7. *Observations of the Deformation of the Earth's Surface at Aburatsubo, Miura Peninsula. Part III.*

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(Read March 29, 1949.—Received Oct. 20, 1949.)

1. As already described in the previous paper[1], the deformation of the earth's surface caused by ocean-tide is clearly observed by the extensometers as well as by the tiltmeters. Hence, it is of interest to study how well the deformation may be explained from the stand-point of the theory of elasticity. The deformation due to ocean-tide, which was observed hitherto only by tiltmeters, was usually interpreted as that of the surface of isotropic and semi-infinite solid under surface tractions. R. Takahasi[2], who observed the tilting motion at the same place, also treated the problem along the above-mentioned line. In the first place, the writers will attempt to interpret the problem with the similar method.

2. It was hitherto considered that when the surface of sea upheaves as high as Δh, the increase of the vertical component of the pressure on the sea-bottom was only effective to the deformation of the earth's surface. It should be borne in mind, however, that there occur the horizontal components in the event of inclined bottom. If we define the increase of the vertical component, the horizontal component to the direction x and y by P_1, P_2 and P_3 respectively, we obtain

$$\left.\begin{aligned} P_1 &= \rho g\Delta h/\sqrt{1+(\partial h/\partial x)^2+(\partial h/\partial y)^2}, \\ P_2 &= -\rho g\Delta h(\partial h/\partial x)/\sqrt{1+(\partial h/\partial x)^2+(\partial h/\partial y)^2}, \\ P_3 &= -\rho g\Delta h(\partial h/\partial y)/\sqrt{1+(\partial h/\partial x)^2+(\partial h/\partial y)^2}, \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(1)$$

where ρ and g denote respectively the density of sea-water and the acceleration of gravity and h is the depth of the sea. When the inclination of the sea-bottom is small, the denominators in the expressions of (1) are regarded as to be equal to unity. As will be shown in the later

1) T. HAGIWARA, T. RIKITAKE, K. KASAHARA and J. YAMADA, *Bull. Earthq. Res. Inst.*, **27** (1949), 35.

2) R. TAKAHASI, *Bull. Earthq. Res. Inst.*, **6** (1929), 86.

paragraph, the horizontal forces acting on the bottom become appreciable in the case of extension or contraction though they can be neglected in the treatment of tilting.

The forces P_1, P_2 and P_3 acting at a point $(x, y, z=0)$ on the surface of semi-infinite elastic body cause some deformations which are easily calculated with the aid of elasticity-theory. The displacements in x, y and z directions due to respective forces are given by

$$\left.\begin{aligned} u_1 &= \frac{P_1}{4\pi\mu}\frac{zx}{R^3} - \frac{P_1}{4\pi(\lambda+\mu)}\frac{x}{R(R+z)}, \\ v_1 &= \frac{P_1}{4\pi\mu}\frac{yz}{R^3} - \frac{P_1}{4\pi(\lambda+\mu)}\frac{y}{R(R+z)}, \\ w_1 &= \frac{P_1}{4\pi\mu}\frac{z^2}{R^3} + \frac{P_1}{4\pi\mu}\frac{\lambda+2\mu}{\lambda+\mu}\frac{1}{R}, \end{aligned}\right\} \quad \ldots\ldots(2)$$

$$\left.\begin{aligned} u_2 &= \frac{P_2}{4\pi\mu}\left(\frac{1}{R}+\frac{x^2}{R^3}\right) + \frac{P_2}{4\pi(\lambda+\mu)}\left\{\frac{1}{R+z} - \frac{x^2}{R(R+z)^2}\right\}, \\ v_2 &= \frac{P_2}{4\pi\mu}\frac{xy}{R^3} - \frac{P_2}{4\pi(\lambda+\mu)}\frac{xy}{R(R+z)^2}, \\ w_2 &= \frac{P_2}{4\pi\mu}\frac{zx}{R^3} + \frac{P_2}{4\pi(\lambda+\mu)}\frac{x}{R(R+z)}, \end{aligned}\right\} \quad \ldots\ldots(3)$$

