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3. *On the Spectrum of Strong Earthquake Motions.*

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1. Introduction.

Investigations concerning differences in earthquake motions in different types of ground were instigated near the end of the nineteenth century, shortly after the invention of the seismograph.

The seismic characteristics of the ground have been made clearer year after year from analytical studies of the records of both small as well as moderate sized earthquakes, theoretical studies of elastic or visco-elastic waves in the layered bodies and statistical studies of the damage to buildings caused by the many past earthquakes.

In the previous investigation, it was found that the maximum accelerations of strong earthquake motions which have been computed by the empirical formula using the epicentral distance, the magnitude and the predominant period of the ground agree with the maximum recorded accelerations of the respective strong earthquake motions which have been obtained in the U.S.A[1].

In the present paper, the relation between the acceleration spectrum of the strong earthquake motions observed in the U.S.A. and the ground vibration characteristics of the seismograph stations will be investigated.

2. The frequency-period relation of the strong earthquake motions observed in the U.S.A.

The frequency-period curves of the strong earthquake motions obtained at sixteen places in California; two places in Washington; two places in Montana; one place in Nevada and at one place in Costa Rica together with the frequency-period curves of microtremors observed at the same places, are shown in Figs. 12–33. In these figures, the

1) K. Kanai, "An Empirical Formula for the Spectrum of Strong Earthquake Motions", *Bull. Earthq. Res. Inst.*, **39** (1961), 85-95.

values next to the dates represent the maximum accelerations.

It will be seen from Figs. 12–33 that the similarity in the shapes of the whole curves of earthquake motions of various intensities, as well as the curve of microtremors at each spot is considerably good.

When the ground has no abrupt change in elasticity, in other words, when the discontinuity of elasticity exists only between the ground and the bed rock, the frequency-period curves of earthquake motions as well as of microtremors on the ground surface, take only one peak. On the other hand, when the ground has one or more abrupt changes in elasticity, the frequency-period curves mentioned above take many peaks and there is a little disagreement between each peak period of the earthquake motions of various intensities and of the microtremors, since the elastic wave reflections at the various boundaries interfere with one another[2].

3. The numerical calculations of the multiple reflection problem in the actual ground.

The present section deals with the case where the primary transversal waves transmit upwards normally to the doubly stratified layers residing on a semi-infinite body.

Let ρ, μ, v and H be the density, the rigidity, the velocity and the thickness of the layers, respectively, and u be the displacement. Suffixes 1, 2 and 3 represent the first layer, the second layer and the bottom medium, respectively.

In order to make the numerical calculation easier, the viscosities of every media will be overlooked. That is, to put the conditions $\xi_1 p/\mu_1 = \xi_2 p/\mu_2 = \xi_3 p/\mu_3 = 0$ into equations (11) and (12) given in the previous paper[3]. Then we obtain the expression of the displacement at the free surface of the ground as follows:

$$\frac{|u_s|}{|u_o|} = \frac{2}{\sqrt{\Phi_1^2 + \Phi_2^2}}, \tag{1}$$

where

$$\left.\begin{aligned} \Phi_1 &= (1+\alpha)\cos(P_2+P_1) + (1-\alpha)\cos(P_2-P_1), \\ \Phi_2 &= (\beta+\gamma)\sin(P_2+P_1) - (\beta-\gamma)\sin(P_2-P_1), \end{aligned}\right\} \tag{2}$$

2) K. Kanai, "The Requisite Conditions for the Predominant Vibration of Ground", *Bull. Earthq. Res. Inst.*, **35** (1957), 457-471.

3) K. Kanai, "Relation between the Nature of the Surface Layer and the Amplitudes of Earthquake Motions", *Bull. Earthq. Res. Inst.*, **30** (1952), 33.

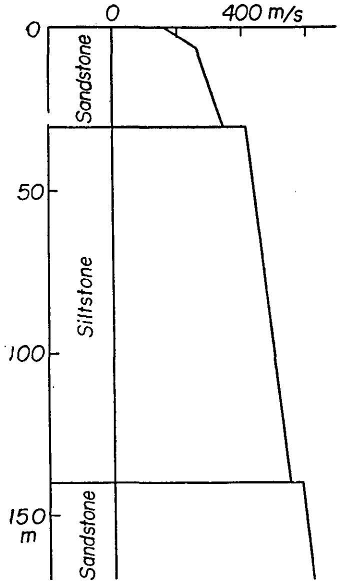

Fig. 1. Velocity distributions of transversal waves to the depth and geological formation at Eureka, California, U.S.A.

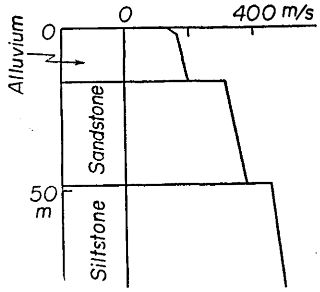

Fig. 2. Velocity distributions of transversal waves to the depth and geological formation at Ferndale, California, U.S.A.

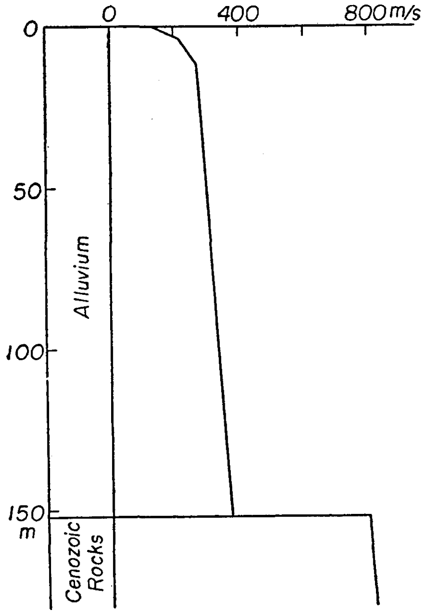

Fig. 3. Velocity distributions of transversal waves to the depth and geological formation at Hollister, California, U.S.A.

