37. *Vibration of a Tank Tower.*

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For the purpose of determining the vibrational characteristics, vibration tests were made on a steel tower with a 100,000 gallons tank at its top. The tower was an independent structure isolated from the neighbouring houses. The tank was supported by six columns and the platform around the tank was of a height of 34.75 m. above the ground (Fig. 1). It has been designed so as to resist the horizontal earthquake motion of an intensity of 0.3 g.

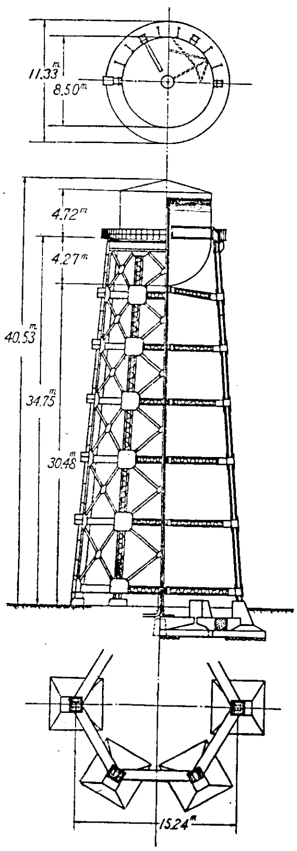


Fig. 1.

Instruments. The instruments mostly used in the tests were the horizontal component seismographs having a period of 1.0 second, magnification of 200 and a damping ratio of about 10 which was secured by an air-damper. The instruments were usually placed on the platform of the tank between two consecutive columns. When the records of two component-motions were required, two sets of instruments were installed at right angles to each other.

For the measurement of small vibrations such as of the foundation, instruments of optical registration were used. These instruments were a set of Wood-Anderson torsion seismometer and a vertical motion seismometer. The magnification of these instruments was 2,000 when the optical distance was kept at 1.0m.

The recording drum of every instument was driven by a small synchronous moter which was motivated by the 50-cycle and 100-volt alternating current.

Vibration by wind. A considerable effect upon the tower is expected, if wind is strong. During the tests, there was one windy day. According to the observation made at 10 m. above the ground, the wind-velocity was between 7.6 and 8.9 m. per second and the wind-direction was mostly from the north. The tower actually oscillated the whole day with its own period. Fig. 2 shows a portion of the records of vibration of the

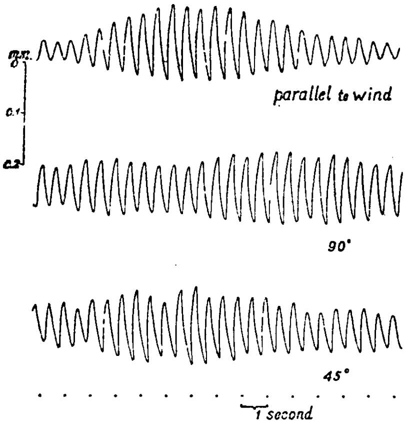


Fig. 2

tower taken on that day. These records were made with a single component seismograph placed first in the direction of wind, and then at 90° and 45° against the wind direction. In all records, the waves represent the fundamental translatory vibration of the tower whose period is nearly constant and 0.57 second. There is no wave with a period differing from this value, When the records were being taken, the tank was kept full of water.

The maximum amplitude is expected to increase in proportion to the square of the wind velocity, so if the velocity reaches 40 m./sec., the maximum range of motion near the top of this tower will become 0.55 cm.

Pull-test. The pull-test is the best method of determining the vibrational property of such tall structure as a tank-tower. It is conducted by attaching a rope to a member near the tank and connecting the other end to a pulling arrangement which is anchored on the ground. In this test, a truck (cargo) was used for this pulling machine and the rope was slowly pulled as it was wound around the winch attached at the head of the truck. Thus, the tension in the rope gradually increases. Instead of attaching the rope directly to the member near the tank, a wire is used between this member and the end of the rope in order to apply the force through this wire. When the tension reaches the ultimate strength of the wire, the wire will naturally break. Thus, the release of the applied force is effected in this moment, and thenceforth the tower will be set into the free vibration which usually lasts for several minutes.