$$\left.\begin{aligned} u_3 &= \frac{P_3}{4\pi\mu}\frac{xy}{R^3} - \frac{P_3}{4\pi(\lambda+\mu)}\frac{xy}{R(R+z)^2}, \\ v_3 &= \frac{P_3}{4\pi\mu}\left(\frac{1}{R}+\frac{y^2}{R^3}\right) + \frac{P_3}{4\pi(\lambda+\mu)}\left\{\frac{1}{R+z} - \frac{y^2}{R(R+z)^2}\right\}, \\ w_3 &= \frac{P_3}{4\pi\mu}\frac{yz}{R_3} + \frac{P_3}{4\pi(\lambda+\mu)}\frac{y}{R(R+z)}, \end{aligned}\right\} \quad \ldots\ldots(4)$$

where
$$R^2 = x^2+y^2+z^2$$

and z is taken to be positive inwards. Hence, the extension, the tilting in x direction and the extension in z direction due to the respective forces become as follows:

$$\left.\begin{aligned} \left(\frac{\partial u_1}{\partial x}\right)_0 &= \frac{P_1}{4\pi(\lambda+\mu)}\frac{x^2-y^2}{r^4}, \\ \left(\frac{\partial w_1}{\partial x}\right)_0 &= -\frac{P_1}{4\pi\mu}\frac{\lambda+2\mu}{\lambda+\mu}\frac{x}{r^3}, \\ \left(\frac{\partial w_1}{\partial z}\right)_0 &= 0, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(5)$$

$$\left.\begin{aligned}
\left(\frac{\partial u_2}{\partial x}\right)_0 &= \frac{P_2}{4\pi\mu}\frac{x(r^2-3x^2)}{r^5} - \frac{P_2}{4\pi(\lambda+\mu)}\frac{3xy^2}{r^5}, \\
\left(\frac{\partial w_2}{\partial x}\right)_0 &= -\frac{P_2}{4\pi(\lambda+\mu)}\frac{x^2-y^2}{r^4}, \\
\left(\frac{\partial w_2}{\partial z}\right)_0 &= \frac{P_2}{4\pi\mu}\frac{\lambda}{\lambda+\mu}\frac{x}{r^3},
\end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots(6)$$

$$\left.\begin{aligned}
\left(\frac{\partial u_3}{\partial x}\right)_0 &= \frac{P_3}{4\pi\mu}\frac{\lambda}{\lambda+\mu}\frac{y(r^2-3x^2)}{r^5}, \\
\left(\frac{\partial w_3}{\partial x}\right)_0 &= -\frac{P_3}{4\pi(\lambda+\mu)}\frac{2xy}{r^4}, \\
\left(\frac{\partial w_3}{\partial z}\right)_0 &= \frac{P_3}{4\pi\mu}\frac{\lambda}{\lambda+\mu}\frac{y}{r^3},
\end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots(7)$$

where $$r^2=x^2+y^2.$$

The suffix denotes the quantities at the surface.

In the next place, the strain-components are obtained by integrating the expressions (5), (6), and (7) with respect to the actual distribution of land and sea. In the practice, the integrations are carried out with a similar method with that of topographical correction of the torsion-balance[3].

3. Assuming that the surface of sea-water upheaves uniformly, the tilting to the direction E 9° S was obtained as shown in Figs. 1 and 2. In the figures, the effect of the sea-water integrated from the observing-point to the respective distance is taken as the ordinates. The effect of the horizontal forces are so small that the scale in Fig. 2 is taken to be ten times larger than that in Fig. 1. In order to explain the observed tilting, it has

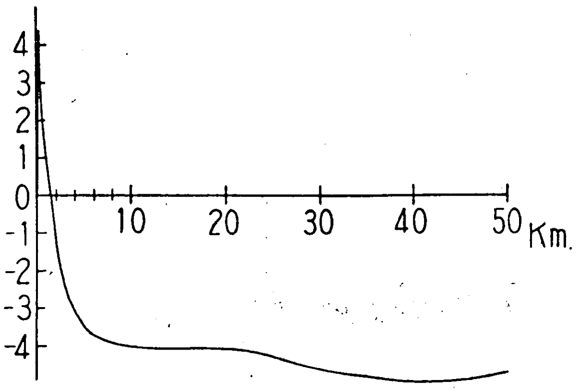

Fig. 1.
Tilting in the direction E 9°S due to the vertical force.
Unit: $100\times\frac{\kappa}{4\pi\mu}\frac{\lambda+2\mu}{\lambda+\mu}\rho g\Delta h$

