34. *On the Elastic Properties of Soil, particularly in relation to its Water Content.*

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1. In his previous papers[1] the writer discussed the results of his experimental studies on the physical properties, elastic and viscous, of substances, such as rocks and soils, of which the present study is a continuation.

As already explained in the preceding paper,[2] the physical properties of soils are greatly affected by the water contained in them, and differ considerably whether it exceeds or is less than a certain water content, the value of which is 30～40 per cent. As, however, we had not yet established these properties for the whole range of water content, we made further experiments with larger and smaller water contents than those in our previous experiments.

Our object was to investigate carefully the relation between the elastic properties of soils and their water contents, to extend the scope of our data, with a view to increasing our present knowledge of the nature of such substances like rocks and soils that compose the superficial layer of the earth's crust.

The varieties of soil collected from various localities for use in these experiments are shown in detail in Table I.

2. Since soils have both elastic and plastic properties, the usual vibration method, as already shown,[3] was adopted in order to obtain their elastic properties alone, and the velocity of elastic waves propagated in soils thus measured, from which results their elastic constants could be determined.

Most of the apparatus and the methods employed were the same as those used previously,[4] although our apparatus is suitable for most

1) K. IIDA, *Bull. Earthq. Res. Inst.*, **15** (1937), 826; **16** (1938), 131; 391; **17** (1939), 59; 79; **18** (1940), 78.

2) K. IIDA, "Sur l'élasticité et la contraction du sol à Maru-no-uti, Tokyo," *Bull. Earthq. Res. Inst.*, **18** (1940), 78～101.

3) M. ISHIMOTO and K. IIDA, *Bull. Earthq. Res. Inst.*, **14** (1936), 632; **15** (1937), 67.

K. IIDA, *loc. cit.*, 1).

4) *loc. cit.*, 3).

soils in the moist state. When the soil was hard through loss of its water, it was difficult to observe the resonance frequency of the higher orders within the limit of our experiment, for which reason it was not possible to investigate the properties of each soil for their smaller water contents. In order to overcome this drawback, the vibration limit of the vibrating plate was doubled by increasing the speed of revolution and the number of pole-pieces of the generator attached to the apparatus.

In these experiments, a soil block as deposited in the natural state was first obtained, and the soil specimens were prepared from them. The soil specimens were cylinders within an original height of from 20 cm to 30 cm, and a diameter of about 4 cm. One of these specimens was tested without delay, while the others were left to dry in a room of constant temperature, about 15°C, the water contained in them thus gradually changing with the period of drying. These specimens could then be tested in various degrees of water content. In order to obtain a certain desired larger water content than that of the original soil, water was added to the original specimen.

Other properties of soils, such as the size of the particles, density, porosity, and shrinkage were also studied, the methods for obtaining which data were the same as those used previously.[5] The values of the size of the particles and the density thus obtained are shown in Table I.

Table I. Size of Soil Particles, Density, Water Content, Critical Water Volume, Elastic Wave-velocity, and Poisson's Ratio of Soils collected from Various Localities.

Soil sample No.	Size of particles			Water cont. w	θ	φ	Critical water volume R_c	Critical water volume R'_c	Density ρ	Wave-velocity		Poisson's ratio σ	Locality	
	Sand	Silt	Clay							longitud. V_l	torsional V_t			
	%	%	%	%	°	°				m/sec.	m/sec.			(deep)
1	73·5	12·1	14·4	28·0	10	42	0·47	0·70	1·73	178	110	0·31	Marunouti	(3·5 m)
2	48·4	23·9	27·7	44·1	3	15	0·43	0·96	1·65	150	92	0·33	〃	(4·5 m)
3	17·1	31·3	51·6	50·6	9	42	0·70	1·20	1·57	92	54	0·45	〃	(6·7 m)
4	10·3	19·8	68·9	54·3	10	39	0·68	1·10	1·51	129	78	0·36	〃	(7·6 m)
5	8·4	15·0	76·6	49·3	10	46	0·72	1·40	1·46	97	57	0·46	〃	(9·1 m)
6	6·6	24·9	68·5	50·2	10	40	0·72	1·21	1·46	92	54	0·47	Hibiya	(6·0 m)
7	8·2	21·6	70·2	50·6	10	48	0·80	1·30	1·32	90	52	0·48	〃	(8·0 m)
8	8·7	23·8	67·5	49·5	9	45	0·60	1·30	1·45	100	63	0·26	Yurakutyo	(5·9 m)

(to be continued.)

5) *loc. cit*, 1), 3).

Table I. (*continued.*)

(Loam is marked*)

