31. *Elasticity of Rocks under the Initial Stresses, with Special Reference to the Fracture Problem.*

By Daisuke SHIMOZURU,

Earthquake Research Institute.

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1. Introduction

The occurrence of earthquakes is considered by many seismologists to be the consequence of the accumulation of stresses in the earth's interior to such an amount as to produce fracture of the earth's crust. If the earth's crust is subjected, in addition to hydrostatic pressure, to such stresses as simple tension, compression or shearing stress, the elastic properties of the crust may differ from those whithout such additional stresses. In effect, some observations in Japan of the earth tides[1] or of the velocity of seismic waves[2] seem to show the change in the elasticity of the earth's crust with time. This, in turn, may be considered to show the state of stress accumulation in the crust, which seems also to have some relations with the occurrence of earthquakes.

Some years ago, the author measured the variation of Young's modulus of some igneous rocks such as granite, basalt, andesite etc. under the uniaxial compression by the method of flexural vibrations. The purpose of the above experiment was to study how the variation due to the stresses of Young's modulus depends upon the fine petrological structure of the rocks. Results obtained are described briefly in the previous paper[3].

The purpose of the present research, which is somewhat different from the experiment cited above, is to trace the variation of elastic behaviour of rocks when subjected to the initial stress, which leads the rock to fracture eventually. Especially, as the elasticity near the fracture point is the essential point in these problems, the experiment has been undertaken with this point in view.

The change in elasticity due to pressure was measured by the

1) E. NISHIMURA, *Trans. Amer. Geophys. Union.*, **31** (1950), 357.
2) M. HAYAKAWA, *Rep. Geol. Surv. Japan.*, *Spec. Number*, (1950).
3) D. SHIMOZURU, *Bull. Earthq. Res. Inst.*, **30** (1952), 63.

velocity of elastic waves in rock specimens. Though for geophysical applications rock samples must be subjected to a differential force under the confining hydrostatic pressures, only compressional initial stress was applied to the specimens.

2. Experimental Procedure

Specimens. White marble specimens produced from Akiyoshidai, Yamaguchi Prefecture were chosen as the test pieces. Each of them was cut into a rectangular prismatic bar of about 7 cm in length. When the length of the sample is considerably large compared with the lateral dimensions, the buckling of the sample will occur at large axial force. Therefore, the shape of the testing samples must not be too long. In order to compare the change in elasticity of polycrystalline aggregates with that of amorphous materials, methyl-methacrylate[4], a kind of plastics, was used as a testing specimen.

Hydraulic press. A thirty-ton hydraulic press of the Amsler type was used to produce the uniaxial stress. The specimens were simply kept compressed between the anvil and the piston of the press. The oil control cock was adjusted so as to compress the specimens with a uniform speed at the whole stage of each experiment. Contraction of the specimens was measured by a dial gauge as a function of the pressure.

Measurements of the velocity of elastic waves in the specimen. Ultrasonic pulse equipment was used for the determination of the velocity of dilatational wave in the specimens. The equipment has been the same with that described in a previous paper[5].

Trigger pulses, delayed about 10 microseconds by a variable delay line, are amplified by the pulse amplifier and are impressed on the driving quartz X-cut crystal. The signal, transmitted through the specimen and received by a similar crystal, is amplified and displayed on the screen of a cathode ray oscilloscope, on which time marks are displayed at every 2.5 microseconds. Velocities of dilatational waves propagated along the axis of the load and those propagated perpendicular to the axis were measured separately. Experimental arrangements for the measurements of the velocity are illustrated in Fig. 1 and Fig. 2. (cf. Figs. 11, 12)

4) Fujikasei Co. Ltd. kindly put methyl-methacrylate at the author's disposal.

5) D. SHIMOZURU and I. MURAI, *Bull. Earthq. Res. Inst.*, **33** (1955), 65.