3) P_2 and P_3 are obtained by making the differences of the depths which are read on the chart. The writers express their hearty thanks to Dr. K. Suda, the Chief of the Hydrographic Bureau, for his kindness of putting the necessary charts at the writers' disposal.

to be assumed that only the water within the range of 100—200 *m* is effective, the conclusion being the same with Takahasi's one.

On the other hand, assuming $\lambda=\mu$, the extension to E 9° S, and N 9° E was calculated as shown in Figs. 3 and 4. The contribution of the horizontal forces are considerable as seen in the figures. The extension to the vertical direction is also shown in Fig. 5 in which the contribution of the vertical force does not exist.

The observation shows that the earth's surface extends remarkably in E 9° S direction, slightly in N 9° E and contracts in vertical direction with the upheaval of the sea-surface. We find the disagreement between the observation and the theory, the sense in the extension in N 9° E component as well as in vertical one being reversed and their relative magnitude considerably differing each other.

It was found out, thus, that the state of the earth's surface must have a com-

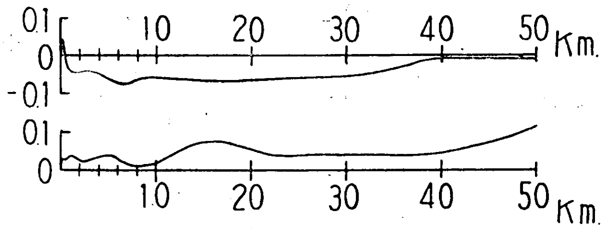

Fig. 2.

Upper: Tilt in the direction E 9°S due to the horizontal force in the same direction.

Lower: Tilt in the direction N 9°E, due to the horizontal force in the perpendicular direction.

Unit: $100\times\dfrac{\kappa}{4\pi(\lambda+\mu)}\rho g\Delta h$

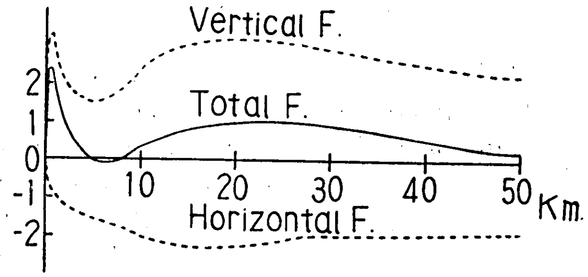

Fig. 3.

Linear strain in the direction E 9°S.

Unit: $100\times\dfrac{\kappa}{8\pi\mu}\rho g\Delta h$

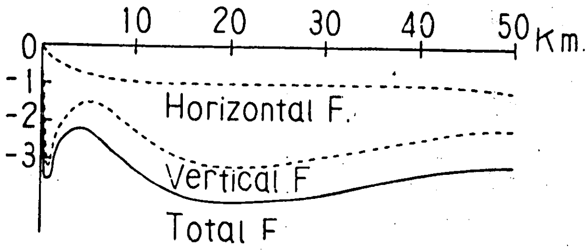

Fig. 4.

Linear strain in the direction N 9°E.

Unit: $100\times\dfrac{\kappa}{8\pi\mu}\rho g\Delta h$

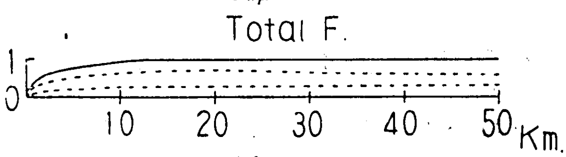

Fig. 5.

Linear strain in the vertical direction.