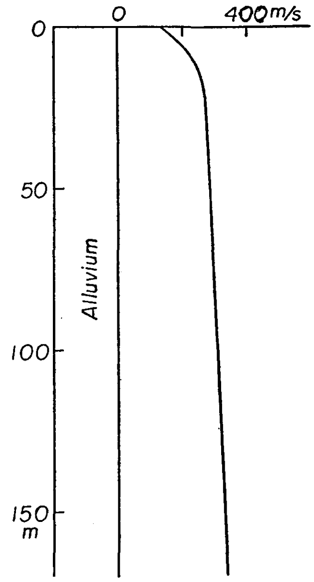

Fig. 4. Velocity distributions of transversal waves to the depth and geological formation at El Centro, California, U.S.A.

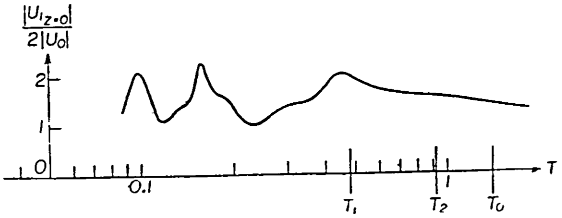

Fig. 5. Calculated displacement spectrum by using the data of Eureka as shown in Fig. 1.

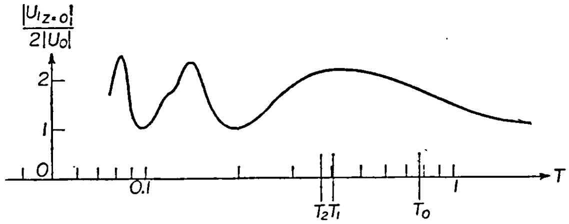

Fig. 6. Calculated displacement spectrum by using the data of Ferndale as shown in Fig. 2.

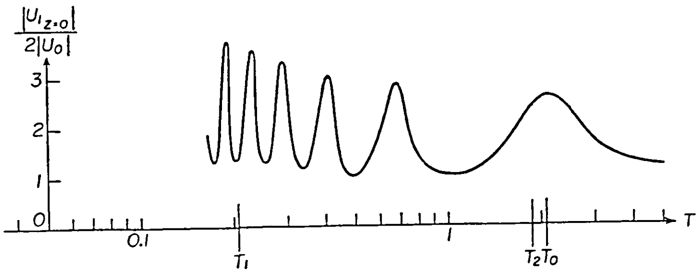

Fig. 7. Calculated displacement spectrum by using the data of Hollister as shown in Fig. 3.

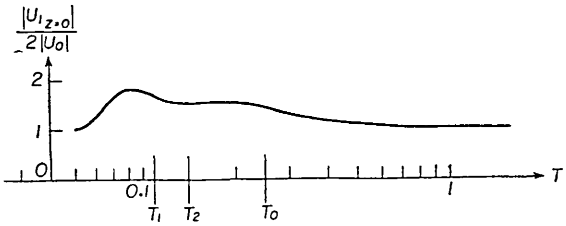

Fig. 8. Calculated displacement spectrum by using the data of El Centro as shown in Fig. 4.

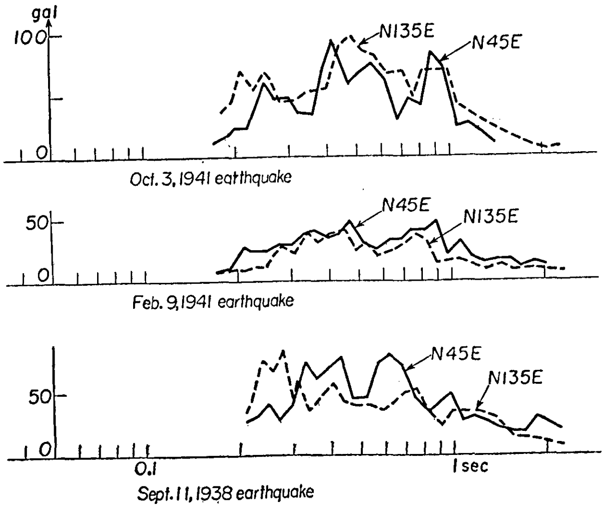

Fig. 9. Acceleration spectrum of the strong earthquake motions observed at Ferndale, California, U.S.A.

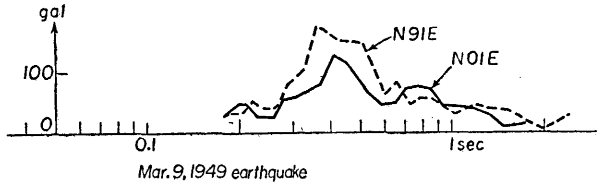

Fig. 10. Acceleration spectrum of the strong earthquake motions observed at Hollister, California, U.S.A.

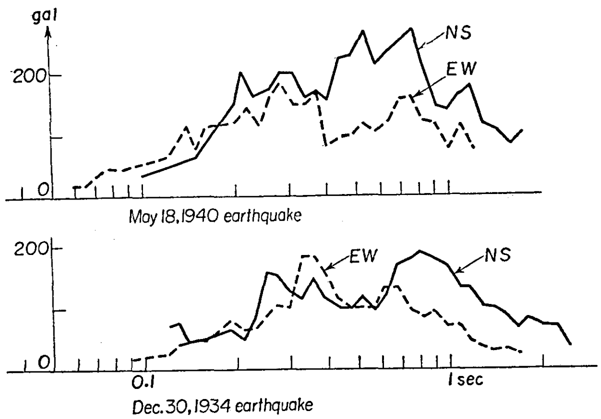

Fig. 11. Acceleration spectrum of the strong earthquake motions observed at El Centro, California, U.S.A.

$$\left.\begin{aligned} &P_1=\frac{pH_1}{v_1}, \quad P_2=\frac{pH_2}{v_2}, \\ &\alpha=\frac{\rho_1 v_1}{\rho_2 v_2}, \quad \beta=\frac{\rho_1 v_1}{\rho_3 v_3}, \quad \gamma=\frac{\rho_2 v_2}{\rho_3 v_3}, \end{aligned}\right\} \qquad (3)$$

in which u_s and u_0 represent the displacement amplitude at ground surface and the amplitude of the incident waves in the lowest medium, respectively.

The velocity distributions of transversal waves to the depth, as well as the geological formation in the four strong-motion seismograph stations, that is, Hollister, Eureka, Ferndale and El Centro, California in the U.S.A. are shown in Figs. 1–4[4].

