an analogy between electromagnetism and ideal fluid. In the latter case, we define a velocity potential ϕ that should satisfy

$$\nabla^2\phi=0 \tag{19}$$

in the fluid, while a boundary condition

$$\frac{\partial\phi}{\partial n}=0 \tag{20}$$

must be fulfilled on the surface of a body placed in the flow, n being a normal to the surface. Turning now to electromagnetic induction, a mangetic potential W is governed by

$$\nabla^2 W=0 \tag{21}$$

outside conductors. At the surface of a perfect conductor, a condition

$$\frac{\partial W}{\partial n}=0 \tag{22}$$

is obviously satisfied. It is thus apparent that, if we could solve a fluid motion problem, we could obtain the solution of the induction problem replacing ϕ by W.

H. Lamb[37] discussed motion of a disk moving with a constant velocity in an infinite mass of liquid as a limiting case of motion of a planetary ellipsoid. Making use of the analogy between liquid motion and electromagnetic induction as discussed in the above, Lamb's result can

37) H. LAMB, *Hydrodynamics*, 6th ed. Cambridge (1932), 144.

be applied to the electromagnetic induction within a perfectly conducting disk by a uniform magnetic field perpendicular to the disk after a little modification. The component perpendicular to the disk plane of the induced magnetic field on an axis passing perpendicularly to the centre of the disk becomes

$$Z=-\frac{2H}{\pi}\left[\cot^{-1}\frac{z}{a}-\left(\frac{z}{a}\right)\Big/\left\{1+\left(\frac{z}{a}\right)^2\right\}\right] \tag{23}$$

at a point on the axis, the distance from the centre being z, while H is the intensity of the external magnetic field as berore.

On the basis of the theory described in the above, the effect of a conducting disk lying parallel to the earth's surface in a non-conducting mantle is estimated. Assuming $a=100\,km$ and $D=10\,km$, the depth of the disk that gives rise to the 10 percent field intensity of the inducing one is also calculated for a nubmer of combinations of conductivity and period. The result is shown in Fig. 5 in which the difficulty of detecting

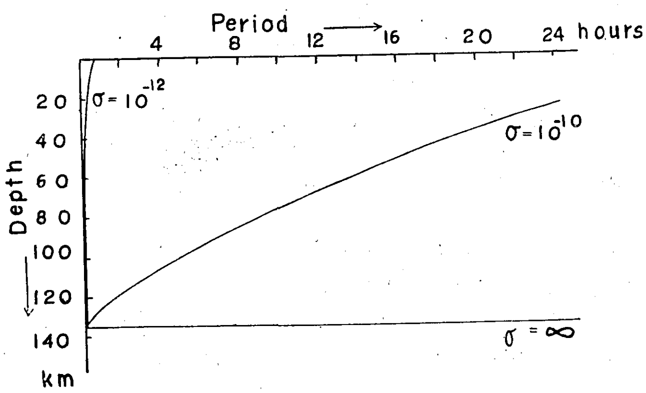

Fig. 5. Curves for a circular disk showing periods and depths of the conductor which give an induced field amounting to 10 percent of the inducing field at the ground surface. The radius and the thickness of the disk are assumed as 100 *km* and 10 *km* respectively.

a deep-seated conductor is well demonstrated. Even if we assume a perfect conductor, it would not be possible to find it when the depth exceeds 130 *km* or so. Taking into account the accuracy of geomagnetic observation, the limiting depth would become much shallower. What we have been dealing with in this section suggests that it is no easy matter to detect conductors in the earth's mantle.

Meanwhile, actual observation tells us that something very anomalous must be existing in the mantle at some parts of the earth, *i.e.* in Japan where the induced field is larger than the inducing field. Such an anomaly does not seem to be accounted for by simple examples discussed in this section, so that quite an unusual configuration of passage of electric currents induced in the deeper part of the mantle must be supposed for the explanation of the anomaly.[26),34)]

5. Concluding remarks

A simple theory of solid state physics leads to a conclusion that a temperature gradient required for the formation of the mantle low-velocity layer would have to result in a high electrical conductivity. If the layer is extensive such that it covers large parts of the earth, it should be detected by geomagnetic observation. Since no such effect has been reported from geomagnetic observation so far, it is strongly suggested that the layer contains inclusions of high conductivity, so that the bulk conductivity is not as large as that which can be detected by geomagnetic observation. The suggestion is supported in Section 3 where a simple theory of bulk conductivity is described in a form appropriate to the electromagnetic induction problem. It is therefore seen that the seismological view that the mantle low-velocity layer is likely to be composed of low-elasticity material mixed as inclusions in a high-elasticity medium is also supported from the standpoint of geomagnetism.

An estimate is made in Section 4 for size and depth of conducting mass which is possibly to be detected by geomagnetic observation. Generally speaking, it is unexpectedly difficult to find such a mass even for a very high conductivity. Although some of the irregularities in the distribution of transient geomagnetic field may be ascribed to the field induced in such conductors embedded in the upper mantle, extremely large anomalies such as are found in Japan cannot be accounted for by assuming such simple configuration of conducting material as considered in this paper.

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22. マントル低速層と地磁気変化

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深さ 100～200 *km* にあるといわれる低速層の存在は，最近のマントル表面波や地球振動の研究結

果からみても，ほぼ確実なようである．この低速層を形成するために必要なマントル上部の温度勾配の見つもりは，1500°C 程度の高温が低速層に存在することを示唆している．マントルを構成する物質の電気伝導度を低速層の温度圧力に対して見つもるときは，10^{-11}～10^{-10} $e.m.u.$ という値を与える．

このように大きな電気伝導度をもつ層が，広範囲にわたつて存在しているとすれば，外部磁場の変動によつてマントル内に誘導される電流のつくる内部磁場は相当の影響をうけるはずである．この影響を定量的に見つもり，従来からの地磁気観測において，そのような影響が見出されていないことから，低速層は地球全面にわたつて高電導性の層が存在していることに対応してはいないことが結論された．

低速層の構成に関し，低弾性の部分が高弾性の媒質中に点在しているとする考え方があるが，高電導性の部分が絶縁性の媒質に点在している場合の見かけの電気伝導度を電磁感応理論により求めた．両部分の体積比を，弾性波速度減少から示唆されているように 10% とすれば，電気伝導度が無限に大きい場合にも，見かけの電気伝導度は小さくなつて，地磁気変化の観測にきいてこないことになる．したがつて，低速層が溶融状態の inclusion を 10% 程度含んでいるという考え方は，地磁気変化の観測結果に矛盾しない．

つぎに球および円板の導体が地下にある場合，地磁気変動の際に発生する異常磁場を調べた．各種電気伝導度，大きさおよび周期に対し，外部磁場の 1/10 に達する内部磁場が発生する条件を求めた．その結果，地磁気変動の際の異常磁場によつて地下の導体を検出することは，比較的浅い導体についてだけ可能であることがわかつた．例えば，半径 2 km の球については，完全導体の場合でさえも，球の頂上の深さが 1.4 km より深くなれば，外部磁場の 1/10 以下の内部磁場しかあらわれない．したがつて，火山体下にあるマグマ溜をこの方法によつて検出することは容易ではないことがわかつた．