

20. *The Vibration due to an Obliquely Incident Longitudinal Wave of Harmonic Type of a Surface Stratum Adhering Closely to the Subjacent Medium, and the Properties of its Resonance Conditions. (Second Paper.)*

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**1. Introduction.** In the present paper, which is a continuation of the previous one<sup>1)</sup>, we shall, with the aid of figures, deal with the vibrations of the surface-layer for various incidence angles  $\theta$  of a primary dilatational wave of harmonic type, and also study the resonance properties of that layer. The case<sup>2)</sup> in which the incidence angle of that wave becomes  $45^\circ$ , has, however, already been studied. The elasticity conditions dealt with in the present paper are [1] the surface-layer adheres closely to the subjacent medium at its bottom surface, the top surface of it being a free surface, [2] both media, the surface-layer and the subjacent medium, satisfy Cauchy's condition, [3] the densities of the materials are exactly the same for both media, [4] the ratio of the rigidity of the surface-layer to that of the subjacent medium is  $1/10$ . With these conditions, the vibrations of the surface-layer are studied, taking the incidence angle of the primary wave as in the following ten cases:

$$\theta = 90^\circ, 80^\circ, 70^\circ, 45^\circ, 30^\circ, 20^\circ, 15^\circ, 10^\circ, 5^\circ, 0^\circ.$$

**2. General Solution.** Now, when the dilatational wave expressed by

$$\phi_0 = \mathcal{U} \exp \left\{ i(fx - ry - pt) \right\}$$

is primarily incident on the bottom surface of the surface-layer, the following two waves are reflected at this surface in the subjacent medium:

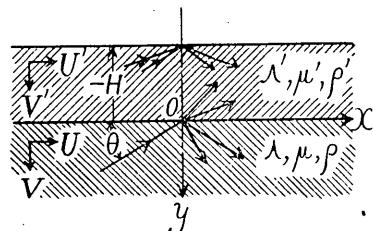


Fig. 1.

- 1) G. NISHIMURA, *B. E. R. I.*, 13 (1935), 540~554.  
G. NISHIMURA and T. TAKAYAMA, *B. E. R. I.*, 15 (1937), 394~440.
- 2) G. NISHIMURA and T. TAKAYAMA, *loc. cit.*

$$\phi = A \exp \left\{ i(fx + ry - pt) \right\}, \quad \psi = B \exp \left\{ i(fx + sy - pt) \right\},$$

the following four waves are also reflected and refracted from the surface-layer:

$$\phi' = C \exp \left\{ i(fx - r'y - pt) \right\} + D \exp \left\{ i(fx + r'y - pt) \right\},$$

$$\psi' = E \exp \left\{ i(fx - s'y - pt) \right\} + F \exp \left\{ i(fx + s'y - pt) \right\}.$$

Then the horizontal and vertical movements of the particle in the surface-layer ( $U'$ ,  $V'$ ), and also those in the subjacent medium ( $U$ ,  $V$ ) are expressed by

$$\left. \begin{aligned} U' &= ifC \exp \left\{ i(fx - r'y - pt) \right\} + ifD \exp \left\{ i(fx + r'y - pt) \right\} \\ &\quad - is'E \exp \left\{ i(fx - s'y - pt) \right\} + is'F \exp \left\{ i(fx + s'y - pt) \right\}, \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} V' &= -ir'C \exp \left\{ i(fx - r'y - pt) \right\} + ir'D \exp \left\{ i(fx + r'y - pt) \right\} \\ &\quad - ifE \exp \left\{ i(fx - s'y - pt) \right\} - ifF \exp \left\{ i(fx + s'y - pt) \right\}, \end{aligned} \right\}$$

$$\left. \begin{aligned} U &= if\mathfrak{A} \exp \left\{ i(fx - ry - pt) \right\} + ifA \exp \left\{ i(fx + ry - pt) \right\} \\ &\quad + isB \exp \left\{ i(fx + sy - pt) \right\}, \end{aligned} \right\} \quad (2)$$

$$\left. \begin{aligned} V &= -ir\mathfrak{A} \exp \left\{ i(fx - ry - pt) \right\} + irA \exp \left\{ i(fx + ry - pt) \right\} \\ &\quad - ifB \exp \left\{ i(fx + sy - pt) \right\}. \end{aligned} \right\}$$

The boundary-conditions at the top surface of the surface-layer  $y = -H$  are expressed as follows:

$$\lambda' \left( \frac{\partial U'}{\partial x} + \frac{\partial V'}{\partial y} \right) + 2\mu' \frac{\partial V'}{\partial y} = 0, \quad \mu' \left( \frac{\partial V'}{\partial x} + \frac{\partial U'}{\partial y} \right) = 0,$$

and those at its bottom surface  $y = 0$  are

$$U = U', \quad V = V',$$

$$\lambda \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) + 2\mu \frac{\partial V}{\partial y} = \lambda' \left( \frac{\partial U'}{\partial x} + \frac{\partial V'}{\partial y} \right) + 2\mu' \frac{\partial V'}{\partial y},$$

$$\mu \left( \frac{\partial V}{\partial x} + \frac{\partial U}{\partial y} \right) = \mu' \left( \frac{\partial V'}{\partial x} + \frac{\partial U'}{\partial y} \right).$$

Then, for satisfying these conditions,  $A, B, C, D, E, F$  are put in the following forms:

$$\left. \begin{aligned} A &= \frac{A_A}{A} \mathfrak{A}, & B &= \frac{A_B}{A} \mathfrak{A}, & C &= \frac{A_C}{A} \mathfrak{A}, \\ D &= \frac{A_D}{A} \mathfrak{A}, & E &= \frac{A_E}{A} \mathfrak{A}, & F &= \frac{A_F}{A} \mathfrak{A}, \end{aligned} \right\} \quad (3)$$

where

$$\begin{aligned} A = & 16 \frac{r's'}{f^2} \left( 1 - \frac{s'^2}{f^2} \right) \left[ -\mu^2 \left( \left( 1 - \frac{s^2}{f^2} \right)^2 + 4 \frac{rs}{f^2} \right) + \mu\mu' \left( 1 - \frac{s^2}{f^2} + 2 \frac{rs}{f^2} \right) \left( 3 - \frac{s'^2}{f^2} \right) \right. \\ & \left. - 2\mu'^2 \left( 1 + \frac{rs}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} \right) \right] \\ & + \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \left[ \mu^2 \left( 1 + \frac{r's'}{f^2} \right) \left( \left( 1 - \frac{s^2}{f^2} \right)^2 + 4 \frac{rs}{f^2} \right) \right. \\ & \left. - \mu\mu' \left\{ \left( 1 + \frac{s^2}{f^2} \right) \left( 1 + \frac{s'^2}{f^2} \right) \left( \frac{sr'}{f^2} + \frac{rs'}{f^2} \right) + 2 \left( 1 - \frac{s^2}{f^2} + 2 \frac{rs}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} + 2 \frac{r's'}{f^2} \right) \right\} \right. \\ & \left. + \mu'^2 \left( 1 + \frac{rs}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \right] \exp \{ i(r' + s')H \} \\ & + \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \left[ -\mu^2 \left( 1 - \frac{r's'}{f^2} \right) \left( \left( 1 - \frac{s^2}{f^2} \right)^2 + 4 \frac{rs}{f^2} \right) \right. \\ & \left. + \mu\mu' \left\{ \left( 1 + \frac{s^2}{f^2} \right) \left( 1 + \frac{s'^2}{f^2} \right) \left( \frac{sr'}{f^2} - \frac{rs'}{f^2} \right) + 2 \left( 1 - \frac{s^2}{f^2} + 2 \frac{rs}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} - 2 \frac{r's'}{f^2} \right) \right\} \right. \\ & \left. - \mu'^2 \left( 1 + \frac{rs}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \right] \exp \{ i(r' - s')H \} \\ & + \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \left[ -\mu^2 \left( 1 - \frac{r's'}{f^2} \right) \left( \left( 1 - \frac{s^2}{f^2} \right)^2 + 4 \frac{rs}{f^2} \right) \right. \\ & \left. + \mu\mu' \left\{ 2 \left( 1 - \frac{s^2}{f^2} + 2 \frac{rs}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} - 2 \frac{r's'}{f^2} \right) + \left( 1 + \frac{s^2}{f^2} \right) \left( 1 + \frac{s'^2}{f^2} \right) \left( \frac{rs'}{f^2} - \frac{r's}{f^2} \right) \right\} \right. \\ & \left. - \mu'^2 \left( 1 + \frac{rs}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \right] \exp \{ -i(r' - s')H \} \end{aligned}$$

3) Although the expressions (3) were already obtained in the previous paper, we rewrite them here, correcting misprints in that paper.

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$$\begin{aligned}
& + \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \left[ \mu^2 \left( 1 + \frac{r's'}{f^2} \right) \left\{ \left( 1 - \frac{s^2}{f^2} \right)^2 + 4 \frac{rs}{f^2} \right\} \right. \\
& + \mu\mu' \left\{ \left( 1 + \frac{s^2}{f^2} \right) \left( 1 + \frac{s'^2}{f^2} \right) \left( \frac{rs'}{f^2} + \frac{sr'}{f^2} \right) - 2 \left( 1 - \frac{s^2}{f^2} + 2 \frac{rs}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} + 2 \frac{r's'}{f^2} \right) \right\} \\
& \left. + \mu'^2 \left( 1 + \frac{rs}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \right] \exp \{ -i(r'+s')H \}, \quad (4)
\end{aligned}$$

$$\begin{aligned}
A_A = & 16 \frac{r's'}{f^2} \left( 1 - \frac{s'^2}{f^2} \right) \left[ \mu^2 \left\{ \left( 1 - \frac{s^2}{f^2} \right)^2 - 4 \frac{rs}{f^2} \right\} - \mu\mu' \left( 1 - \frac{s^2}{f^2} - 2 \frac{rs}{f^2} \right) \left( 3 - \frac{s'^2}{f^2} \right) \right. \\
& + 2\mu'^2 \left( 1 - \frac{rs}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} \right) \left. \right] \\
& + \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \left[ -\mu^2 \left( 1 + \frac{r's'}{f^2} \right) \left\{ \left( 1 - \frac{s^2}{f^2} \right)^2 - 4 \frac{rs}{f^2} \right\} \right. \\
& + \mu\mu' \left\{ \left( 1 + \frac{s^2}{f^2} \right) \left( 1 + \frac{s'^2}{f^2} \right) \left( \frac{r's}{f^2} - \frac{rs'}{f^2} \right) + 2 \left( 1 - \frac{s^2}{f^2} - 2 \frac{rs}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} + 2 \frac{r's'}{f^2} \right) \right\} \\
& \left. - \mu'^2 \left( 1 - \frac{rs}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \right] \exp \{ i(r'+s')H \} \\
& + \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \left[ \mu^2 \left( 1 - \frac{r's'}{f^2} \right) \left\{ \left( 1 - \frac{s^2}{f^2} \right)^2 - 4 \frac{rs}{f^2} \right\} \right. \\
& - \mu\mu' \left\{ \left( 1 + \frac{s^2}{f^2} \right) \left( 1 + \frac{s'^2}{f^2} \right) \left( \frac{sr'}{f^2} + \frac{s'r}{f^2} \right) + 2 \left( 1 - \frac{s^2}{f^2} - 2 \frac{rs}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} - 2 \frac{r's'}{f^2} \right) \right\} \\
& \left. + \mu'^2 \left( 1 - \frac{rs}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \right] \exp \{ i(r'-s')H \} \\
& + \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \left[ -\mu^2 \left( 1 + \frac{r's'}{f^2} \right) \left\{ \left( 1 - \frac{s^2}{f^2} \right)^2 - 4 \frac{rs}{f^2} \right\} \right. \\
& + \mu\mu' \left\{ \left( 1 + \frac{s^2}{f^2} \right) \left( 1 + \frac{s'^2}{f^2} \right) \left( \frac{rs'}{f^2} - \frac{r's}{f^2} \right) + 2 \left( 1 - \frac{s^2}{f^2} - 2 \frac{rs}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} + 2 \frac{r's'}{f^2} \right) \right\} \\
& \left. - \mu'^2 \left( 1 - \frac{rs}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \right] \exp \{ -i(r'+s')H \} \\
& + \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \left[ \mu^2 \left( 1 - \frac{r's'}{f^2} \right) \left\{ \left( 1 - \frac{s^2}{f^2} \right)^2 - 4 \frac{rs}{f^2} \right\} \right.
\end{aligned}$$

$$+ \mu\mu' \left\{ \left( 1 + \frac{s^2}{f^2} \right) \left( 1 + \frac{s'^2}{f^2} \right) \left( \frac{r's}{f^2} + \frac{rs'}{f^2} \right) - 2 \left( 1 - \frac{s^2}{f^2} - 2 \frac{rs}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} - 2 \frac{r's'}{f^2} \right) \right\} \\ + \mu'^2 \left( 1 - \frac{rs}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \exp \{-i(r' - s')H\}, \quad (5)$$

$$\begin{aligned} A_n = & 16 \frac{rr's'}{f^3} \left( 1 - \frac{s'^2}{f^2} \right) \left[ 4\mu^2 \left( 1 - \frac{s^2}{f^2} \right) - \mu\mu' \left( 3 - \frac{s^2}{f^2} \right) \left( 3 - \frac{s'^2}{f^2} \right) + 4\mu'^2 \left( 1 - \frac{s'^2}{f^2} \right) \right] \\ & + 2 \frac{r}{f} \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \left[ -2\mu^2 \left( 1 - \frac{s^2}{f^2} \right) \left( 1 + \frac{r's'}{f^2} \right) \right. \\ & \left. + \mu\mu' \left( 3 - \frac{s^2}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} + 2 \frac{r's'}{f^2} \right) - \mu'^2 \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \right] \exp \{i(r' + s')H\} \\ & + 2 \frac{r}{f} \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \left[ 2\mu^2 \left( 1 - \frac{s^2}{f^2} \right) \left( 1 - \frac{r's'}{f^2} \right) \right. \\ & \left. - \mu\mu' \left( 3 - \frac{s^2}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} - 2 \frac{r's'}{f^2} \right) + \mu'^2 \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \right] \exp \{i(r' - s')H\} \\ & + 2 \frac{r}{f} \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \left[ -2\mu^2 \left( 1 - \frac{s^2}{f^2} \right) \left( 1 + \frac{r's'}{f^2} \right) \right. \\ & \left. + \mu\mu' \left( 3 - \frac{s^2}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} + 2 \frac{r's'}{f^2} \right) - \mu'^2 \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \right] \exp \{-i(r' + s')H\} \\ & + 2 \frac{r}{f} \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \left[ 2\mu^2 \left( 1 - \frac{s^2}{f^2} \right) \left( 1 - \frac{r's'}{f^2} \right) \right. \\ & \left. - \mu\mu' \left( 3 - \frac{s^2}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} - 2 \frac{r's'}{f^2} \right) + \mu'^2 \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \right] \exp \{-i(r' - s')H\}, \end{aligned} \quad (6)$$

$$\begin{aligned} A_c = & 8 \frac{rs'}{f^2} \left( 1 + \frac{s^2}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} \right) \left[ -\mu^2 \left( 2 \frac{r's}{f^2} - 1 + \frac{s^2}{f^2} \right) + \mu\mu' \left( 2 \frac{r's}{f^2} - 1 + \frac{s'^2}{f^2} \right) \right] \\ & + 2 \frac{r}{f} \left( 1 + \frac{s^2}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \left[ \mu\mu' \left\{ \frac{s}{f} \left( 1 - \frac{s'^2}{f^2} \right) + 2 \frac{s'}{f} \right\} \right. \\ & \left. - \mu^2 \left\{ \frac{s'}{f} \left( 1 - \frac{s^2}{f^2} \right) + 2 \frac{s}{f} \right\} \right] \exp \{-i(r' - s')H\} \\ & + 2 \frac{r}{f} \left( 1 + \frac{s^2}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \left[ \mu\mu' \left\{ -\frac{s}{f} \left( 1 - \frac{s'^2}{f^2} \right) + 2 \frac{s'}{f} \right\} \right. \\ & \left. + \mu^2 \left\{ -\frac{s'}{f} \left( 1 - \frac{s^2}{f^2} \right) + 2 \frac{s}{f} \right\} \right] \exp \{-i(r' + s')H\}, \end{aligned} \quad (7)$$

$$\begin{aligned}
 A_d = & 8 \frac{rs'}{f^2} \left( 1 + \frac{s^2}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} \right) \left[ -\mu^2 \left\{ 2 \frac{r's}{f^2} + 1 - \frac{s^2}{f^2} \right\} + \mu\mu' \left\{ 2 \frac{r's}{f^2} + 1 - \frac{s'^2}{f^2} \right\} \right] \\
 & + 2 \frac{r}{f} \left( 1 + \frac{s^2}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \left[ -\mu\mu' \left\{ \frac{s}{f} \left( 1 - \frac{s'^2}{f^2} \right) + 2 \frac{s'}{f} \right\} \right. \\
 & \left. + \mu^2 \left\{ \frac{s'}{f} \left( 1 - \frac{s^2}{f^2} \right) + 2 \frac{s}{f} \right\} \right] \exp \{ i(r' + s')H \} \\
 & + 2 \frac{r}{f} \left( 1 + \frac{s^2}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \left[ \mu\mu' \left\{ \frac{s}{f} \left( 1 - \frac{s'^2}{f^2} \right) - 2 \frac{s'}{f} \right\} \right. \\
 & \left. + \mu^2 \left\{ \frac{s'}{f} \left( 1 - \frac{s^2}{f^2} \right) - 2 \frac{s}{f} \right\} \right] \exp \{ i(r' - s')H \}, \tag{8}
 \end{aligned}$$

$$\begin{aligned}
 A_x = & 8 \frac{rr'}{f^2} \left( 1 + \frac{s^2}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} \right) \left[ \mu^2 \left\{ \frac{s'}{f} \left( 1 - \frac{s^2}{f^2} \right) + 2 \frac{s}{f} \right\} - \mu\mu' \left\{ \frac{s}{f} \left( 1 - \frac{s'^2}{f^2} \right) + 2 \frac{s'}{f} \right\} \right] \\
 & + 2 \frac{r}{f} \left( 1 + \frac{s^2}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \left[ \mu\mu' \left( 1 - \frac{s'^2}{f^2} + 2 \frac{r's}{f^2} \right) \right. \\
 & \left. - \mu^2 \left( 1 - \frac{s^2}{f^2} + 2 \frac{r's}{f^2} \right) \right] \exp \{ -i(r' + s')H \} \\
 & + 2 \frac{r}{f} \left( 1 + \frac{s^2}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \left[ -\mu\mu' \left( 1 - \frac{s'^2}{f^2} - 2 \frac{r's}{f^2} \right) \right. \\
 & \left. + \mu^2 \left( 1 - \frac{s^2}{f^2} - 2 \frac{r's}{f^2} \right) \right] \exp \{ i(r' - s')H \}, \tag{9}
 \end{aligned}$$

$$\begin{aligned}
 A_r = & 8 \frac{rr'}{f^2} \left( 1 + \frac{s^2}{f^2} \right) \left( 1 - \frac{s'^2}{f^2} \right) \left[ \mu^2 \left\{ \frac{s'}{f} \left( 1 - \frac{s^2}{f^2} \right) - 2 \frac{s}{f} \right\} + \mu\mu' \left\{ \frac{s}{f} \left( 1 - \frac{s'^2}{f^2} \right) - 2 \frac{s'}{f} \right\} \right] \\
 & + 2 \frac{r}{f} \left( 1 + \frac{s^2}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 + 4 \frac{r's'}{f^2} \right\} \left[ \mu\mu' \left( 1 - \frac{s'^2}{f^2} - 2 \frac{r's}{f^2} \right) \right. \\
 & \left. - \mu^2 \left( 1 - \frac{s^2}{f^2} - 2 \frac{r's}{f^2} \right) \right] \exp \{ i(r' + s')H \} \\
 & + 2 \frac{r}{f} \left( 1 + \frac{s^2}{f^2} \right) \left\{ \left( 1 - \frac{s'^2}{f^2} \right)^2 - 4 \frac{r's'}{f^2} \right\} \left[ -\mu\mu' \left( 1 - \frac{s'^2}{f^2} + 2 \frac{r's}{f^2} \right) \right. \\
 & \left. + \mu^2 \left( 1 - \frac{s^2}{f^2} + 2 \frac{r's}{f^2} \right) \right] \exp \{ -i(r' - s')H \}. \tag{10}
 \end{aligned}$$

3. Now, for graphically studying the stationary vibrations of the particles in the surface-layer, the Poisson ratios of both media are assumed to be 1/4, and the ratio of the rigidity of the surface-layer to that of the subjacent medium 1/10, the densities of the materials being exactly the same for both media. Then, the horizontal and vertical vibrations in the surface-layer are generally expressed by

$$\frac{LU'}{2\pi\mathfrak{U}} \Big|_{x=0} = \sqrt{\alpha^2 + \beta^2} \sin \left\{ z + \tan^{-1} \frac{\beta}{\alpha} \right\} \cos \theta, \quad (11)$$

$$\frac{LV'}{2\pi\mathfrak{U}} \Big|_{x=0} = \sqrt{\alpha'^2 + \beta'^2} \sin \left\{ z + \tan^{-1} \frac{\beta'}{\alpha'} \right\} \cos \theta, \quad (12)$$

where

$$\begin{aligned} \alpha &= \left\{ (C'' - D'') \sin a + (C' + D') \cos a \right\} \\ &\quad + \frac{s'}{f} \left\{ -(E'' + F'') \sin b + (-E' + F') \cos b \right\}, \end{aligned} \quad (13)$$

$$\begin{aligned} \beta &= \left\{ (C' - D') \sin a - (C'' + D'') \cos a \right\} \\ &\quad + \frac{s'}{f} \left\{ -(E' + F') \sin b + (E'' - F'') \cos b \right\}, \end{aligned} \quad (14)$$

$$\begin{aligned} \alpha' &= \frac{r'}{f} \left\{ -(C'' + D'') \sin a + (-C' + D') \cos a \right\} \\ &\quad + \left\{ -(E'' - F'') \sin b - (E' + F') \cos b \right\}, \end{aligned} \quad (15)$$

$$\begin{aligned} \beta' &= \frac{r'}{f} \left\{ -(C' + D') \sin a + (C'' - D'') \cos a \right\} \\ &\quad + \left\{ -(E' - F') \sin b + (E'' + F'') \cos b \right\}, \end{aligned} \quad (16)$$

$$z = \frac{2\pi}{T} t, \quad a = \frac{r'}{f}, \quad b = \frac{s'}{f},$$

and the phase-differences such that  $\tan^{-1}(\beta/\alpha)$ ,  $\tan^{-1}(\beta'/\alpha')$  shall satisfy the conditions that

$$\sqrt{\alpha^2 + \beta^2} \sin \gamma = \beta, \quad \sqrt{\alpha^2 + \beta^2} \cos \gamma = \alpha, \quad \gamma = \tan^{-1}(\beta/\alpha), \quad (17)$$

$$\sqrt{\alpha'^2 + \beta'^2} \sin \gamma' = \beta', \quad \sqrt{\alpha'^2 + \beta'^2} \cos \gamma' = \alpha', \quad \gamma' = \tan^{-1}(\beta'/\alpha'). \quad (18)$$

Now the quantities  $s'/f$ ,  $r'/f$  in (13)~(16) are here merely varied with the two values  $\theta$ ,  $H/L$  for the present case, while  $C'$ ,  $C''$ ,  $D'$ ,  $D''$ ,  $E'$ ,  $E''$ ,  $F'$ ,  $F''$  are connected with  $C$ ,  $D$ ,  $E$ ,  $F$  by the expressions

$$\left. \begin{array}{l} C = C' + iC'', \\ E = E' + iE'', \end{array} \quad \begin{array}{l} D = D' + iD'', \\ F = F' + iF'', \end{array} \right\} \quad (19)$$

all transcendental functions, of which the variables for the present example are also  $\theta$  and  $H/L$ .

