21. *Vibrations due to Obliquely Incident Longitudinal Waves of Shock Type of a Surface-Layer of an Elastic Earth's Crust. (1).*

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1. Introduction. The present paper is a continuation of the previous one[1], in which one of the present writers obtained general expressions for the free wave motion in a surface-layer of an elastic earth, when a dilatational wave of any wave form, such as $F(x, y)$, where x, y are the rectangular co-ordinates, is obliquely incident on the bottom surface of the surface-layer, using the Fourier double integral formula, without any actual example. Two years ago, K. Sezawa[2] and K. Kanai, using our mathematical results[3], obtained the general results of free wave motions in the surface-layer for the purpose of obtaining the damping moduli of the waves in both media, the surface-layer and the subjacent medium.

In the present paper, we shall graphically show the following two cases: [1] The time variations in the movements of particles on the top and bottom surfaces of the surface-layer when a dilatational wave of shock type, such that

$$F(x, y) = \exp\left\{-(x\cos\theta - y\sin\theta)^2/c^2\right\},$$

is obliquely incident on the bottom surface of it, the incidence angle of that wave being assumed to be 45°, and [2] the loci of the vibrations of the same particles on the same two positions in the surface-layer, as in case [1].

2. General expressions. Let the axes of x, y be on the bottom plane

1) G. NISHIMURA, *B. E. R. I.*, **13** (1935), 540~554.

In recent papers, we corrected as far as possible the misprints contained in our paper that was published in 1935.

G. NISHIMURA and T. TAKAYAMA, *B. E. R. I.*, **15** (1937), 394~440; **17** (1939), 200~220.

2) K. SEZAWA and K. KANAI, *B. E. R. I.*, **15** (1937), 370~380.

3) G. NISHIMURA. *loc. cit.*, 1).

surface of a surface-layer of thickness H, and let it be drawn vertically downwards from it. Also let ρ, λ, μ, ρ', λ', μ' be the densities, and elastic Láme constants of the subjacent medium and the surface-layer, respectively.

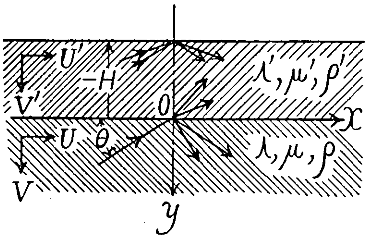

Fig. 1.

Now, when the dilatational wave ϕ_0, expressed by

$$\phi_0 = a \exp\left\{i(fx - ry - pt)\right\}, \qquad (1)$$

is primarily incident on the bottom surface of the surface-layer, two waves, such as

$$\phi = A\exp\left\{i(fx+ry-pt)\right\}, \qquad \psi = B\exp\left\{i(fx+sy-pt)\right\} \qquad (2)$$

are reflected at this surface in the subjacent medium, and four waves, such that

$$\left.\begin{aligned} \phi' &= C\exp\left\{i(fx-r'y-pt)\right\} + D\exp\left\{i(fx+r'y-pt)\right\}, \\ \psi' &= E\exp\left\{i(fx-s'y-pt)\right\} + F\exp\left\{i(fx+s'y-pt)\right\} \end{aligned}\right\} \qquad (3)$$

are reflected and refracted in the surface-layer.

In expressions (1), (2), (3),

$$\left.\begin{aligned} r^2 &= \rho p^2/(\lambda+2\mu) - f^2, \qquad s^2 = \rho p^2/\mu - f^2, \\ r'^2 &= \rho' p^2/(\lambda'+2\mu') - f^2, \quad s'^2 = \rho' p^2/\mu' - f^2, \end{aligned}\right\} \qquad (4)$$

and ϕ_0, ϕ, ϕ' are the dilatational plane waves and ψ, ψ' show the distortional plane waves respectively.