Soil sample No.	Size of particles			Water cont. w	θ	φ	Critical water volume R_c	Critical water volume R'_c	Density ρ	Wave-velocity		Poisson's ratio σ	Locality	
	Sand	Silt	Clay							longitud. V_l	torsional V_t			
	%	%	%	%	°	°				m/sec.	m/sec.			(deep)
9	8·7	22·9	68·4	50·4	12	44	0·67	1·30	1·46	120	75	0·28	Hamamatutyo	(5·0 m)
10	14·1	20·7	65·2	48·3	7	44	0·63	1·15	1·47	130	81	0·30	Yurakutyo	(4·8 m)
11	14·1	25·4	60·5	49·9	12	44	0·61	1·20	1·39	100	60	0·38	Hamamatutyo	(7·0 m)
12	9·0	15·7	75·3	51·0	8	45	0·73	1·30	1·47	100	62	0·31	Sibaura	(6·7 m)
13*	43·4	21·8	34·8	45·1	11	4	0·15		1·29	180	120	0·13	Hongō	(3·5 m)
14*	44·1	23·0	32·9	46·6	10	3	0·15		1·34	171	109	0·23	Teidai Rigakubu	(4·0 m)
15*	46·5	20·5	33·0	48·5	12	2	0·14		1·29	200	130	0·19	Teidai Igakubu	(4·0 m)
16*	45·7	25·1	29·2	46·0	3	10	0·27		1·25	179	112	0·28	Ginza	(4·7 m)
17	14·4	20·3	65·3	32·3	12	45	0·55	1·22	1·69	92	52	0·45	Komatugawa	(3·8 m)
18	10·1	19·7	69·4	34·7	8	45	0·56	1·40	1·59	95	56	0·41	Hukagawa	(3·0 m)
19	9·0	13·8	77·2	57·8	10	44	0·70	1·60	1·43	80	47	0·43	Honzyo	(6·0 m)
20	9·2	13·2	77·6	53·0	9	44	0·72	1·25	1·43	86	51	0·42	〃	(5·1 m)
21*	36·4	27·6	36·0	58·0	30	8	0·40		1·23	179	108	0·17	Ueno	(4·2 m)
22*	35·0	30·0	35·0	56·9	29	7	0·37		1·26	190	123	0·20	〃	(5·1 m)
23	45·1	25·1	29·8	36·3	5	42	0·45	0·70	1·64	130	78	0·40	Tokyo station	(3·0 m)
24	43·6	24·9	31·5	35·9	5	40	0·43	0·72	1·63	112	67	0·39	〃	(4·0 m)
25	7·0	8·9	84·1	50·5	11	47	0·85	1·45	1·82	96	57	0·42	Marunouti	(8·0 m)
26*	23·2	30·7	46·1	34·5	15	4	0·15		1·64	230	147	0·22	Otyanomizu	(5·7 m)
27	19·8	40·2	40·0	45·0	11	40	0·52	1·04	1·70	128	79	0·33	Kanda	(6·0 m)
28	29·1	38·9	32·0	46·2	9	43	0·45	1·00	1·50	115	71	0·31	Tamati	(5·8 m)
29	17·3	30·7	52·0	29·0	9	43	0·68	1·13	1·61	161	99	0·37	Sitaya	(3·0 m)
30	16·9	27·1	56·0	32·6	12	44	0·53	1·20	1·49	122	73	0·39	〃	(4·1 m)
31	20·0	44·0	36·0	49·0	10	43	0·69	1·10	1·53	88	52	0·44	Hukagawa	(5·7 m)
32	18·0	42·0	40·0	48·5	9	42	0·60	1·12	1·40	110	60	0·40	Marunouti	(7·0 m)
33*	14·8	10·2	75·0	55·0	17	10	0·35		1·33	190	120	0·25	Turumi	(6·0 m)
34*	30·1	9·9	60·0	40·8	15	8	0·32		1·40	200	123	0·30	〃	(5·0 m)
35*	33·9	15·9	50·2	52·6	20	12	0·23		1·36	188	118	0·27	Mitaka	(5·3 m)
36*	16·5	11·5	72·0	50·8	21	11	0·37		1·47	179	113	0·26	〃	(4·9 m)
37*	23·4	29·6	47·0	49·0	19	10	0·27		1·50	153	94	0·32	Yokohama	(5·3 m)
38*	57·2	30·3	42·5	39·0	25	11	0·22		1·28	206	134	0·18	Kamakura	(3·2 m)
39*	39·0	42·0	19·0	27·9	20	11	0·18		1·80	250	161	0·21	Tabata	(2·5 m)
40*	44·0	38·5	17·5	32·1	18	8	0·15		1·67	220	138	0·28	Yokosuka	(2·7 m)
41*	9·9	12·1	78·0	52·3	19	13	0·50		1·36	168	106	0·25	Koisikawa	(6·1 m)
42*	10·2	10·3	79·5	57·2	22	13	0·38		1·29	170	109	0·22	Teidai Nōgakubu	(5·0 m)
43*	17·1	17·9	65·0	52·4	21	7	0·30		1·46	103	64	0·31	Akabane	(5·3 m)
44*	23·2	20·3	56·5	38·0	16	11	0·28		1·37	116	74	0·21	〃	(4·1 m)

(*to be continued.*)

Table I. (*continued.*)

Soil sample No.	Size of particles			Water cont. w	θ	φ	Critical water volume R_c	Critical water volume R'_c	Density ρ	Wave-velocity		Poisson's ratio σ	Locality	
	Sand	Silt	Clay							longitud. V_l	torsional V_t			
	%	%	%	%	°	°				m/sec.	m/sec.			(deep)
45*	30·9	30·6	38·5	32·0	14	8	0·21		1·44	108	66	0·33	Kamakura	(2·0 m)
46*	30·0	28·5	41·5	46·0	13	9	0·25		1·57	125	79	0·27	Kawasaki	(3·0 m)
47*	13·3	19·7	67·0	41·0	12	8	0·43		1·49	117	74	0·25	〃	(2·1 m)
48*	15·0	30·0	55·0	57·8	16	9	0·33		1·53	120	75	0·28	Miura-misaki	(3·0 m)
49*	39·4	29·6	31·0	36·3	10	5	0·20		1·36	140	91	0·19	Kokubunzi	(3·2 m)
50*	41·4	28·6	30·0	45·3	9	5	0·18		1·27	128	82	0·21	Tatikawa	(3·5 m)
51*	20·0	30·7	49·3	47·7	23	10	0·30		1·30	141	85	0·22	Komaba	(3·8 m)
52	12·4	7·6	80·0	55·8	10	45	0·80	1·45	1·46	79	47	0·40	Mukōzima	(5·1 m)
53	9·5	8·0	82·5	54·9	11	45	0·80	1·50	1·44	85	51	0·41	〃	(4·7 m)
54	48·5	23·8	27·7	32·9	10	40	0·41	0·90	1·50	93	55	0·40	Edogawa	(4·0 m)
55	37·2	27·8	35·0	20·1	12	41	0·50	0·97	1·62	89	53	0·42	Sinagawa	(2·0 m)
56	38·8	20·0	41·2	41·0	10	40	0·52	1·07	1·47	115	69	0·39	〃	(3·9 m)
57	38·8	13·7	47·5	42·8	12	43	0·55	1·12	1·81	208	127	0·35	Hibiya	(2·9 m)
58	45·8	28·9	25·3	50·0	12	41	0·43	0·86	1·42	88	51	0·48	Kayabatyo	(6·9 m)
59	30·3	18·5	51·2	51·1	10	44	0·60	1·15	1·60	91	54	0·42	〃	(5·7 m)
60	74·6	12·2	13·2	28·5	8	36	0·32	0·62	1·68	192	120	0·28	Susaki	(3·0 m)
61	34·9	20·1	45·0	48·3	9	44	0·58	1·10	1·62	77	45	0·48	〃	(6·9 m)
62	33·7	19·7	56·4	51·6	10	45	0·63	1·16	1·72	130	78	0·39	Tukisima	(5·1 m)
63	14·6	22·6	62·8	59·2	10	45	0·69	1·30	1·61	150	90	0·39	〃	(6·0 m)
64	81·4	10·6	8·0	20·3	6	38	0·31	0·50	1·70	189	118	0·28	Oku	(2·6 m)
65	2·5	28·1	69·4	60·4	10	45	0·70	1·36	1·41	99	59	0·42	〃	(7·2 m)
66	47·4	27·6	25·0	49·2	11	39	0·40	0·90	1·76	105	63	0·39	Kanda	(7·0 m)
67	12·4	10·2	77·4	57·3	11	45	0·78	1·32	1·47	109	65	0·41	Sibaura	(6·0 m)
68	58·3	16·9	14·8	28·2	8	42	0·22	0·65	1·80	200	120	0·38	Sumidakōen	(3·0 m)