Now the quantities  $A, A_A, A_B, A_C, A_D, A_E, A_F$  expressed by (4)~(10) are next rewritten for the present practical example in order to facilitate the numerical calculations of constants  $A, B, C, D, E, F$ , as follows:

$$\begin{aligned} \frac{A}{\Re \mu^2} &= \left\{ \mathfrak{Q} + \Re \cos\left(\xi \frac{H}{L}\right) + \mathfrak{S} \cos\left(\eta \frac{H}{L}\right) \right\} \\ &\quad + i \left\{ \mathfrak{T} \sin\left(\xi \frac{H}{L}\right) + \mathfrak{W} \sin\left(\eta \frac{H}{L}\right) \right\}, \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{A_A}{\Re \mu^2} &= \left\{ \mathfrak{Q}_A + \Re_A \cos\left(\xi \frac{H}{L}\right) + \mathfrak{S}_A \cos\left(\eta \frac{H}{L}\right) \right\} \\ &\quad + i \left\{ \mathfrak{T}_A \sin\left(\xi \frac{H}{L}\right) + \mathfrak{W}_A \sin\left(\eta \frac{H}{L}\right) \right\}, \end{aligned} \quad (20)$$

$$\frac{A_B}{\Re \mu^2} = \left\{ \mathfrak{Q}_B + \Re_B \cos\left(\xi \frac{H}{L}\right) + \mathfrak{S}_B \cos\left(\eta \frac{H}{L}\right) \right\}, \quad (21)$$

$$\begin{aligned} \frac{A_C}{\Re \mu^2} &= \left\{ \mathfrak{Q}_C + \Re_C \cos\left(\xi \frac{H}{L}\right) + \mathfrak{S}_C \cos\left(\eta \frac{H}{L}\right) \right\} \\ &\quad + i \left\{ \mathfrak{T}_C \sin\left(\xi \frac{H}{L}\right) + \mathfrak{W}_C \sin\left(\eta \frac{H}{L}\right) \right\}, \end{aligned} \quad (22)$$

$$\begin{aligned} \frac{A_D}{\Re \mu^2} &= \left\{ \mathfrak{Q}_D + \Re_D \cos\left(\xi \frac{H}{L}\right) + \mathfrak{S}_D \cos\left(\eta \frac{H}{L}\right) \right\} \\ &\quad + i \left\{ \mathfrak{T}_D \sin\left(\xi \frac{H}{L}\right) + \mathfrak{W}_D \sin\left(\eta \frac{H}{L}\right) \right\}, \end{aligned} \quad (23)$$

$$\begin{aligned} \frac{A_E}{\Re \mu^2} &= \left\{ \mathfrak{Q}_E + \Re_E \cos\left(\xi \frac{H}{L}\right) + \mathfrak{S}_E \cos\left(\eta \frac{H}{L}\right) \right\} \\ &\quad + i \left\{ \mathfrak{T}_E \sin\left(\xi \frac{H}{L}\right) + \mathfrak{W}_E \sin\left(\eta \frac{H}{L}\right) \right\}, \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{A_F}{\Re \mu^2} &= \left\{ \mathfrak{Q}_F + \Re_F \cos\left(\xi \frac{H}{L}\right) + \mathfrak{S}_F \cos\left(\eta \frac{H}{L}\right) \right\} \\ &\quad + i \left\{ \mathfrak{T}_F \sin\left(\xi \frac{H}{L}\right) + \mathfrak{W}_F \sin\left(\eta \frac{H}{L}\right) \right\}, \end{aligned} \quad (25)$$

where  $\mathfrak{P}$ ,  $\mathfrak{Q}$ ,  $\mathfrak{R}$ ,  $\mathfrak{S}$ ,  $\mathfrak{T}$ ,  $\mathfrak{W}$ , etc. are functions of  $\theta$  and  $H/L$ , the numerical values of these quantities being calculated for various values of  $H/L$  and  $\theta$ , as shown in Table I. Therefore, by using Table I, the quantities  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $F$  for various values of  $\theta$  and  $H/L$  are numerically calculated, with results as given in Tables II~IX. The numerical values of constants  $A$ ,  $B$  are also shown in Tables II~IX, which are useful in studying the vibrations of particles in the subjacent medium.

Now, using expressions (11), (12), and with the aid of Tables II ~IX, we calculate the amplitudes of both the horizontal and vertical vibrations on the top and the bottom surfaces of the surface-layer for various values of  $\theta$  and  $H/L$ , as given in Tables X a~XVII b, the results being shown in Figs. 2 a~10 b.

From Tables X a~XVII b and the curves shown in Figs. 2 a~10 b, we obtain a number of facts helpful in studying the forced vibrations of the particles in the surface-layer. When a dilatational wave of harmonic type is obliquely incident on the bottom surface of the surface-layer, forced vibrations of the particles in it are generally excited. When the incidence angle of the primary wave is  $90^\circ$ , i. e., when it is vertically upward incident on the bottom surface of the surface-layer, merely vertical vibration is excited in it, because no distortional plane-wave is propagated in that layer in this case. When the incidence angle becomes zero, i. e., when the direction of propagation of the primary dilatational wave is parallel to the bottom surface of the surface-layer, there is no vibration in that layer. This may be seen from Tables X a, X b which show that all the constants, excepting  $A$ , become zero when  $\theta=0$ . In other words, when  $\theta=0$ , it is not possible for any bodily wave to be propagated in the surface-layer, and only surface-waves, such as Rayleigh- and Love-waves can be propagated in the surface-layer. When  $0 < \theta < 90^\circ$ , vertical and horizontal forced vibrations are excited in the surface-layer and the particles in it usually pursue elliptic orbital motions, because when the dilatational plane-wave is obliquely incident on the bottom surface of the surface-layer, usually not only longitudinal, but also transversal plane-waves are generated to be propagated in that layer.

Generally speaking, when a plane wave of harmonic dilatational type is obliquely incident on the bottom surface of the surface-layer, forced vibrations of particles in it are excited only for the incidence angle of this wave, such that

$$0 < \theta \leq 90^\circ,$$

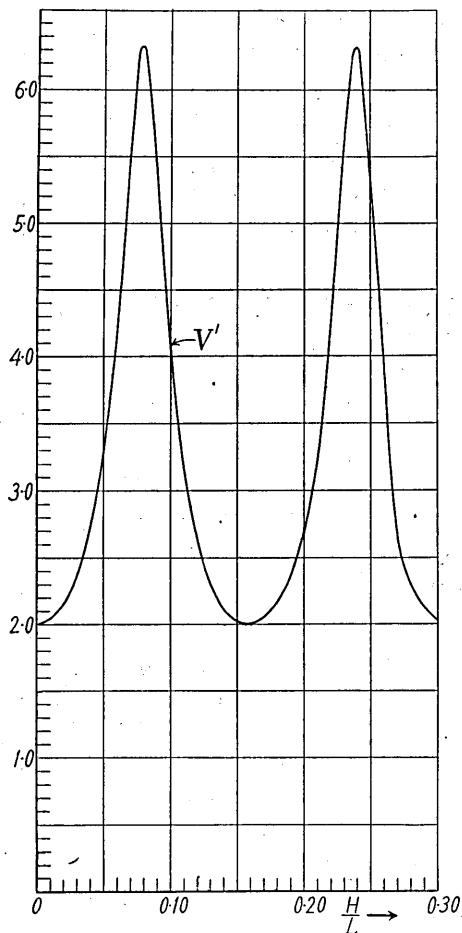


Fig. 2a. Displacement amplitude on the top surface when  $\theta=90^\circ$ .  $V'$  shows the vertical displacement curve.

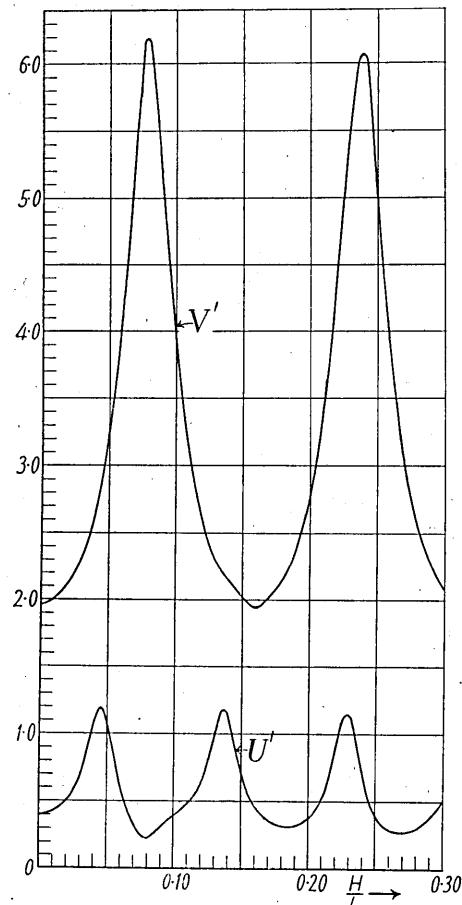


Fig. 3a.  $U'$  and  $V'$  show respectively the horizontal and vertical displacement curves on the top surface when  $\theta=80^\circ$ .

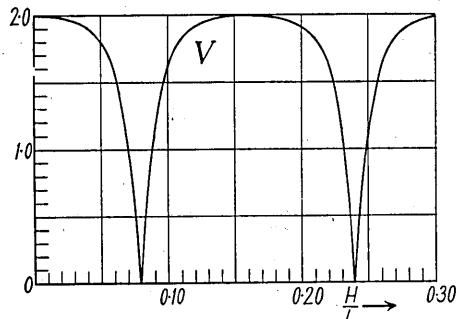


Fig. 2b. Displacement amplitude on the bottom surface when  $\theta=90^\circ$ .  $V$  shows horizontal displacement curve.

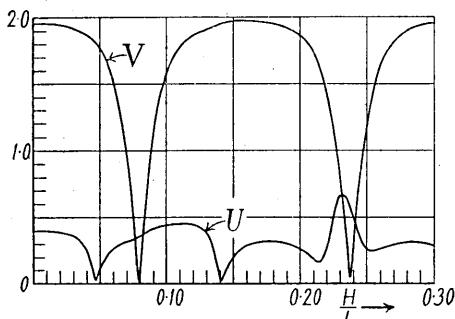


Fig. 3b.  $U$  and  $V$  show respectively the horizontal and vertical displacement curves on the bottom surface when  $\theta=80^\circ$ .

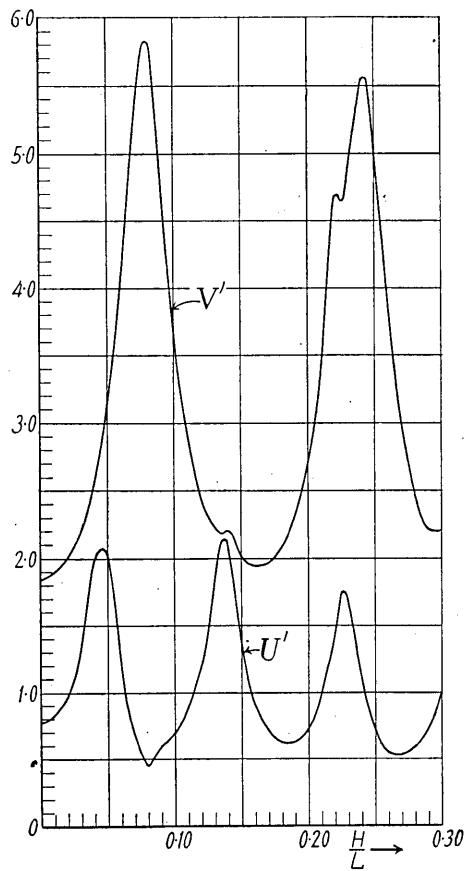


Fig. 4a.  $U'$  and  $V'$  show respectively the horizontal and vertical displacement curves on the top surface when  $\theta=70^\circ$ .

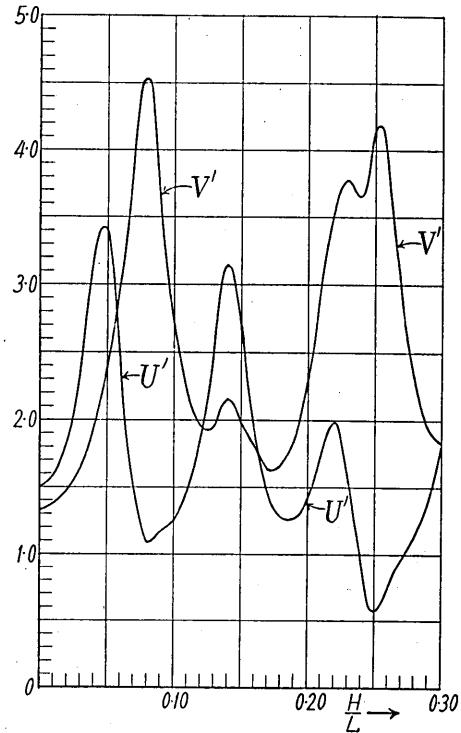


Fig. 5a.  $U'$  and  $V'$  show respectively the horizontal and vertical displacement curves on the top surface when  $\theta=45^\circ$ .

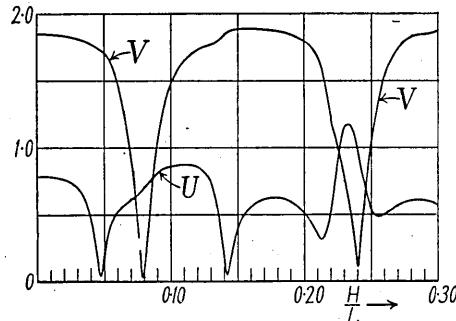


Fig. 4b.  $U$ ,  $V$  show respectively the horizontal and vertical displacement curves on the bottom surface when  $\theta=70^\circ$ .

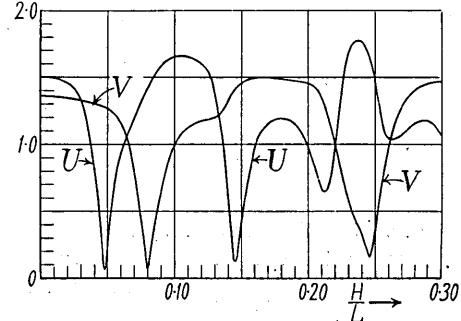


Fig. 5b.  $U$ ,  $V$  show respectively the horizontal and vertical displacement curves on the bottom surface when  $\theta=45^\circ$ .

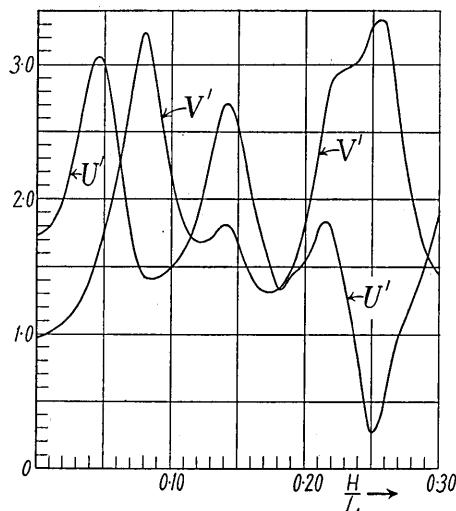


Fig. 6a.  $U'$ ,  $V'$  show respectively the horizontal and vertical displacement curves on the top surface when  $\theta=30^\circ$ .

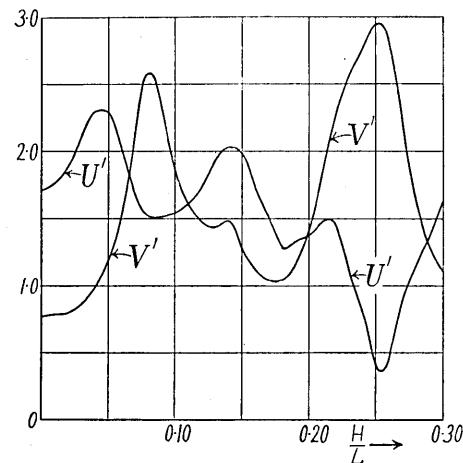


Fig. 7a.  $U'$ ,  $V'$  show respectively the horizontal and vertical displacement curves on the top surface when  $\theta=20^\circ$ .

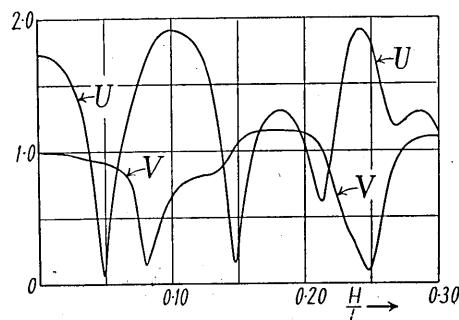


Fig. 6b.  $U$ ,  $V$  show respectively the horizontal and vertical displacement curves on the bottom surface when  $\theta=30^\circ$ .

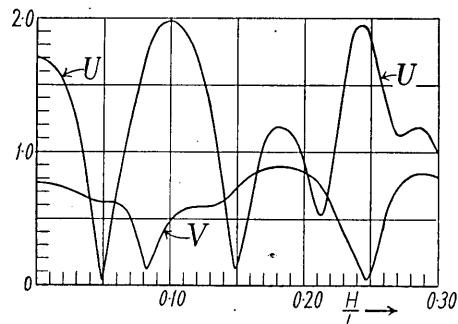


Fig. 7b.  $U$ ,  $V$  show respectively the horizontal and vertical displacement curves on the bottom surface when  $\theta=20^\circ$ .

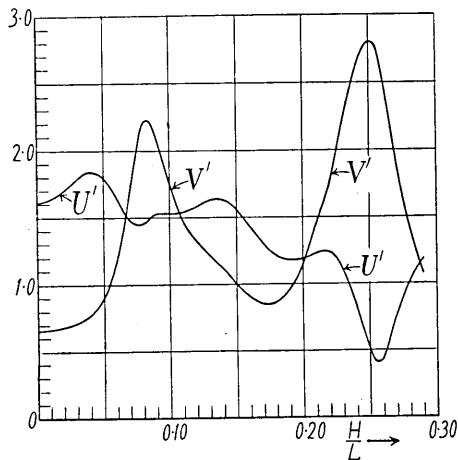


Fig. 8a.  $U'$ ,  $V'$  show respectively the horizontal and vertical displacement curves on the top surface when  $\theta=15^\circ$ .

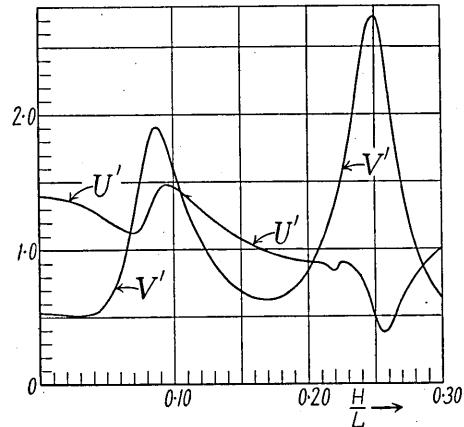


Fig. 9a.  $U'$ ,  $V'$  show respectively the horizontal and vertical displacement curves on the top surface when  $\theta=10^\circ$ .