Unit: $100\times\dfrac{\kappa}{8\pi\mu}\rho g\Delta h$

plicated structure which is considered to be far from the simple one such as semi-infinite elastic solid. However, it is of great difficulty to decide an adequate model of the earth's surface with which we can satisfactorily explain the observed deformations.

4. We shall next examine the influences of the topography around the observing point. For this purpose, the elastic deformation of a body, the section of which is shown in Fig. 6, were studied in an approximate method, where normal pressure acting on the surface except the raised part. As a result of two dimensional treatment, it was proved without any serious errors that the deformation nearly agrees with that of semi-infinite body under the same traction, provided the height of the raised part is small in proportion to its breadth.

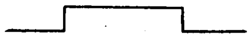

Fig. 6.

If it is supposed that the pressure $-Q$ acts on the surface, which is taken as $z=0$, except the region $-a<x<a$, the x and z components of the displacement of the body are respectively given by

$$u=\frac{Q}{2\pi(\lambda+\mu)}\left\{(a+x)\tan^{-1}\frac{a+x}{z}+|a-x|\tan^{-1}\frac{|a-x|}{z}\right.$$
$$\left.-\frac{z}{2}\log[z^2+(a+x)^2][z^2+(a-x)^2]\right\}$$
$$+\frac{Q}{4\pi\mu}z\log[z^2+(a+x)^2][z^2+(a-x)^2]$$

$$w=-\frac{Q}{2\pi(\lambda+\mu)}\left\{z\tan^{-1}\frac{a+x}{z}+z\tan^{-1}\frac{|a-x|}{z}\right.$$
$$\left.+\frac{a+x}{2}\log[z^2+(a+x)^2]+\frac{|a-x|}{2}\log[z^2+(a-x)^2]\right\}$$
$$-\frac{Q}{4\pi\mu}\{(a+x)\log[z^2+(a+x)^2]+|a-x|\log[z^2+(a-x)^2]\}.$$

Hence, we get at $z=0$

$$u_0=0,$$

$$w_0=-\frac{Q}{2\pi\mu}\frac{\lambda+2\mu}{\lambda+\mu}\{(a+x)\log(a+x)+|a-x|\log|a-x|\}.$$

At $x=0$, it becomes

$$w_{00}=-\frac{Q}{\pi\mu}\frac{\lambda+2\mu}{\lambda+\mu}a\log a.$$

The relative deformation is shown in Fig. 7 in an arbitrary scale. In order to check the above-mentioned theory, an experiment was made by use of agar-agar. An agar-agar model of suitable size, having the cross-section as shown in Fig. 6, was prepared. The surface, except the raised part, was pressed by wooden plates uniformly. Many marks, written on the side of the agar-agar, were photographed and the displacement of the marks were read off from the photograph. As shown in Fig. 8, the relative deformation agrees within limits of errors of the experiment with the theoretically calculated one.

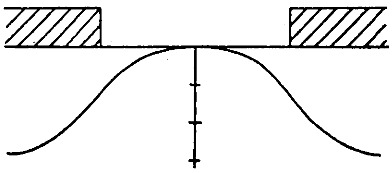

Fig. 7.

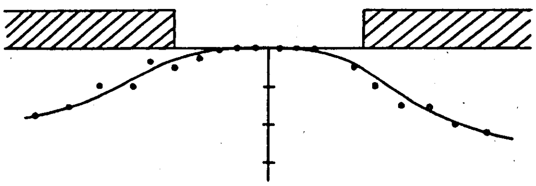

Fig. 8.

Summarizing the theoretical and experimental studies described in this section, we are able to say approximately that the deformation in the raised part agrees with that of semi-infinite body under the same pressure. Hence, we may be able to conclude that the actual topography around the observing point approximately satisfies the condition of plain surface and the influence of the topography is small. And the conclusion obtained in the foregoing section will not be seriously affected by the topography.

5. In summary, the deformations due to the tidal load are so complicated that the explanation is not possible by the simple theory in which the earth is treated as a semi-infinite elastic body. A more complete interpretation of the observed results will be made after measuring the distribution of the deformation at various places nearer to the present observing point.