Substituting the data of Figs. 1–4 in equation (1), on an assumption of $\rho_1=\rho_2=\rho_3$, we obtained the displacement of waves at the ground surface as shown in Figs. 5–8. In Figs. 5–8, T_1, T_2 and T_0 represent the natural periods of the first layer, the second layer and the layer between the free surface and the bottom of the second layer, respectively, that is, $T_1=4H_1/v_1$, $T_2=4H_2/v_2$ and $T_0=4H_1/v_1+4H_2/v_2$.

It can be seen from these figures that the amplitude reaches its maximum at about T_1, while at about T_2 and T_0 amplitudes do not always sustain maximum.

The acceleration spectra of the strong earthquake motions observed at Hollister, Ferndale and El Centro, California in the U.S.A. are shown in Figs. 9–11.

In comparing the observational results shown in Figs. 9–11 with the calculated results shown in Figs. 5–8, it is found that the essential shape of the calculated displacement spectrum at one place is similar to that of the acceleration spectrum of the actual earthquake motions at the same place.

Next, the reason why the former, as mentioned above, is similar to the latter, will be discussed.

The expression of the displacement spectrum at ground surface can be rewritten from (1) as follows:

$$u_s=aG(T), \qquad (4)$$

in which a is a constant.

From the previous paper[5] and others, it can be assumed that

4) J. H. Wiggins, "The Effect of Soft Surface Layering on Earthquake Intensity", *Civil Engg. Studies, Struct. Res.* Series No. 216, Univ. of Illinois, May 1961, pp. 122 and 123.

5) K. Kanai, *loc. cit.*, 1).

seismic waves, excepting considerably short and long periods in the bed rock, satisfy the nature of energy equipartition, that is

$$a=bT\,, \tag{5}$$

in which b is also a constant.

On the other hand, from the theoretical studies[6] of visco-elastic waves in layered ground, the vibration characteristics of the ground may be assumed roughly as

$$cT^{n}G(T) \tag{6}$$

instead of $G(T)$ in equation (4), in which c and n are constants. From (4), (5) and (6), a semi-empirical formula of the displacement spectrum of seismic waves at ground surface may be written as follows:

$$\bar{u}_s=bcT^{1+n}G(T)\,. \tag{7}$$

Acceleration spectrum may be calculated easily from (7), on the assumption of a simple harmonic motion in each period of waves, that is

$$\bar{\alpha}_s=kT^{n-1}G(T)\,, \tag{8}$$

in which k is also a constant. And, from the result of the previous papers, it can be assumed that n takes about 1, then

$$\bar{\alpha}_s=kG(T)\,. \tag{9}$$

Consequently, from (4) and (9), it may be said roughly that the shape of the displacement spectrum of earthquake motions at ground surface, calculated from the assumption that incident waves of unit amplitude transmit into layered ground of purely elastic media and perform multiple reflection phenomena in the ground, is similar to the shape of the acceleration spectrum which is expectant in actual earthquake motions at the same ground surface.

It must be born in mind that, in the case of single-layered ground, the acceleration spectra of earthquake motions appear quite similar not only to the theoretical result mentioned above but also to the frequency-period curve of the same earthquake motions, as well as to the frequency-period curve of microtremors at the same place. (This will, for instance, be seen in Figs. 3, 30 and 33.)

On the other hand, in the case of plurally layered ground, the similarity stated above becomes somewhat complex, nevertheless, the

6) K. Kanai, "The Effect of the Solid Viscosity of the Surface Layer on the Earthquake Movements", *Bull. Earthq. Res. Inst.*, **28** (1950), 31-35.

general patterns are similar, because the conditions of the superposition of the waves reflected at more than two boundaries, are very complex.

At any rate, it is a noteworthy fact that the vibration characteristics of actual earthquake motions, including destructive ones, were explained by the multiple reflection theory of elastic waves.

In general, the seismic characteristics of the ground can be expressed by the following formula:

$$G(T)=f\left(\frac{H_n}{v_n}\ ,\ \frac{\rho_{n+1}v_{n+1}}{\rho_n v_n}\ ,\ T\right), \tag{10}$$

in which H, v and ρ represent the thickness, the velocity of the transversal waves and the density of the layer, respectively, and the suffix n represents the number of layers from the free surface, and T is the period of the incident waves.

It will be seen from equation (10) that the seismic characteristics of the ground depend largely on the ratios of the constants of the ground and relate somewhat to the absolute values of the constants of the ground itself. The results of some actual cases investigated by us, in which the seismic characteristics of soft ground are similar to those of hard ground may be explained by the above conclusion.

Therefore, it may be born in mind that discrimination between good or bad ground as considered from the vibrational point of view, differs when considered from the statical point of view, namely, the bearing power of the ground, etc.

4. Principles for the adopting of seismic force coefficients in earthquake-proof construction design.

Various principles to be adopted in determining the seismic force coefficients in earthquake proof construction design are considered as follows:

(1) Consideration of largest acceleration among the past earthquake motions visited at a construction site.

(2) Consideration of the value of acceleration expected from seismicity.

(3) Consideration of the spectrum calculated from an empirical formula for strong earthquake motions, using the values of the magnitude, the epicentral distance of the past largest earthquake motions sustained at the construction site and the predominant period of the

same place.

(4) Utilization of information concerning the largest observed earthquake motion record in the world.

(5) Utilization of information from records of a strong earthquake motion observed at a place where the seismic characteristics of the ground are similar to those at the construction site. The seismic characteristics of the ground can be obtained easily by microtremor observation. This principle is concerned in the results of the present investigation.

This is one way of utilizing the strong earthquake motion records obtained from any where in the world.

In general, the intensity of the earthquake motions obtained by following many of the principles mentioned above will become very large, which will result in an uneconomical design. Therefore, the practical seismic force coefficients in earthquake-proof construction design are or will be, based a great deal on engineering judgement and they differ somewhat in the strictness of the requirements, after reflection on the public safety, the relation between the term of the use of the construction and the seismicity, the degree of damage to be tolerated and so forth.

5. Conclusion.

From this investigation, it was ascertained that the similarity in the shapes of the acceleration-period and frequency-period curves of any earthquake motions including destructive ones, and the frequency-period curve of microtremors at each spot is close.

It was also ascertained that the vibration characteristics of earthquake motions, including destructive ones, can be explained by the multiple reflection theory of elastic waves in the surface layer.