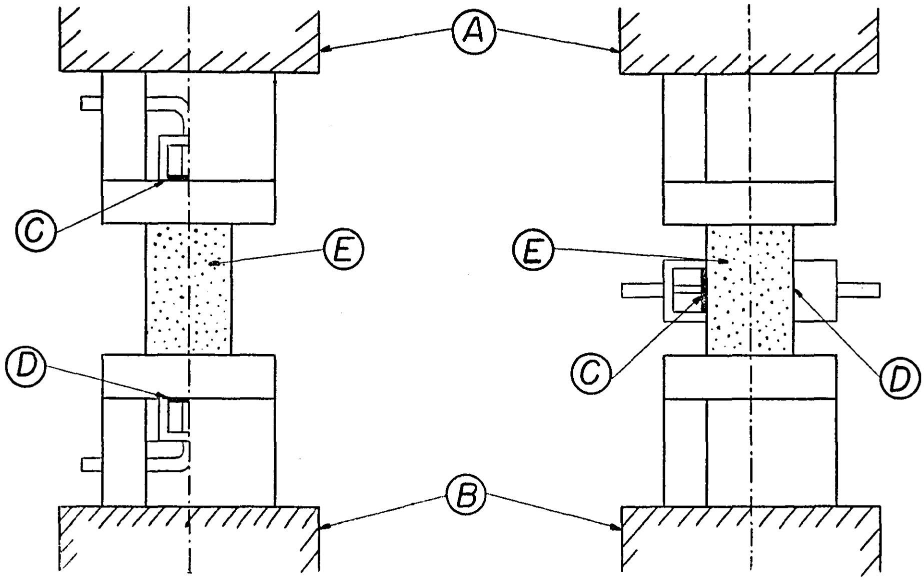

Fig. 1. Experimental arrangement for the determination of the velocity propagated parallel to the axis.

Fig. 2. Experimental arrangement for the determination of the velocity propagated perpendicular to the axis.

(A) The Anvil of the hydraulic press. (B) The piston. (C) Pulse-generating crystal. (D) Pulse-receiving crystal. (E) Specimen.

3. Experimental Results

Three marble specimens were compressed and led to fracture finally in every experiment. Velocities of dilatational waves propagated parallel to the axis were measured for one specimen and those propagated perpendicular to the axis were measured for two specimens. Every experiment was carried out at the same stress rate, that is, about 2.5 minutes per ton. Tables I, II and III are the tabulated values of the velocities and the corresponding axial loads for three marble specimens. These values are also illustrated in Fig. 3 and Fig. 4. In every case, three specimens fractured at the axial load of about 520 kg/cm^2.

Velocities of dilatational wave for methyl-methacrylate were measured only for the direction parallel to the axis of load. They are tabulated in Table IV and are illustrated in Fig. 5. Up to the critical axial load, the marble specimens keep up the resisting strength, until they suddenly fracture and break into fragments much smaller than the individual crystal grains leaving two pyramids at both the upper and lower ends of the specimens. These pyramids of the wrecked specimens are shown in Fig. 13.

Table I. Uniaxial pressure versus velocity of dilatational wave propagated parallel to the axis of a prismatic bar of the marble specimen (originated) from Akiyoshidai.

Pressure (kg/cm²)	Velocity (m/sec)	Pressure (kg/cm²)	Velocity (m/sec)
0.0	5500	297.7	5732
22.9	5493	320.6	5747
45.8	5497	332.1	5721
68.7	5530	343.5	5710
91.6	5538	366.4	5695
114.5	5577	389.3	5699
137.4	5583	412.2	5681
160.3	5601	435.1	5666
183.2	5644	458.0	5544
206.1	5662	480.9	5304
229.0	5655	503.8	4987
251.9	5736	515.3	4717
274.8	5721	527.0	fractured

Tables II, III. Uniaxial pressure versus velocity of dilatational wave propagated perpendicular to the axis of a prismatic bar of the marble specimen (originated) from Akiyoshidai.

Table II.

Pressure (kg/cm²)	Velocity (m/sec)
0.0	5578
57.5	5613
115.0	5683
172.5	5750
230.0	5822
287.5	5802
345.0	5817
402.5	5682
460.0	5304
517.5	4375
520.0	fractured

Table III.