3. *Soil-density and water content.* The bulk density of soils varies as a function of the water content. As already discussed in the previous paper,[6] soil density gradually increases in the initial stage of its drying, the maximum value being reached at a certain water content, after which it declines somewhat rapidly, attaining eventually a certain value. In the present case, every soil that was collected was examined.

The density of the soil was determined as usual by measuring its mass and volume. Although to measure its mass is not difficult, to measure its volume is not so easy. In order to measure accurately the volume of the soil specimen, a glass constant volume bottle, shown in

6) K. IIDA, *loc. cit.*, 2).

Fig. 1 was made. The top of this bottle is closed by a stopper and its bottom by a circular glass disc held by a brass washer and screw cap, the sealing of which is perfectly accomplished by the rubber plate. Insertion and withdrawal of the soil specimen is permitted by removing this disc. The volume was measured by weighing the mercury that was required to fill the bottle, with and without the soil specimen, and calculating the displaced volume, in doing which the temperature corrections were duly made. In order to preserve the soil specimen from damage when the mercury is poured into the bottle, and to prevent any mercury from entering the pores of the soil, careful manipulation is necessary in order to get rid of the air bubbles that cling to both the surface of the soil and that of the glass.

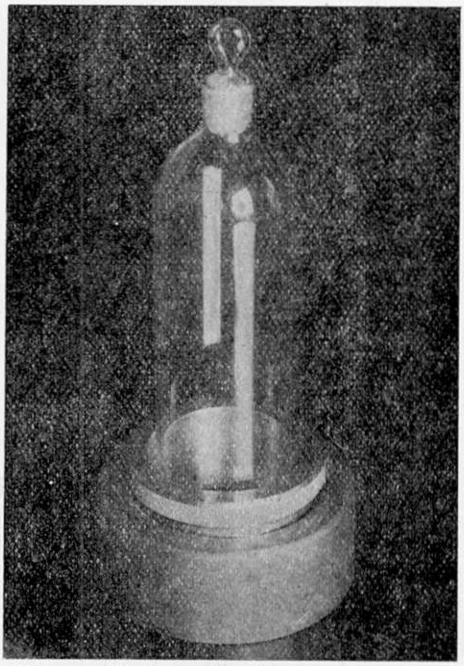

Fig. 1. Photograph of a glass constant volume bottle. (Actual size $\times\frac{1}{3}$.)

The soil specimens were weighed at the same time that their volumes were determined. The water content w may be deduced from the final weight W_0 of the dry soil by means of the formula

$$w=\frac{W_a-W_0}{W_a}, \tag{1}$$

in which W_a is the weight of the soil.

We now introduce a new unit for expressing the quantity of water that is contained in the soil. It is expressed in terms of the volume of water present as referred to unit volume of oven-dry soil, although it is customary to show the water in terms of unit weight of material.

Let W_a be the weight of the soil in moist condition, W_0, V_0 the respective weight and the volume of the oven-dried soil. Then the new unit is expressed by

$$\frac{W_a-W_0}{V_0}(\equiv R), \tag{2}$$

which we shall call the "water volume."

In order to observe the relation between the customary water content w and the new water volume R, we take w as ordinate and R as abscissa, a diagram of which is shown in Fig. 2. The relations between these two quantities are not linear, but exponential, such that

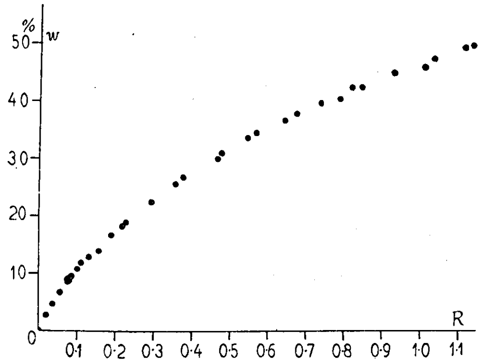

Fig. 2. Relation between water content w and water volume R.