By changing the thickness of the wire, the maximum applied force may be rendered to any desired value. In our tests, galvanized wires 4.05 and 2.75 mm. in diameters were used; the wire having the former diameter is usually called "wire No. 8" and the wire of the latter diameter "wire No. 12." The ultimate tension in the former wire is 440 kg. and in the latter 110 kg. These wires shall be denoted by wire ♯8 and wire ♯12 in the following descriptions.

In the actual test, the maximum force applied by wire ♯8 was sometimes too large and that applied by wire ♯12 was too small to cause the vibration recorded by the seismometer of a convenient scale. It was found then that double wires of

♯12 were the most convenient for the present test, and wire ♯8 should be specially used for applying such strong force as to cause the torsional vibration of the tower.

It is desirable to make this pull test in various directions of the tower. Usually the tower is pulled in the directions radial and tangential to the tank (Fig. 3). The radial pull, however, may be given in two ways; first, the pull is in a direction parallel to one of the diagonals of a hexagon which is formed by joining the six columns; secondly, the pull is in a direction normal to one of the sides of this hexagon. In these two cases, the translatory vibration of the tower will be caused. The another pull which is applied to one of the columns and directed tangentially to the tank will cause the torsional vibration.

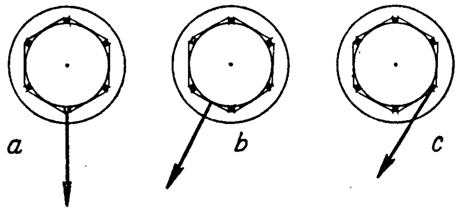


Fig. 3.

Translatory vibration. The tower was pulled in the two directions mentioned above. The tests were made for the various heights of the water in the tank. Initially the tank was full, when the water-level was set at a height of 7.7 m. from the hemispherical bottom of the tank. Thereafter, the water-level was lowered stepwise to various heights and finally to zero (empty).

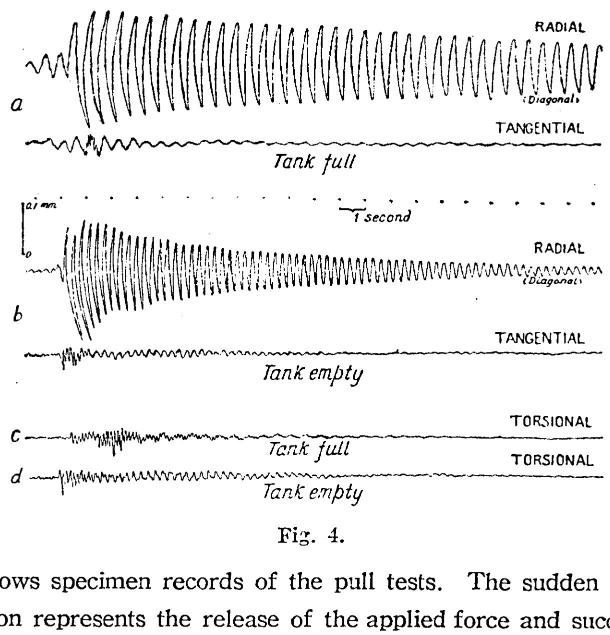


Fig. 4.

Fig. 4. shows specimen records of the pull tests. The sudden departure from the rest position represents the release of the applied force and succeeding to this,

the free vibration of the tower can be seen which gradually decreases in amplitude. Of course, the motion on the tangential component is far smaller than that in the direction of the pull.

It is noteworthy that none of the vibrograms so far made of this tower shows beating effect which is due to the union of two vibrations, that is the natural vibration of the tank-tower system and the vibration of the water surging from one side to the other. It should be kept in mind that the body of water constitutes a separate vibrating system. If the surge period of the water is nearly equal to the natural period of the tower, the beating effect will be enormously displayed.