Pressure (kg/cm²)	Velocity (m/sec)
0.0	5455
88.2	5480
176.4	5523
264.6	5536
352.8	5505
441.0	5331
470.4	5183
499.8	4821
517.4	fractured

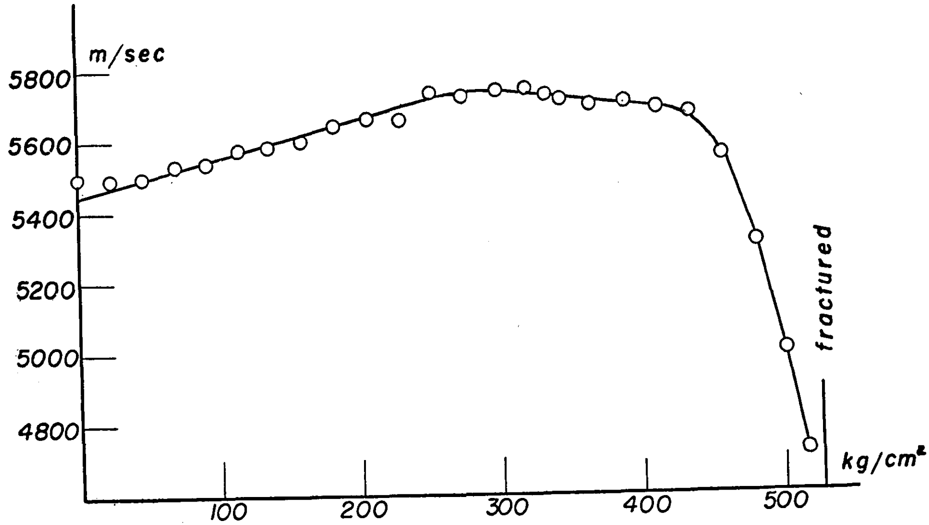

Fig. 3. Uniaxial pressure versus velocity of dilatational waves propagated along the axis of the pressure.

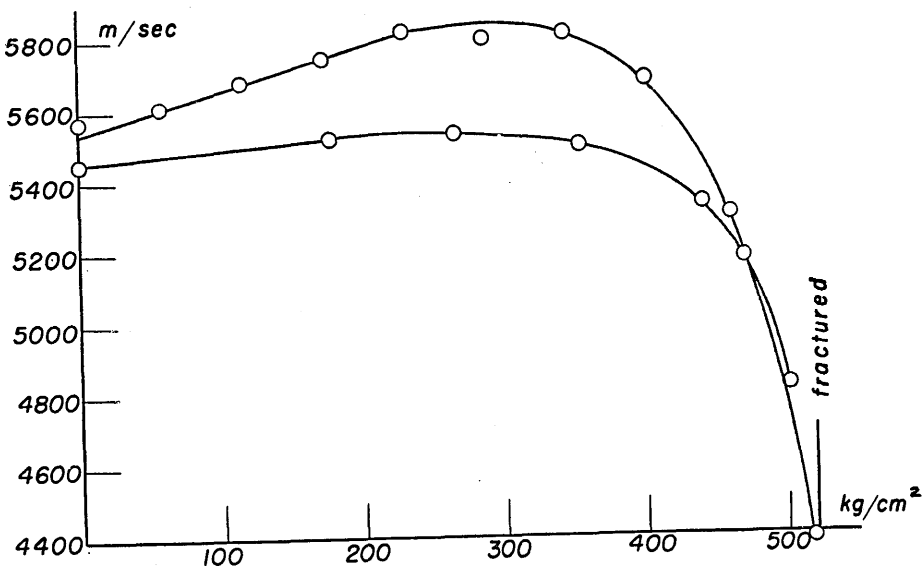

Fig. 4. Uniaxial pressure versus velocity of dilatational waves propagated parallel to the axis of the pressure.

Methyl-methacrylate did not fracture up to a considerable high pressure, but extraordinary flow began to occur at a certain pressure. This change in shape is shown in Fig. 14.

Compression curves of the marble and the methyl-methacrylate specimens are illustrated in Fig. 6 and Fig. 7.

Table IV.

Pressure (kg/cm^2)	Velocity (m/sec)
0.0	2740
108.5	2772
217.0	2777
325.5	2812
434.1	2830
542.6	2823
651.1	2876
759.7	2900
868.2	2920
976.7	2925
1085.3	2918
1193.8	2926

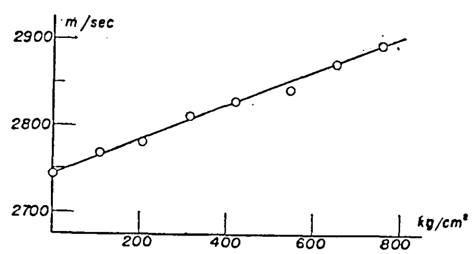

Fig. 5. Velocity of dilatational wave in methyl-methacrylate versus pressure.