Soil specimen No. 3.

$$w = A(1 - e^{-BR}), \qquad (3)$$

in which A, B are constants depending on the kind of soil. Hereinafter we shall take R as the unit that shows the water contained in soil. Figs. 3, 4 show the relation between the foregoing water and the soil density, the former showing that of silty-clay

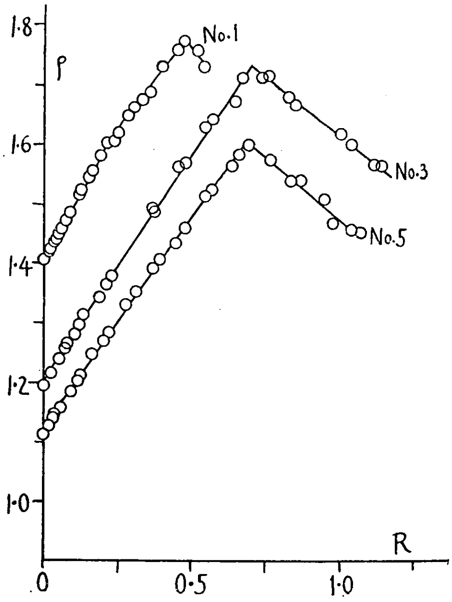

Fig. 3. Relation between density and water volume R. (Silty-clay)

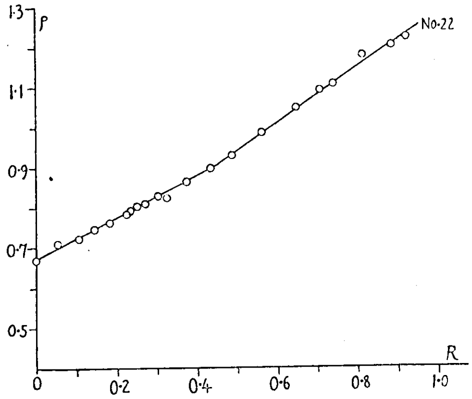

Fig. 4. Relation between density and water volume R. (Loam)

and the latter that of loam. As will be seen from these figures, the curve in the case of silty-clay, obtained by plotting as just indicated, changes discontinuously at a certain value of the water volume.

As already pointed out,[7] the soil density varies greately with the

7) *loc. cit.*, 2).

water. That is, at the beginning, with decrease of the water the density gradually increases, reaching maximum at a certain value of the water content, after which it decreases discontinuously, provided the water is less than this certain value. These certain water values we shall call the critical water volume R_c. Consequently,

$$\text{when } R > R_c, \quad \rho \text{ decreases,}$$

$$\text{when } R < R_c, \quad \rho \text{ increases.}$$

The values of R_c differ with the kind of soil, the constituents of the material, and the size of the soil particles. R_c assumes an increasing larger value with increase in the clay particles forming the soil, and a decreasing smaller value with increase in the sand particles contained in it.

In the case of loam, although its density varies with the water, it was not possible toobserve the critical water volume, where in the density changes discontinuously as in the case of silty-clay when it is dried from the original moist conditions. The reason for this will be explained later.

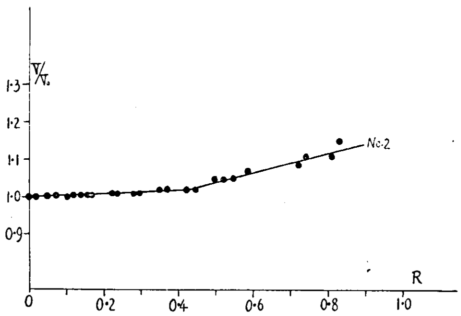

Fig. 5. Shrinkage curve of soil. (Sandy soil)

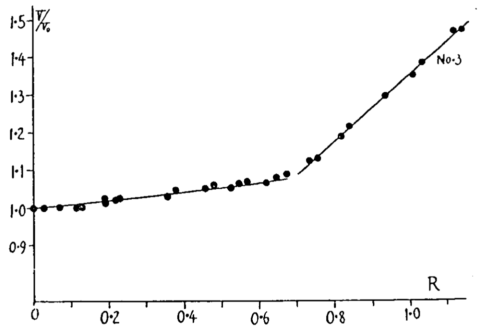

Fig. 6. Shrinkage curve of soil. (Clayey soil)

4. *Shrinkage of soil.* It is possible to find the shrinkage of soil from the relations between the volume of the soil and the variation in the amount of water in the soil, as obtained in Section 3. For this purpose the soil volume was plotted against the water volume R, as shown in Figs. 5~7, from which result it

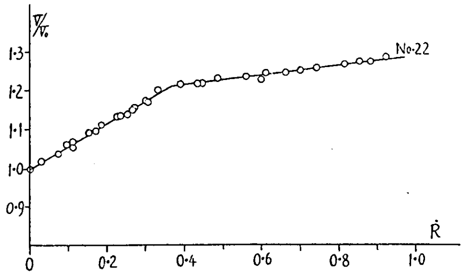

Fig. 7. Shrinkage curve of soil. (Loam)

was found that the volume of soil diminishes with decrease in the water contained in it, and that the rate of this decrease changes discontinuously at a certain value of the water content. This value of water corresponds to the critical water volume R_c, shown in the preceding paragraph 3.

The shrinkage curve thus obtained consists of two straight lines with different inclinations to the axis. As already pointed out, the values of R_c differ with the kind of soil. Moreover the inclination of the straight line differs also with the kind of soil, so that this inclination may be regarded as an element specifying the property of the soil. The nature of these straight lines will be considered next.

In general, soil is composed of solid particles and pore spaces, part of which latter is occupied by air and the other by water. If it is assumed that the air in the pore spaces between the solid particles of soil are displaced by water, and that any change in volume caused by compressing the adosorbed water is negligible, then the increment of volume must be accompanied by an equal increment of water, with the result that the curve representing these relations is represented by a straight line taking an upward course at an inclination of 45° (Fig. 8, the AB part).

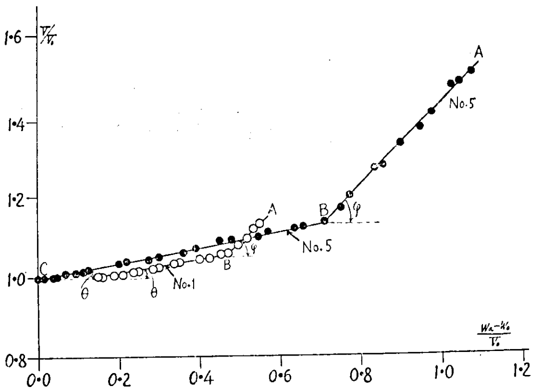

Fig. 8. Shrinkage of soil.
No. 1, sandy soil. No. 5, clayey soil.