Now, the displacements, and the stresses of both media should be continuous at the common boundary, and the top surface of the surface-layer must be a free surface. Thut, the boundary conditions when $y = -H$, are expressed as follows:

$$\left.\begin{aligned} &\left\{\lambda'\frac{\partial^2}{\partial x^2} + (\lambda'+2\mu')\frac{\partial^2}{\partial y^2}\right\}(\phi') - 2\mu'\frac{\partial^2\psi'}{\partial x\partial y} = 0, \\ &2\mu'\frac{\partial^2\phi'}{\partial x\partial y} - \mu'\left\{\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right\}(\psi') = 0, \end{aligned}\right\} \qquad (5)$$

and also when $y=0$,

$$\frac{\partial}{\partial x}(\phi_0+\phi) + \frac{\partial\psi}{\partial y} = \frac{\partial\phi'}{\partial x} + \frac{\partial\psi'}{\partial y}, \qquad \frac{\partial}{\partial y}(\phi_0+\phi) - \frac{\partial\psi}{\partial x} = \frac{\partial\phi'}{\partial y} - \frac{\partial\psi'}{\partial x},$$

$$\left.\begin{aligned}&\left\{\lambda\frac{\partial^2}{\partial x^2}+(\lambda+2\mu)\frac{\partial^2}{\partial y^2}\right\}(\phi_0+\phi)-2\mu\frac{\partial^2\psi}{\partial x\,\partial y}\\&\qquad\qquad=\left\{\lambda'\frac{\partial^2}{\partial x^2}+(\lambda'+2\mu')\frac{\partial^2}{\partial y^2}\right\}(\phi')-2\mu'\frac{\partial^2\psi'}{\partial x\partial y},\\&2\mu\frac{\partial^2}{\partial x\,\partial y}(\phi_0+\phi)-\mu\left\{\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right\}(\psi)=2\mu'\frac{\partial^2}{\partial x\,\partial y}\phi'-\mu'\left\{\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right\}(\psi').\end{aligned}\right\}\quad(6)$$

Then, solving the six simultaneous equations of the first degree, containing six unknown quantities (5), (6), we obtain

$$\begin{aligned}\frac{A}{\mathfrak{a}}=&\frac{16a'b'd'}{\varDelta}\exp\{i(a'+b')fH\}\\&\cdot\Big[\mu^2(d^2-4ab)-\mu\mu'(d-2ab)(3-b')+2\mu'^2(1-ab)(d')\Big]\\&+\frac{(d'^2+4a'b')}{\varDelta}\exp\{i2(a'+b')fH\}\\&\cdot\begin{bmatrix}-\mu^2(1+a'b')(d^2-4ab)\\+\mu\mu'\{cc'(a'b-ab')+2(d-2ab)(d'-2a'b')\}\\-\mu'^2(1-ab)(d'^2+4a'b')\end{bmatrix}\\&+\frac{(d'^2-4a'b')}{\varDelta}\exp\{i2a'fH\}\\&\cdot\begin{bmatrix}\mu^2(1-a'b')(d^2-4ab)\\-\mu\mu'\{cc'(ab'+a'b)+2(d-2ab)(d'-2a'b')\}\\+\mu'^2(1-ab)(d'^2-4a'b')\end{bmatrix}\\&+\frac{(d'^2+a'b')}{\varDelta}\begin{bmatrix}-\mu^2(1+a'b')(d^2-4ab)\\+\mu\mu'\{cc'(ab'-a'b)+2(d-2ab)(d'-2a'b')\}\\-\mu'^2(1-ab)(d'^2+4a'b')\end{bmatrix}\\&+\frac{(d'^2-4a'b')}{\varDelta}\exp\{i2b'fH\}\\&\cdot\begin{bmatrix}\mu^2(1-a'b')(d^2-4ab)\\+\mu\mu'\{cc'(ab'+a'b)-2(d-2ab)(d'-2a'b')\}\\+\mu'^2(1-ab)(d'^2-4a'b')\end{bmatrix},\end{aligned}\quad(7)$$