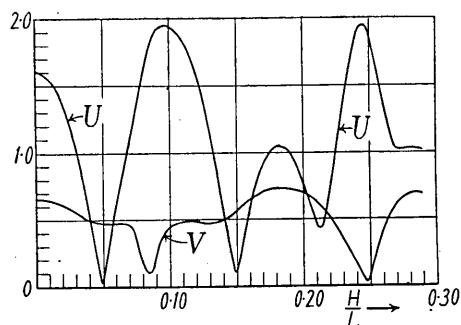


Fig. 8b.  $U$ ,  $V$  show respectively the horizontal and vertical displacement curves on the bottom surface when  $\theta=15^\circ$ .

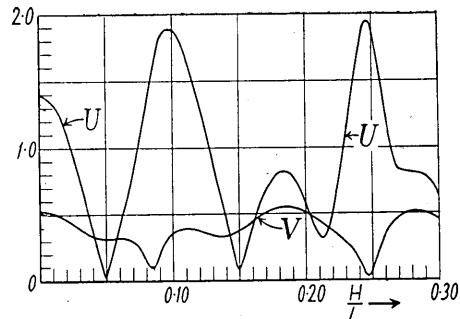


Fig. 9b.  $U$ ,  $V$  show respectively the horizontal and vertical displacement curves on the bottom surface when  $\theta=10^\circ$ .

while for the case when the primary dilatational plane-wave is propagated in the direction parallel to its bottom surface, no forced vibration is ever excited in its particles.

Now, upon comparing the curves shown in Figs. 2a~10b, it will be seen that both the horizontal and vertical amplitudes of vibration vary not only with the periods of the primary wave, but also with the incidence angle of that wave; the vertical amplitude on the top surface becomes small when the incidence angle of the primary wave becomes small, and the horizontal amplitude on the same surface usually becomes large. Moreover, it has a maximum when the incidence angle varies from zero to  $80^\circ$ .

From Figs. 8a, 8b, it will be seen that when the incidence angle of the primary wave is comparatively large, i. e., when it is between  $45^\circ$  and  $90^\circ$  for the present example, the curves showing respectively the horizontal and vertical amplitudes of vibration on the top surface for the variation in  $H/L$  become fairly regular. Even when it becomes relatively small, the curve of the vertical vibration on the top surface is also fairly regular for variation in  $H/L$ , while the curve of the horizontal vibration on the top surface becomes less regular than the vertical vibration, as will be seen by comparing Figs. 2a~10b. In contrast to this, the curve for variation in the vertical amplitude on the bottom surface with variation in  $H/L$ , which is very regular for large incidence angles, becomes very irregular when the incidence angle becomes small. For all incidence angles of the primary wave, variation in the horizontal amplitude on the bottom surface with variation in  $H/L$ , however, is not so regular as the former

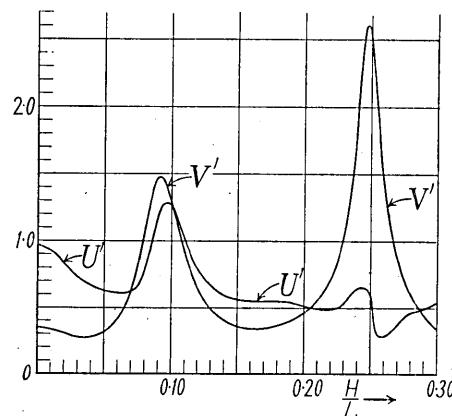


Fig. 10a.  $U'$ ,  $V'$  show respectively the horizontal and vertical displacement curves on the top surface when  $\theta=5^\circ$ .

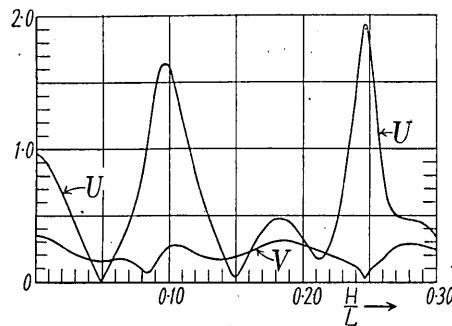


Fig. 10b.  $U$ ,  $V$  show respectively the horizontal and vertical displacement curves on the bottom surface when  $\theta=5^\circ$ .

amplitude for small incidence angles, but a certain similarity in the variation with variation in  $H/L$ , which is independent of any variation in the incidence angle, is found by comparing them with one another (compare Figs. 2a~10b).

In other words, the positions of the curves at which the amplitude of the vertical vibration on the bottom surface is nearly zero, become rather vague with decrease in the incidence angle, and the curve becomes flat. In contrast to this, sharpness in variation in the curve, giving nearly zero amplitude to the horizontal vibration on the bottom surface, is maintained fairly well notwithstanding the variation in the incidence angle (see Figs. 9b, 10b). The positions on the curves, showing nearly maximum amplitude for the vertical vibration on the top surface, where minimum or zero amplitude are shown for that on the bottom surface, are comparatively sharp notwithstanding the variation in the incidence angle, whereas the positions showing maximum horizontal amplitude on the top surface where the horizontal amplitude on the bottom surface becomes zero or almost nil, are comparatively clear and sharp when the incidence angle becomes comparatively large. However, when the incidence angle becomes small, these positions become not only vague, but also the curve becomes comparatively flat. The horizontal amplitude on the top surface on the positions where the horizontal amplitude on the bottom surface also becomes zero or almost nil, does not necessarily become maximum when the incidence angle becomes small. In other words, the amplitudes on the top surface vary not only with the period of the primary wave, but also with its incidence angle. The latter becomes, moreover, the main factor in varying the horizontal amplitude on the top surface when the incidence angle becomes small.

It is not easy, however, to show, physically, the reason for the occurrence of these phenomena in the amplitude curve with variations in  $\theta$  and  $H/L$ , but there is no doubt that these phenomena are caused by the interference effect of reflections and refractions of both the dilatational and distortional waves that are propagated in the surface-layer.

Next, before discussing the problem of resonance periods of the

surface-layer, some of the forced vibration modes in the surface-layer will be studied for several values of  $\theta$  and  $H/L$ , as shown in Table XVIII. Calculating both the amplitudes of horizontal and vertical vibrations of particles at

Table XVIII.

$\theta$	$H/L$
5°	0.05, 0.084, 0.15, 0.21, 0.246
15°	0.05, 0.084, 0.15
80°	0.05, 0.079, 0.14

various positions in the surface-layer, and also their phase-differences for the cases shown in Table XVIII, we obtain the results given in Table XIX and shown in Figs. 11a~21b. The values of  $H/L$  shown in Table XVIII correspond to the cases for which one of the horizontal and vertical amplitudes on the bottom surface of the surface-layer becomes nearly zero, as will be seen from Figs. 2a~10b.

From Figs. 11a~21b, it will be seen that the vibration modes of the horizontal or the vertical vibrations, for cases when the amplitudes of the horizontal or the vertical vibrations on the bottom surface become nearly zero, become approximately equal to the free vibration modes of the horizontal or the vertical vibrations that may be excited in the surface-layer, when it may be regarded as an isolated elastic solid. When the horizontal vibration modes become nearly the free vibration modes for a certain value of  $H/L$ , the vertical vibration modes differ entirely from the vertical free vibration modes<sup>4)</sup>, and conversely, when the vertical modes become nearly the free vibration modes for a certain value of  $H/L$ , the horizontal modes differ completely from the horizontal free vibration modes of the surface-layer. From these facts we may assume that the vibrations in the surface-layer are in resonance condition with the primary wave when the vibration mode in that layer is nearly equal to its free vibration mode, and the surface-layer has its resonance periods.

It will be seen from Figs. 2a~10b that even if the periods of the primary wave becomes synchronous with the resonance periods of the surface-layer, the vibration amplitudes in that layer do not become infinite as in the case of an isolated elastic pendulum, because the energy accumulated in the said layer is dispersed in the subjacent medium as wave energy. The amplitudes of the forced oscillation on the top surface of the surface-layer become relatively large at such periods as are synchronous with the resonance periods of vibration of that layer when the incidence angle of the primary wave becomes relatively large such that  $\theta = 45^\circ \sim 90^\circ$ , but when it becomes relatively small such that  $\theta = 5^\circ \sim 10^\circ$ , the horizontal amplitudes on the top surface at resonance conditions become large (not necessarily), but they become comparatively small compared with the amplitudes under non-resonance conditions (see Figs. 9a, 10a).

It may, therefore, be very difficult to obtain the resonance periods of horizontal vibrations from the resonance curve when the incidence

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4) The free vibration modes of the surface-layer, in this paper, mean those vibrations that may be excited in it in the case when it may be regarded as an elastic solid, isolated from the subjacent semi-infinite elastic solid.

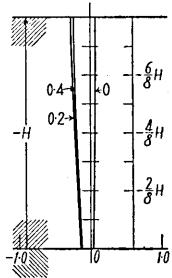


Fig. 11a. Horizontal vibration in the surface-layer when  $\theta=5^\circ$ ,  $H/L=0.05$ . Numerals on the curves show the values of  $t/T$ .

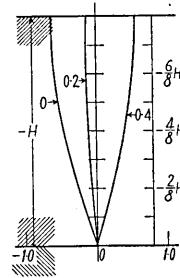


Fig. 11b. Vertical vibration in the surface-layer when  $\theta=5^\circ$ ,  $H/L=0.05$ . Numerals on the curves show the values of  $t/T$ .

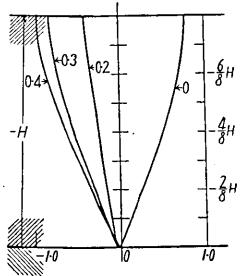


Fig. 12a. Horizontal vibration in the surface-layer when  $\theta=5^\circ$ ,  $H/L=0.084$ . Numerals on the curves show the values of  $t/T$ .

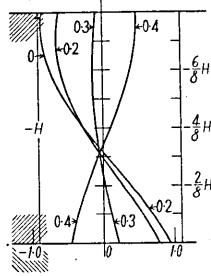


Fig. 12b. Vertical vibration in the surface-layer when  $\theta=5^\circ$ ,  $H/L=0.084$ . Numerals on the curves show the values of  $t/T$ .

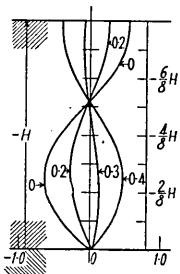


Fig. 13a. Horizontal vibration in the surface-layer when  $\theta=5^\circ$ ,  $H/L=0.15$ . Numerals on the curves show the values of  $t/T$ .

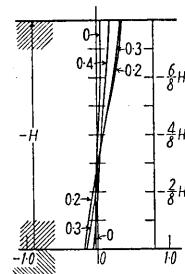


Fig. 13a. Vertical vibration in the surface-layer when  $\theta=5^\circ$ ,  $H/L=0.15$ . Numerals on the curves show the values of  $t/T$ .

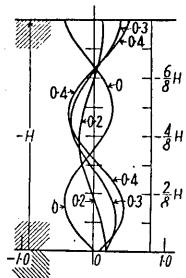


Fig. 14a. Horizontal vibration in the surface-layer when  $\theta=5^\circ$ ,  $H/L=0.21$ . Numerals on the curves show the values of  $t/T$ .

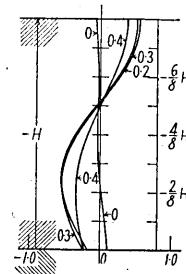


Fig. 14b. Vertical vibration in the surface-layer when  $\theta=5^\circ$ ,  $H/L=0.21$ . Numerals on the curves show the values of  $t/T$ .

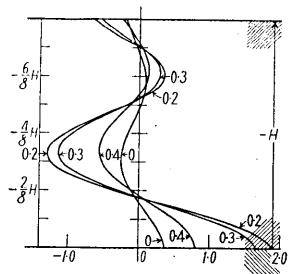


Fig. 15a. Horizontal vibration in the surface-layer when  $\theta=5^\circ$ ,  $H/L=0.246$ . Numerals on the curves show the values of  $t/T$ .

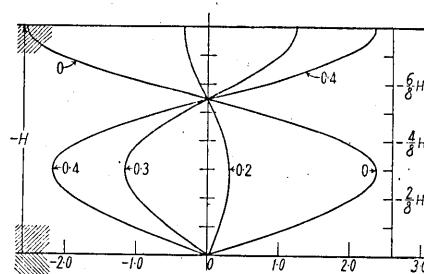


Fig. 15b. Vertical vibration in the surface-layer when  $\theta=5^\circ$ ,  $H/L=0.246$ . Numerals on the curves show the values of  $t/T$ .

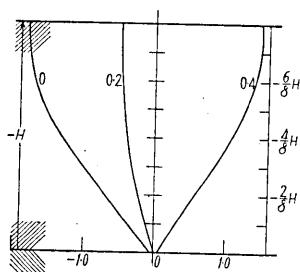


Fig. 16a. Horizontal vibration in the surface-layer when  $\theta=15^\circ$ ,  $H/L=0.05$ . Numerals on the curves show the values of  $t/T$ .

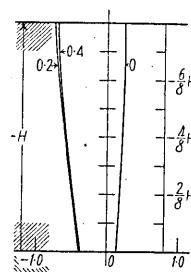


Fig. 16b. Vertical vibration in the surface-layer when  $\theta=15^\circ$ ,  $H/L=0.05$ . Numerals on the curves show the values of  $t/T$ .

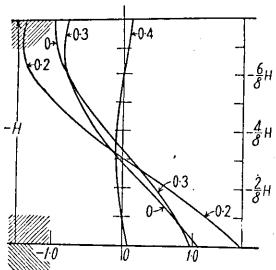


Fig. 17a. Horizontal vibration in the surface-layer when  $\theta=15^\circ$ ,  $H/L=0.084$ . Numerals on the curves show the values of  $t/T$ .

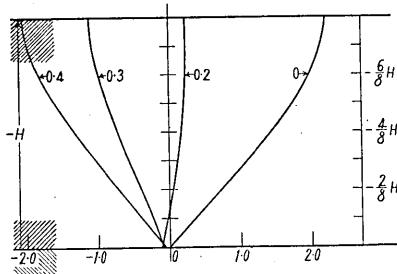


Fig. 17b. Vertical vibration in the surface-layer when  $\theta=15^\circ$ ,  $H/L=0.084$ . Numerals on the curves show the values of  $t/T$ .

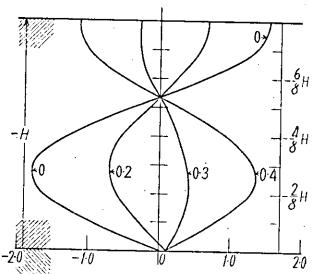


Fig. 18a. Horizontal vibration in the surface-layer when  $\theta=15^\circ$ ,  $H/L=0.15$ . Numerals on the curves show the values of  $t/T$ .

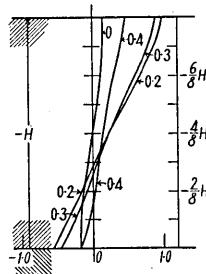


Fig. 18b. Vertical vibration in the surface-layer when  $\theta=15^\circ$ ,  $H/L=0.15$ . Numerals on the curves show the values of  $t/T$ .

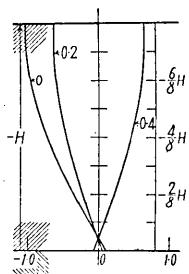


Fig. 19a. Vertical vibration in the surface-layer when  $\theta=80^\circ$ ,  $H/L=0.05$ . Numerals on the curves show the values of  $t/T$ .

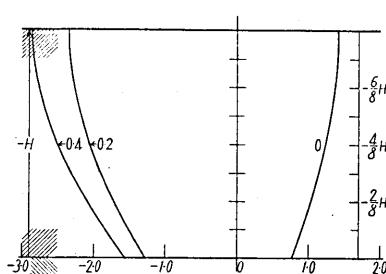


Fig. 19b. Horizontal vibration in the surface-layer when  $\theta=80^\circ$ ,  $H/L=0.05$ . Numerals on the curves show the values of  $t/T$ .

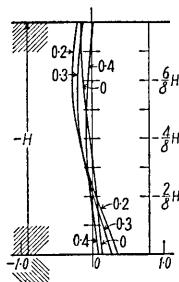


Fig. 20a. Horizontal vibration in the surface-layer when  $\theta=20^\circ$ ,  $H/L=0.079$ . Numerals on the curves show the values of  $t/T$ .

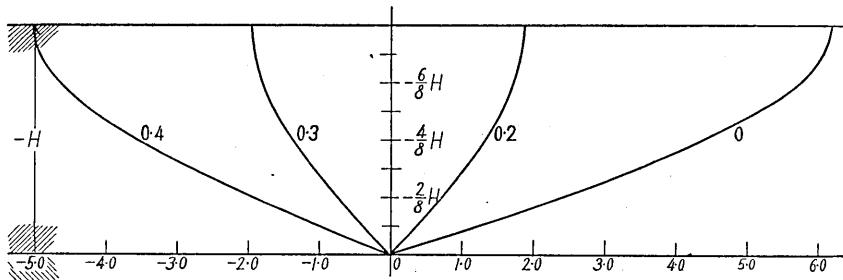


Fig. 20b. Vertical vibration in the surface-layer when  $\theta=20^\circ$ ,  $H/L=0.079$ . Numerals on the curves show the values of  $t/T$ .

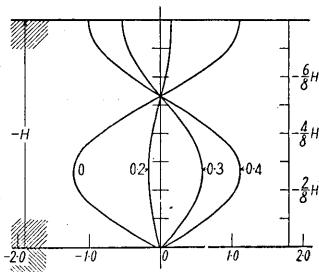


Fig. 21a. Horizontal vibration in the surface-layer when  $\theta=80^\circ$ ,  $H/L=0.14$ . Numerals on the curves show the values of  $t/T$ .

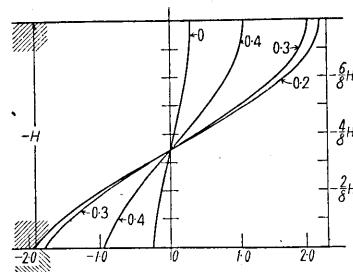


Fig. 21b. Vertical vibration in the surface-layer when  $\theta=80^\circ$ ,  $H/L=0.14$ . Numerals on the curves show the values of  $t/T$ .

angle of the primary wave is small, but for this case it may be possible to obtain from the horizontal amplitude curves on the bottom surface of the surface-layer, because the positions at which the horizontal vibration amplitudes at that surface become minimum and very small, may be clearly determined, as will be seen from Figs. 9a~10b. The vibration modes at these positions of  $H/L$ , of course, become nearly free vibration modes. When the incidence angle becomes relatively large, we may obtain the resonance periods of horizontal vibration fairly easily from the resonance curves for the top surface.

Obviously, the resonance periods of vertical vibration of the surface-layer may easily be obtained from the resonance curves on the top surface of it for all kinds of incidence angle, excepting  $\theta=0$ .

From these discussions, it will be seen that there exist in the surface-layer resonance periods of vibration for all kinds of incidence angles of primary waves, i. e., for  $0 < \theta \leq 90^\circ$ . The vibration amplitudes on the top surface of the surface-layer usually become large under resonance conditions when the incidence angle becomes relatively large, but they do not become large in its resonance condition when the incidence angle is relatively small, so that in this case, resonance periods have little effect on the vibration amplitudes of the surface-layer.

From Figs. 2a~10b, we obtain the resonance periods of the horizontal and vertical vibrations of the surface-layer for the gravest and second modes of vibration for various incidence angles as shown in Table XX. From Table XX, it will be seen that the resonance periods of vibration of the surface-layer usually differ in their horizontal and vertical vibrations, and the fundamental resonance period of horizontal vibration becomes longer than that of vertical vibration. The same features as in the gravest mode above described will also be found in

Table XX. Resonance Periods of the Horizontal  
and Vertical Vibrations.

( $T_1$ : the period of horizontal vibration,  $T_2$ : the period of vertical vibration.)

$\theta$	5°		10°		15°		20°		30°
	$T_1$	$T_2$	$T_1$	$T_2$	$T_1$	$T_2$	$T_1$	$T_2$	$T_1$
1st mode	$H$ $0\cdot050V_1$	$H$ $0\cdot085V_1$	$H$ $0\cdot050V_1$	$H$ $0\cdot085V_1$	$H$ $0\cdot049V_1$	$H$ $0\cdot082V_1$	$H$ $0\cdot049V_1$	$H$ $0\cdot082V_1$	$H$ $0\cdot049V_1$
2nd mode	$H$ $0\cdot150V_1$	$H$ $0\cdot247V_1$	$H$ $0\cdot150V_1$	$H$ $0\cdot247V_1$	$H$ $0\cdot149V_1$	$H$ $0\cdot250V_1$	$H$ $0\cdot149V_1$	$H$ $0\cdot247V_1$	$H$ $0\cdot148V_1$

(to be continued.)

Table XX. (*continued.*)

$\theta$	30°		45°		70°		80°		90°	
	$T_2$	$T_1$	$T_2$	$T_1$	$T_2$	$T_1$	$T_2$	$T_1$	$T_2$	
1st mode	$H$	non-existence	$H$	$0.078 V_1$						
	$0.081 V_1$	$0.048 V_1$	$0.080 V_1$	$0.047 V_1$	$0.080 V_1$	$0.045 V_1$	$0.079 V_1$			
2nd mode	$H$	non-existence	$H$	$0.240 V_1$						
	$0.248 V_1$	$0.145 V_1$	$0.245 V_1$	$0.142 V_1$	$0.242 V_1$	$0.137 V_1$	$0.241 V_1$			

the resonance periods of higher modes.