Let φ, θ be the angles between the two straight lines AB, BC and the axis, P the pore space in unit volume of dry soil, R the weight of water (consequently volume) added to unit volume of dry soil. Then

$$\left.\begin{aligned} \tan\varphi &= \frac{V}{RV_0}, \\ \tan\theta &= \frac{R_c - P}{R_c}, \\ \text{and} \qquad P &= \frac{\rho_t - \rho}{\rho_t}, \end{aligned}\right\} \tag{4}$$

where V is the soil volume and ρ_t, ρ the real and the apparent specific gravity of soil.

The general expressions for soil volume are given by

$$\left.\begin{aligned} V&=V_0(1+R\tan\theta), \text{ for values of } R<R_c, \\ V&=V_0(1+R-P), \text{ for values of } R>R_c. \end{aligned}\right\} \qquad (5)$$

The pore space may therefore be determined by the equation

$$P=R_c(1-\tan\theta), \text{ or } P=1-\rho/\rho_t. \qquad (6)$$

It was found that the length of AB (Fig. 8) depends on the ability of the soil to absorb water without turning into a virtual fluid, because in the case of clayey soil AB is long, while in the case of sandy soil it is short, since the sand becomes fluid immediately its pores are filled with water. This is chiefly due to the reason that the surface tension of the liquid films between the soil particles ceases to hold them together.

It was found also that the shrinkage is not marked after air began to replace the water in the soil pores. In this last state the soil particles contact with one another, showing that the structure of the soil is somewhat similar to that of a sponge.

5. *The values of θ, φ.* Since the values of the angles θ, φ, the values of which are shown in Table I, differ with the kind of soil, it is supposed that these also depend on the constituent particles. In order to clarify the relation of these two quantities, the quantities of clay particles were plotted as abscissae and the angles as ordinates, as shown in Figs. 9, 10.

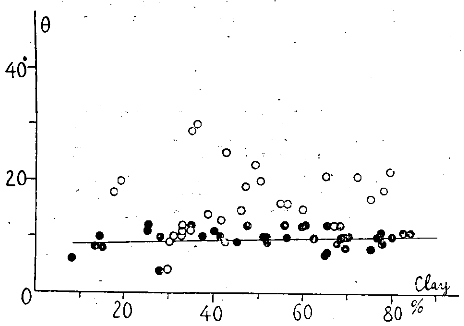

Fig. 9. Relation between the angle θ and the clay particles of soil.

● Silty-clay ○ Loam

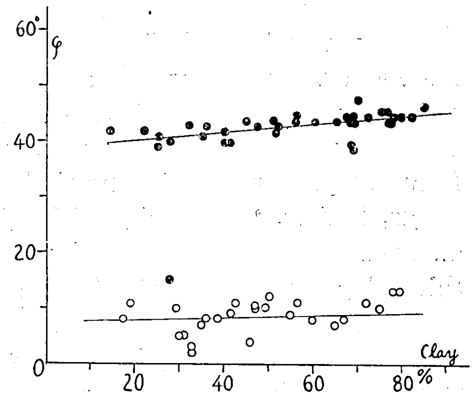

Fig. 10. Relation between the angle φ and the clay particles of soil.

● Silty-clay ○ Loam

As will be seen from these figures, the soils may be divided into two kinds, that is, in one it is assumed that θ is small and φ is large, and in the other that θ is large and φ is small. Silty-clay belongs in the former and the loam in the latter class. In the case of silty-clay, θ is about 10°, and φ about 45°, while in the case of the loam, θ is about 20° and φ about 10°. Further, θ seems to be independent of the proportion of clay particles, while φ increases linearly with increase in the amount (A in percentage) of clay, the relations between them being expressed by

$$\left.\begin{aligned} \varphi &= p + qA, \\ \theta &= r + sA, \end{aligned}\right\} \tag{7}$$

where p, q, r, s are constants depending on the nature of the soil. In the case of silty-clay, $p=38°$, $q=0{\cdot}08$, $r=8°$, $s=0{\cdot}03$, and φ becomes nearly 45° if there is more clay than about 70 per cent, whence it follows that in such a state, soil would behave like a fluid. In the case of loam, $p=7°$, $q=0{\cdot}03$, $r=17°$, $s=0{\cdot}005$.

From the foregoing data it was concluded that soils may be divided approximately into two groups, according to differences in their physical properties and in the geologic conditions under which they were deposited.

6. *Velocity of elastic waves propagated through soils.* With the same methods that were described in our previous papers,[8] two kinds of elastic wave-velocities, longitudinal and torsional, were determined. The values of these various soils thus obtained are shown in Table I. According to these results, the velocities differ with the geologic conditions of the localities whence the soils were obtained.

In order to observe the effect of water on the velocities of soils, the wave-velocities obtained by the experiments were plotted against the water volume, examples of them being shown in Figs. 11, 12, from which it will be seen that the velocities decrease with increase of water, as already described in our previous paper.[9] It was found moreover from the present experiments that the rate of decrease changes in three stages, that is, the velocity of longitudinal wave V_l also decreases gradually at the first stage, rapidly at the second, finally increasing at the third stage, whereas that of the torsional wave V_t also decreases gradually at the first stage, rapidly at the second and third stages, and then disappears. Since, in the foregoing third stage, velocity V_l seems to approach the velocity of sound in a liquid, soil in such a state behaves like a liquid. If the values of the water volume that corresponds to the two boundaries of the three stages are denoted by R_c and R'_c, the general expressions for the velocities in the soil are

8), 9) K. IIDA, *loc. cit.*, 1).

$$\left.\begin{array}{ll}\text{(a)}\ V_l=A-BR, \quad V_t=C-DR, & \text{for values of } R<R_c, \\ \text{(b)}\ V_l=A'e^{-B'R}, \quad V_t=C'e^{-D'R}, & \text{for values of } R'_c>R>R_c, \\ \text{(c)}\ V_l=A''(1-e^{-B''R}), \quad V_t=C''e^{-D''R}, & \text{for values of } R>R'_c,\end{array}\right\} \tag{8}$$

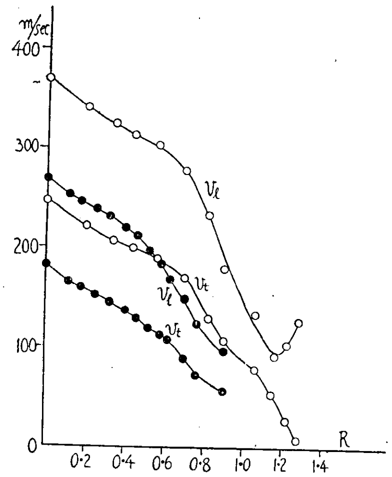

Fig. 11. Wave-velocities vary accoding to R.