$$\begin{aligned}\frac{B}{\mathfrak{a}}=&\frac{16aa'b'd'}{\varDelta}\exp\{i(a'+b')fH\}\\&\cdot\Big[4\mu^2d-\mu\mu'(2+d)(2+d')+4\mu'^2d'\Big]\end{aligned}$$

$$+\frac{2a(d'^2+4a'b')}{\Delta}\exp\{i2(a'+b')fH\}$$

$$\cdot\begin{bmatrix}-2\mu^2 d(1+a'b')+\mu\mu'(2+d)(d'+2a'b')\\ -\mu'^2(d'^2+4a'b')\end{bmatrix}$$

$$+\frac{2a(d'^2-4a'b')}{\Delta}\exp\{i2a'fH\}$$

$$\cdot\begin{bmatrix}2\mu^2 d(1-a'b')-\mu\mu'(2+d)(d'-2a'b')\\ +\mu'^2(d'^2-4a'b')\end{bmatrix}$$

$$+\frac{2a(d'^2+4a'b')}{\Delta}\begin{bmatrix}-2\mu^2 d(1+a'b')+\mu\mu'(2+d)(d'+a'b')\\ -\mu'^2(d'^2+4a'b')\end{bmatrix}$$

$$+\frac{2a(d'^2-4a'b')}{\Delta}\exp\{i2b'fH\}$$

$$\cdot\begin{bmatrix}2\mu^2 d(1-a'b')-\mu\mu'(2+b)(d'-2a'b')\\ +\mu'^2(d'^2-4a'b')\end{bmatrix}, \tag{8}$$

$$\frac{C}{\mathfrak{a}}=\frac{8ab'cd'}{\Delta}\exp\{i(a'+b')fH\}\Big[-\mu^2(2a'b-d)+\mu\mu'(2a'b-d')\Big]$$

$$+\frac{2ac(d'^2-4a'b')}{\Delta}\exp\{i2b'fH\}\Big[\mu\mu'(bd'+2b')-\mu^2(b'd+2b)\Big]$$

$$+\frac{2ac(d'^2+4a'b')}{\Delta}\Big[\mu\mu'(-bd'+2b')+\mu^2(-b'd+2b)\Big], \tag{9}$$

$$\frac{D}{\mathfrak{a}}=\frac{8ab'cd}{\Delta}\exp\{i(a'+b')fH\}\Big[-\mu^2(2a'b+d)+\mu\mu'(2a'b+d')\Big]$$

$$+\frac{2ac(d'^2+4a'b')}{\Delta}\exp\{i2(a'+b')fH\}\Big[-\mu\mu'(bd'+2b')+\mu^2(b'd+2b)\Big]$$

$$+\frac{2ac(d'^2-4a'b')}{\Delta}\exp\{i2a'fH\}\Big[\mu\mu'(bd'-2b')+\mu^2(b'd-2b)\Big], \tag{10}$$

$$\frac{E}{\mathfrak{a}}=\frac{8aa'cd'}{\Delta}\exp\{i(a'+b')fH\}\Big[\mu^2(b'd+2b)-\mu\mu'(bd'+2b')\Big]$$

$$+\frac{2ac(d'^2+4a'b')}{\Delta}\exp\{i2(a'+b')fH\}\Big[\mu\mu'(d'+2a'b)-\mu^2(d+2a'b)\Big]$$

$$+\frac{2ac(d'^2-4a'b')}{\Delta}\exp\{i2a'fH\}\Big[-\mu\mu'(d-2a'b)+\mu^2(d-2a'b)\Big], \tag{11}$$

$$\frac{F}{\mathfrak{a}}=\frac{8aa'cd'}{\varDelta}\exp\{i(a'+b')fH\}\Big[\mu^2(b'd-2b)+\mu\mu'(bd'-2b')\Big]$$
$$+\frac{2ac(d'^2+4a'b')}{\varDelta}\exp\{i2(a'+b')fH\}\Big[\mu\mu'(d'-2a'b)-\mu^2(d-2a'b)\Big]$$
$$+\frac{2ac(d'^2-4a'b')}{\varDelta}\exp\{i2b'fH\}\Big[-\mu\mu'(d'+2a'b)+\mu^2(d+2a'b)\Big], \quad (12)$$