It may be worth noting that the resonance periods of vibration (horizontal and vertical) are not necessarily constant, but they have a tendency to become slightly short when the incidence angle becomes small. For the present example, however, from Table XX, we find that the resonance periods of the gravest mode for incidence angle 5° become short, only about 7·5 percent and 11 percent of those for incidence angle 80° for the vertical and horizontal vibrations respectively, while the same properties as in the gravest mode above described will also be seen in the resonance periods of higher modes. There is good reason, therefore, for saying that, from the practical point of view, the resonance periods of vibration of the surface-layer are nearly constant, and do not depend on the incidence angle of the primary wave at the bottom surface of the surface-layer, in other words, it has been ascertained that the resonance periods are practically the proper periods of the surface-layer. It was shown, moreover, in the other paper<sup>5)</sup> that the resonance periods of the surface-layer are also practically independent of the kinds of primary wave incident on its bottom surface. It is noteworthy, moreover, that although the vibration modes in the surface-layer under resonance conditions with the primary wave are nearly equal to the free vibration modes, as already discussed, the vibration amplitudes on the bottom surface under resonance conditions usually do not become zero.<sup>6)</sup> Moreover, the nodal points become unsteady, fluctuating within a small range, as will be seen from Figs. 11a~21b.

Now, from Figs. 2a~10b, we obtain Fig. 22, which shows variations in the amplitudes (horizontal and vertical) of vibrations on the

5) G. NISHIMURA and T. TAKAYAMA, *B. E. R. I.*, 17 (1939), 300.

6) When the incidence angle become 90°, the bottom surface becomes a nodal surface, as will be seen from Fig. 2a. This fact has already been shown by K. Sezawa and K. Kanai by methods different to ours.

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top surface of the surface-layer corresponding to the longest resonance periods, and those for the case when  $H/L=0$  with variation in the incidence angle of the primary dilatational wave. The case  $H/L=0$ , is, of course, a statical problem. It will be seen from Fig. 22 that, as

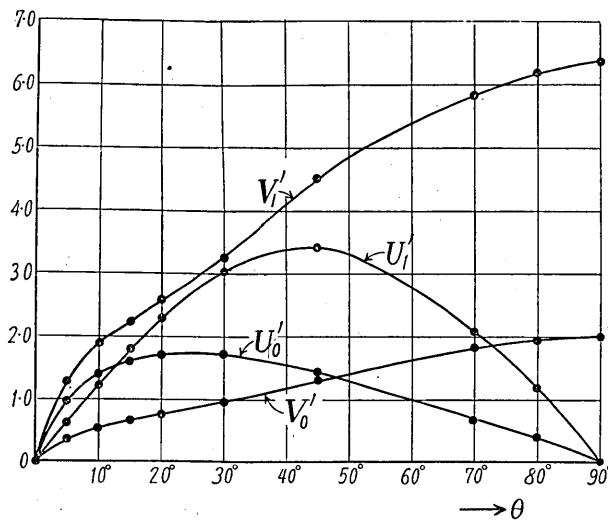


Fig. 22.  $U_0'$ : horizontal displacement on the top surface when  $H/L=0$ .

$V_0'$ : vertical " " " " "

$U_1'$ : horizontal " " " "  $H/L=0.048$  (gravest).

$V_1'$ : vertical " " " "  $H/L=0.079$ .

already discussed, when  $\theta=0$ , there are no statical and vibrational movements on the top surface of the surface-layer, and the horizontal amplitudes for both the gravest mode and the statical problem become maximum for certain values of the incidence angle, the vertical amplitudes for both cases also becoming largest when  $\theta=90^\circ$ .

Generally speaking, the vibrational amplitudes on the top surface of the surface-layer under resonance conditions usually become larger than the statical displacements on the same surface.

Table I.

$\theta$	0°	5°	10°	15°	20°	30°	45°	70°	80°	90°
$\Phi$	10 <sup>4</sup>	10 <sup>4</sup>	10 <sup>3</sup>	10 <sup>3</sup>	10 <sup>4</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>8</sup>	10 <sup>10</sup>	
$\Sigma$	-1.56333	-0.824757	-0.625134	8.83739	2.26256	8.64076	7.40496	3.77367	9.1851	
$\Re$	4.49836	6.39007	92.3213	138.240	21.7560	65.5850	68.0261	131.076	1215.97	308652
$\Im$	2.88095	3.62859	49.8749	74.0059	11.8209	38.0645	44.8222	102.341	983.141	252534
$\mathcal{G}$	-6.48071	-7.49659	-93.0499	-124.185	-17.8784	-47.4875	-43.8376	-76.9600	-702.766	-177462
$\mathcal{T}$	5.49371	5.09824	50.8591	54.5996	6.31772	10.7513	4.80220	1.45035	3.23534	
$\Sigma_A$	1.56333	2.41426	35.3557	51.6120	7.69093	19.2017	12.1020	4.28713	9.50538	
$\Re_A$	-4.49836	-3.12753	-20.3459	-10.9279	-0.30891	-1.43699	-15.0573	-87.2560	-946.408	-252534
$\Im_A$	-2.88095	-2.55366	-26.1781	-32.1856	-4.81406	-17.7647	-31.0615	-112.047	-1169.20	-308652
$\mathcal{G}_A$	6.48071	6.00609	59.6768	63.6429	7.30087	12.1421	5.20046	1.47692	3.25043	-177462
$\mathcal{T}_A$	-5.49371	-6.36343	-79.3012	-106.525	-15.4709	-42.0479	-40.5270	-75.5757	-699.504	
$\Sigma_B$		0.171102	4.20842	8.67970	1.73931	7.32913	9.17154	10.7019	51.0894	
$\Re_B$		0.819807	19.1603	37.0364	6.9738	27.0289	33.4488	44.9239	225.552	
$\Im_B$		1.14051	25.9001	48.0185	8.61856	30.4730	34.5154	44.8125	225.319	
$\Sigma_C$		0.190943	4.24545	7.60053	1.29965	4.01120	3.36645	1.84177	4.56415	
$\Re_C$		0.614290	13.9571	25.9162	4.66833	16.8272	20.8609	47.6382	455.523	116791
$\Im_C$		-0.0095939	-0.0034167	0.629518	0.264009	2.33350	5.74416	22.1415	230.262	60671.7
$\mathcal{G}_C$		-0.614290	-13.9571	-25.9162	-4.66833	-16.8272	-20.8609	-47.6382	-455.523	-116791
$\mathcal{T}_C$		0.0095939	0.0034167	-0.629518	-0.264009	-2.31350	-5.74416	-22.1415	-230.262	-60671.7

(to be continued.)

Table I. (continued.)

	$\theta$	0°	5°	10°	15°	20°	30°	45°	70°	80°	90°
	$\mathfrak{P}$	$10^4$	$10^4$	$10^3$	$10^3$	$10^4$	$10^4$	$10^5$	$10^8$	$10^{10}$	
$\Sigma_D$		0.307193	6.73198	11.7767	1.95352	5.58959	4.13626	1.92430	4.61502		
$\Re_D$		0.0113023	0.0040091	-0.733379	-0.305093	-2.61278	-6.22049	-22.5470	-231.336	-60671.7	
$\Im_D$		-0.521437	-11.89484	-22.2308	-4.03968	-14.8997	-19.2635	-46.7813	-453.409	-116791	
$\mathfrak{S}_D$		0.0113023	0.0040091	-0.733879	-0.305093	-2.61278	-6.22049	-22.5470	-231.336	-60671.7	
$\mathfrak{B}_D$		-0.521437	-11.89484	-22.2308	-4.03968	-14.8997	-19.2935	-46.7813	-453.409	-116791	
$\Sigma_E$		-0.0044662	-0.0015663	0.281901	0.114383	0.910555	1.79836	3.22850	16.9068		
$\Re_E$		-0.423785	-9.61017	-17.1231	-2.91571	-9.01960	-8.11769	-7.72848	-36.4214		
$\Im_E$		-0.228873	-5.16506	-9.4792	-1.67856	-5.73120	-6.10075	-7.26396	-35.8528		
$\mathfrak{S}_E$		0.433785	9.61017	17.1231	2.91571	9.01960	8.11769	7.72848	36.4214		
$\mathfrak{B}_E$		-0.228873	-5.16506	-9.47952	-1.67856	-5.73120	-6.10075	-7.26396	-35.8528		
$\Sigma_F$		0.242415	5.45293	9.95503	1.75020	5.86429	6.03094	6.82130	33.2912		
$\Re_F$		0.268629	6.06056	11.0510	1.93977	6.47262	6.60665	7.39702	36.0200		
$\Im_F$		0.368216	8.19019	14.6881	2.52307	7.98643	7.49609	7.58946	36.2524		
$\mathfrak{S}_F$		0.268629	6.06056	11.0510	1.93977	6.47262	6.60665	7.39702	36.0200		
$\mathfrak{B}_F$		-0.368216	-8.19019	-14.6881	-2.52307	-7.98643	-7.49609	-7.58946	-36.2524		
$\xi$		52.6979	52.7346	52.7947	52.8759	53.0910	53.4921	54.0999	54.2393	54.2836	
$\eta$		-14.9829	-14.9724	-15.9555	-14.9325	-14.8720	-14.7607	-14.5946	-14.5579	-14.5452	

Table II. ( $\theta = 90^\circ$ .)

$\frac{H}{L}$	A	B	C	D	E	F
0	-1.000		0.3162	-0.3162		
0.01	-0.9919 $-i\cdot 0\cdot 1268$		0.3190 $-i\cdot 0\cdot 04336$	-0.3109 $-i\cdot 0\cdot 08345$		
0.02	-0.9654 $-i\cdot 0\cdot 2609$		0.3281 $-i\cdot 0\cdot 08919$	-0.2934 $-i\cdot 0\cdot 1717$		
0.03	-0.9120 $-i\cdot 0\cdot 4102$		0.3463 $-i\cdot 0\cdot 1403$	-0.2583 $-i\cdot 0\cdot 2699$		
0.04	-0.8119 $-i\cdot 0\cdot 5838$		0.3805 $-i\cdot 0\cdot 1996$	-0.1924 $-i\cdot 0\cdot 3842$		
0.05	-0.6186 $-i\cdot 0\cdot 7868$		0.4467 $-i\cdot 0\cdot 2687$	-0.0652 $-i\cdot 0\cdot 1571$		
0.06	-0.2256 $-i\cdot 0\cdot 9742$		0.5809 $-i\cdot 0\cdot 3331$	0.1933 $-i\cdot 0\cdot 6403$		
0.07	0.5028 $-i\cdot 0\cdot 8645$		0.8300 $-i\cdot 0\cdot 2955$	0.6728 $-i\cdot 0\cdot 5689$		
0.078	0.9912 $-i\cdot 0\cdot 1322$					
0.07908	1.000 $+i\cdot 0\cdot 003$					
0.08	0.9930 $+i\cdot 0\cdot 1183$		0.9976 $+i\cdot 0\cdot 04039$	0.9954 $+i\cdot 0\cdot 07775$		
0.09	0.3441 $+i\cdot 0\cdot 9389$		0.7758 $+i\cdot 0\cdot 3210$	0.5684 $+i\cdot 0\cdot 6179$		
0.10	-0.3228 $+i\cdot 0\cdot 9465$		0.5478 $+i\cdot 0\cdot 3236$	0.1294 $+i\cdot 0\cdot 6229$		
0.11	-0.6659 $+i\cdot 0\cdot 7461$		0.4305 $+i\cdot 0\cdot 2551$	-0.09633 $+i\cdot 0\cdot 4910$		
0.12	-0.8359 $+i\cdot 0\cdot 5488$		0.3723 $+i\cdot 0\cdot 1876$	-0.2082 $+i\cdot 0\cdot 3612$		
0.13	-0.9248 $+i\cdot 0\cdot 3804$		0.3419 $+i\cdot 0\cdot 1301$	-0.2667 $+i\cdot 0\cdot 2503$		
0.14	-0.9721 $+i\cdot 0\cdot 2346$		0.3258 $+i\cdot 0\cdot 08024$	-0.2979 $+i\cdot 0\cdot 1544$		
0.15	-0.9947 $+i\cdot 0\cdot 1027$		0.3180 $+i\cdot 0\cdot 03508$	-0.3128 $+i\cdot 0\cdot 06751$		
0.16	-0.9997 $-i\cdot 0\cdot 02373$		0.3163 $-i\cdot 0\cdot 00811$	-0.3160 $-i\cdot 0\cdot 01562$		
0.17	-0.9885 $-i\cdot 0\cdot 1513$		0.3202 $-i\cdot 0\cdot 05173$	-0.3087 $-i\cdot 0\cdot 09958$		

Table III. ( $\theta = 80^\circ.$ )

$\frac{H}{L}$	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
0	-0.9537	0.2273	0.3126	-0.3080	-0.02507	0.04780
0.01	-0.9464 <i>-i</i> 0.1189	0.2237 <i>+i</i> 0.04016	0.3154 <i>-i</i> 0.04307	-0.3028 <i>-i</i> 0.08087	-0.02589 <i>+i</i> 0.00722	0.04584 <i>+i</i> 0.02002
0.02	-0.9223 <i>-i</i> 0.2437	0.2099 <i>+i</i> 0.08598	0.3246 <i>-i</i> 0.08863	-0.2860 <i>-i</i> 0.1661	-0.02923 <i>+i</i> 0.01600	0.03807 <i>+i</i> 0.04341
0.03	-0.8763 <i>-i</i> 0.3811	0.1701 <i>+i</i> 0.1438	0.3434 <i>-i</i> 0.1395	-0.2525 <i>-i</i> 0.2606	-0.04004 <i>+i</i> 0.02811	0.01455 <i>+i</i> 0.07398
0.04	-0.8010 <i>-i</i> 0.5452	0.04516 <i>+i</i> 0.1814	0.3796 <i>-i</i> 0.1973	-0.1923 <i>-i</i> 0.3706	-0.07752 <i>+i</i> 0.03216	-0.06293 <i>+i</i> 0.08997
0.045	-0.7387 <i>-i</i> 0.6549	-0.03551 <i>+i</i> 0.1151	0.4075 <i>-i</i> 0.2278	-0.1446 <i>-i</i> 0.4359	-0.1010 <i>+i</i> 0.00466	-0.1123 <i>+i</i> 0.04118
0.046	-0.7029 <i>-i</i> 0.6796	-0.04184 <i>+i</i> 0.09345				
0.047	-0.7009 <i>-i</i> 0.7046	-0.04322 <i>+i</i> 0.07177				
0.048	-0.6783 <i>-i</i> 0.7297	-0.05906 <i>+i</i> 0.05146				
0.049	-0.6533 <i>-i</i> 0.7545	-0.03316 <i>+i</i> 0.03372				
0.05	-0.6259 <i>-i</i> 0.7788	-0.02369 <i>+i</i> 0.01929	0.4440 <i>-i</i> 0.2604	-0.07233 <i>-i</i> 0.5078	-0.09135 <i>-i</i> 0.03336	-0.09872 <i>-i</i> 0.02762
0.06	-0.2196 <i>-i</i> 0.9734	0.04902 <i>+i</i> 0.0066	0.5724 <i>-i</i> 0.5239	0.1851 <i>-i</i> 0.6330	-0.04658 <i>-i</i> 0.04747	-0.03041 <i>-i</i> 0.04592
0.07	0.4997 <i>-i</i> 0.8659	0.0116 <i>+i</i> 0.0112	0.8147 <i>-i</i> 0.2886	0.6558 <i>-i</i> 0.5636	-0.02227 <i>-i</i> 0.04048	-0.01770 <i>-i</i> 0.03739
0.075	0.8713 <i>-i</i> 0.4890	-0.01354 <i>-i</i> 0.02820	0.9404 <i>-i</i> 0.1599	0.8995 <i>-i</i> 0.3152	-0.01162 <i>-i</i> 0.03427	-0.01488 <i>-i</i> 0.04358
0.078	0.9849 <i>-i</i> 0.1500	-0.01440 <i>-i</i> 0.06379				
0.079	0.9944 <i>-i</i> 0.02780	-0.01121 <i>-i</i> 0.07620	0.9811 <i>-i</i> 0.00243	0.9793 <i>-i</i> 0.1118	-0.00457 <i>-i</i> 0.02662	-0.00702 <i>-i</i> 0.05106
0.0795	0.9934 <i>+i</i> 0.03341	-0.00895 <i>-i</i> 0.08230				
0.08	0.9885 <i>+i</i> 0.0947	-0.0063 <i>-i</i> 0.0883	0.9787 <i>-i</i> 0.0394	0.9750 <i>+i</i> 0.0695	-0.0032 <i>-i</i> 0.0244	-0.0041 <i>-i</i> 0.0526
0.0805	0.9800 <i>+i</i> 0.1549	-0.00312 <i>-i</i> 0.09415				
0.085	0.7600 <i>+i</i> 0.6229	0.03992 <i>-i</i> 0.1345	0.8986 <i>+i</i> 0.2189	0.8223 <i>+i</i> 0.4169	0.00038 <i>-i</i> 0.01266	0.01418 <i>-i</i> 0.05538
0.09	0.8654 <i>+i</i> 0.9011	0.0990 <i>-i</i> 0.1462	0.7622 <i>+i</i> 0.3124	0.5606 <i>+i</i> 0.5987	-0.00021 <i>-i</i> 0.00239	0.0323 <i>-i</i> 0.0487
0.10	-0.2802 <i>+i</i> 0.9181	0.1835 <i>-i</i> 0.1061	0.5403 <i>+i</i> 0.3144	0.1336 <i>+i</i> 0.6061	-0.00519 <i>+i</i> 0.01176	0.05403 <i>-i</i> 0.02170
0.11	-0.6133 <i>+i</i> 0.7322	0.2199 <i>-i</i> 0.04186	0.4263 <i>+i</i> 0.2470	-0.08617 <i>+i</i> 0.4795	-0.01015 <i>+i</i> 0.02351	0.06053 <i>+i</i> 0.01051
0.12	-0.7793 <i>+i</i> 0.5524	0.2210 <i>+i</i> 0.03462	0.3711 <i>+i</i> 0.1804	-0.1941 <i>+i</i> 0.3558	-0.01825 <i>+i</i> 0.03963	0.05251 <i>+i</i> 0.05104

(to be continued.)

Table III. (*continued.*)

$\frac{H}{L}$	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
0·13	-0·8775 +i0·4099	0·1430 +i0·1225	0·3454 +i0·1252	-0·2508 +i0·2552	-0·04905 +i0·06142	-0·00314 +i0·1009
0·135	-0·9259 +i0·3427	0·04535 +i0·1112	0·3381 +i0·1039	-0·2735 +i0·2126	-0·08383 +i0·05425	-0·06925 +i0·09008
0·14	-0·9659 +i0·2583	-0·00195 +i0·01105	0·3290 +i0·0848	-0·2952 +i0·1668	-0·1015 +i0·01683	-0·1021 +i0·02052
0·141	-0·9706 +i0·2404	0·00270 -i0·00944				
0·142	-0·9741 +i0·2229	0·01081 -i0·02742				
0·143	-0·9767 +i0·2056	0·02112 -i0·04208				
0·145	0·9796 +i0·1729	0·04512 -i0·06255	0·3203 +i0·06301	-0·3084 +i0·1180	-0·08664 -i0·01187	-0·07217 -i0·03182
0·15	-0·9810 +i0·1002	0·1003 -i0·07602	0·3149 +i0·04005	-0·3142 +i0·07195	-0·06848 -i0·02003	-0·03633 -i0·04433
0·16	-0·9755 -i0·02685	0·1578 -i0·04857	0·3116 -i0·00459	-0·3159 -i0·01306	-0·04916 -i0·01743	0·00143 -i0·03301
0·17	-0·9606 -i0·1507	0·1765 -i0·01309	0·3149 -i0·04887	-0·3079 -i0·09671	-0·04222 -i0·01201	0·01437 -i0·01629
0·18	-0·0299 -i0·2824	0·1770 +i0·02056	0·3250 -i0·09565	-0·2880 -i0·1854	-0·04049 -i0·00761	0·01628 -i0·00118
0·19	-0·8729 -i0·4321	0·1630 +i0·05244	0·3452 -i0·1482	-0·2497 -i0·2855	-0·04222 -i0·00470	0·01010 +i0·01186
0·20	-0·7962 -i0·6103	0·1296 +i0·07983	0·3836 -i0·2098	-0·1773 -i0·4037	-0·04780 -i0·00474	-0·00612 +i0·02050
0·21	-0·5475 -i0·8255	0·06483 +i0·08352	0·4621 -i0·2816	-0·02943 -i0·5439	-0·05808 -i0·01449	-0·03704 +i0·01180
0·213	-0·4487 -i0·8874	0·03900 +i0·07037				
0·216	-0·3176 -i0·9463	0·01429 +i0·04376				
0·22	-0·08974 -i0·9957	0·00033 -i0·01885	0·6199 -i0·3290	0·2738 -i0·6452	-0·05606 -i0·05703	-0·05543 -i0·06364
0·225	0·2717 -i0·9475	0·07204 -i0·1051	0·7335 -i0·3054	0·5022 -i0·6062	-0·01390 -i0·08308	0·00991 -i0·1173
0·23	0·6387 -i0·7189	0·2012 -i0·05069	0·8439 -i0·2343	0·7291 -i0·4625	0·04753 -i0·05344	0·1130 -i0·07010
0·235	0·9181 -i0·3002	0·1764 +i0·08411	0·9389 -i0·1073	0·9128 -i0·2026	0·05361 +i0·01251	0·1113 +i0·03930
0·237	0·9712 -i0·07908	0·1306 +i0·1087				
0·238	0·9782 +i0·03740	0·1112 +i0·1172				
0·239	0·9709 +i0·1558	0·08290 +i0·1101				

(to be continued.)