○ Soil sample: No. 3.
● Soil sample: No. 2.

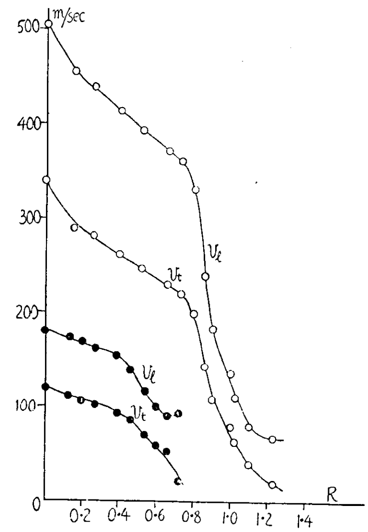

Fig. 12. Wave-velocities vary according to R.

○ Soil sample: No. 5.
● Soil sample: No. 1.

where A, B; A', B'; A'', B''; C, D; C', D'; C'', D'' are constants depending on the nature of the soil. The values of R_c correspond to that of the critical water volume that is found in the case of shrinkage. It may thus be concluded that the physical properties of soils change according as it is larger or smaller than $R=R_c$ and $R=R'_c$, and that the soil behaves like an elastic solid, a plastic body, and a fluid in each limit of the condition of the water volume R shown by equation (8). The values of R_c, R'_c that are shown in Table I vary with the kind of soil, its constituent particles, and the manner in which the soils were deposited.

It ought to be possible now to know how the velocities decrease with increase of water. For this purpose the relations of the elastic constants of soils to the water contained in them were investigated. For the reason that soils behave like an elastic solid, it was assumed that the relations between the velocities and the elastic constants at water volume ranging from $R=0$ to $R=R'_c$ are given by

$$E=\rho V_l^2, \quad \mu=\rho V_t^2, \quad \sigma=\frac{1}{2}\left(\frac{E}{\mu}-2\right). \tag{9}$$

The values of E, μ, σ were determined from the values of the foregoing velocities and densities, the results being shown in Table II.

Table II. Variation of Wave-velocities with increase in Water Content.

Soil sample	Water cont. w	Water volume R	Density ρ	Wave-velocity longitud. V_l	Wave-velocity torsional. V_t	Young's mod. E (C.G.S.)	Rigidity μ (C.G.S.)	Poisson's ratio σ
	%			m/sec.	m/sec.	$\times 10^8$	$\times 10^8$	
No. 1	0·0	0·00	1·40	180	120	4·52	2·02	0·12
	9·4	0·13	1·54	172	111	4·56	1·90	0·20
	13·0	0·20	1·59	168	106	4·43	1·80	0·23
	17·0	0·27	1·64	161	102	4·27	1·71	0·25
	22·2	0·39	1·72	154	93	4·07	1·59	0·28
	25·1	0·46	1·76	138	86	3·38	1·30	0·30
	28·2	0·54	1·74	116	70	2·34	0·85	0·37
	30·2	0·60	1·69	100	59	1·69	0·59	0·43
	35·0	0·66	1·66	90	52	1·34	0·45	0·48
	37·0	0·72	1·62	94	22	1·43	0·08	
No. 2	0·0	0·00	1·08	269	181	7·81	3·55	0·10
	9·8	0·11	1·18	553	165	7·55	3·20	0·18
	14·4	0·17	1·23	246	159	7·45	3·10	0·20
	19·6	0·25	1·30	238	152	7·40	3·01	0·23
	24·2	0·33	1·38	231	145	7·30	2·88	0·27
	28·0	0·40	1·44	220	137	7·00	2·70	0·29
	31·0	0·46	1·49	212	130	6·70	2·50	0·34
	33·8	0·52	1·53	197	120	5·93	2·20	0·35
	36·0	0·58	1·56	184	112	5·30	1·95	0·36
	37·4	0·62	1·58	168	101	4·45	1·60	0·39
	39·6	0·70	1·63	148	88	3·60	1·27	0·42
	41·4	0·76	1·62	124	73	2·48	0·85	0·46
	44·4	0·90	1·59	96	56	1·48	0·50	0·48
	45·3	0·96	1·57	110	30			
No. 3	0·0	0·00	1·19	370	246	16·4	7·25	0·13
	16·7	0·20	1·35	341	221	15·7	6·60	0·19
	24·3	0·34	1·46	325	207	15·4	6·27	0·23
	29·0	0·44	1·54	314	200	15·2	6·03	0·26
	34·0	0·56	1·63	304	189	15·0	5·82	0·29
	39·0	0·70	1·73	279	170	13·5	5·00	0·35
	42·7	0·82	1·69	213	129	7·67	2·80	0·37
	44·6	0·90	1·65	179	107	5·30	1·90	0·39

(*to be continued.*)

Table II. (*continued.*)