where

$$\varDelta=\alpha+\beta\exp\Big\{i(a'+b')fH\Big\}+\gamma\exp\Big\{i2(a'+b')fH\Big\}$$
$$+\delta\exp\Big\{i2a'fH\Big\}+\varepsilon\exp\Big\{i2b'fH\Big\}, \quad (13)$$

and

$$\left.\begin{aligned}
\alpha&=(d'^2+4a'b')\begin{bmatrix}\mu'^2(1+ab)(d'^2+4a'b')\\ +\mu\mu'\{cc'(ab'+a'b)-2(d+2ab)(d'+2a'b')\}\\ +\mu^2(1+a'b')(d^2+4ab)\end{bmatrix},\\
\beta&=16a'b'd'\Big[-2\mu'^2(1+ab)d'+\mu\mu'(d+2ab)(3-b')-\mu^2(d^2+4ab)\Big],\\
\gamma&=(d'^2+4a'b')\begin{bmatrix}\mu'^2(1+ab)(d'^2+4a'b')\\ -\mu\mu'\{cc'(ab'+a'b)+2(d+2ab)(d'+2a'b')\}\\ +\mu^2(1+a'b')(d^2+4ab)\end{bmatrix},\\
\delta&=(d'^2-4a'b')\begin{bmatrix}-\mu'^2(1+ab)(d'^2-4a'b')\\ -\mu\mu'\{cc'(ab'-a'b)-2(d+2ab)(d'-2a'b')\}\\ -\mu^2(1-a'b')(d^2+4ab)\end{bmatrix},\\
\varepsilon&=(d'^2-4a'b')\begin{bmatrix}-\mu'^2(1+ab)(d'^2-4a'b')\\ +\mu\mu'\{cc'(ab'-a'b)+2(d+2ab)(d'-2a'b')\}\\ -\mu^2(1-a'b')(d^2+4ab)\end{bmatrix}.
\end{aligned}\right\} \quad (14)$$

In expressions (7) $\sim$ (14), a, b, c, d, and a', b', c', d' are expressed by

$$\left.\begin{aligned}
a&=\tan\theta, & b&=\left\{\left(\frac{V_1}{V_2}\right)^2\sec^2\theta-1\right\}^{1/2},\\
a'&=\left\{\left(\frac{V_1}{V_1'}\right)^2\sec^2\theta-1\right\}^{1/2}, & b'&=\left\{\left(\frac{V1}{V_2'}\right)^2\sec^2\theta-1\right\}^{1/2},\\
c&=\left(\frac{V_1}{V_2}\right)^2\sec^2\theta, & d&=2-\left(\frac{V_1}{V_2}\right)^2\sec^2\theta,\\
c'&=\left(\frac{V_1}{V_2'}\right)^2\sec^2\theta, & d'&=2-\left(\frac{V_1}{V_1'}\right)^6\sec^2\theta,
\end{aligned}\right\} \quad (15)$$

.

where θ is the incidence angle of the primary wave, and V_1, V_2, V_1' and V_2' are the velocities of the dilatational and distortional waves in both media respectively, such that

$$V_1=\sqrt{\frac{\lambda+2\mu}{\rho}}, \quad V_2=\sqrt{\frac{\mu}{\rho}}, \quad V_1'=\sqrt{\frac{\lambda'+2\mu'}{\rho'}}, \quad V_2'=\sqrt{\frac{\mu'}{\rho'}}.$$

