1. *Motion of the Bay Water caused by Seismic Sea Waves.*

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It is well known[1] that when a bay, of which the depth is $h(x)$, the breadth $b(x)$, the length l, is subjected to the disturbance $\zeta = f(t)$ at its mouth $x = 0$, the elevation of the sea surface $\zeta(x, t)$ in the bay is given by

$$\zeta = gb(0)h(0) \sum_{s=1}^{\infty} \left(\frac{d\chi_s}{dx}\right)_0 \frac{\chi_s}{\lambda_s} \int_0^t f(\tau) \sin \lambda_s(t-\tau) d\tau \,, \qquad (1)$$

where χ_s's are normalized normal functions satisfying the equations:

$$\frac{d}{dx}\left[b(x)h(x)\frac{d\chi_s}{dx}\right] + \frac{\lambda_s^2}{g} b(x)\chi_s = 0, \qquad (2)$$

$$\chi_s = 0 \qquad \text{at} \qquad x = 0 \,, \qquad (3)$$

$$d\chi_s/dx = 0 \qquad \text{at} \qquad x = l \,, \qquad (4)$$

and

$$\int_0^l b(x)\chi_s^2 dx = 1 \,. \qquad (5)$$

In the case of a bay having a uniform depth H and breadth B, we have

$$\chi_s = \sqrt{2/Bl} \sin\left(s - \frac{1}{2}\right)\frac{\pi}{l}x \,, \qquad \lambda_s = \frac{\sqrt{gH}}{l}\left(s - \frac{1}{2}\right)\pi \,, \qquad (6)$$

so that

$$\zeta = \frac{2c}{l} \sum_{s=1}^{\infty} \sin\left(s - \frac{1}{2}\right)\frac{\pi x}{l} \int_0^t f(\tau) \sin c\left(s - \frac{1}{2}\right)\frac{\pi}{l}(t-\tau) d\tau \,, \qquad (7)$$

where $c = \sqrt{gH}$. Writing that $t' = ct/l$, $\tau' = c\tau/l$, we have

1) *Bull. Earthq. Res. Inst. Suppl. Vol.* **1** (1934), 182.

$$\zeta = 2\sum_{s=1}^{\infty} \sin\left(s-\frac{1}{2}\right)\frac{\pi x}{l}\int_0^{t'} f(l\tau'/c)\sin\left(s-\frac{1}{2}\right)\pi(t'-\tau')d\tau' \tag{7'}$$

$$= \frac{2}{l}\sum_{s=1}^{\infty} \sin\left(s-\frac{1}{2}\right)\frac{\pi x}{l}\int_0^{lt'} f(t-\xi/c)\sin\left(s-\frac{1}{2}\right)\frac{\pi \xi}{l}d\xi\,. \tag{7''}$$

Since we assume that $f(t)$ vanishes when $t<0$, the upper limit of the integral in (7″) may be replaced by ∞. Now devide the range of integration $0\sim\infty$ into $0\sim l$, $l\sim 2l$, $2l\sim 3l$, ... and change the variable ξ to ξ', $2l-\xi'$, $2l+\xi'$, ... respectively. Then, noting that

$$f(x) = \frac{2}{l}\sum_{s=1}^{\infty} \sin\left(s-\frac{1}{2}\right)\frac{\pi x}{l}\int_0^{l} f(\xi)\sin\left(s-\frac{1}{2}\right)\frac{\pi \xi}{l}d\xi, \tag{8}$$

because $\sqrt{2/Bl}\sin\left(s-\frac{1}{2}\right)\pi x/l$ is a normal function, we obtain the following expression:

$$\zeta = f(t-x/c)-f(t-2l/c-x/c)+f(t-4l/c-x/c)$$
$$+f(t-2l/c+x/c)-f(t-4l/c+x/c)+ \tag{9}$$

The same expression will be obtained more easily by the method of operational calculus. It can easily be seen from the above expression that ζ is composed of the direct wave and the waves reflected at the head and mouth of the bay. The coefficients of reflection are 1 and -1 in this case. When they are respectively α, $(-1<\alpha<0)$ at the mouth and β, $(1>\beta>0)$ at the head of the bay owing to such causes as the scattering of the energy, the friction at the bottom, etc., we have the following expression instead of (9):

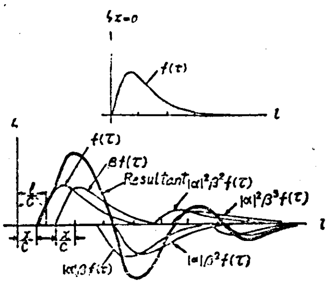

Fig. 1. $\zeta(t)$ at the mouth of the bay (above) and $\zeta(t)$ at any place in the bay (below), which is composed of the direct wave and reflected waves.

$$\zeta = \sum_{n=0}^{\infty}\alpha^n\beta^n[f(t-2nl/c-x/c)$$
$$+\beta f(t-2\overline{n+1}\,l/c+x/c)] \tag{10}$$

It can be seen from (10) that in the bay there develop oscillations of the period $4l/c$ and their harmonics of odd orders, irrespective of the shape of $f(t)$.

It will be worthy of notice that from (10) we can deduce the following important relations:

$$\Phi(t) \equiv \zeta(t) - \alpha\beta\zeta(t-2l/c) \equiv f(t-x/c) + \beta f(t-2l/c+x/c), \tag{11}$$

and

$$f(t) = \sum_{n=0}^{\infty} (-1)^n \beta^n \Phi(t-2n\overline{l-x}/c+x/c). \tag{12}$$

By means of this expression, we can calculate the original shape of the seismic sea waves from the observation made at any point in the bay, provided that α and β are known. In most cases β is nearly equal to unity. As for α, it depends on the shape of the bay, but we can estimate it from the period-amplitude diagram of the record of the seismic sea waves. Assuming, for the sake of simplicity, that $\beta = 1$, $x = l$, and $f = e^{ipt}$, we have

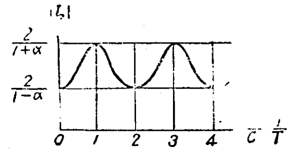

Fig. 2. Theoretical period-amplitude diagram, from which α may be estimated.

$$|\zeta| = 2/\sqrt{(1-\alpha)^2\cos^2 pl/c + (1+\alpha)^2 \sin^2 pl/c}, \tag{13}$$

so that the ratio $|\zeta|_{max}/|\zeta|_{min.}$ is equal to $(1-\alpha)/(1+\alpha)$.