Soil sample	Water cont. w	Water volume R	Density ρ	Wave-velocity longitud. V_l	Wave-velocity torsional. V_t	Young's mod. E (C.G.S.)	Rigidity μ (C.G.S.)	Poisson's ratio σ
	%			m/sec.	m/sec.	$\times 10^8$	$\times 10^8$	
No. 3	48·7	1·06	1·59	134	79	2·84	1·00	0·42
	50·7	1·16	1·55	92	54	1·35	0·47	0·45
	51·7	1·22	1·53	103	27			
	53·4	1·28	1·50	127	9			
No. 5	0·0	0·00	1·11	340	504	29·3	12·8	0·14
	12·8	0·15	1·23	289	454	26·1	11·0	0·19
	19·1	0·26	1·30	282	439	25·0	10·4	0·21
	27·0	0·40	1·41	262	414	24·1	9·64	0·25
	32·0	0·52	1·49	247	394	23·1	9·07	0·28
	37·4	0·66	1·60	231	372	22·1	8·52	0·30
	39·8	0 73	1·59	220	362	20·8	7·70	0·35
	42·0	0·80	1·56	200	332	17·1	6·24	0·37
	44·0	0·86	1·54	144	240	8·85	3·16	0·40
	45·0	0·90	1·52	108	183	5·07	1·77	0·43
	47·6	1·00	1·48	80	136	2·74	0·95	0·45
	48·0	1·02	1·47	64	110	1·78	0·60	0·48
	50·0	1·10	1·44	40	80	0·92	0·23	
	52·0	1·23	1·38	20	68	0·63	0·05	

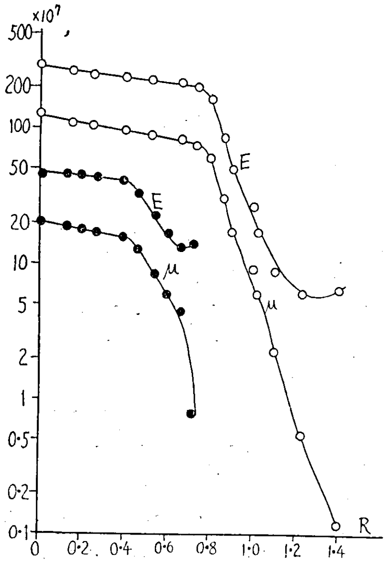

Fig. 13. Relation E or μ and R.
○ Soil sample No. 5.
● Soil sample No. 1.

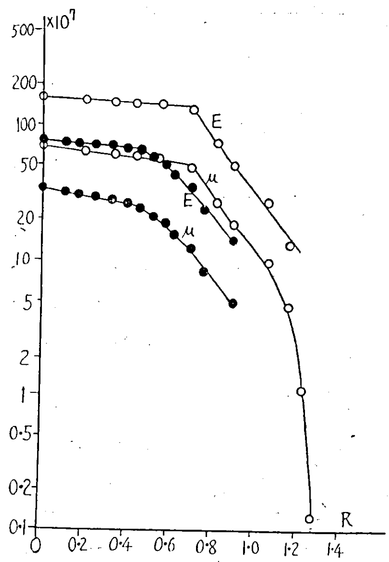

Fig. 14. Relation E or μ and R.
○ Soil sample No. 3.
● Soil sample No. 2.

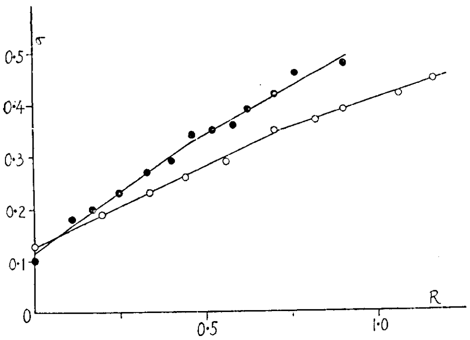

Fig. 15. Relation between σ and R.

○ Soil sample No. 3.

● Soil sample No. 2.

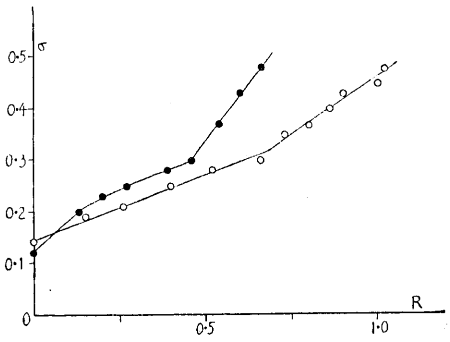

Fig. 16. Relation between σ and R.

○ Soil sample No. 5.

● Soil sample No. 1.

Some examples showing the relations between the values of E, μ, σ and water may be seen in the diagrams, Figs. 13∼16, from which it was ascertained that the elastic constants vary also with the increasing water. The variation in the values of the coefficients, such as E, μ, is linear as referred to water at the water volume R, ranging from $R=0$ to $R=R_c$, whereas the values E and μ rapidly diminish exponentially with increase in water at ranges of R between $R=R_c$ and $R=R'_c$. If R is larger than R'_c, E becomes great again, while μ, which becomes very small, finally becomes zero.

The values of σ vary discontinuously at the foregoing first critical water volume R_c, and

$$\left.\begin{array}{ll} \sigma<0\cdot3, & \text{for water volume of } R<R_c, \\ 0\cdot5>\sigma>0\cdot3, & \text{for water volume of } R'_c>R>R_c. \end{array}\right\} \qquad (10)$$

It is interesting to note that the value of σ which corresponds to that of R_c is about 0·3.

7. In the foregoing experiments, silty-clay showed three stages of change (a), (b), (c), in its elastic property according to the quantity of water contained in it. The critical water volume R_c in this case agrees with that in the shrinkage experiment. Loam did not show changes in three stages, so far as the present experiment was concerned. The critical value R_c, as referred to loam in the case of the shrinkage experiment, was not clearly observed in the case of the elasticity experiment, so that R_c as referred to loam differs from that as referred to silty-clay. In the case of silty-clay, R_c is the criterion showing the change of state, whereas it seems to be not so in the case of loam. As to the main reason for this difference, it is believed that because the loam has many pore spaces, it appears to shrink much more in that state wherein the water left in the pore spaces is practically replaced by air, with the result that the value of R_c appears very small, although the true value of R_c of the loam may be large.