For the purpose of estimating the surge period, let us take a simple case of a rectangular tank. The period in this case, T_w will be given by equation,

$$T_w=\frac{2L}{v}=\frac{2L}{\sqrt{gh}} \tag{1}$$

where L is the length of the tank, h the average depth of water and g the acceleration due to gravity. In other words, T_w is the time which the gravitational wave takes to reach the opposite side of the tank and come back again with velocity v. In a tank of circular or any other section, the evalution of T_w will not be as simple as in this case, and if h varies, as is the case when the bottom is hemispherical, the relationship becomes more complicated. However, the fact remains that the gravitational period of the water in the tank increases with the decrease in the depth of water.

An estimation of the surge period was made by using the formula given above for a rectangular tank which has the same cross-section and capacity as the actual tank in which the water-level was 7.7 m. from the bottom. The period in this case became 1.92 seconds. Further calculation[1] was made for the fundamental period of the surging water in a circular tank of which the diameter was 8.5 m. and the water-level was 7.7 m, the results being 3.05 seconds. Thus, the gravitational period of the water is at least longer than these calculated values and of course far longer than that of the tower system. Consequently, the beating effect is quite small in the present tower.

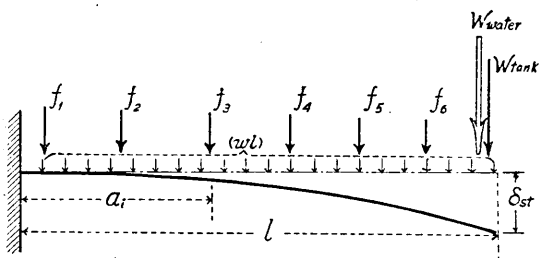
Computation and measurement of period of tower. The method for computing the natural period of vibration of the tower is based on the relation between the period and the deflection of the tower under the action of a horizontal load equal to its own weight. The period of vibration is then given by the equation

1) Takuji SUZUKI calculated.

$$T=2\pi\sqrt{\frac{\delta_{st}}{g}}\doteqdot 0.200\sqrt{\delta_{st}}, \tag{2}$$

where T is the period in second, δ_{st} the deflection in cm. of the top of thet ower under a static horizontal load equal to its own weight and load.

Let us assume that the deflection of the tower during vibration is equivalent to that of a heavy mass (W) supported on a weightless spring having one degree of freedom. In this case, the deflection may be calculated simply for a horizontal beam which is loaded as shown in Fig. 5.



Load (ton)		a_i (m.)
Water	< 346.4	< 34.96
Tank	40.0	34.25
(wl)	86.0	———
f_1	6.65	1.70
f_2	6.30	7.50
f_3	6.00	14.00
f_4	5.71	19.85
f_5	5.36	25.20
f_6	5.08	30.00

$l = 35.50$ m.

Fig. 5.

Supposing that the weight of the six columns and the bracings be distributed uniformly along the length of the column, this load is shown by wl in this figure. The weights of the horizontal beams and tie rods act as loads designated as $f_1, f_2 \ldots\ldots f_6$ and the weights of the tank and the water act at their respective points as shown in the figure.

Let E be the Young's modulus and I the moment of inertia of the section of a beam to which the tower is assumed to be equivalent. Then, the total deflection,

$$\delta_{st}=\frac{(wl)l^3}{8\,EI}+\sum\frac{f_i}{6\,EI}(3a_i^2l-a_i^3)+\frac{W water}{6\,EI}(3a_w^2l-a_w^3)+\frac{W tank}{6EI}(3a_t^2l-a_t^3), \tag{3}$$

in which l denotes the length of the column and a the distance of the point of application of the load measured from the foundation in general.

According to the assumption of a weightless spring, the deflection, δ_{st}, becomes as

$$\delta_{st}=\frac{[w]}{3EI}l^3. \tag{4}$$

By the calculation using the values as shown in the figure, the equivalent mass, when the tank is empty, was found to be

$$[W]=91.68 \text{ tons.}$$

From this value and the measured period of the tower with empty tank (period= 0.30 second), EI can be deduced as follows:

$$EI=6.05\cdot 10^{17} \text{ gram. cm}^2.$$

As previously mentioned, δ_{st} may be calculated by equation (2), if the period is known. Then, the deflection of the empty tank tower becomes as 2.25 cm.