Table III. (continued.)

$\frac{H}{L}$	A	B	C	D	E	F
0.24	0.9491 +i 0.2719	0.06152 +i 0.1035	0.9594 +i 0.0811	0.9433 +i 0.1673	0.01717 +i 0.04809	0.03801 +i 0.08141
0.245	0.6615 +i 0.7487	0.00187 +i 0.03267	0.8693 +i 0.2457	0.7618 +i 0.4835	-0.01723 +i 0.04900	-0.01567 +i 0.05990
0.25	0.2514 +i 0.9666	0.01179 -i 0.03641	0.7322 +i 0.3243	0.4944 +i 0.6313	-0.03486 +i 0.03727	-0.03037 +i 0.02615
0.26	-0.3662 +i 0.9171	0.08636 -i 0.08243	0.5204 +i 0.3120	0.08646 +i 0.6033	-0.04171 +i 0.01953	-0.01367 -i 0.00636
0.27	-0.6730 +i 0.7111	0.1385 -i 0.06657	0.4138 +i 0.2422	-0.1176 +i 0.4680	-0.04012 +i 0.01426	0.00444 -i 0.00666
0.28	-0.8248 +i 0.5204	0.1634 -i 0.03487	0.3611 +i 0.1760	-0.2185 +i 0.3412	-0.03963 +i 0.01502	0.01284 +i 0.00414
0.29	-0.9061 +i 0.3602	0.1680 +i 0.00174	0.3338 +i 0.1195	-0.2716 +i 0.2339	-0.04225 +i 0.01895	0.01166 +i 0.01971
0.30	-0.9537 +i 0.2215	0.1481 +i 0.04143	0.3197 +i 0.0699	-0.3008 +i 0.1402	-0.05126 +i 0.02500	-0.003765 +i 0.03841

Table IV. ( $\theta = 70^\circ$ .)

$\frac{H}{L}$	A	B	C	D	E	F
0	-0.8222	0.4235	0.3020	-0.2842	-0.0496	0.0919
0.01	-0.8168 -i 0.09563	0.4163 +i 0.07631	0.3048 -i 0.04217	-0.2795 -i 0.07334	-0.05121 +i 0.01378	0.08799 +i 0.03873
0.02	-0.8007 -i 0.1939	0.3893 +i 0.1630	0.3145 -i 0.08691	-0.2646 -i 0.1502	-0.05778 +i 0.03040	0.07251 +i 0.08372
0.03	-0.7766 -i 0.2973	0.3126 +i 0.2702	0.3347 -i 0.1367	-0.2360 -i 0.2339	-0.07843 +i 0.05269	0.02644 +i 0.1412
0.04	-0.7709 -i 0.4361	0.08248 +i 0.3354	0.3757 -i 0.1840	-0.1914 -i 0.3322	-0.1466 +i 0.05910	-0.1182 +i 0.1687
0.045	-0.7576 -i 0.5768	-0.05355 +i 0.2181	0.4046 -i 0.2130	-0.1572 -i 0.3992	-0.1890 +i 0.0108	-0.2090 +i 0.08074
0.047	-0.6821 -i 0.5392	-0.09038 +i 0.1168				
0.048	-0.6766 -i 0.5970	-0.07963 +i 0.08874				
0.0485	-0.6715 -i 0.6280	-0.07313 +i 0.02574				
0.05	-0.6495 -i 0.7552	-0.0501 +i 0.0433	0.4353 -i 0.2377	-0.0920 -i 0.4796	-0.1748 -i 0.0582	-0.1899 -i 0.0471
0.06	-0.2063 -i 0.9716	0.08534 +i 0.00966	0.5473 -i 0.2979	0.1608 -i 0.6086	-0.09255 -i 0.08868	-0.06070 -i 0.08972
0.07	0.4959 -i 0.8680	0.01992 +i 0.01888	0.7727 -i 0.2677	0.6113 -i 0.5456	-0.04593 -i 0.07612	-0.03273 -i 0.07366

(to be continued.)

Table IV. (continued.)

$\frac{H}{L}$	$A$	$B$	$C$	$D$	$E$	$F$
0.075	0.8522 $-i0.5173$	-0.02561 $-i0.05250$	0.8891 $-i0.1488$	0.8420 $-i0.3139$	-0.02596 $-i0.06442$	-0.02634 $-i0.08414$
0.078	0.9658 $-i0.2016$	-0.02711 $-i0.1176$				
0.079	0.9778 $-i0.08770$	-0.02125 $-i0.1403$				
0.0795	0.9783 $-i0.03035$	-0.01708 $-i0.1515$				
0.08	0.9754 $+i0.0265$	-0.01213 $-i0.1625$	0.9243 $+i0.03579$	0.9166 $+i0.04553$	-0.01034 $-i0.04603$	-0.00569 $-i0.09992$
0.085	0.7788 $+i0.5234$	0.07248 $-i0.2467$	0.8500 $+i0.2017$	0.7787 $+i0.3710$	-0.00358 $-i0.02438$	0.02832 $-i0.1041$
0.09	0.4256 $+i0.7935$	0.1807 $-i0.2684$	0.7238 $+i0.2877$	0.5387 $+i0.5436$	-0.00453 $-i0.00540$	0.06182 $-i0.09112$
0.10	-0.1610 $+i0.8379$	0.3357 $-i0.1944$	0.5185 $+i0.2887$	0.1446 $+i0.5582$	-0.01351 $+i0.02087$	0.1018 $-i0.04008$
0.11	-0.4665 $+i0.6935$	0.4012 $-i0.07646$	0.4142 $+i0.2245$	-0.05796 $+i0.4469$	-0.02272 $+i0.04250$	0.1130 $+i0.02013$
0.12	-0.6224 $+i0.5618$	0.4008 $+i0.06130$	0.3668 $+i0.1609$	-0.1553 $+i0.3401$	-0.03754 $+i0.07125$	0.09733 $+i0.09409$
0.13	-0.7447 $+i0.4862$	0.2651 $+i0.2133$	0.3526 $+i0.1119$	-0.2075 $+i0.2658$	-0.08962 $+i0.1095$	-0.00040 $+i0.1826$
0.135	-0.8436 $+i0.4433$	0.09854 $+i0.2027$	0.3508 $+i0.1007$	-0.2397 $+i0.2409$	-0.1487 $+i0.1023$	-0.1152 $+i0.1716$
0.14	-0.9419 $+i0.3317$	-0.00314 $+i0.03621$	0.3382 $+i0.09499$	-0.2830 $+i0.2008$	-0.1883 $+i0.04207$	-0.1891 $-i0.05557$
0.141	-0.9530 $+i0.3030$	0.00052 $-i0.00268$				
0.142	-0.9602 $+i0.2741$	0.01127 $-i0.03773$				
0.143	-0.9642 $+i0.2461$	0.02746 $-i0.06803$				
0.144	-0.9766 $+i0.2166$	0.04826 $-i0.09364$				
0.145	-0.9647 $+i0.1945$	0.06917 $-i0.1125$	0.3189 $+i0.07849$	-0.3101 $+i0.1416$	-0.1689 $-i0.01527$	-0.1457 $-i0.05181$
0.15	-0.9464 $+i0.09485$	0.1754 $-i0.1482$	0.3063 $+i0.05420$	-0.3177 $+i0.08567$	-0.1354 $-i0.03533$	-0.07685 $-i0.08418$
0.16	-0.9089 $-i0.03717$	0.2927 $-i0.09901$	0.2978 $+i0.00555$	-0.3151 $-i0.00559$	-0.09683 $-i0.03308$	0.00086 $-i0.06615$
0.17	-0.8817 $-i0.1495$	0.3317 $-i0.02960$	0.2995 $-i0.04055$	-0.30500 $-i0.08815$	-0.08261 $-i0.02293$	0.02817 $-i0.03342$
0.18	-0.8508 $-i0.2681$	0.3343 $+i0.03660$	0.3084 $-i0.08798$	-0.2862 $-i0.1740$	-0.07893 $-i0.01432$	0.03285 $-i0.003334$
0.19	-0.8028 $-i0.4066$	0.3086 $+i0.09983$	0.3274 $-i0.1410$	-0.2517 $-i0.2718$	-0.08208 $-i0.00849$	0.02137 $-i0.02291$

(to be continued.)

Table IV. (continued)

$\frac{H}{L}$	A	B	C	D	E	F
0.20	-0.7143 $-i0.5812$	0.24490 $+i0.1554$	0.3653 $-i0.2037$	-0.1850 $-i0.3908$	-0.09276 $-i0.00835$	-0.01010 $+i0.04074$
0.21	-0.5191 $-i0.8099$	0.1182 $+i0.1650$	0.4464 $-i0.2751$	-0.0404 $-i0.5354$	-0.1122 $-i0.02701$	-0.07077 $+i0.02427$
0.213	-0.4154 $-i0.8854$	0.06813 $+i0.1392$				
0.214	-0.3731 $-i0.9098$	0.05166 $+i0.1252$				
0.215	-0.3269 $-i0.9328$	0.03612 $+i0.1079$				
0.216	-0.2759 $-i0.9536$	0.02220 $+i0.08720$				
0.22	0.1912 $-i0.9805$	0.00293 $-i0.02852$	0.6831 $-i0.2943$	0.4245 $-i0.6072$	-0.07961 $-i0.1009$	-0.07339 $-i0.1147$
0.225	0.3328 $-i0.9012$	0.1280 $-i0.1620$	0.7045 $-i0.2473$	0.4901 $-i0.5343$	-0.02902 $-i0.1333$	0.01938 $-i0.1911$
0.23	0.6113 $-i0.6586$	0.3134 $-i0.09066$	0.7379 $-i0.1854$	0.6311 $-i0.3954$	0.05261 $-i0.08914$	0.1639 $-i0.1222$
0.235	0.8263 $-i0.3479$	0.3150 $+i0.09694$	0.8232 $-i0.1226$	0.7652 $-i0.2347$	0.07333 $-i0.00382$	0.1861 $+i0.02687$
0.24	0.9423 $+i0.1012$	0.1596 $+i0.1759$	0.8850 $+i0.00089$	0.8653 $+i0.03192$	0.03783 $+i0.06208$	0.09941 $+i0.1210$
0.241	0.9367 $+i0.2049$	0.1252 $+i0.1702$				
0.2415	0.9292 $+i0.2571$	0.1088 $+i0.1648$				
0.242	0.9186 $+i0.3090$	0.08185 $+i0.1385$				
0.245	0.7904 $+i0.5989$	0.02334 $+i0.09250$	0.8619 $+i0.1669$	0.7947 $+i0.3577$	-0.01716 $+i0.08407$	-0.00154 $+i0.1175$
0.25	0.4191 $+i0.9063$	0.00692 $-i0.04153$	0.7515 $+i0.2835$	0.5656 $+i0.5737$	-0.05707 $+i0.07299$	-0.04853 $+i0.06317$
0.26	-0.2463 $+i0.9261$	0.1421 $-i0.1582$	0.5255 $+i0.3094$	0.1258 $+i0.6068$	-0.07937 $+i0.04001$	-0.02882 $-i0.00850$
0.27	-0.5757 $+i0.7229$	0.2510 $-i0.1336$	0.4065 $+i0.2432$	-0.09943 $+i0.4755$	-0.07790 $+i0.02820$	0.00739 $-i0.01304$
0.28	-0.73302 $+i0.5356$	0.3035 $-i0.07205$	0.3496 $+i0.1756$	-0.2071 $+i0.3475$	-0.07703 $+i0.02895$	0.02502 $+i0.00735$
0.29	-0.8204 $+i0.3848$	0.3146 $-i0.00032$	0.3210 $+i0.1174$	-0.2637 $+i0.2405$	-0.08195 $+i0.03616$	0.02340 $+i0.03775$
0.30	-0.8850 $+i0.2578$	0.2780 $+i0.07706$	0.3076 $+i0.06549$	-0.2975 $+i0.1474$	-0.09895 $+i0.04731$	-0.00604 $+i0.07405$

Table V. ( $\theta = 30^\circ$ .)

$\frac{H}{L}$	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
0	0	0.5773	0.2062	-0.1062	-0.1232	0.1810
0.01	-0.02464 +i0.1142	0.5564 +i0.1387	0.2092 -i0.02737	-0.1055 -i0.01769	-0.1271 +i0.02098	0.1669 +i0.08498
0.02	-0.1145 +i0.2314	0.4826 +i0.2797	0.2192 -i0.05500	-0.1083 -i0.03568	-0.1410 +i0.04240	0.1169 +i0.1715
0.03	-0.3253 +i0.3209	0.3201 +i0.4016	0.2392 -i0.08078	-0.1177 -i0.05597	-0.1728 +i0.05981	0.004541 +i0.2444
0.04	-0.7053 +i0.1999	0.04808 +i0.3897	0.2686 -i0.09576	-0.1440 -i0.0980	-0.2286 +i0.04758	-0.1880 +i0.2187
0.045	-0.8677 -i0.04653	-0.06663 +i0.2779				
0.046	-0.8847 -i0.1104	-0.08021 +i0.2488				
0.047	-0.8948 -i0.1773	-0.08978 +i0.2187				
0.048	-0.8975 -i0.2460	-0.09527 +i0.1886				
0.049	-0.8925 -i0.3155	-0.09676 +i0.1590				
0.05	-0.8798 -i0.3846	-0.09445 +i0.1309	0.2879 -i0.1025	-0.1481 -i0.2110	-0.2556 -i0.02643	-0.2848 -i0.00152
0.06	-0.4570 -i0.8891	0.01603 -i0.00305	0.3178 -i0.1375	-0.03136 -i0.3491	-0.2077 -i0.08368	-0.1621 -i0.1490
0.07	0.1783 -i0.9840	-0.0012 -i0.00072	0.4224 -i0.1531	0.2266 -i0.3875	-0.1493 -i0.09242	-0.06484 -i0.1636
0.076	0.5716 -i0.7997	-0.05446 -i0.09146				
0.077	0.6257 -i0.7344	-0.05598 -i0.1155				
0.078	0.6756 -i0.6891	-0.05454 -i0.1412				
0.079	0.7181 -i0.6270	-0.04895 -i0.1673				
0.08	0.7536 -i0.5616	-0.03943 -i0.1933	0.5245 -i0.02454	0.4708 -i0.1336	-0.09812 -i0.06702	0.01239 -i0.1852
0.09	0.7574 +i0.03871	0.1908 -i0.3244	0.4501 +i0.1117	0.3678 +i0.1625	-0.08074 -i0.02035	0.1226 -i0.1480
0.10	0.5442 +i0.3351	0.3829 -i0.2248	0.3543 +i0.1254	0.1965 +i0.2341	-0.08654 +i0.01485	0.1787 -i0.05140
0.11	0.3662 +i0.5079	0.4454 -i0.06560	0.3062 +i0.1005	0.09806 +i0.2300	-0.09863 +i0.04283	0.1790 +i0.05305
0.12	0.1758 +i0.6788	0.4018 +i0.08942	0.2908 +i0.07665	0.04278 +i0.2270	-0.1197 +i0.07033	0.1266 +i0.1552
0.13	-0.1486 +i0.8394	0.2387 +i0.1848	0.2932 +i0.07061	-0.01111 +i0.2470	-0.1622 +i0.09245	-0.00489 +i0.2272
0.14	-0.6719 +i0.7327	0.01908 +i0.05976	0.2833 +i0.09469	-0.1177 +i0.2617	-0.2276 +i0.07320	-0.1963 +i0.1500

(to be continued.)

Table V. (continued.)

$\frac{H}{L}$	A	B	C	D	E	F
0·144	-0·8025 +i0·5262	-0·00532 -i0·06317				
0·146	-0·8786 +i0·4254	0·01331 -i0·1255				
0·147	-0·8916 +i0·3629	0·02758 -i0·1538				
0·148	-0·8968 +i0·3044	0·04487 -i0·1797				
0·15	-0·8893 +i0·1938	0·08846 -i0·2226	0·2271 +i0·1027	-0·2344 +i0·1774	-0·2429 +i0·00174	-0·1998 -i0·07383
0·16	-0·6751 -i0·1386	0·3294 -i0·2579	0·1818 +i0·06240	-0·2598 +i0·06241	-0·2050 -i0·03208	-0·04908 -i0·1458
0·17	-0·4837 -i0·2051	0·4709 -i0·1400	0·1655 +i0·01363	-0·2485 -i0·01730	-0·1765 -i0·02960	0·05003 -i0·1007
0·18	-0·4008 -i0·1970	0·5165 +i0·00808	0·1636 -i0·03333	-0·2366 -i0·08134	-0·1654 -i0·01790	0·08553 -i0·02855
0·19	-0·4036 -i0·2023	0·4897 +i0·1599	0·1718 -i0·08358	-0·2255 -i0·1523	-0·1676 -i0·00718	0·04253 +i0·07230
0·20	-0·4682 -i0·2929	0·3762 +i0·3003	0·1987 -i0·1434	-0·1995 -i0·2517	-0·1821 -i0·00530	0·00753 +i0·09436
0·21	-0·5062 -i0·5714	0·1419 +i0·3449	0·2691 -i0·2013	-0·1081 -i0·3818	-0·2030 -i0·02815	-0·1103 +i0·06938
0·211	-0·4688 -i0·6285	0·1165 +i0·3389				
0·212	-0·4492 -i0·6725	0·0856 +i0·3276				
0·214	-0·3924 -i0·7617	0·03905 +i0·2951				
0·22	-0·07688 -i0·9652	-0·04449 +i0·1374	0·3967 -i0·1951	0·1457 -i0·4428	-0·1764 -i0·08044	-0·1278 -i0·09473
0·23	0·5740 -i0·8049	0·07518 -i0·04370	0·4591 -i0·1112	0·3479 -i0·2975	-0·1065 -i0·07880	0·03716 -i0·1544
0·24	0·9104 -i0·3068	0·1595 +i0·01582	0·4841 -i0·05295	0·4423 -i0·1281	-0·06868 -i0·02763	0·1331 -i0·04337
0·244	0·9662 -i0·07292	0·1343 +i0·04859				
0·246	0·9754 +i0·05278	0·1099 +i0·05677				
0·248	0·9682 +i0·1106	0·07977 +i0·05600				
0·25	0·9415 +i0·3181	0·04679 +i0·4388	0·5256 +i0·03524	0·5057 +i0·1055	-0·07409 +i0·03312	0·0863 +i0·07368
0·26	0·4692 +i0·8261	-0·00751 -i0·1749	0·4687 +i0·1856	0·3438 +i0·4051	-0·1220 +i0·06463	-0·01934 +i0·05384
0·27	-0·04622 +i0·7714	0·2194 -i0·2934	0·3155 +i0·2157	0·03802 +i0·4373	-0·1541 +i0·05072	0·00292 -i0·01169

(to be continued.)

Table V. (*continued.*)

$\frac{H}{L}$	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
0.28	-0.2458 +i0.6067	0.3937 -i0.1885	0.2277 +i0.1559	-0.1198 +i0.3248	-0.1599 +i0.04303	0.05275 +i0.01094
0.29	-0.3644 +i0.5353	0.4391 -i0.02980	0.1945 +i0.09311	-0.1870 +i0.2245	-0.1677 +i0.04806	0.05469 +i0.07464
0.30	-0.5442 +i0.4949	0.3725 +i0.1192	0.1881 +i0.03830	-0.2278 +i0.1410	-0.1896 +i0.05523	-0.00687 +i0.1389

Table VI. ( $\theta = 20^\circ.$ )

$\frac{H}{L}$	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
0	0.07165	0.4836	0.1739	-0.06672	-0.1250	0.1734
0.01	0.02354 +i0.1900	0.4553 +i0.1385	0.1760 -i0.01719	-0.06824 -i0.00711	-0.1282 +i0.01355	0.1540 +i0.08821
0.02	-0.1335 +i0.3542	0.3644 +i0.2620	0.1825 -i0.03329	-0.07398 -i0.01512	-0.1388 +i0.02537	0.09150 +i0.1661
0.03	-0.4225 +i0.4207	0.2028 +i0.3322	0.1926 -i0.04643	-0.08737 -i0.02832	-0.01579 +i0.03074	-0.02102 +i0.2066
0.04	-0.7833 +i0.237	0.0114 +i0.2783	0.2017 -i0.0560	-0.1092 -i0.0607	-0.1816 +i0.0200	-0.1569 +i0.1567
0.048	-0.9352 -i0.1390	-0.06730 +i0.1428				
0.049	-0.9353 -i0.1949	-0.06878 +i0.1248				
0.05	-0.9323 -i0.2489	-0.06846 +i0.1074	0.2058 -i0.07216	-0.1177 -i0.1330	-0.1914 -i0.00941	-0.2134 +i0.01194
0.051	-0.9239 -i0.3039	-0.06649 +i0.09088				
0.06	-0.6785 -i0.7343	-0.01121 +i0.00406	0.2247 -i0.1103	-0.06430 -i0.2383	-0.1770 -i0.03681	-0.1519 -i0.1029
0.07	-0.1522 -i0.9879	-0.01294 -i0.00509	0.3034 -i0.1450	0.1059 -i0.3154	-0.1506 -i0.04984	-0.07770 -i0.1431
0.08	0.5186 -i0.7901	-0.05228 -i0.1494	0.4159 -i0.0683	0.3411 -i0.1807	-0.1180 -i0.04404	0.00057 -i0.1786
0.082	0.6173 -i0.6774	-0.03095 -i0.1916				
0.084	0.6892 -i0.5517	+0.00247 -i0.2277				
0.09	0.7562 -i0.1802	0.1379 -i0.2721	0.3888 +i0.06242	0.3238 +i0.07606	-0.1006 -i0.01692	0.1184 -i0.1458
0.10	0.6252 +i0.2569	0.3091 -i0.1792	0.3133 +i0.09688	0.1973 +i0.1710	-0.1022 +i0.00775	0.1753 -i0.04245
0.11	0.4183 +i0.5477	0.3501 -i0.02933	0.2691 +i0.09091	0.1106 +i0.1907	-0.1114 +i0.02669	0.1639 +i0.06513
0.12	0.1322 +i0.7719	0.2854 +i0.09704	0.2478 +i0.08278	0.05036 +i0.2005	-0.1277 +i0.04162	0.09454 +i0.1529

(to be continued.)