It is a noteworthy fact that the nature of earthquake motions, as concluded from this investigation, is utilizable in the adopting of both reasonable and economical seismic force coefficients, in earthquake-proof construction design.

In conclusion, the writer wishes to express his sincere thanks to the U. S. Coast and Geodetic Survey and especially to Mr. William K. Cloud, for cooperation in making their strong motion seismograph records available to him. The writer's sincere thanks are also due to Professor C. Martin Duke and Mr. David J. Leeds of the University of California at Los Angeles, Professors George W. Housner and Donald E. Hudson of the California Institute of Technology, Messers Charles F. Knudson and Kenneth W. King of the U. S. Coast and Geodetic Survey, who gave him important help in microtremor observations, without which, successful results could not have been obtained. The writer is much obliged to Dr. Teiji Tanaka and Miss Shizuyo Yoshizawa without whose help this work could not have been done.

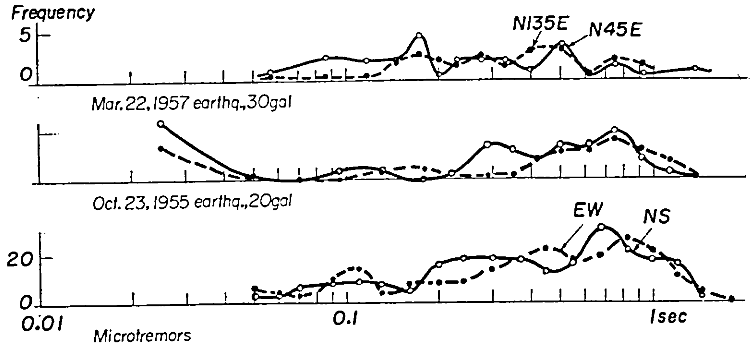

Fig. 12. Frequency-period curves of earthquakes and microtremors at Southern Pacific Bldg. in San Francisco, California, U.S.A.

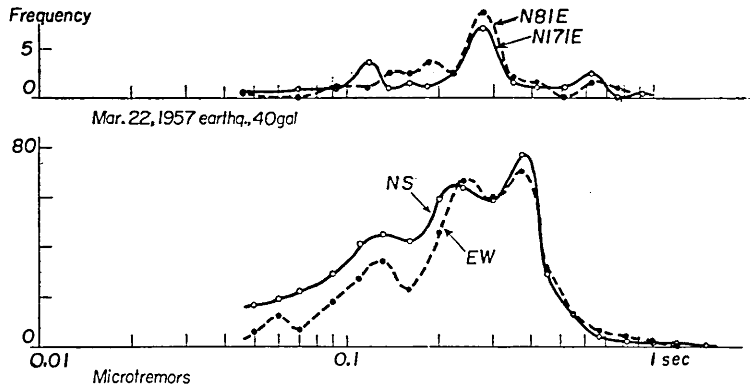

Fig. 13. Frequency-period curves of earthquakes and microtremors at Alexander Bldg. in San Francisco, California, U.S.A.

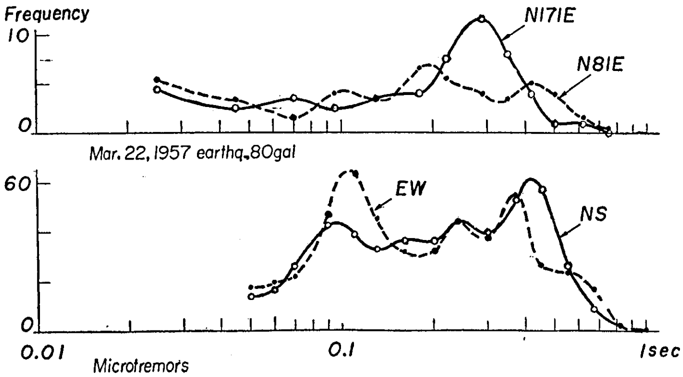

Fig. 14. Frequency-period curves of earthquakes and microtremors at State Bldg. in San Francisco, California, U.S.A.

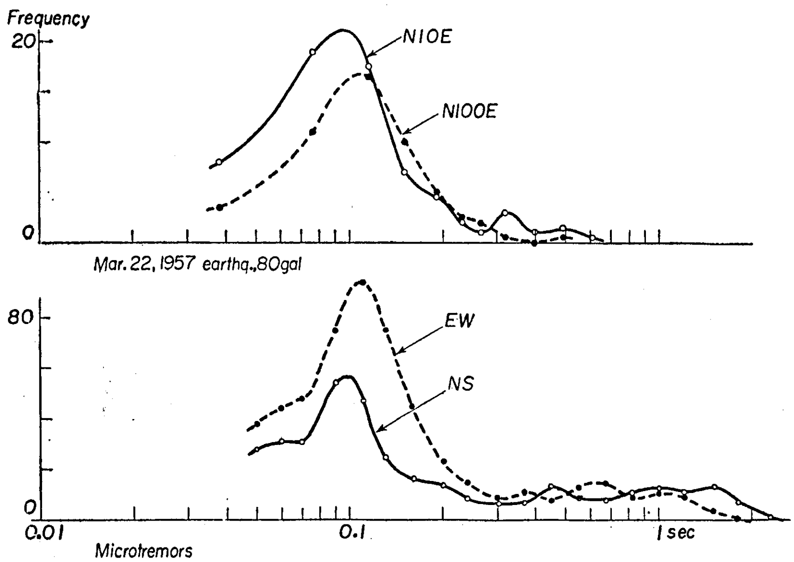

Fig. 15. Frequency-period curves of earthquakes and microtremors at Golden Gate Park in San Francisco, California, U.S.A.

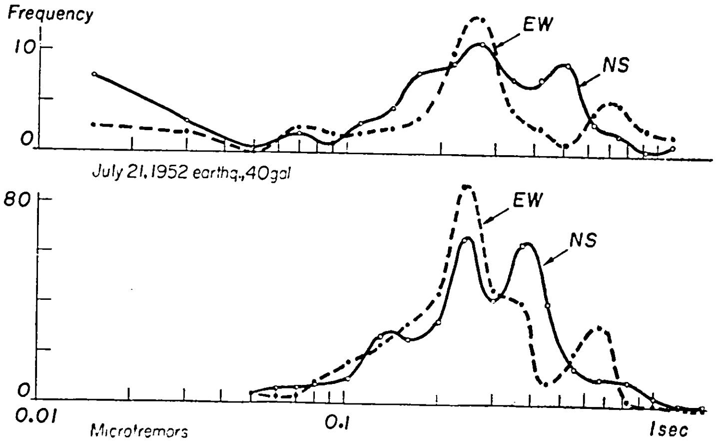

Fig. 16. Frequency-period curves of earthquakes and microtremors at Hollywood Storage in Los Angeles, California, U.S.A.