The actual deflection in the pull-test, however, is far smaller than this calculated value. When wire ♯8 was used for the empty tank, the deflection at the top of the column measured 0.01075 cm., which could be deduced from the first motion after the release of the applied force. This value must be proportional to the ratio between [W] and the force applied by the wire, that is, 208:1. Hence, from the measured value, the deflection is worked out as 2.24 cm. In the case of the full tank, the deflection becomes 8.45 cm. for a period of 0.578 second and the ratio mentioned above is 815:1, whereas from the actually measured amplitude of 0.0104 cm., δ_{st} becomes 8.48 cm. Thus there is a good agreement between the calculated and measured deflections.

The period of the translatory vibration of the tower varies considerably with the water-height (Fig. 6 and Table I).

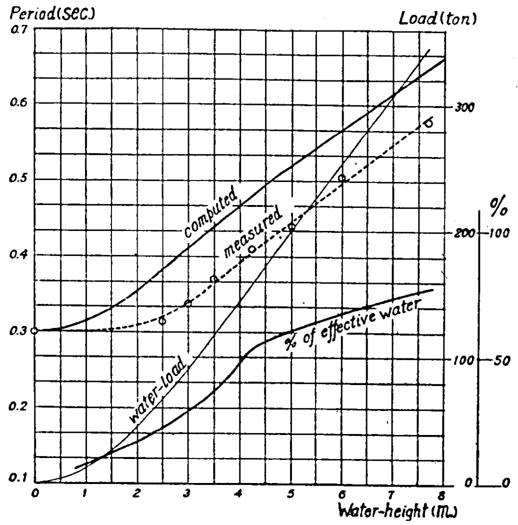


Fig. 6.

Table I. Period and damping of tower vibration,

Water-level in tank	Direction of pull	Period		Damping ratio (v)		ξ/E
		normal to side	diagonal			
7.7 m	b	0.580 sec.	0.575 sec.	1.026		
	a	0.580	0.570		1.026	$7.66.10^{-3}$
6.0	b	0.507	0.500	1.030		7.47
5.0	a	0.440	0.440		1.035	7.77
4.3	b	0.420	0.400	1.039		7.89
3.5	a	0.370	0.370		1.043	7.76
3.0	a	0.334	0.340	1.050		7.82
2.5	b	0.310	0.325		1.047	7.84
0	b	0.305	0.295	1.053		7.86
	a	0.300	0.300		1.053	

(Direction of pull; see Fig 3.)

As the water does not all act as a rigid mass, the computed period becomes longer than the measured period. Calculations show that the equivalent translational mass of the water increases rapidly with the increase in depth, When the tank is nearly full, about 78 per cent of the mass is apparently effective. But, when the water is in the hemispherical bottom of the tank, the rate of the effective water is considerably small, probably due to greater movement of the water relative to the tank than when the tank is more nearly full.

A close examination of the period revealed that excepting the case of the empty tank, one or two waves after the release of the pull had usually slightly longer periods than the subsequent waves. For example, when the tank was full, the period of the first wave was 0.590 second, the period of the second wave 0.585 second and that of the third wave 0.582 second, whereas the periods of the waves elsewhere were 0.578 second in the mean. This suggests that at the instant of the release of the applied force, more mass of the water was effective than in the subsequent vibrations. Contrariwise when the tower was continuously set into vibration, such as by the wind, the period was slightly shorter than any period mentioned above. The period determined in the wind records was 0.570 second. Perhaps in this case, the effective mass of water became smaller than in the case of any pull-test made on the full tank.