Table VI. (*continued.*)

$\frac{H}{L}$	A	B	C	D	E	F
0·13	-0·2863 +i0·8691	0·1392 +i0·1374	0·2336 +i0·08399	-0·01221 +i0·2119	-0·1529 +i0·04766	-0·02967 +i0·1845
0·14	-0·7459 +i0·6645	0·00589 +i0·02160	0·2091 +i0·09156	-0·09441 +i0·2032	-0·1799 +i0·03404	-0·1540 +i0·1043
0·148	-0·9128 +i0·2941	0·02459 -i0·1352				
0·149	-0·9141 +i0·2462	0·03627 -i0·1519				
0·15	-0·9114 +i0·1996	0·04959 -i0·1673	0·1702 +i0·08446	-0·1692 +i0·1447	-0·1870 +i0·00521	-0·1583 -i0·04503
0·151	-0·9049 +i0·1548	0·06430 -i0·1811				
0·16	-0·7409 -i0·1326	0·2242 -i0·2272	0·1383 +i0·05479	-0·2004 +i0·06928	-0·1733 -i0·01322	-0·05696 -i0·1192
0·17	-0·5288 -i0·2196	0·3679 -i0·1506	0·1227 +i0·01676	-0·2041 +i0·0034	-0·1585 -i0·01539	0·03670 -i0·09776
0·18	-0·4175 -i0·1878	0·4308 -i0·01593	0·1186 -i0·02266	-0·1981 -i0·05259	-0·1511 -i0·00991	0·08032 -i0·03381
0·19	-0·4219 -i0·1490	0·4131 +i0·1318	0·1237 -i0·06531	-0·1908 -i0·1129	-0·1518 -i0·00372	0·07188 +i0·03692
0·20	-0·5185 -i0·2012	0·3087 +i0·2590	0·1436 -i0·1147	-0·1704 -i0·1923	-0·1597 -i0·00265	0·01206 +i0·08561
0·21	-0·5913 -i0·4631	0·1205 +i0·2965	0·1945 -i0·1659	-0·1012 -i0·2951	-0·1688 -i0·01474	-0·08138 +i0·06484
0·212	-0·5783 -i0·5440	0·07947 +i0·2839				
0·214	-0·5470 -i0·6309	0·04200 +i0·2692				
0·22	-0·3275 -i0·8786	-0·03771 +i0·1643	0·2854 -i0·1843	0·06166 -i0·3636	-0·1601 -i0·03944	-0·1136 -i0·05272
0·23	0·3346 -i0·9421	-0·00564 +i0·00457	0·3732 -i0·1413	0·2589 -i0·3054	-0·1259 -i0·04726	-0·00507 -0·1323
0·24	0·8735 -i0·4672	0·06599 -i0·00249	0·4298 -i0·06854	0·3970 -i0·1491	-0·09671 -i0·02357	0·1034 -i0·06770
0·246	0·9944 -i0·02756	0·04600 +i0·01734				
0·248	0·9890 +i0·1312	0·02853 +i0·01561				
0·25	0·9568 +i0·2901	0·00824 +i0·0·620	0·4611 +i0·04103	0·4522 +i0·09027	-0·09420 +i0·01477	0·08973 +i0·04408
0·26	0·4244 +i0·8440	-0·02309 -i0·01572	0·3923 +i0·1768	0·2963 +i0·3490	-0·1219 +i0·03791	0·0000 +i0·05092
0·27	-0·1164 +i0·7666	0·1651 -i0·2578	0·2503 +i0·1983	0·02964 +i0·3737	-0·1436 +i0·03112	0·00600 -i0·00450
0·28	-0·3035 +i0·5937	0·3233 -i0·1614	0·1707 +i0·1410	-0·1085 +i0·2705	-0·1485 +i0·02520	0·04912 +i0·01341
0·29	-0·4204 +i0·5345	0·3553 -i0·01063	0·1429 +i0·08282	-0·1619 +i0·1772	-0·1534 +i0·02657	0·04661 +i0·07150
0·30	-0·6117 +i0·4875	0·2787 +i0·1130	0·1390 +i0·03534	-0·1859 +i0·1012	-0·1650 +i0·02777	-0·01440 +i0·1194

Table VII. ( $\theta=15^\circ.$ )

$\frac{H}{L}$	A	B	C	D	E	F
0	0.03844	0.4240	0.1545	-0.05061	-0.1191	0.1615
0.01	-0.02862 +i0.2279	0.3896 +i0.1367	0.1554 -i0.00981	-0.05248 -i0.00380	-0.1211 +i0.0073	0.1384 +i0.0871
0.02	-0.2282 +i0.3959	0.2888 +i0.2429	0.1578 -i0.01879	-0.05872 -i0.00934	-0.1273 +i0.01278	0.07023 +i0.1535
0.03	-0.5332 +i0.4200	0.1391 +i0.2788	0.1600 -i0.02681	-0.07052 -i0.02056	-0.1367 +i0.01382	-0.03203 +i0.1719
0.04	-0.8368 +i0.2160	-0.00187 +i0.2134	0.1597 -i0.03669	-0.08626 -i0.04606	-0.1460 +i0.00796	-0.1304 +i0.1184
0.048	-0.9523 -i0.09610	-0.05119 +i0.1117				
0.049	-0.9544 -i0.1396	-0.05211 +i0.09900				
0.05	-0.9537 -i0.1839	-0.05199 +i0.08668	0.1582 -i0.05647	-0.09443 -i0.09668	-0.1495 -i0.00352	-0.1665 +i0.01410
0.051	-0.9499 -i0.2280	-0.05087 -i0.07495				
0.052	-0.9488 -i0.2795	-0.04956 +i0.06405				
0.06	-0.7966 -i0.6035	-0.01577 +i0.00748	0.1704 -i0.09484	-0.06765 -i0.1771	-0.1452 -i0.01489	-0.1313 -i0.07233
0.07	-0.3935 -i0.9185	-0.01580 -i0.00398	0.2321 -i0.1393	0.04808 -i0.2617	-0.1351 -i0.02287	-0.08086 -i0.1167
0.078	0.1392 -i0.9597	-0.06218 -i0.08301				
0.08	0.2841 -i0.9046	-0.05967 -i0.1210	0.3501 -i0.09890	0.2649 -i0.1984	-0.1189 -i0.02395	-0.01483 -i0.1650
0.082	0.4170 -i0.8196	-0.04496 -i0.1605				
0.084	0.5291 -i0.7095	-0.01792 -i0.1966	0.3739 -i0.04436	0.3140 -i0.1042	-0.1122 -i0.02002	0.03370 -i0.1729
0.09	0.7047 -i0.3153	0.1073 +i0.2474	0.3599 +i0.03259	0.3011 +i0.03474	-0.1063 -i0.01084	0.1093 -i0.1454
0.10	0.6238 +i0.2386	0.2746 -i0.1558	0.2929 +i0.08743	0.1931 +i0.1474	-0.1063 +i0.00404	0.1708 -i0.03682
0.11	0.3625 +i0.6036	0.3012 -i0.00583	0.2442 +i0.09439	0.1046 +i0.1763	-0.1125 +i0.01493	0.1495 +i0.07305
0.12	-0.00770 +i0.8181	0.2218 +i0.1015	0.2135 +i0.09203	0.03788 +i0.1832	-0.1226 +i0.02156	0.06915 +i0.1458
0.13	-0.4509 +i0.8256	0.09130 +i0.1114	0.1883 +i0.08996	-0.02263 +i0.1806	-0.1353 +i0.02186	-0.04220 +i0.1510
0.14	-0.8212 +i0.5705	0.00157 +i0.00741	0.1609 +i0.08490	-0.08350 +i0.1601	-0.1455 +i0.01427	-0.1269 +i0.07457
0.148	-0.9325 +i0.2536	0.01917 -i0.1074				

(to be continued.)

Table VII. (continued.)

$\frac{H}{L}$	A	B	C	D	E	F
0.149	-0.9327 +i0.2143	0.02732 -i0.1200				
0.15	-0.9301 +i0.1761	0.03658 -i0.1317	0.1328 +i0.07000	-0.1325 +i0.1173	-0.1478 +i0.00278	-0.1256 -i0.03436
0.152	-0.9173 +i0.1034	0.05806 -i0.1525				
0.16	-0.7937 -i0.1146	0.1669 -i0.1907	0.1111 +i0.04479	-0.1594 +i0.06368	-0.1429 -i0.00516	-0.05268 -i0.09595
0.17	-0.5984 -i0.2094	0.2959 -i0.1419	0.09884 +i0.01432	-0.1686 +i0.01172	-0.1364 -i0.00693	0.02691 -i0.08694
0.18	-0.4804 -i0.1768	0.3635 -i0.02606	0.09479 -i0.01854	-0.1690 -i0.03757	-0.1327 -i0.00471	0.07066 -i0.03328
0.19	-0.4823 -i0.1229	0.3514 +i0.1094	0.09847 -i0.05456	-0.1634 -i0.09068	-0.1329 -i0.00173	0.06521 +i0.03143
0.20	-0.5839 -i0.1566	0.2575 +i0.2188	0.1136 -i0.09549	-0.1450 -i0.1558	-0.1365 -i0.00124	0.01231 +i0.07407
0.208	-0.6677 -i0.3195	0.1371 +i0.2503				
0.21	-0.6741 -i0.3830	0.1048 +i0.2464	0.1496 -i0.1391	-0.09371 -i0.2347	-0.1406 -i0.00653	-0.06443 +i0.05754
0.212	-0.6712 -i0.4545	0.07328 +i0.2373				
0.214	-0.6549 -i0.5327	0.04374 +i0.2232				
0.22	-0.5056 -i0.7795	-0.02258 +i0.1553	0.2171 -i0.1700	0.01931 -i0.3024	-0.1380 -i0.01749	-0.09779 -i0.03109
0.23	0.08467 -i0.9924	-0.02600 +i0.02749	0.3086 -i0.1609	0.1905 -i0.2989	-0.1234 -i0.02433	-0.02539 -i0.1124
0.24	0.7951 -i0.6029	0.02750 -i0.00370	0.3999 -i0.09108	0.3695 -i0.1712	-0.1051 -i0.01503	0.08643 -i0.07703
0.246	0.9959 -i0.08147	0.01567 +i0.00500				
0.248	0.9935 +i0.09704	0.00246 +i0.00124				
0.25	0.9504 +i0.3089	-0.01277 -i0.00921	0.4368 +i0.05028	0.4324 +i0.09336	-0.1017 +i0.00756	0.08862 +i0.03471
0.252	0.8680 +i0.4883	-0.02715 -i0.02730				
0.26	0.2944 +i0.8858	-0.01908 -i0.1510	0.3384 +i0.1849	0.2473 +i0.3308	-0.1181 +i0.02018	0.00383 +i0.04623
0.27	-0.2448 +i0.7421	0.1464 -i0.2206	0.2018 +i0.1857	0.00861 +i0.3248	-0.1295 +i0.01577	0.00720 -i0.00156
0.28	-0.4057 +i0.5604	0.2776 -i0.1295	0.1343 +i0.1281	-0.1040 +i0.2265	-0.1319 +i0.01236	0.04323 +i0.01468
0.29	-0.5136 +i0.5014	0.2954 +i0.00551	0.1120 +i0.07494	-0.1439 +i0.1412	-0.1343 +i0.01253	0.03733 +i0.06511

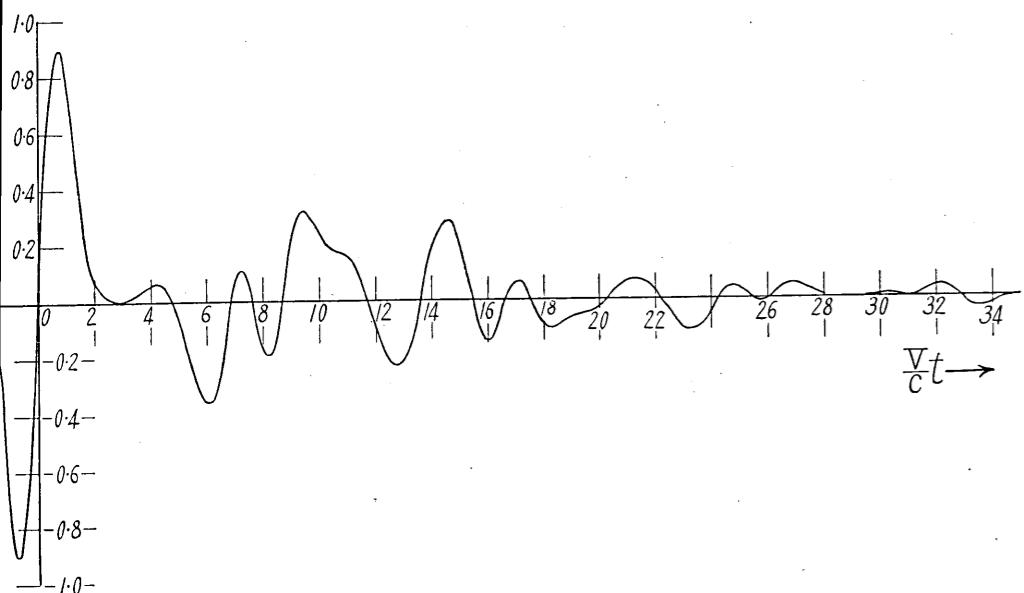


Fig. 6. The horizontal vibration on the bottom surface when  $H/c=0.80$ .

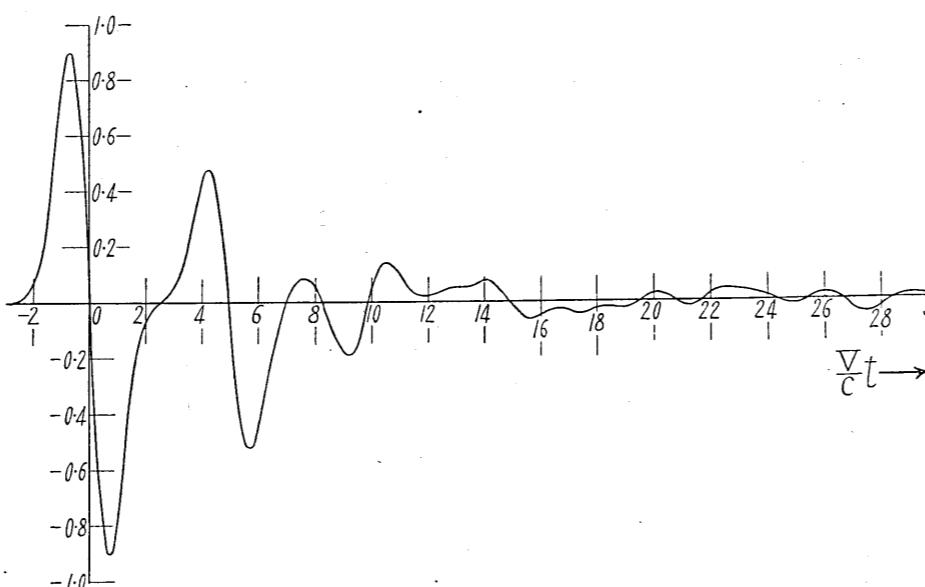


Fig. 7. The vertical vibration on the bottom surface when  $H/c=0.80$

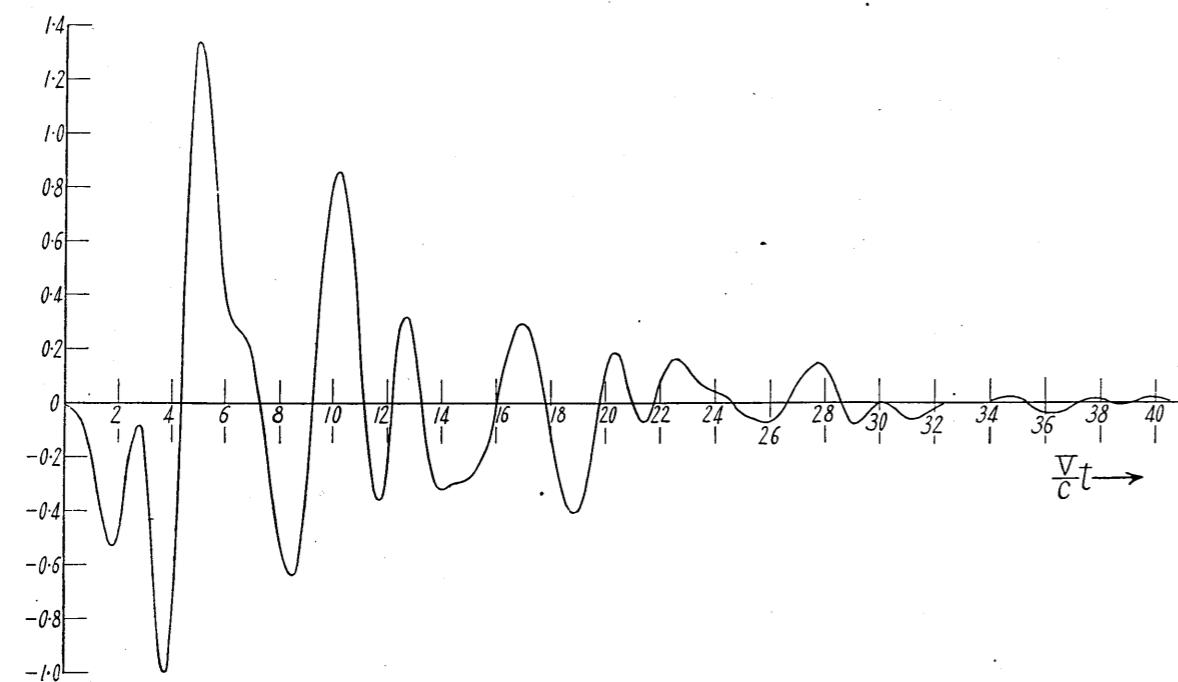


Fig. 8. The horizontal vibration on the top surface when  $H/c=0.80$ .

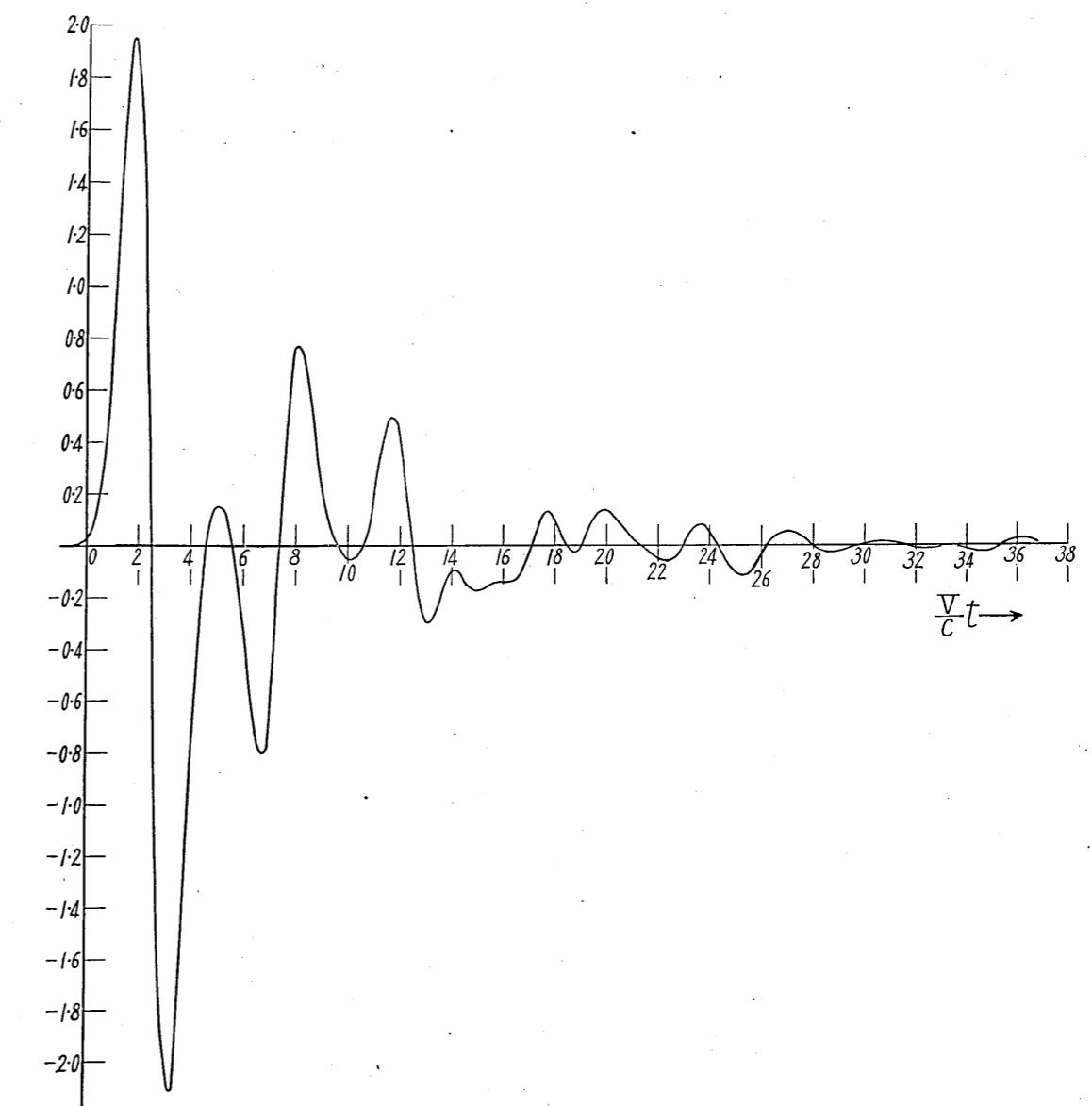


Fig. 9. The vertical vibration on the top surface when  $H/c=0.80$ .