Now, using the Fourier double integral formula, we shall generalize expressions (1), (2), (3) as follows. Let the wave form of the primary dilatational wave be $F(x, y)$, when the primary wave ϕ_0 becomes

$$\phi_0=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\exp\left\{-i\frac{V_1 t}{\cos\theta}f\right\}df\int_{-\infty}^{\infty}dr\int_{-\infty}^{\infty}d\xi\int_{-\infty}^{\infty}F(\xi,\eta)\exp\Big[i\{f(x-\xi)-r(y-\eta)\}\Big]d\eta, \tag{16}$$

so that the reflected and refracted waves ϕ, ψ, ϕ', ψ' in both media are generally written

$$\phi=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\exp\left\{-i\frac{V_1 t}{\cos\theta}f\right\}\frac{A}{\alpha}df\int_{-\infty}^{\infty}dr\int_{-\infty}^{\infty}d\xi\int_{-\infty}^{\infty}F(\xi,\eta)$$
$$\cdot\exp\Big[i\{f(x-\xi)+r(y+\eta)\}\Big]d\eta, \tag{17}$$

$$\psi=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\exp\left\{-i\frac{V_1 t}{\cos\theta}f\right\}\frac{B}{\alpha}df\int_{-\infty}^{\infty}dr\int_{-\infty}^{\infty}d\xi\int_{-\infty}^{\infty}F(\xi,\eta)$$
$$\cdot\exp\left[i\left\{f(x-\xi)+r\left(\frac{\sqrt{(V_1/V_2)^2-\cos^2\theta}}{\sin\theta}y+\eta\right)\right\}\right]d\eta, \tag{18}$$

$$\phi'=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\exp\left\{-i\frac{V_1 t}{\cos\theta}f\right\}\frac{C}{\alpha}df\int_{-\infty}^{\infty}dr\int_{-\infty}^{\infty}d\xi\int_{-\infty}^{\infty}F(\xi,\eta)$$
$$\cdot\exp\left[i\left\{f(x-\xi)-r\left(\frac{\sqrt{(V_1/V_1')^2-\cos^2\theta}}{\sin\theta}y+\eta\right)\right\}\right]d\eta$$
$$+\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\exp\left\{-i\frac{V_1 t}{\cos\theta}f\right\}\frac{D}{\alpha}df\int_{-\infty}^{\infty}dr\int_{-\infty}^{\infty}d\xi\int_{-\infty}^{\infty}F(\xi,\eta)$$
$$\cdot\exp\left[i\left\{f(x-\xi)+r\left(\frac{\sqrt{(V_1/V_1')^2-\cos^2\theta}}{\sin\theta}y+\eta\right)\right\}\right]d\eta, \tag{19}$$

$$\psi'=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\exp\left\{-i\frac{V_1t}{\cos\theta}f\right\}\frac{E}{\mathfrak{a}}df\int_{-\infty}^{\infty}dr\int_{-\infty}^{\infty}d\xi\int_{-\infty}^{\infty}F(\xi,\eta)$$

$$\cdot\exp\left[i\left\{f(x-\xi)-r\left(\frac{\sqrt{(V_1/V_2')^2-\cos^2\theta}}{\sin\theta}y-\eta\right)\right\}\right]d\eta$$

$$+\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\exp\left\{-i\frac{V_1t}{\cos\theta}f\right\}\frac{F}{\mathfrak{a}}df\int_{-\infty}^{\infty}dr\int_{-\infty}^{\infty}d\xi\int_{-\infty}^{\infty}F(\xi,\eta)$$

$$\cdot\exp\left[i\left\{f(x-\xi)+r\left(\frac{\sqrt{(V_1/V_2')^2-\cos^2\theta}}{\sin\theta}y+\eta\right)\right\}\right]d\eta. \quad (20)$$