Damping of tower vibration. According to K. Suyehiro[2], the decay of the free vibration of a beam is given by $e^{-\frac{kt}{2}}$

2) K. SUEHIRO, *Bull. Earthq. Res. Inst.*, **6**(1929), 63.

in which k stands for $\frac{\xi}{E}\left(\frac{2\pi}{Tr}\right)^2$, ξ the coefficient of normal viscosity of the material composing the beam (in actual structures, the dissipation of vibrational energy through the foundation and that due to other causes is to be included in this), E Young's modulus, Tr the undamped natural period of the beam, e the base of the Napilian logarithm. Thus, the decay constant $\frac{k}{2}$ is inversely proportional to the square of the natural frequency, n, if the material of the beam is known. Therefore, from the logarithmic decrement of the amplitude and the natural period of a vibration, we can deduce the value of $\frac{\xi}{E}$ as shown in the last column of Table I. These values of $\frac{\xi}{E}$ are comparativery larger than those which Suyehiro calculated for the brick chimneys and reinforced concrete frame, that is,

$$\text{for brick chimneys, } \frac{\xi}{E}=4.78\sim1.95\times10^{-3},$$

$$\text{for reinforced concrete frame, } \frac{\xi}{E}=1.85\times10^{-3}.$$

This may be due to the greater dissipation of the vibrational energy through the large number of joints existing in the tower than in the case of the brick chimneys or concrete frame for which the above values were calculated.

The effect of the water in the tank upon the tower vibration, like the effect of the water in the Schlingertank[3] for stabilizing ship cannot be considered in this case, because the damping is stronger for the empty tank than for the tank partially empty or full.

Torsional vibration. When the tower is pulled in such a direction as shown in Fig. 3-c, torsional vibration will be caused in the tower after the release of the pull. The records of such vibrations are shown in Fig. 4-c and-d. It was found that the period of the torsional vibration did not vary according to the water-load as in the case of the translatory vibration and was nearly constant throughout the tests. The period was 0.14 second. Thus, in the torsional vibration, the effective mass of water was quite small. By the way, waves of smaller periods of 0.04～0.08 second which are seen in the earlier stage of vibration, are due to the vibration of the steel members struck by the end of the rope cut down at the release of the applied force.

Movement of foundation. As the movement of the foundation was very small, the optical instruments were set on the concrete foundation. For recording a hori-

3) S. TIMOSHENKO, *Vibration Problems in Engineering.*

zontal component motion, one set of Wood-Anderson torsion seismometer was used. For the vertical motion, a vertical component seismograph specially designed for the field work was used. The period of the former seismometer was 1.0 second and that of the latter 0.7 second. In both instruments, the static magnification was 2,000 when the optical distance was set at 1.0 m.

A comparative observation was made on the platform of the tower and the foundation with the result that the maximum range of motion of the platform was 0.230 mm., and against this value, the maximum range of motion on the foundation was 0.001 mm. Further, the motions of the top of the tower and the foundation were found to be in the same phase. A slight rotatory yielding of the tower's foundation was suggested by a vertical motion which was observed to be 0.001 mm. at its maximum.

The ratio of the displacements of the platform and the foundation stated above is 230:1. This exceeds the value of 200:1 reported[4] to be the value determined for towers standing on firm grounds and of course it is greater than the value of 150:1 determined for towers on yielding grounds where piling was required.

The ground on which the present tower is standing is of a hard loam whose bearing power is estimated to be greater than 50 tons per square meter. Thus, the ground may be classed as "good" or "firm" and the yielding effect of the foundation upon the vibration will be very slight.

In conclusion it must be added that the most probable cause of the seismic damage of the tank tower is the beating effect between the tower and the water in the tank, so the tower should be designed so as to make its natural period of vibration quite different from that of the surging water. From this point of view, the present tower may be said to be faultless.

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37. 水槽鐵塔の振動について

那 須 信 治

100,000 ガロンの水槽を有する鐵塔の振動試驗を行つた. (1) 風に對する振動は規則正しい波動の連續であつて, 弛緩部分による不規則な振動は認められなかつた. (2) 引張り試驗により, 水の容量と塔の振動週期との關係を調査した. 水槽内の水の動搖週期は塔の週期より遙に大であつて共鳴の現象は實際上認められなかつた. 水の固體として振動に寄與する有效率を求めた。又捩れ振動に對して水の影響は殆んどないものと思われる. 塔の振動減衰は比較的早いようである. これは多數の接合點に於て振動エネルギーが逸散される結果であろう. 基礎の yielding は少なく, 振動週期其他に及ぼす影響もこの場合少い.

4) *Special Publication, Coast and Geodetic Survey*, No. 201 (1934-1935). 95.