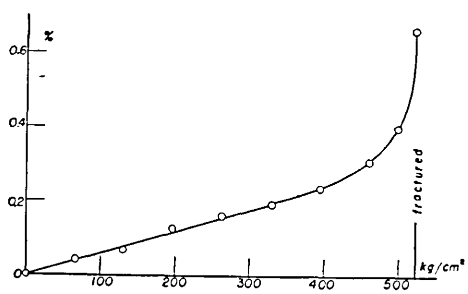

Fig. 6. Deformation curve of marble specimen.

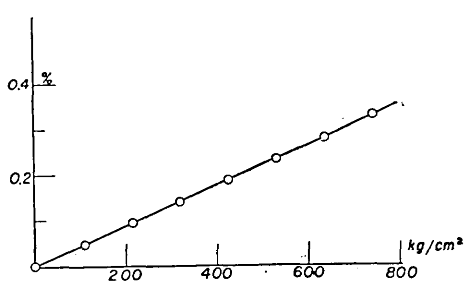

Fig. 7. Deformation curve of methyl-methacrylate specimen.

4. Discussions

In every case, the velocities of dilatational wave for marble specimens vary extraordinarily with pressure. At low pressure, the velocity increases with pressure at a considerable rate, until it ceases to increase and begins to decrease slightly with pressure. Then the velocity begins to decrease with pressure by a considerable amount and fracture occurs. It seems that the change of velocity due to pressure in marble until fracture occurs is composed of three different regions in nature. Accordingly, we will divide the pattern of this velocity change into three parts, namely, (A) elastic region, (B) plastic region, (C) fracture region as shown in Fig. 8. The following discussions on the velocity variation in marble specimen due to pressure are made in contrast with the results obtained in plastics with reference to the above divisions by turns.

(A) Elastic region

In this region, namely, from 0 kg/cm^2 to 300 kg/cm^2 in pressure, the velocity increases largely with pressure. The compression curve, illustrated in Fig. 6, shows that the deformation is almost elastic in this region holding a linear relation between stress and strain. However, this large increase in velocity can not be considered only as the consequence of finite strain of the specimen, because the velocity increase computed from the theory of finite strain is much smaller than that obtained by the present experiment. Therefore, the velocity increase in this region may be due partly to the finite compression of the specimen and partly to the vanishment of void spaces due to pressure which existed *a priori* between each crystal grains[6].

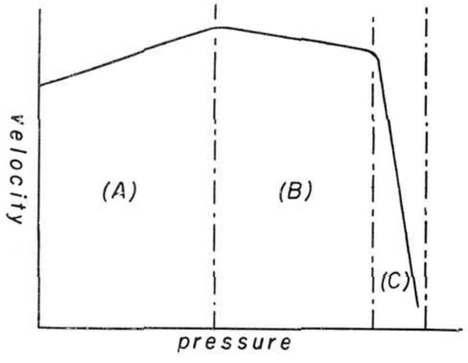

Fig. 8.

The calculation of stress distribution in the specimen is somewhat complicated owing to the shape of the specimen and to the existence of void space. L. N. G. Filon[7] has calculated the stress distribution in a cylindrical isotropic elastic bar compressed parallel to the axis of which both ends are restricted not to expand laterally. His results, as shown graphically in Fig. 9, indicates that the stress distribution is not uniform but is intricate with the exception of the centre part of the cylinder. In this complicated stress field, pressure will be efficacious on the velocity of elastic wave as the mean stress in the specimen.

6) D. SHIMOZURU, *Bull. Earthq. Res. Inst.*, **32** (1954), 73.
7) L. N. G. FILON, *Phil. Trans. Roy. Soc. London*, **198** (1902), 209.

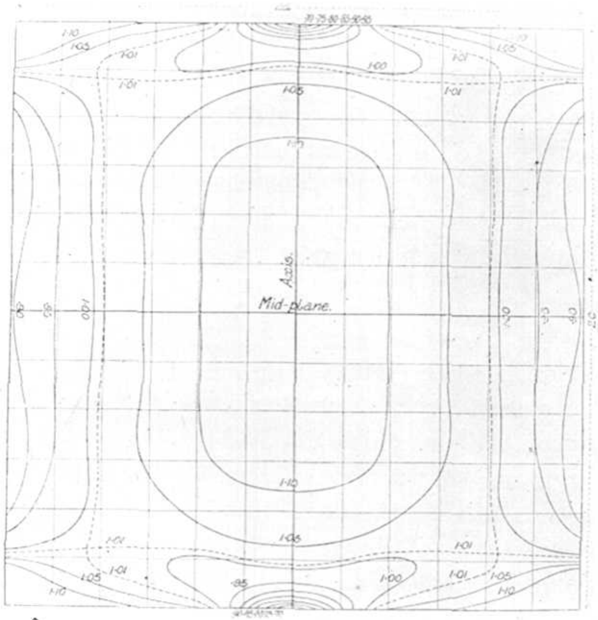

Fig. 9. Distribution of principal stress $\widehat{ZZ}$ inside the cylinder (case of compression between rough rigid plane) after Filon.