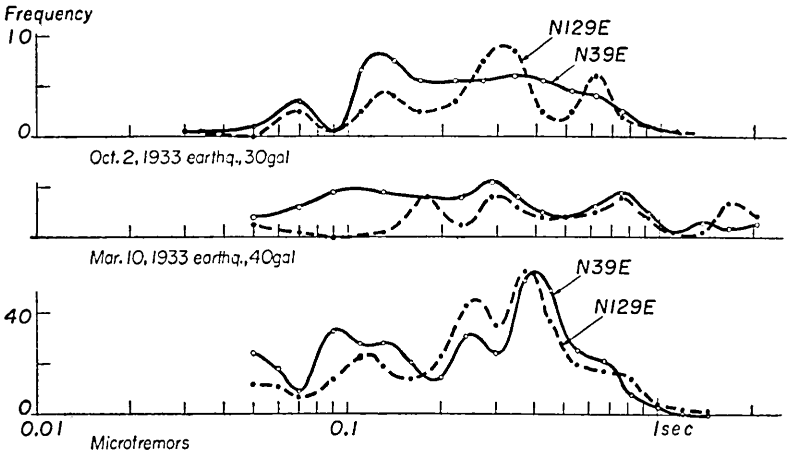

Fig. 17. Frequency-period curves of earthquakes and microtremors at Subway Terminal in Los Angeles, California, U.S.A.

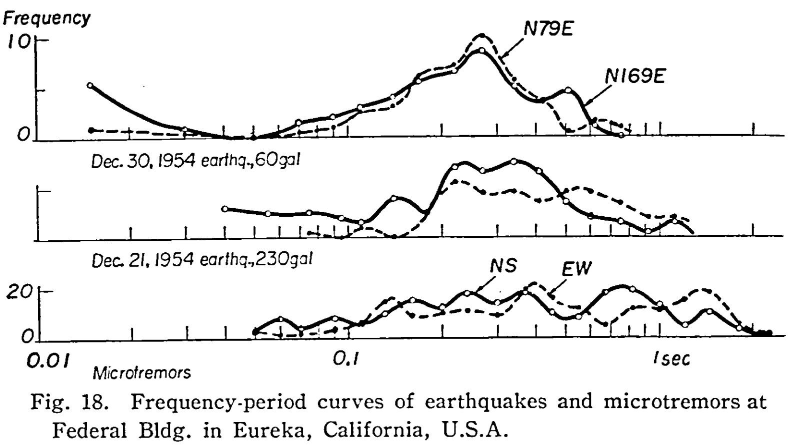

Fig. 18. Frequency-period curves of earthquakes and microtremors at Federal Bldg. in Eureka, California, U.S.A.

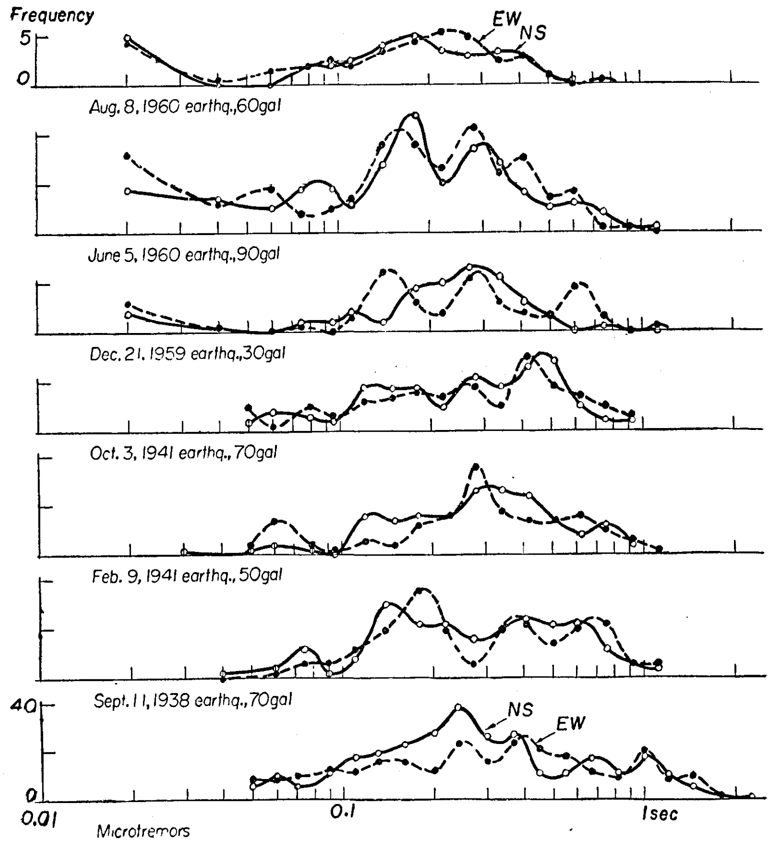

Fig. 19. Frequency-period curves of earthquakes and microtremors at City Hall in Ferndale, California, U.S.A.

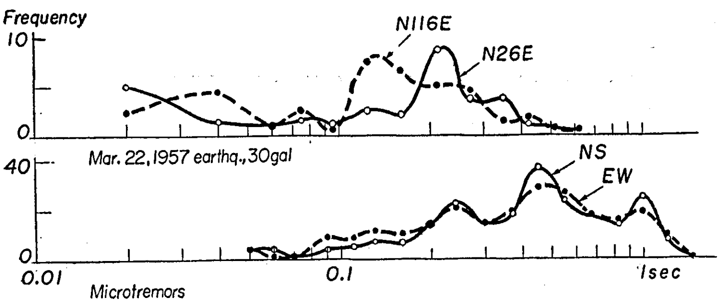

Fig. 20. Frequency-period curves of earthquakes and microtremors at City Hall in Oakland, California, U.S.A.