For evaluating the integrals in expressions (17)$\sim$(20), the reciprocal of expression (13) will be expanded in power series of two quantities $\exp\{ia'fH\}$, $\exp\{ib'fH\}$, as follows:

$$\frac{1}{\Delta}=\Bigg[\frac{1}{\alpha}-\left\{\frac{\delta}{\alpha^2}\exp\{i2a'fH\}+\frac{\beta}{\alpha^2}\exp\{i(a'+b')fH\}+\frac{\varepsilon}{\alpha^2}\exp\{i2b'fH\}\right\}$$

$$+\left\{\frac{\delta}{\alpha^3}\exp\{i4a'fH\}+2\frac{\beta\delta}{\alpha^3}\exp\{i(3a'+b')fH\}+\left(\frac{\beta}{\alpha^3}+2\frac{\delta\varepsilon}{\alpha^3}-\frac{\gamma}{\alpha^2}\right)\right.$$

$$\left.\cdot\exp\{i2(a'+b')fH\}+2\frac{\beta\varepsilon}{\alpha^3}\exp\{i(a'+3b')fH\}+\frac{\varepsilon}{\alpha^3}\exp\{i4b'fH\}\right\}$$

$$-\left\{\frac{\delta^3}{\alpha^4}\exp\{i6a'fH\}+3\frac{\beta\delta}{\alpha^4}\exp\{i(5a'+b')fH\}+\left(3\frac{\beta^2\delta}{\alpha^4}+3\frac{\varepsilon\delta^2}{\alpha^4}-2\frac{\gamma\delta}{\alpha^3}\right)\right.$$

$$\cdot\exp\{i2(2a'+b')fH\}+\left(6\frac{\beta\delta g}{\alpha^4}+\frac{\beta^3}{\alpha^4}-2\frac{\beta\gamma}{\alpha^3}\right)\exp\{i3(a'+b')fH\}$$

$$+\left(3\frac{\beta^2\varepsilon}{\alpha^4}+3\frac{\varepsilon^2\delta}{\alpha^4}-2\frac{\gamma\varepsilon}{\alpha^3}\right)\exp\{i2(a'+2b')fH\}+3\frac{\beta\varepsilon^2}{\alpha^4}\exp\{i(a'+3b')fH\}$$

$$\left.+\frac{\varepsilon^3}{\alpha^4}\exp\{i6b'fH\}\right\}+\left\{\frac{\delta^4}{\alpha^5}\exp\{i8a'fH\}+4\frac{\beta\delta^3}{\alpha^5}\exp\{i(7a'+b')fH\}\right.$$

$$+\left(4\frac{\delta^3\varepsilon}{\alpha^5}+6\frac{\delta^2\beta^2}{\alpha^5}-3\frac{\delta^2\gamma}{\alpha^4}\right)\exp\{i2(3a'+b')fH\}+\left(4\frac{\delta\beta^3}{\alpha^5}+12\frac{\delta^2\beta\varepsilon}{\alpha^5}-6\frac{\gamma\delta\beta}{\alpha^4}\right)$$

$$\cdot\exp\{i(5a'+3b')fH\}+\left(12\frac{\varepsilon\beta^2\delta}{\alpha^5}+6\frac{\varepsilon^2\delta^2}{\alpha^5}+\frac{\beta^4}{\alpha^5}+\frac{\gamma^2}{\alpha^3}-\frac{3\beta^2\gamma}{\alpha^4}-\frac{6\gamma\delta\varepsilon}{\alpha^4}\right)$$

$$\cdot\exp\{i4(a'+b')fH\}+\left(\frac{\varepsilon\beta^3}{\alpha^5}+12\frac{\delta\beta\varepsilon^2}{\alpha^5}-6\frac{\varepsilon\beta\gamma}{\alpha^3}\right)\exp\{i(3a'+5b')fH\}$$

$$\left.+\left(4\frac{\delta\varepsilon^3}{\alpha^5}+6\frac{\varepsilon^2\beta^2}{\alpha^5}-3\frac{\varepsilon^2\gamma}{\alpha^4}\right)\exp\{i2(a'+b')fH\}+\frac{\varepsilon^3\beta}{\alpha^5}\exp\{i(a'+7b')fH)\right\}$$

$$+\frac{\varepsilon^4}{\alpha^5}\exp\{i8b'fH\}+\cdots\cdots\Bigg]. \quad (21)$$

It will be seen from expression (21) that, since α is greater than the other quantities β, γ, δ, ε, the coefficients of higher terms of the power series $1/\Delta$ decrease as the number of terms n increases.