(B) Plastic Region

In this region which ranges from about 300 kg/cm² to 400 kg/cm² in pressure plastic deformation seems to have taken place. At present, metallurgists hold that many crystals deform plastically through translational slip which is caused by the movement of dislocations. According to them the critical shear stress necessary to move a dislocation is approximately

$$\sigma = \frac{2\mu}{(1-\nu)} e^{-2\pi/(1-\nu)} ,$$

where σ ...... critical shear stress,
ν ...... Poisson's ratio,
μ ...... *Rigidity modulus.*

If we put $\nu=0.27$, $\mu=2\times 10^{11}$ dynes/cm², we obtain $\sigma=100$ kg/cm² as the critical shear stress needed to move a dislocation in a calcite crystal along a crystallographic gliding plane. Considering the orientation of gliding plane in a crystal, however, the above shear stress must be multiplied by the orientation factor. Therefore, we infer that the shear stress in this region is sufficient to move a dislocation in a composing crystal of the specimen. Then what will happen in the specimen?

If a slip plane contains an obstacle or a crystal grain boundary to the passage of dislocations, they will pile up into an equilibrium distribution against the obstacle, and thus large stress concentration will occur near a piled-up group of dislocations. The normal tensile stress σ acting on the plane making an angle θ with the slip plane is[8)9)]

$$\sigma=\frac{3}{2}\left(\frac{L_0}{r}\right)^{1/2}\sigma_0 \sin\theta\cos\frac{1}{2}\theta\ ,$$

where

L_0 .... length of the slip plane,

σ_0 .... applied shear stress,

r .... length of the crack.

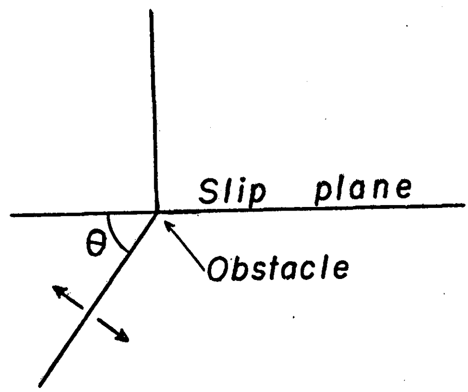

Fig. 10.

Therefore, θ is maximum when $\theta=70.5°$ of which the value is

$$\sigma_{\max}=\left(\frac{2}{\sqrt{3}}\right)\left(\frac{L_0}{r}\right)^{1/2}\sigma_0\ .$$

On the other hand, according to Griffith's theory of crack of length r subjected to a tensile stress σ, the condition for the initiation of the crack is

$$\sigma^2 r > 16\gamma\mu/\pi(1-\nu)\ ,$$

where γ ...... surface energy per unit area,

μ ...... rigidity modulus,

ν ...... Poisson's ratio.

Thus we obtain the condition of initiation of crack as

$$n > 12\gamma/b\sigma_0\ ,$$

where n ...... number of edge dislocations packed into a length L_0,

b ...... atomic spacing.

Since the approximate order of magnitude of γ, b, σ_0 are 10^3, 10^{-8} and 10^8 respectively n becomes of the order of 10^3 which is natural for many crystals. Thus, we conclude that, though it is difficult to

8) J. S. KOEHLER, *Phy. Rev.*, **85** (1952), 480.

9) A. N. STROH, *Proc. Roy. Soc.*, **223** (1954), 404.

demonstrate at the present stage, large stress concentration round the piled-up group of dislocations at the end of a slip line as the consequence of plastic flow is sufficient to initiate a crack, and these cracks may reduce the velocity of dilatational wave in the specimen.