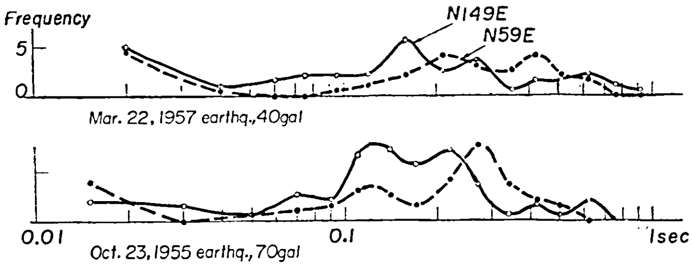

Fig. 21. Frequancy-period curves of earthquakes at Suisum Bay Bridge in Martinz, California, U.S.A.

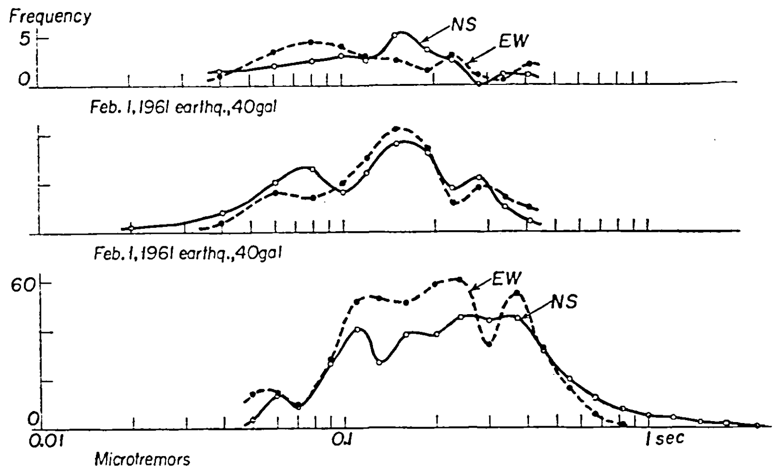

Fig. 22. Frequancy-period curves of earthquakes and microtremors at Los Angeles Water Dept. in Bishop, California, U.S.A.

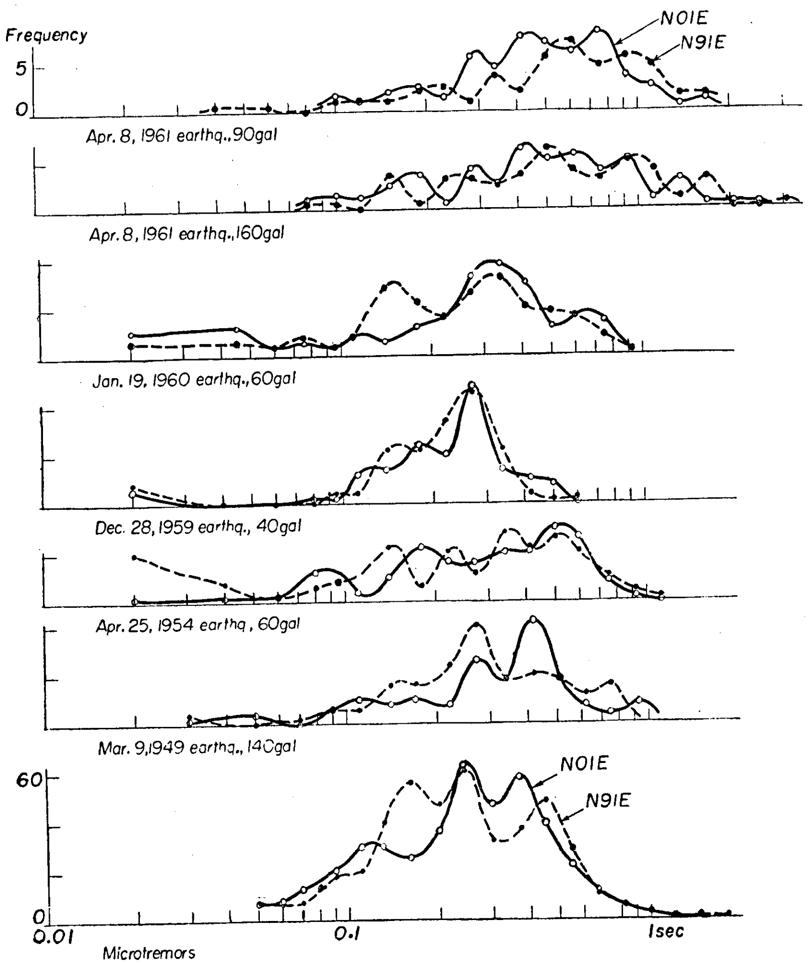

Fig. 23. Frequency-period curves of earthquakes and microtremors at City Library in Hollister, California, U.S.A.

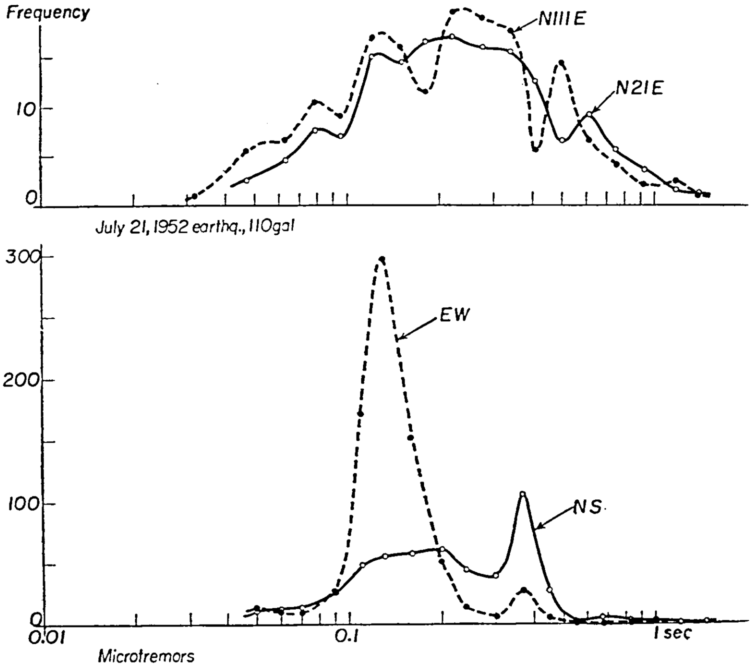

Fig. 24. Frequency-period curves of earthquakes and microtremors at Lincoln School Tunnel in Taft, California, U.S.A.