Table VIII. (*continued.*)

$\frac{H}{L}$	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
0·145	-0·9512 +i·0·2755	0·00356 -i·0·04841	0·09823 +i·0·06076	-0·08081 +i·0·09519	-0·1027 -i·0·00051	-0·09602 +i·0·00854
0·149	-0·9560 +i·0·1601	0·01975 -i·0·08345				
0·15	-0·9532 +i·0·1328	0·02547 -i·0·09134	0·09076 +i·0·05143	-0·09264 +i·0·08099	-0·1027 -i·0·00024	-0·08706 -i·0·02433
0·151	-0·9490 +i·0·1063	0·03180 -i·0·09879				
0·152	-0·9430 +i·0·08045	0·03873 -i·0·1058				
0·16	-0·8562 -i·0·08605	0·1099 -i·0·1399	0·07913 +i·0·03148	-0·1115 +i·0·05029	-0·1029 +i·0·00017	-0·04163 -i·0·06749
0·17	-0·7025 -i·0·1761	0·2100 -i·0·1177	0·07163 +i·0·00957	-0·1240 +i·0·01592	-0·1032 +i·0·00035	0·01629 -i·0·06763
0·18	-0·5929 -i·0·1519	0·2743 -i·0·03127	0·06859 -i·0·01463	-0·1293 -i·0·02256	-0·1034 +i·0·00029	0·05465 -i·0·02895
0·19	-0·5908 -i·0·09427	0·2680 +i·0·08005	0·07097 -i·0·04146	-0·1256 -i·0·06535	-0·1034 +i·0·00017	0·05223 +i·0·02363
0·20	-0·6836 -i·0·1119	0·1920 +i·0·1629	0·08056 -i·0·07128	-0·1099 -i·0·1126	-0·1032 +i·0·00019	0·01074 +i·0·05644
0·21	-0·7750 -i·0·2844	0·08314 +i·0·1786	0·1010 -i·0·1046	-0·07703 -i·0·1647	-0·1030 +i·0·00051	-0·04433 +i·0·04497
0·212	-0·7800 -i·0·3394	0·06225 +i·0·1727				
0·214	-0·7772 -i·0·4005	0·04268 +i·0·1640				
0·216	-0·7650 -i·0·4671	0·02489 +i·0·1528				
0·22	-0·7061 -i·0·6119	-0·00393 +i·0·1243	0·1407 -i·0·1404	-0·01451 -i·0·2201	-0·1032 +i·0·00113	-0·07354 -i·0·01194
0·225	-0·5503 -i·0·7982	-0·02447 +i·0·08203	0·1728 -i·0·1571	0·03579 -i·0·2457	-0·1034 +i·0·00149	-0·06586 -i·0·04909
0·23	-0·2797 -i·0·9499	-0·02647 +i·0·04080	0·2177 -i·0·1684	0·1058 -i·0·2631	-0·1039 +i·0·00178	-0·03841 -i·0·08219
0·235	0·1169 -i·0·9918	-0·01401 +i·0·01110	0·2784 -i·0·1656	0·2003 -i·0·2584	-0·1047 +i·0·00186	0·00823 -i·0·09898
0·24	0·5914 -i·0·8064	-0·00022 +i·0·00006	0·3514 -i·0·1317	0·3141 -i·0·2055	-0·1056 +i·0·00152	0·06359 -i·0·08422
0·246	0·9857 -i·0·1662	-0·00873 -i·0·00222				
0·248	0·9911 +i·0·1174	-0·01928 -i·0·00924				
0·249	0·9652 +i·0·2489	-0·02405 -i·0·01452				
0·25	0·9193 +i·0·3795	-0·02910 -i·0·02176	0·4144 +i·0·06905	0·4125 +i·0·1079	-0·1062 -i·0·00071	0·08576 +i·0·03055

(to be continued.)

Table VIII. (*continued.*)

$\frac{H}{L}$	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
0·26	0·01577 +i0·9209	-0·00296 -i0·1359	0·2583 +i0·1947	0·1689 +i0·3048	-0·1045 -i0·00172	0·00229 +i0·03938
0·27	-0·4601 +i0·6610	0·1269 -i0·1635	0·1383 +i0·1612	-0·01939 +i0·2529	-0·1036 -i0·00122	0·00687 +i0·00070
0·28	-0·5694 +i0·4803	0·2170 -i0·08477	0·09111 +i0·1060	-0·09372 +i0·1661	-0·1034 -i0·00089	0·03312 +i0·01473
0·29	-0·6554 +i0·4205	0·2181 +i0·02091	0·07670 +i0·06187	-0·1162 +i0·09631	-0·1032 -i0·00079	0·02459 +i0·05268
0·30	-0·7973 +i0·3503	0·1483 +i0·08626	0·07627 +i0·02913	-0·1163 +i0·04470	-0·1030 -i0·00067	-0·01940 +i0·07268

Table IX. ( $\theta = 5^\circ$ .)

$\frac{H}{L}$	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
0	-0·3553	0·2318	0·08654	-0·02207	-0·07256	0·09574
0·01	-0·4631 +i0·2259	0·1865 +i0·1020	0·08195 +i0·00736	-0·02269 -i0·00233	-0·06909 -i0·00733	0·06832 +i0·06035
0·02	-0·6743 +i0·2916	0·09859 +i0·1364	0·07267 +i0·00796	-0·02438 -i0·00549	-0·06230 -i0·00950	0·01496 +i0·07986
0·03	-0·8486 +i0·2229	0·02832 +i0·1156	0·06427 +i0·00239	-0·02698 -i0·01013	-0·05671 -i0·00734	-0·02814 +i0·06572
0·04	-0·9498 +i0·09331	-0·00853 +i0·07357	0·05809 -i0·00717	-0·03056 -i0·01739	-0·05347 -i0·00327	-0·05154 +i0·03789
0·048	-0·9845 -0·02879	-0·01706 +i0·03932				
0·049	-0·9863 -i0·04452	-0·01703 +i0·03550				
0·05	-0·9876 -i0·06037	-0·01681 +i0·03184	0·05402 -i0·02115	-0·03474 -i0·03024	-0·05234 +i0·00151	-0·05802 +i0·00822
0·051	-0·9883 -i0·07635	-0·01640 +i0·02832				
0·06	-0·9731 -i0·2267	-0·00773 +i0·00599	0·05363 -i0·04256	-0·03624 -i0·05425	-0·05282 +i0·00653	-0·05311 -i0·01592
0·07	-0·9022 -i0·4300	-0·00876 +i0·00101	0·06671 -i0·07734	-0·01969 -i0·09747	-0·05487 +i0·01247	-0·04777 -i0·03446
0·08	-0·6464 -i0·7028	-0·05033 -i0·03636	0·1260 -i0·1173	0·05967 -i0·1434	-0·06197 +i0·02119	-0·04416 -i0·07345
0·082	-0·5376 -i0·7761	-0·05664 -i0·05897				
0·084	-0·4006 -i0·8143	-0·05652 -i0·08751	0·1720 -i0·1174	0·1183 -i0·1379	-0·06906 +i0·02430	-0·02677 -i0·1021
0·086	-0·2366 -i0·8218	-0·04646 -i0·1199				
0·09	0·1270 -i0·6883	0·01189 -i0·1767	0·2443 -i0·06589	0·2015 -i0·06285	-0·08489 +i0·02131	0·04422 -i0·1287

(to be continued.)

Table IX. (*continued.*)

$\frac{H}{L}$	A	B	C	D	E	F
0.10	0.3360 $+i0.2912$	0.2024 $-i0.09169$	0.2199 $+i0.08193$	0.1492 $+i0.1135$	-0.09240 $-i0.00776$	0.1397 $-i0.01417$
0.11	-0.2714 $+i0.6841$	0.1586 $+i0.05511$	0.1253 $+i0.1046$	0.03508 $+i0.1254$	-0.07443 $-i0.02027$	0.07237 $+i0.07820$
0.12	-0.6883 $+i0.5936$	0.06919 $+i0.07689$	0.07971 $+i0.08136$	-0.01219 $+i0.09366$	-0.06166 $-i0.01781$	0.00171 $+i0.07984$
0.13	-0.8919 $+i0.4100$	0.01636 $+i0.04444$	0.06039 $+i0.05901$	-0.03049 $+i0.06923$	-0.05536 $-i0.01224$	-0.03658 $+i0.05158$
0.14	-0.9732 $+i0.2292$	-0.00006 $-i0.00239$	0.05113 $+i0.04183$	-0.04028 $+i0.05292$	-0.05287 $-i0.00664$	-0.04995 $+i0.01762$
0.148	-0.9819 $+i0.1011$	0.00892 $-i0.03874$				
0.149	-0.9801 $+i0.08645$	0.01132 $-i0.04285$				
0.15	-0.9777 $+i0.07217$	0.01398 $-i0.04683$	0.04578 $+i0.02813$	-0.04861 $+i0.04073$	-0.05285 $-i0.00174$	-0.04453 $-i0.01345$
0.151	-0.9747 $+i0.05131$	0.01690 $-i0.05066$				
0.16	-0.9256 $-i0.04760$	0.05428 $-i0.07566$	0.04203 $+i0.01603$	-0.05791 $+i0.02878$	-0.05466 $+i0.00196$	-0.02369 $-i0.03504$
0.17	-0.8390 $-i0.1091$	0.1107 $-i0.07226$	0.03923 $+i0.00391$	-0.06786 $+i0.01313$	-0.05761 $+i0.00372$	0.006602 $-i0.03863$
0.18	-0.7648 $-i0.09958$	0.1557 $-i0.02583$	0.03790 $-i0.00945$	-0.07465 $-i0.00896$	-0.06010 $+i0.00309$	0.03117 $-i0.01872$
0.19	-0.7600 $-i0.05793$	0.1547 $+i0.04306$	0.03910 $-i0.02426$	-0.07301 $-i0.03543$	-0.06023 $+i0.00139$	0.03113 $+i0.01309$
0.20	-0.8221 $-i0.06283$	0.1081 $+i0.08962$	0.04313 $-i0.04035$	-0.06264 $-i0.06097$	-0.05808 $+i0.00114$	0.00688 $+i0.03184$
0.21	-0.8876 $-i0.1593$	0.05043 $+i0.09457$	0.05013 $-i0.05927$	-0.04733 $-i0.08532$	-0.05574 $+i0.00391$	-0.02219 $+i0.02605$
0.212	-0.8954 $-i0.1900$	0.04027 $+i0.09140$				
0.214	-0.9003 $-i0.2244$	0.03090 $+i0.08718$				
0.216	-0.9018 $-i0.2626$	0.02239 $+i0.08209$				
0.22	-0.8923 $-i0.3511$	0.00814 $+i0.06986$	0.06297 $-i0.08557$	-0.02631 $-i0.1157$	-0.05528 $+i0.00966$	-0.04002 $+i0.00017$
0.23	-0.7409 $-i0.6562$	-0.01055 $+i0.03395$	0.09617 $-i0.1285$	0.01759 $-i0.1657$	-0.05953 $+i0.01879$	-0.03542 $-i0.03923$
0.24	-0.01569 $-i0.9985$	-0.01062 $+i0.00758$	0.2261 $-i0.1813$	0.1811 $-i0.2291$	-0.08058 $+i0.02892$	0.02337 $-i0.07787$
0.246	0.9232 $-i0.3716$	-0.02438 $-i0.00324$	0.3987 $-i0.0655$	0.4000 $-i0.08207$	-0.1077 $+i0.01081$	0.09385 $-i0.03148$
0.248	0.9782 $+i0.1514$	-0.03185 $-i0.01553$				

(to be continued.)

Table IX. (*continued.*)

$\frac{H}{L}$	$A$	$B$	$C$	$D$	$E$	$F$
0.25	0.7663 $+i0.6137$	-0.03413 $-i0.03303$	0.3738 $+i0.1207$	0.3698 $+i0.1556$	-0.1031 $-i0.01756$	0.07712 $+i0.03617$
0.26	-0.5105 $+i0.7747$	0.02108 $-i0.09014$	0.1272 $+i0.1680$	0.05303 $+i0.2217$	-0.06646 $-i0.02180$	-0.00370 $+i0.02569$
0.27	-0.7517 $+i0.4434$	0.08790 $-i0.08328$	0.06227 $+i0.1065$	-0.03626 $+i0.1435$	-0.05997 $-i0.01224$	0.00428 $+i0.00164$
0.28	-0.7862 $+i0.3093$	0.1283 $-i0.03403$	0.04299 $+i0.06645$	-0.06511 $+i0.08751$	-0.05927 $-i0.00872$	0.01811 $+i0.01106$
0.29	-0.8322 $+i0.2611$	0.1188 $+i0.02485$	0.03784 $+i0.03870$	-0.07052 $+i0.04588$	-0.05786 $-i0.00777$	0.01046 $+i0.03142$
0.30	-0.9085 $+i0.2021$	0.07362 $+i0.05303$	0.03882 $+i0.01898$	-0.06415 $+i0.01786$	-0.05531 $-i0.00627$	-0.01433 $+i0.03824$

Table X a. (Top Surface,  $\theta=90^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0		2.00	0.06		4.23	0.12		2.64
0.01		2.04	0.07		5.57	0.13		2.31
0.02		2.15	0.08		6.33	0.14		2.12
0.03		2.36	0.09		5.31	0.15		2.02
0.04		2.72	0.10		4.02	0.16		2.00
0.05		3.30	0.11		3.16	0.17		2.05

Table X b. (Bottom surface,  $\theta=90^\circ$ )

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0		2.000	0.07		0.997	0.12		1.917
0.01		2.00	0.078		0.133	0.13		1.962
0.02		1.98	0.079057		0	0.14		1.986
0.03		1.96	0.08		0.119	0.15		1.997
0.04		1.91	0.09		1.145	0.16		2.000
0.05		1.81	0.10		1.626	0.17		1.994
0.06		1.57	0.11		1.825			

Table XI a. (Top Surface,  $\theta=80^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\tan^{-1} \frac{\beta}{\alpha}$	$\tan^{-1} \frac{\beta'}{\alpha'}$
0	0.399	1.958		
0.01	0.422	1.995	3.072	3.08
0.02	0.499	2.11	2.970	3.01
0.03	0.681	2.32	2.767	2.934
0.04	1.057	2.67	2.259	2.835
0.045	1.200	2.91		
0.05	1.075	3.25	1.264	2.692
0.06	0.615	4.16	0.641	2.471
0.07	0.333	5.44	0.415	2.096
0.075	0.244	6.04		
0.08	0.222	6.19	0.793	1.515
0.085	0.262	5.71		
0.09	0.311	5.20	0.912	0.967
0.10	0.389	3.94	0.711	0.630
0.11	0.477	3.10	0.485	0.434
0.12	0.642	2.58	0.193	0.309
0.13	0.982	2.29	2.799	0.223
0.135	1.165	2.21		
0.14	1.126	2.14	1.753	0.131
0.145	0.903	2.075		
0.15	0.698	2.02	0.970	0.0495
0.16	0.462	1.935	0.559	3.13
0.17	0.357	2.03	0.242	3.065
0.18	0.315	2.15	3.07	2.999
0.19	0.316	2.39	2.711	2.918
0.20	0.368	2.78	2.273	2.813
0.21	0.510	3.43	1.681	2.656
0.22	0.856	4.43	0.738	2.400
0.225	1.077	4.99		
0.23	1.151	5.50	2.201	1.989
0.235	0.936	5.94		
0.24	0.700	6.08	0.523	1.441
0.245	0.511	5.72		
0.25	0.387	5.06	2.237	0.909
0.26	0.286	3.85	0.253	0.586
0.27	0.275	3.04	0.537	0.398
0.28	0.302	2.55	0.0933	0.274
0.29	0.374	2.26	2.895	0.184
0.30	0.517	2.08	2.566	0.111

Table XI b. (Bottom Surface,  $\theta=80^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0	0.399	1.963	0.142	0.0246	
0.01	0.398	1.956	0.143	0.0546	
0.02	0.390	1.945	0.145	0.112	1.965
0.03	0.364	1.919	0.15	0.209	1.967
0.04	0.245	1.867	0.16	0.289	1.973
0.045	0.086	1.831	0.17	0.314	1.965
0.046	0.0553		0.18	0.317	1.950
0.047	0.0226		0.19	0.303	1.922
0.048	0.0596		0.20	0.266	1.864
0.049	0.0730		0.21	0.190	1.742
0.05	0.105	1.773	0.213	0.166	
0.06	0.271	1.542	0.216	0.168	
0.07	0.319	0.987	0.22	0.259	1.450
0.075	0.330	0.492	0.225	0.488	1.068
0.078		0.137	0.23	0.666	0.799
0.079		0.0145	0.235	0.643	0.329
0.0795		0.0474	0.237		0.109
0.08	0.361	0.109	0.238		0.0443
0.0805		0.170	0.239		0.140
0.085	0.394	0.681	0.24	0.498	0.257
0.09	0.418	1.116	0.245	0.346	0.804
0.10	0.442	1.587	0.25	0.259	1.209
0.11	0.449	1.781	0.26	0.259	1.641
0.12	0.447	1.868	0.27	0.295	1.816
0.13	0.389	1.910	0.28	0.313	1.897
0.135	0.267	1.928	0.29	0.312	1.940
0.14	0.0638	1.952	0.30	0.285	1.960
0.141	0.0272				

Table XII a. (Top Surface,  $\theta=70^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0	0.771	1.838	0.045	2.254	2.755
0.01	0.822	1.887	0.05	2.047	3.111
0.02	0.969	1.995	0.06	1.208	3.964
0.03	1.310	2.197	0.07	0.670	5.185
0.04	1.997	2.536	0.08	0.546	5.70

(to be continued.)

Table XII a. (*continued.*)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0.08	0.447	5.83	0.20	0.720	2.692
0.085	0.515	5.49	0.21	0.984	3.360
0.09	0.601	4.90	0.22	1.405	4.675
0.10	0.689	3.71	0.225	1.746	4.65
0.11	0.896	2.92	0.23	1.678	4.82
0.12	1.187	2.45	0.235	1.432	5.17
0.13	1.777	2.24	0.24	1.06	5.55
0.135	2.115	2.18	0.245	0.894	5.53
0.14	2.130	2.204	0.25	0.708	5.09
0.145	1.715	2.14	0.26	0.537	3.91
0.15	1.382	2.024	0.27	0.533	3.04
0.16	0.912	1.941	0.28	0.592	2.501
0.17	0.707	1.956	0.29	0.726	2.21
0.18	0.621	2.077	0.30	1.00	2.06
0.19	0.624	2.300			

Table XII b. (Bottom Surface,  $\theta=70^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0	0.780	1.855	0.09	0.815	1.030
0.01	0.776	1.847	0.10	0.858	1.477
0.02	0.758	1.839	0.11	0.869	1.660
0.03	0.704	1.812	0.12	0.860	1.733
0.04	0.472	1.770	0.13	0.753	1.770
0.045	0.175	1.740	0.135	0.542	1.794
0.047	0.0448		0.14	0.175	1.847
0.048	0.0589		0.141	0.107	
0.0485	0.0869		0.142	0.0442	
0.05	0.188	1.718	0.143	0.0666	
0.06	0.523	1.478	0.144	0.131	
0.07	0.606	0.950	0.145	0.179	1.880
0.075	0.646	0.482	0.15	0.385	1.893
0.078		0.151	0.16	0.558	1.893
0.079		0.0368	0.17	0.612	1.885
0.0795		0.0279	0.18	0.628	1.870
0.08	0.707	0.0827	0.19	0.595	1.841
0.085	0.744	0.592	0.20	0.518	1.797

(to be continued.)

Table XII b. (*continued.*)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0.21	0.365	1.680	0.241		0.169
0.213	0.322		0.2415		0.212
0.214	0.318		0.242		0.265
0.215	0.321		0.245	0.746	0.570
0.216	0.336		0.25	0.552	1.020
0.22	0.563	1.187	0.26	0.501	1.529
0.225	0.892	1.036	0.27	0.572	1.725
0.23	1.147	0.753	0.28	0.609	1.840
0.235	1.157	0.471	0.29	0.610	1.851
0.24	1.984	0.115	0.30	0.556	1.875

Table XIII a. (Top Surface,  $\theta=30^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0	1.73	0.966	0.16	2.02	1.42
0.01	1.80	1.02	0.17	1.65	1.32
0.02	2.02	1.08	0.18	1.33	1.33
0.03	2.42	1.21	0.19	1.43	1.48
0.04	2.95	1.42	0.20	1.54	1.80
0.05	3.03	1.72	0.21	1.78	2.34
0.06	2.42	2.18	0.22	1.78	2.89
0.07	1.75	2.74	0.23	1.35	2.97
0.08	1.42	3.24	0.24	0.797	3.03
0.09	1.42	2.76	0.25	0.264	3.32
0.10	1.49	2.17	0.26	0.510	3.32
0.11	1.63	1.82	0.27	0.984	2.63
0.12	1.88	1.68	0.28	1.24	1.99
0.13	2.30	1.71	0.29	1.50	1.62
0.14	2.70	1.81	0.30	1.88	1.45
0.15	2.54	1.68			

Table XIII b. (Bottom Surface,  $\theta=30^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0	1.73	1.000	0.03	1.380	0.958
0.01	1.706	0.994	0.04	0.824	0.925
0.02	1.613	0.991	0.045	0.376	

(to be continued.)