(C) Fracture region

The rate of deformation of the specimen becomes large in this region, especially just before the fracture. Fracture stress in every experiment is about 520 kg/cm^2 which is roughly of the order of 10^{-3} of the elastic moduli of the marble.

Velocity of dilatational wave decreases rapidly with stress before fracture occurs to such low value as 4300 m per second. In methylmethacrylate, neither plastic flow nor velocity decrease occur in such range of stress. Therefore, this large decrease in velocity may be a peculiar character for such brittle and polycrystalline aggregates as marble.

The cause of this large decrease in velocity in this region is considered to be quite different from that in plastic region. If the applied stress becomes considerably large, e.g. more than 400 kg/cm^2 in the present case, besides the formation of narrow cracks as a result of plastic flow, large cracks are produced along the grain boundary of the specimen which can be seen with the naked eye at the surface. As a result of formation of such cracks, which loosen the mechanical structure and reduce the resisting strength against the applied stress, the specimen begins to yield and finally breaks down. Thus these cracks which are wider than those produed by plastic flow will slacken greatly the velocity of elastic waves.

The total amount of deformation up to the fracture stress reaches to 10^{-3}, however, the actual elastic strain of the specimen will be much smaller than this value. Total work done till the specimen fractures which are obtained by the method of graphical integration of stress-strain curve is as follows:

$$\text{Total Work Done} = 10^6 \text{ ergs per unit volume.}$$

This value is much larger than that expected from the elasticity theory. In reality, the above value will depend not only on the mechanical structure of the specimen but on the rate of strain of the specimen, the temperature and the confining pressure. If we assume that rocks can store such amount of strain energy in their unit volume, the effective volume which can store a strain energy of 10^{25} ergs, which is the energy of the earthquake of magnitude 8.7, is estimated to be approximately a sphere with a radius of about 24 km.[10)]

Summary and Acknowledgment

Velocity of dilatational wave in marble specimens compressed by the hydraulic press vary markedly with applied pressure. The following table shows briefly the results obtained by the present experiment.

Pressure in kg/cm^2	Region classified by the author	$\overline{dv}/\overline{dp}$ *	Miscellaneous facts
0–300	(A) Elastic	+0.1	void vanishes, elastic finite strain
300–400	(B) Plastic	−0.05	plastic flow, dislocations, narrow cracks
400–520	(C) Fracture	−1.6	large cracks, large flow

* Average rate of change in velocity with pressure.

It was found that symptoms of fracture of rocks could be regarded as the velocity decrease of dilatational wave starting from a point far below the fracture stress.

The author wishes to express his sincere thanks to Prof. R. Takahasi who offered him valuable suggestions regarding the present study. His thanks are also due to the members of the Department of Precise Engineering, Chûô University who accommodated him with a hydraulic press and operated it for the present experiment. He is also greatly indepted to the Grant in Aid for the Miscellaneous Scientific Researches of the Department of Education given for the series of the present work.

10) K. E. BULLEN, *Bull. Seis. Soc. Amer.* **45** (1955), 43.

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31. 応力と岩石の弾性，特に破壊領域に就いて

地震研究所　下　鶴　大　輔

地殻に応力が蓄積して，或る限界を超えると遂いに破壊に至るという現象は浅い地震の発震機巧を考える場合に根本的問題となつて来る．此の実験は岩石にその様な応力が作用して遂いに破壊に至るまでの弾性的な性質の変化を超音波速度というものを媒介にして研究したものである．岩石の

試料としては，秋吉台産の大理石を角柱に切つたものを用い，材料試験機で軸圧をかけながら，軸方向及び軸に直角な方向の縦波の速度を測定した．又，岩石のように brittle な多結晶体と比較するために一種の有機ガラスについても同様な実験をした．その結果によると，速度の変化に三段階あるようにみえる．すなわち，圧力の小さいところでは速度はほぼ直線的に増加する．(弾性領域)．つぎに，速度は少しづつ減少しはじめる (塑性領域)．速度の減少の割合が急に激しくなつて遂いに破壊する (破壊領域)．この説明として，弾性領域では，void space の圧縮に伴う減少と結晶自体の finite strain による速度の増加がみられ，塑性領域では結晶中の dislocation が動いて，その結果粒界に応力集中が起きて，非常に小さい crack が沢山入るために速度が少しづつ減少する．破壊領域では結晶粒界の結合が loose になつて，大きな crack があちこちに入るため，変形も大きくなり速度の減少が甚しい．有機ガラスでは，同程度の応力でははとんど弾性的性質を保つと考えられる．