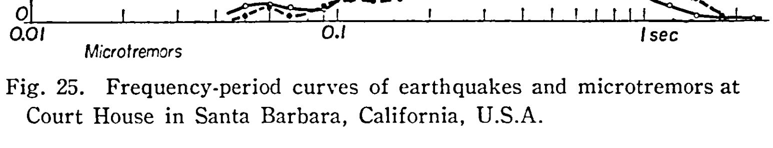

Fig. 25. Frequency-period curves of earthquakes and microtremors at Court House in Santa Barbara, California, U.S.A.

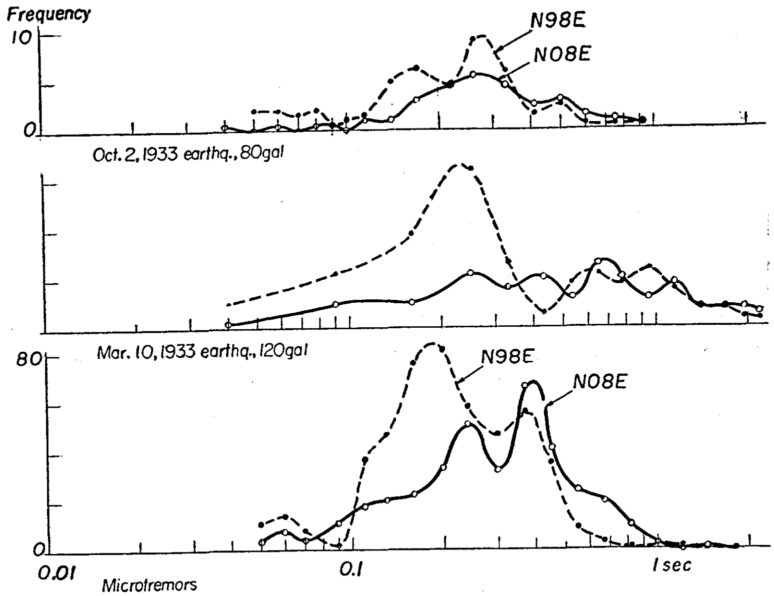

Fig. 26. Frequency-period curves of earthquakes and microtremors at C.M.D. Bldg. in Vernon, California, U.S.A.

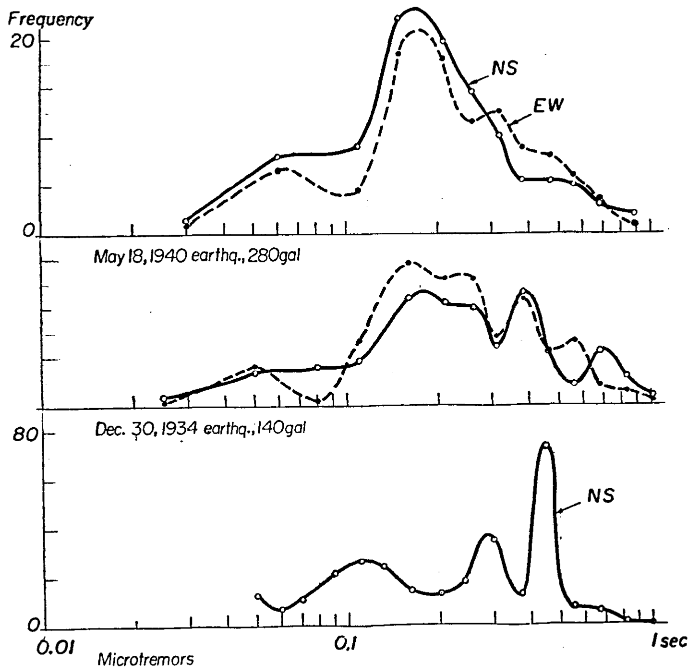

Fig. 27. Frequency-period curves of earthquakes and microtremors at I.V. Irrig. Dist. Sub-Station in El Centro, California, U.S.A.

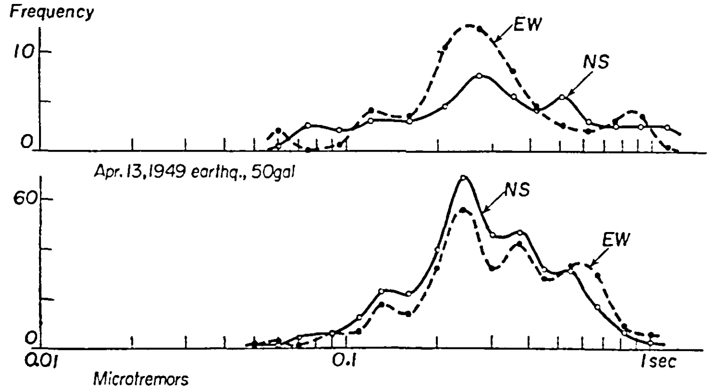

Fig. 28. Frequency-period curves of earthquakes and microtremors at Federal Office Bldg. in Seattle, Washington, U.S.A.

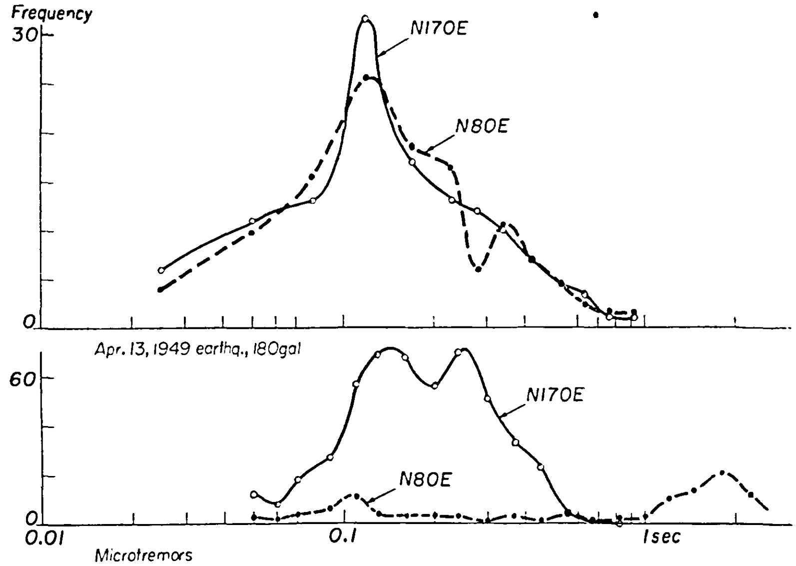

Fig. 29. Frequency-period curves of earthquakes and microtremors at Hiway Test Lab. in Olympia, Washington, U.S.A.