Putting (21) in expressions (7)$\sim$(12), the quantities $A/\mathfrak{a}$, $B/\mathfrak{a}$, $C/\mathfrak{a}$, $D/\mathfrak{a}$, $E/\mathfrak{a}$, $F/\mathfrak{a}$ are expressed as the polinomials of $\exp(ia'fH)$ and $\exp(ib'fH)$. Therefore all waves expressed by (17)$\sim$(20), excited by the primary wave as expressed by (16), are put in the form of a summation of the following wave form:

$$F.W.=\sum_n Q_n \int_{-\infty}^{\infty} \exp\left\{i\left(q_n H-\frac{V_1 t}{\cos\theta}\right)f\right\}df \int_{-\infty}^{\infty} dr \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} F(\xi,\eta)$$

$$.\exp\Big[i\,\{f(x-\xi)-r(\pm wy\pm\eta)\}\Big]d\eta\,. \tag{22}$$

w, in (22), should, of course, correspond to the reflected and refracted waves of dilatational and distortional types, as will be seen from (17)$\sim$(20), while Q_n, q_n in expression (22) should also satisfy the relations expressed by (17)$\sim$(20) and (21). The quantity q_n is usually composed of a', b', such that

$$q_n = ma' + m'b'\,, \tag{23}$$

where m, m' are positive integers, such as 0, 1, 2, 3, being determined by the relations (7)$\sim$(12) and (21), while a', b' are positive, as will be seen from (15), whence we know that as the number of terms in the power series (21) is increased, quantity q_n also increases. The sign of Q_n becomes alternately positive and negative with increase in n; it becomes small as the number of terms in the power series increases, as will be seen from expression (21). Therefore, all the waves excited in the surface-layer and in the subjacent medium have the properties of multiple reflection and refraction. Their intensities thus decrease with time-variation.

Now, the integrals in (22) can easily be evaluated when the wave form of the primary wave is of shock type, such as

$$F(x,y)=\exp\left\{-(x\cos\theta-y\sin\theta)^2/c^2\right\}, \tag{23}$$

and the evaluated result becomes

$$F.W.=\sum Q_n \exp\left\{-(x\cos\theta\pm y\sin\theta-V_1 t+q_n H)^2\Big/c^2\right\}. \tag{24}$$

The horizontal and vertical displacements U, V, U', V' in the surface-layer and in the subjacent medium are then obtained from the following relations:

$$U=\frac{\partial\phi_0}{\partial x}+\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y}, \tag{25}$$

$$V=\frac{\partial\phi_0}{\partial y}+\frac{\partial\phi}{\partial y}-\frac{\partial\psi}{\partial x}, \tag{26}$$

$$U'=\frac{\partial\phi'}{\partial x}+\frac{\partial\psi'}{\partial y}, \tag{27}$$

$$V'=\frac{\partial\phi'}{\partial y}-\frac{\partial\psi'}{\partial x}. \tag{28}$$

3. **An example.** To get an idea of the properties of the vibrations of particles on the top and bottom surfaces of the surface-layer, excited by the shock wave incident on the bottom surface of it, let $\lambda=\mu$, $\lambda'=\mu'$, $\rho=\rho'$, and $\mu'/\mu=1/10$, $\theta=45°$ for numerical calculations. Then, applying the method shown in section 2, we numerically calculated the vibrational motions of the particles on the top and bottom surfaces for the three cases

$$H/c=0{\cdot}48,\quad 0{\cdot}80,\quad 1{\cdot}46, \tag{29}$$

with results as shown in Figs. 2∼13.