Table XIII b. (*continued.*)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0.046	0.278		0.16	0.926	1.133
0.047	0.180		0.17	1.217	1.150
0.048	0.0884		0.18	1.304	1.152
0.049	0.0626		0.19	1.252	1.150
0.05	0.142	0.911	0.20	1.043	1.135
0.06	0.918	0.863	0.21	0.644	1.052
0.07	1.328	0.640	0.211	0.636	
0.076		0.362	0.212	0.618	
0.077		0.301	0.214	0.624	
0.078		0.250	0.216	0.815	
0.079		0.195	0.22	0.966	0.781
0.08	1.653	0.144	0.23	1.660	0.458
0.09	1.864	0.415	0.24	1.908	
0.10	1.913	0.666	0.244		0.154
0.11	1.881	0.768	0.246	1.884	0.109
0.12	1.773	0.804	0.248	1.832	0.0850
0.13	1.485	0.823	0.25	1.785	0.134
0.14	0.788	0.908	0.26	1.340	0.621
0.144	0.396		0.27	1.177	0.958
0.146	0.220		0.28	1.267	1.070
0.147	0.159		0.29	1.280	1.101
0.148	0.157		0.30	1.130	1.103
0.15	0.282	1.060			

Table XIV a. (Top Surface,  $\theta = 20^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0	1.71	0.772	0.16	1.72	1.12
0.01	1.76	0.784	0.17	1.51	1.04
0.02	1.89	0.796	0.18	1.28	1.04
0.03	2.11	0.862	0.19	1.33	1.16
0.04	2.30	1.00	0.20	1.38	1.41
0.05	2.29	1.19	0.21	1.48	1.81
0.06	2.01	1.54	0.22	1.47	2.26
0.07	1.68	2.12	0.23	1.10	2.56
0.08	1.51	2.58	0.24	0.823	2.76
0.09	1.52	2.32	0.25	0.402	2.95
0.10	1.54	1.87	0.26	0.440	2.85
0.11	1.61	1.60	0.27	0.886	2.21
0.12	1.74	1.47	0.28	1.15	1.64
0.13	1.93	1.43	0.29	1.36	1.29
0.14	2.03	1.48	0.30	1.62	1.11
0.15	1.98	1.27			

Table XIV b. (Bottom Surface,  $\theta=20^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0	1.710	0.772	0.15	0.165	0.736
0.01	1.670	0.765	0.151	0.217	0.728
0.02	1.522	0.740	0.16	0.604	0.823
0.03	1.213	0.698	0.17	1.067	0.872
0.04	0.605	0.646	0.18	1.191	0.891
0.048	0.0855	0.625	0.19	1.146	0.892
0.049	0.0393	0.625	0.20	0.921	0.857
0.05	0.0855	0.625	0.21	0.559	0.789
0.051	0.155	0.625	0.212	0.521	0.763
0.06	0.741	0.619	0.214	0.526	0.737
0.07	1.217	0.506	0.22	0.823	0.618
0.08	1.657	0.174	0.23	1.525	0.395
0.082	1.731	0.122	0.24	1.930	0.189
0.084	1.805	0.111	0.246	1.941	0.0519
0.09	1.935	0.288	0.248	1.917	0.0430
0.10	1.977	0.491	0.25	1.828	0.096
0.11	1.903	0.570	0.26	1.423	0.470
0.12	1.715	0.590	0.27	1.125	0.737
0.13	1.339	0.594	0.28	1.170	0.829
0.14	0.700	0.637	0.29	1.167	0.842
0.148	0.142	0.714	0.30	0.990	0.816
0.149	0.134	0.726			

Table XV a. (Top Surface,  $\theta=15^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0	1.612	0.658	0.06	0.599	0.478
0.01	1.553	0.646	0.07	1.055	0.465
0.02	1.372	0.611	0.078		0.233
0.03	1.035	0.555	0.08	1.577	0.173
0.04	0.531	0.496	0.082		0.122
0.048	0.0724		0.084	1.745	0.104
0.049	0.0317		0.09	1.916	0.238
0.05	0.061	0.473	0.10	1.962	0.420
0.051	0.115		0.11	1.841	0.483
0.052	0.179		0.12	1.582	0.486

(to be continued.)

Table XV a. (*continued.*)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0.13	1.150	0.475	0.212	0.434	
0.14	0.588	0.492	0.214	0.441	
0.148	0.129		0.22	0.691	0.508
0.149	0.110		0.23	1.365	0.354
0.15	0.121	0.562	0.24	1.867	0.172
0.152	0.202		0.246	1.950	0.0305
0.16	0.583	0.643	0.248	1.931	0.0241
0.17	0.910	0.704	0.25	1.876	0.0889
0.18	1.045	0.735	0.252		0.153
0.19	1.005	0.736	0.26	1.378	0.405
0.20	0.788		0.27	1.020	0.615
0.208	0.520	0.705	0.28	1.034	0.687
0.21	0.465	0.632	0.29	1.020	0.688

Table XV b. (Bottom Surface,  $\theta = 15^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0	1.61	0.658	0.15	1.56	1.00
0.01	1.64	0.664	0.16	1.42	0.903
0.02	1.70	0.685	0.17	1.30	0.850
0.03	1.79	0.719	0.18	1.21	0.864
0.04	1.84	0.778	0.19	1.18	0.960
0.05	1.79	0.905	0.20	1.19	1.15
0.06	1.64	1.19	0.21	1.23	1.45
0.07	1.48	1.70	0.22	1.24	1.84
0.08	1.45	2.21	0.23	1.108	2.24
0.09	1.53	2.11	0.24	0.834	2.61
0.10	1.53	1.74	0.25	0.481	2.80
0.11	1.54	1.47	0.2	0.435	2.56
0.12	1.58	1.32	0.27	0.784	1.92
0.13	1.63	1.21	0.28	1.014	1.40
0.14	1.63	1.11	0.29	1.190	1.07

Table XVI a. (Top Surface;  $\theta = 10^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\tan^{-1} \frac{\beta}{\alpha}$	$\tan^{-1} \frac{\beta'}{\alpha'}$
0	1.402	0.533		
0.01	1.391	0.527	2.830	3.03
0.02	1.370	0.519	2.524	2.937
0.03	1.348	0.517	2.220	2.880
0.04	1.289	0.535	1.923	2.871
0.045	1.248	0.540		
0.05	1.232	0.611	1.638	2.891
0.06	1.165	0.806	1.378	2.876
0.07	1.124	1.208	1.178	2.690
0.075	1.151	1.484	1.099	2.494
0.08	1.248	1.742	0.995	2.224
0.085	1.372	1.900		
0.09	1.464	1.890	0.612	1.589
0.10	1.475	1.596	0.162	1.094
0.11	1.390	1.295	2.897	0.766
0.12	1.302	1.070	2.520	0.530
0.13	1.217	0.904	2.154	0.353
0.135	1.180	0.833	1.970	0.284
0.14	1.145	0.783	1.789	0.217
0.145	1.107	0.736	1.605	0.169
0.15	1.080	0.695	1.418	0.127
0.16	1.029	0.640	1.033	0.0743
0.17	0.988	0.634	0.646	0.206
0.18	0.950	0.649	0.232	0.0385
0.19	0.924	0.717	2.044	0.0085
0.20	0.909	0.845	2.512	3.08
0.21	0.901	0.041	2.057	2.960
0.22	0.846	1.336	1.582	2.774
0.225	0.908	1.537		
0.23	0.898	1.780	1.064	2.493
0.235	0.850	2.080	0.751	2.294
0.24	0.806	2.402	0.469	2.039
0.25	0.491	2.730	3.13	1.361
0.26	0.401	2.165	0.384	0.715
0.27	0.628	1.496	0.234	0.316
0.28	0.798	1.056	3.00	0.0950
0.29	0.917	0.794	2.619	3.12
0.30	1.006	0.640	2.252	3.09

Table XVI b. (Bottom Surface,  $\theta = 10^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0	1.402	0.530	0.15	0.0824	0.381
0.01	1.326	0.510	0.151	0.102	
0.02	1.106	0.462	0.152	0.132	
0.03	0.777	0.395	0.16	0.411	0.447
0.04	0.381	0.339	0.17	0.683	0.509
0.045	0.175	0.324	0.18	0.814	0.547
0.049	0.0230		0.19	0.785	0.548
0.05	0.0385	0.319	0.20	0.597	0.514
0.051	0.0759		0.21	0.340	0.450
0.052	0.115		0.212	0.317	
0.06	0.421	0.326	0.214	0.323	
0.07	0.798	0.301	0.216	0.360	
0.075	1.079	0.247	0.22	0.511	0.371
0.08	1.317	0.152	0.225	0.793	0.329
0.083		0.101	0.23	1.105	0.284
0.085	1.612	0.100	0.235	1.445	0.231
0.087		0.130	0.24	1.755	0.157
0.09	1.820	0.212	0.246	1.942	0.0274
0.10	1.886	0.358	0.248	1.934	0.0343
0.11	1.671	0.395	0.249		0.0602
0.12	1.317	0.373	0.25	1.880	0.0886
0.13	0.885	0.341	0.26	1.224	0.338
0.135	0.656	0.334	0.27	0.825	0.468
0.14	0.428	0.338	0.28	0.813	0.514
0.145	0.209	0.255	0.29	0.787	0.505
0.149	0.0813		0.30	0.622	0.458

Table XII a. (Top Surface,  $\theta = 5^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\tan^{-1} \frac{\beta}{\alpha}$	$\tan^{-1} \frac{\beta'}{\alpha'}$
0	0.970	0.349		
0.01	0.927	0.334	2.731	2.941
0.02	0.830	0.303	2.381	2.815
0.03	0.729	0.278	2.076	2.782
0.04	0.672	0.275	1.869	2.830
0.05	0.629	0.309	1.666	2.922
0.06	0.606	0.410	1.492	2.978
0.07	0.618	0.634	1.379	2.882
0.08	0.784	1.038	1.286	2.526
0.084	0.926	1.242		

(to be continued.)

Table XII a. (continued.)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\tan^{-1} \frac{\beta}{\alpha}$	$\tan^{-1} \frac{\beta'}{\alpha'}$
0.09	1.192	1.476	0.834	1.840
0.10	1.270	1.295	0.0463	1.108
0.11	1.012	0.924	1.754	0.551
0.12	0.797	0.650	2.258	0.300
0.13	0.666	0.495	1.954	0.175
0.14	0.592	0.406	1.677	0.118
0.15	0.558	0.358	1.391	0.100
0.16	0.548	0.339	1.075	0.100
0.17	0.554	0.342	0.706	0.104
0.18	0.555	0.366	0.283	0.085
0.19	0.541	0.407	2.968	0.0345
0.20	0.512	0.466	2.530	3.105
0.21	0.489	0.557	2.126	3.03
0.22	0.490	0.715	1.749	2.928
0.23	0.533	1.043	1.368	2.765
0.24	0.646	1.864	0.841	2.358
0.25	0.506	2.565	0.105	1.215
0.26	0.280	1.407	0.0547	0.440
0.27	0.373	0.872	0.0388	0.390
0.28	0.464	0.606	2.865	0.0389
0.29	0.485	0.444	2.442	0.186
0.30	0.547	0.341	2.148	2.159

Table XVII b. (Bottom Surface,  $\theta=5^\circ$ .)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0	0.970	0.349	0.15	0.0424	0.194
0.01	0.880	0.324	0.151	0.0534	
0.02	0.670	0.268	0.16	0.216	
0.03	0.430	0.212	0.17	0.380	0.277
0.04	0.201	0.174	0.18	0.474	0.309
0.048	0.0284		0.19	0.458	0.311
0.049	0.0121		0.20	0.336	0.283
0.05	0.0188	0.161	0.21	0.185	0.240
0.051	0.0378		0.212	0.172	
0.06	0.218	0.168	0.214	0.175	
0.07	0.435	0.161	0.216	0.195	
0.08	0.802	0.0968	0.22	0.277	0.205
0.082		0.0781	0.23	0.664	0.168
0.084		0.0676	0.24	1.378	0.123
0.086		0.0779	0.246	1.918	0.0340
0.09	1.474	0.145	0.248	1.933	0.0413
0.10	1.625	0.284	0.25	1.800	0.0874
0.11	1.216	0.269	0.26	0.826	0.171
0.12	0.810	0.218	0.27	0.493	0.269
0.13	0.489	0.181	0.28	0.472	0.290
0.14	0.226	0.173	0.29	0.446	0.278
0.148	0.0552		0.30	0.338	0.242
0.149	0.0440				

Table XIX.

$y$	Horizontal amplitude	Vertical amplitude	$\tan^{-1} \frac{\beta}{\alpha}$	$\tan^{-1} \frac{\beta'}{\alpha'}$	$y$	Horizontal amplitude	Vertical amplitude	$\tan^{-1} \frac{\beta}{\alpha}$	$\tan^{-1} \frac{\beta'}{\alpha'}$
$\theta = 5^\circ \quad H/L = 0.05$									
-H	0.629	0.309	1.666	2.922	-H	0.489	0.557	2.126	3.025
-3/4 H	0.592	0.248	1.654	2.905	-15/16 H	0.411	0.547	2.192	3.04
-2/4 H	0.464	0.247	1.640	2.891	-14/16 H	0.264	0.496	2.298	3.05
-1/4 H	0.248	0.204	1.628	2.388	-13/16 H	0.0862	0.405	3.095	3.05
0	0.0188	0.161	0.919	2.911	-12/16 H	0.120	0.279	1.861	3.03
$\theta = 5^\circ \quad H/L = 0.084$									
-H	0.926	1.242	1.166	2.294	-11/16 H	0.263	0.129	2.137	2.944
-7/8 H	0.948	1.194	1.133	2.284	-10/16 H	0.345	0.0496	2.272	0.701
-6/8 H	0.847	1.100	1.096	2.278	-9/16 H	0.349	0.202	2.422	0.176
-5/8 H	0.634	0.966	1.050	2.275	-8/16 H	0.287	0.346	2.668	0.108
-4/8 H	0.330	0.801	0.915	2.277	-7/16 H	0.202	0.460	0.0656	0.0764
-3/8 H	0.0648	0.617	2.024	2.291	-6/16 H	0.204	0.538	0.991	0.0499
-2/8 H	0.424	0.422	1.188	2.328	-5/16 H	0.307	0.570	1.557	0.0196
-1/8 H	0.779	0.227	1.109	2.453	-4/16 H	0.402	0.560	1.825	3.12
0	1.074	0.0676	1.066	0.242	-3/16 H	0.436	0.513	2.006	3.07
$\theta = 5^\circ \quad H/L = 0.15$									
-H	0.558	0.358	1.391	0.100	-2/16 H	0.394	0.436	2.192	2.991
-7/8 H	0.504	0.325	1.418	0.113	-1/16 H	0.286	0.341	2.497	2.873
-6/8 H	0.273	0.248	1.470	0.119	0	0.185	0.240	0.134	2.675
-5/8 H	0.0774	0.162	1.182	0.0895	$\theta = 5^\circ \quad H/L = 0.246$				
-4/8 H	0.415	0.0811	1.398	3.06	-H	0.658	2.60	0.278	1.753
-3/8 H	0.620	0.0311	1.438	1.966	-15/16 H	0.346	2.50	0.344	1.754
-2/8 H	0.614	0.0688	1.475	0.745	-14/16 H	0.0371	2.15	2.196	1.754
-1/8 H	0.401	0.128	1.544	0.449	-13/16 H	0.280	1.585	0.171	1.754
0	0.0424	0.194	3.02	0.277	-12/16 H	0.356	0.867	0.260	1.751
$\theta = 5^\circ \quad H/L = 0.21$									
-H	0.489	0.557	2.126	3.025	-11/16 H	0.204	0.0748	0.432	1.656
-15/16 H	0.411	0.547	2.192	3.04	-10/16 H	0.169	0.704	2.962	1.773
-14/16 H	0.264	0.496	2.298	3.05	-9/16 H	0.706	1.400	0.0832	1.765
-13/16 H	0.0862	0.405	3.095	3.05	-8/16 H	1.037	1.938	0.150	1.762
-12/16 H	0.120	0.279	1.861	3.03	-7/16 H	1.287	2.280	0.178	1.761
-11/16 H	0.263	0.129	2.137	2.944	-6/16 H	1.261	2.417	0.202	1.758
-10/16 H	0.345	0.0496	2.272	0.701	-5/16 H	0.934	2.338	0.232	1.755
-9/16 H	0.349	0.202	2.422	0.176	-4/16 H	0.344	2.084	0.343	1.751
-8/16 H	0.287	0.346	2.668	0.108	-3/16 H	0.400	1.687	0.0592	1.746
-7/16 H	0.202	0.460	0.0656	0.0764	-2/16 H	1.122	1.187	0.156	1.737
-6/16 H	0.204	0.538	0.991	0.0499	-1/16 H	1.672	0.622	0.181	1.717
0	0.185	0.240	0.134	2.675	0	1.918	0.0340	0.197	1.028

(to be continued.)

Table XIX. (continued)

$y$	Horizontal amplitude	Vertical amplitude	$\tan^{-1} \frac{\beta}{\alpha}$	$\tan^{-1} \frac{\beta'}{\alpha'}$	$y$	Horizontal amplitude	Vertical amplitude	$\tan^{-1} \frac{\beta}{\alpha}$	$\tan^{-1} \frac{\beta'}{\alpha'}$
$\theta = 15^\circ \quad H/L = 0.05$									
-H	1.79	0.905	1.611	2.872	-H	1.075	3.25	1.264	2.692
-3/4 H	1.70	0.838	1.612	2.856	-3/8 H	1.019	3.147	1.258	"
-2/4 H	1.32	0.730	1.587	2.844	-2/8 H	0.773	2.850	1.252	"
-1/4 H	0.700	0.603	1.557	2.841	-1/8 H	0.376	2.382	1.235	"
0	0.061	0.473	2.086	2.862	0	0.105	1.773	1.338	"
$\theta = 15^\circ \quad H/L = 0.084$									
-H	1.487	2.252	0.712	1.783	-H	0.220	6.20	0.738	1.578
-7/8 H	1.522	2.180	0.668	1.776	-7/8 H	0.260	6.08	0.571	"
-6/8 H	1.365	2.012	0.625	1.770	-6/8 H	0.270	5.73	0.445	"
-5/8 H	1.026	1.768	0.586	1.767	-5/8 H	0.245	5.16	0.328	"
-4/8 H	0.543	1.465	0.452	1.769	-4/8 H	0.181	4.39	0.180	"
-3/8 H	0.114	1.122	1.691	1.782	-3/8 H	0.0886	3.45	2.938	"
-2/8 H	0.683	0.758	0.734	1.818	-2/8 H	0.0756	2.38	1.068	"
-1/8 H	1.265	0.388	0.639	1.947	-1/8 H	0.209	1.22	0.443	1.580
0	1.745	0.104	0.588	0.061	0	0.354	0.0145	0.394	1.812
$\theta = 15^\circ \quad H/L = 0.15$									
$\theta = 80^\circ \quad H/L = 0.05$									
-H	1.075	3.25	1.264	2.692	-H	0.220	6.20	0.738	1.578
-3/8 H	1.019	3.147	1.258	"	-7/8 H	0.260	6.08	0.571	"
-2/8 H	0.773	2.850	1.252	"	-6/8 H	0.270	5.73	0.445	"
-1/8 H	0.376	2.382	1.235	"	-5/8 H	0.245	5.16	0.328	"
0	0.105	1.773	1.338	"	-4/8 H	0.181	4.39	0.180	"
$\theta = 80^\circ \quad H/L = 0.079$									
$\theta = 80^\circ \quad H/L = 0.14$									
-H	1.126	2.14	1.753	0.131	-H	1.126	2.14	1.753	0.131
-7/8 H	0.966	2.013	1.751	"	-7/8 H	0.966	2.013	1.751	"
-6/8 H	0.457	1.641	1.747	"	-6/8 H	0.457	1.641	1.747	"
-5/8 H	0.230	1.081	1.760	"	-5/8 H	0.230	1.081	1.760	"
-4/8 H	0.860	0.402	1.751	0.133	-4/8 H	0.860	0.402	1.751	0.133
-3/8 H	1.214	0.321	1.749	0.125	-3/8 H	1.214	0.321	1.749	0.125
-2/8 H	1.169	1.002	1.746	0.128	-2/8 H	1.169	1.002	1.746	0.128
-1/8 H	0.736	1.566	1.739	0.128	-1/8 H	0.736	1.566	1.739	0.128
0	0.0638	1.952	1.611	0.126	0	0.0638	1.952	1.611	0.126

20. 斜めの入射波（調和波型縦波）による表面層の  
振動とその共振性質（第2報）

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第1報に於ては入射角を一定 ( $45^\circ$ ) として種々の周期で調和波型縦波が地殻表面層に入射した場合この表面層に生ずる振動の性質を研究した。第2報（本論文）では、入射角と周期を色々變へた場合、表面層内に生ずる振動を研究した。