このように，破壊は，一瞬にして起るけれども，破壊の芽生えは，破壊応力に達する相当以前から除々にでてくる．このことは，中を通る弾性波の速度に非常に鋭敏にきいてくるようであつて，応力が増すにもかかわらず速度が減少するという結果となつて表われることがわかつた．なお，大理石の変形曲線から外力のなした仕事を計算すると 10^6 ergs/cm^3 となる．つまり単位体積の中にこれだけのエネルギーを蓄えられるということである．大地震のエネルギーを 10^{25} ergs とすると，どの位の effective volume になるかというと，半径約 24 km の球となる．このこと自体も，地震の本性の研究上大変面白いが，岩石は完全弾性体ではないので，strain rate を変えたり，温度を変えたり，また，静水圧下で，軸圧を変えたりしなければ，本当の所は判らない．このことも将来に残された問題である．

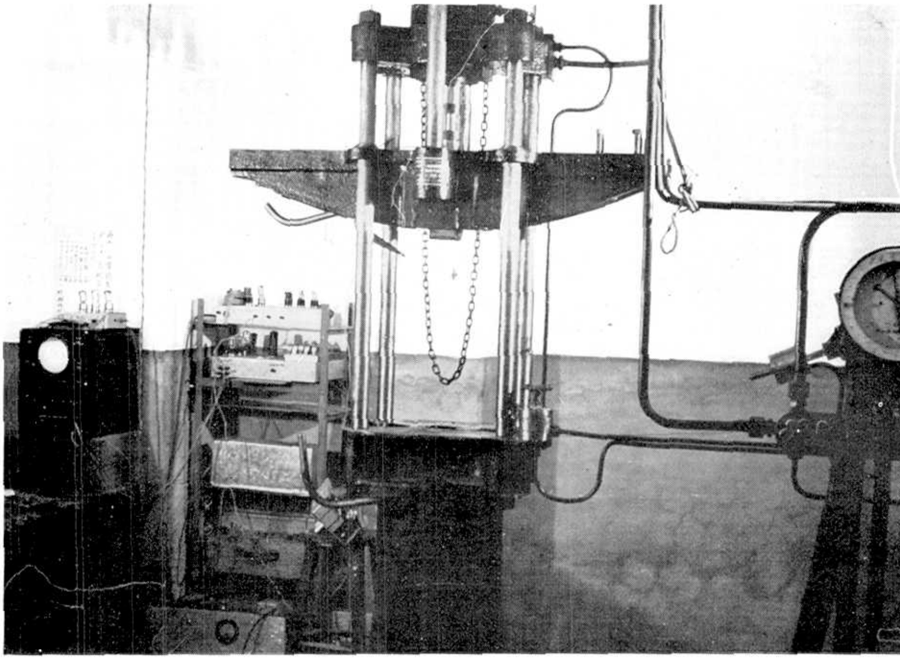

Fig. 11. General view of the experimental equipments.