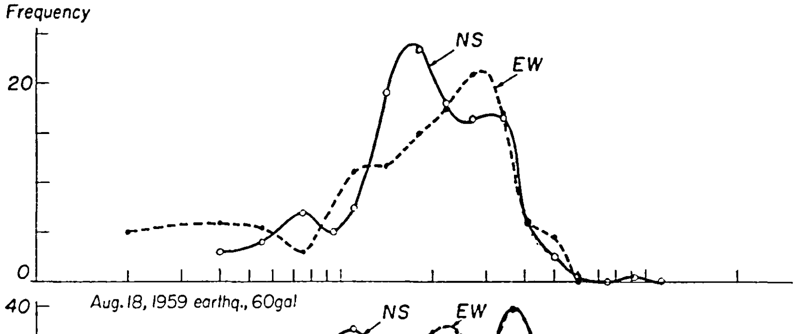

Fig. 30. Frequency-period curves of earthquakes and microtremors at State College Engg. Bldg. in Bozeman, Montana, U.S.A.

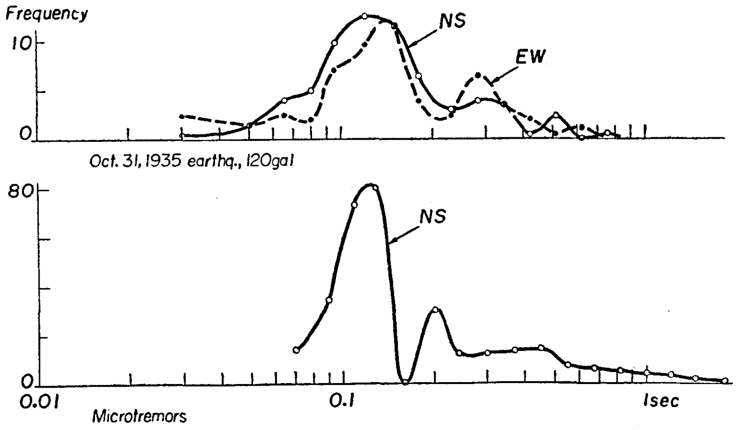

Fig. 31. Frequency-period curves of earthquakes and microtremors at Carroll College in Helena, Montana, U.S.A.

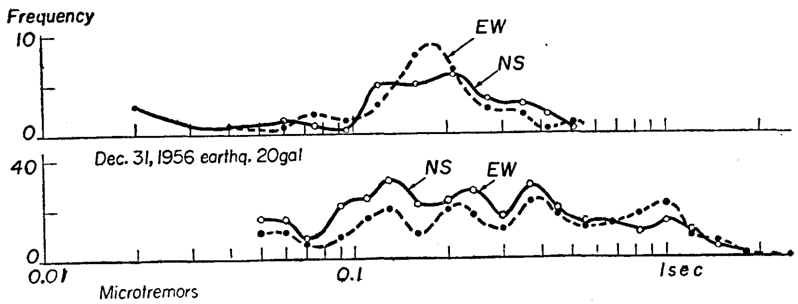

Fig. 32. Frequency-period curves of earthquakes and microtremors at Naval Ammunition Depot in Hawthorne, Nevada, U.S.A.

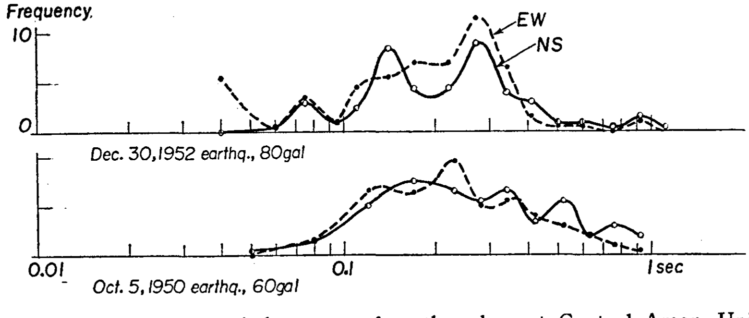

Fig. 33. Frequency-period curves of earthquakes at Central Amer. Univ. of Costa Rica in San Jose, Costa Rica.

3. 強震動の周期特性

地震研究所 金 井 清

アメリカの強震計記録を 20 あまり解析した結果，これまで普通の大きさの地震動の研究から得られた地盤の振動性状に関する結論が，強震動にも同じようにあてはまることが明かになつた．また，強震動の加速度スペクトルが，同一場所での常時微動の周期頻度分布に類似することも，かなりよく確かめられた．

これらのことは，世界中のどこかで観測された強震動の記録を，耐震設計用の資料として利用する場合の 1 つの方法を示唆するものである．即ち，世界中で得られた強震動記録のうち，構造物の建設予定地の地盤の振動性状と類似した地盤の観測所で得られた記録を採用する．地盤の振動性状をしらべるのには，普通の大きさの地震動の観測をするか，場合によつては，常時微動の測定で間に合わせるという方法が考えられるわけである．

また，この研究で，地震動の振動性状が，表面層内での弾性波の重複反射の理論で説明できることを，更に明かにした．また，いわゆる硬質地盤上の地震動の周期特性と軟弱地盤上のそれとが類似した，いくつかの実例を，地震動の振動性状が，

$$G(T)=f\left(\frac{H_n}{v_n},\ \frac{\rho_{n+1}v_{n+1}}{\rho_n v_n},\ T\right) \tag{10}$$

で表わし得ることで，理論的に説明した．それで，最後に一言ふれておきたいのは，(10) 式は，地震工学上での地盤の良否の判断が静的な立場からと動的な立場からとで，ちがう場合があり得ることを示すものであるということである．