Since the critical values of the water volume R_c seem to be related to the component particles of the soil, we shall now discuss these relations. To obtain such a relation, the values of R_c and R'_c were plotted as ordinates and the clay particles of the soil as abscisseae (Figs. 17, 18). As will be seen from the former figure, the relation between the two quantities is linear, that is, the greater the quantity (q in percentage) of clay the larger the value of R_c, whence the relation may be assumed to have the form

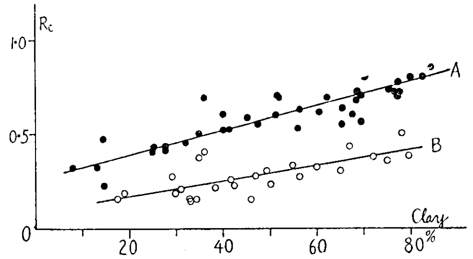

Fig. 17. Relation between R_c and the amount of clay particles of soil.
● Silty-clay ○ Loam

$$R_c = a + bq, \tag{11}$$

where a and b are constants depending on the nature of the soil. In the case of loam, $a=0{\cdot}09$, $b=0{\cdot}004$, while in the case of silty-clay, $a=0{\cdot}25$, $b=0{\cdot}007$. By means of this empirical formula it is possible to determine the critical value of R_c that shows the changes of state, provided the amount of clay contained in the soil is known. From Fig. 18, which shows the relation between R'_c and the amount of clay particles, the conclusion is that the larger the amount of clay the greater the value of R'_c, a relation that may approximately have the form

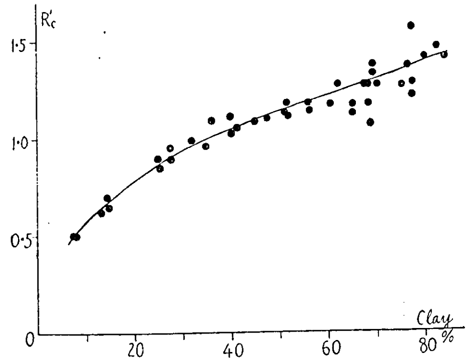

Fig. 18. Relation between the second critical value R'_c and the amount of clay particles of soil.

$$R'_c = C(1-e^{-Aq}), \tag{12}$$

in which C and A are constants, and $C=1{\cdot}6$, $A=0{\cdot}03$ in the case of silty-clay.

In this way, it is possible to determine approximately the critical value at which the velocity or elastic constant becomes great according to the empirical formula (12), provided the amount of clay particles of the soil is known.

It was found that when the pore spaces of clayey soil are completely filled with water, its solid state changes into the fluid state. Probably the transition from one stage to the other occurs at or near the critical water volume R_c. Even in this state that the water present exceeds the value that corresponds to R_c, but does not exceed R'_c, whence soil seems to be an elastic solid from its appearance and from the fact that it does not flow, although the wave-velocity varies in a marked manner. This is due to the presence of water, a film of which separates the particles, but the affinity of the soil particles for water is fully satisfied, overcoming the cohesion of the clay mass. Owing, however, to the presence of colloidal matter around the solid particle, the transition from sol to gel takes place during the vibration at or near the critical water value. It seems that thixotropic phenomena may occur, in this way, as a result of the presence of these colloidal substances. Addition of any further water in excess of the second critical value R'_c, will tend to float the particles away from one another, inducing a state of fluidity.

8. *Conclusion.* The longitudinal and torsional wave-velocities through soil collected from various places were obtained by means of the vibration method. It was found the elastic properties of soil are greatly affected by the water contained in them. In order to ascertain the relations between the physical properties of the soil and water, a new unit of water was invoked. By means of this new unit, it was found that a first and second critical value of water exists, through the boundary of which soil changes from the state of an elastic solid to that of a fluid. Moreover, it is believed that the transition from sol to gel takes place at or near the foregoing critical value of water. At these critical values, the wave-velocity, as well as the elastic constant and the density, change discontinuously.

A certain relation between the critical value of water and the amount of clay particles of the soil was established, and empirical formulae showing the relation between these two elements were derived.

The curve for soil shrinkage consists of two straight lines with different inclinations to the axis. Finally these relations between the inclination and the constituent particles of the soil are discussed.

In conclusion, the writer wishes to express his hearty thanks to Professor K. Terazawa, to Professor K. Sezawa, and also to Professor T. Matuzawa for their kind encouragement.

He also wishes to express his cordial thanks to the Foundation for the Promotion of Scientific and Industrial Research of Japan, with whose grant parts of the present study were made.

34. 土の彈性的性質特に水分との關係に就いて

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土の物理的性質特に彈性及び粘性的性質に關する實驗的研究の結果に就いては既に數回に亙つて報告した．此處に報告する研究も其の續研究であつて，今回は特に水分の變化により土の彈性的性質が如何に變るかを詳細に研究し，今迄の研究に對する不足を補ひ，且又新に土の性質に關する研究を進める積りで行つたものである．

用ひた試料は赤土，泥土等約 70 種であつて種々な場所から採集したものである．

此の研究によつて新に知識を得た事は，土は水分の變化に對して三つの異なつた狀態を呈する事で，此等の各狀態に於ては彈性體，粘彈性體（或ひは準液體），液體等の性質を表はすのであつて，又各狀態の限界水量と土の組成分との間には一定の關係の存在する事を確めた．

以上の如き土の性質の變化は今回新に用ひた含有水分を表はす單位を用ふる事によつて一層明瞭となつた．而して性質の變化が急に起る限界水量は土の種類例へば赤土と沈泥との如きでは著しく相異してゐる事も知り得た．

土の收縮曲線は二本の直線によつて示され，其の直線の分岐點の水量は彈性的性質の變化を示す限界水量 R_c と全く同じである．

土中を傳播する彈性波の速度は其の水分の增加と共に減少し，或る水量以上の增加に對しては急激に減少する．而して更に水分が增加すれば或る水量以上にて再び縱波の速度は增加する．但し横波の速度は更に小となる．斯様に 2 つの限界水量を有するが，彈性波の速度より，密度を用ひて彈性常數を求めた結果，彈性常數も又速度の場合と同じ限界水量を境として變化してゐるのである．