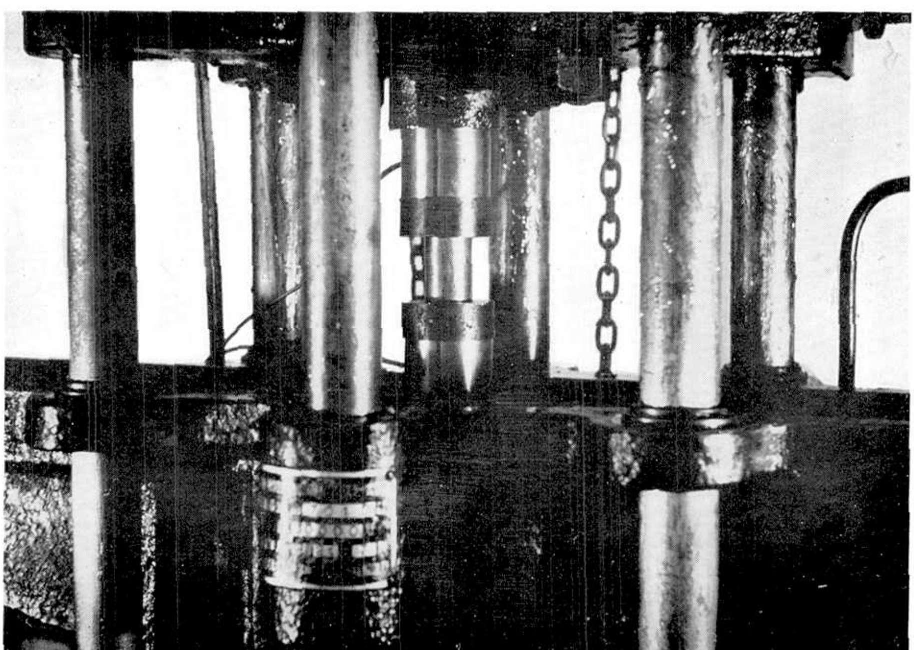

Fig. 12. Close-up view of the specimen compressed by the hydraulic press.

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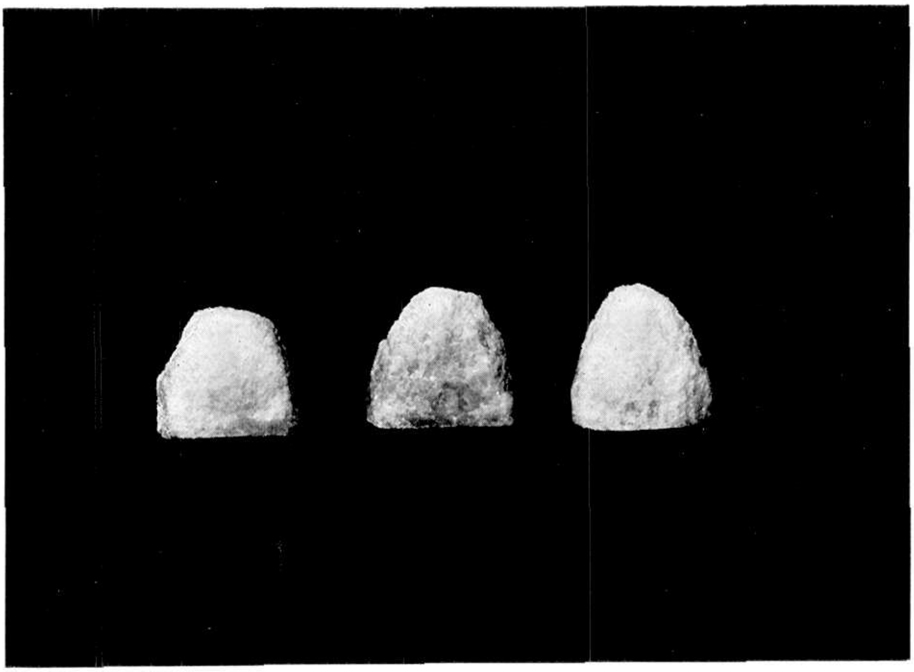

Fig. 13. Pyramids of marble specimen after fractured.

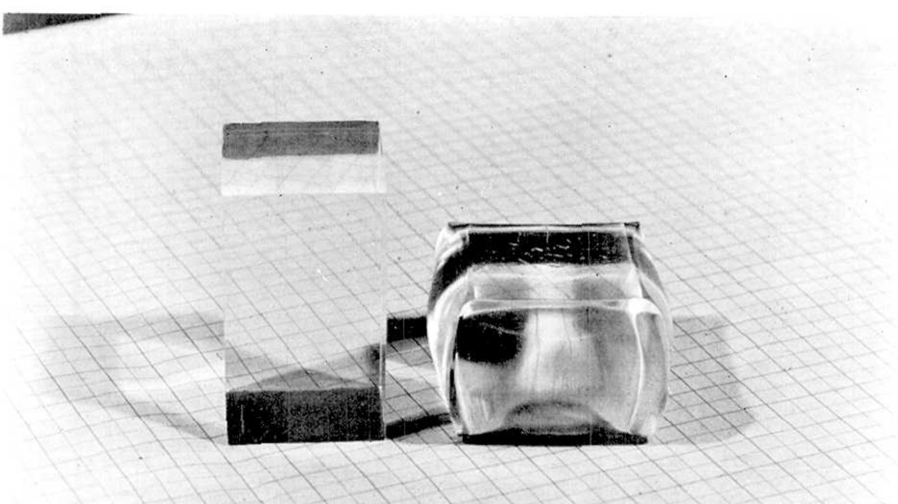

Fig. 14. Methyl-methacrylate specimen, original shape (left) and deformed one (right), respectively.

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