Obviously, when $F(x,y)$ is expressed by (23),

$$\phi_0=\exp\left\{-(x\cos\theta-y\sin\theta-V_1t)^2/c^2\right\}, \tag{30}$$

the horizontal and vertical vibrations due to ϕ_0 being respectively expressed by

$$\begin{aligned}cu_0&=-2\cos\theta\left(\frac{x}{c}\cos\theta-\frac{y}{c}\sin\theta-\frac{V_1t}{c}\right)\exp\left\{-(x\cos\theta-y\sin\theta-V_1t)^2/c^2\right\},\\ cv_0&=2\sin\theta\left(\frac{x}{c}\cos\theta-\frac{y}{c}\sin\theta-\frac{V_1t}{c}\right)\exp\left\{-(x\cos\theta-y\sin\theta-V_1t)^2/c^2\right\}.\end{aligned} \tag{31}$$

Therefore, when $\theta=45°$, (31) becomes

$$\begin{aligned}cu_0&=-2{\cdot}828\left(1{\cdot}414\frac{x}{c}-1{\cdot}414\frac{y}{c}-\frac{V_1t}{c}\right)\exp\left\{-\left(1{\cdot}414\frac{x}{c}-1{\cdot}414\frac{y}{c}-\frac{V_1t}{c}\right)^2\right\},\\ cv_0&=2{\cdot}828\left(1{\cdot}414\frac{x}{c}-1{\cdot}414\frac{y}{c}-\frac{V_1t}{c}\right)\exp\left\{-\left(1{\cdot}414\frac{x}{c}-1{\cdot}414\frac{y}{c}-\frac{V_1t}{c}\right)^2\right\}.\end{aligned} \tag{32}$$

Even though the vibration due to the primary wave is very simple, as will be seen from (32), the vibrations in the surface-layer, however, become very complicated, as will be seen from Figs. 2~13. From these figures, it will be seen that when the pulse of a dilatational wave of shock type is obliquely incident on the bottom surface of the surface-layer, complicated damped vibrations are excited in the surface-layer, the duration of this damped vibration increasing with increase in the quantity H/c; the shorter the apparent period of the primary shock wave, the longer the duration of vibration in the surface-layer. When H/c becomes relatively large, i. e., when the apparent period of the primary shock wave becomes short, the vibrations of particles on the top surface of the surface-layer are usually superposed with vibrations of relatively short periods (Figs. 4, 12).

The damped vibrational movement of the surface-layer is caused, of course, by the interference effect of refraction and reflection of both the dilatational and distortional waves of shock type excited in the surface-layer by the primarily incident wave of shock type, while damping of the vibrations is caused by dispersion of the wave energy into the subjacent medium.

It seems that the duration of vibration of the horizontal movement in the surface-layer is longer than that of the vertical movement. Generally speaking, the vibrational amplitudes of both horizontal and vertical movements on the top surface are larger than those on the bottom surface, but it is remarkable that the horizontal amplitude of the damped vibration at its commencement occasionally becomes smaller than that on the bottom surface of the surface-layer (Figs. 10, 12).

Figs. 14~19 show the vibration loci of particles on the top and the bottom surfaces of the surface-layer for the three cases $H/c=0{\cdot}48$, $0{\cdot}80$, $1{\cdot}46$. From these figures, it will be seen that the loci of vibrations of the surface-layer become also very complicated, even when a simple impulse is applied to the bottom surface of it and the push-pull relation holds in the initial motion of the vibrations excited on the top and bottom surfaces. It will also be seen from these figures that, at a later stage of vibration, the horizontal vibration in the surface-layer dominates the vertical vibration.

21. 斜めの入射波（衝撃波型縦波）による表面層の振動（第1報）

地震研究所 { 西村源六郎 / 高山威雄 }

地殻表面層に入射する地震波が衝撃型である場合に生ずる表面層の地震動を理論的に研究せんとする試みの一つである．本論文ではこの衝撃型の入射地震波が縦波であつて，而も入射角が45°である場合に就て，圖をもつて地震動を研究した．

Errata

$\frac{V}{c}t$ in Figs. 2~13 should be $\frac{V_1}{c}t$.

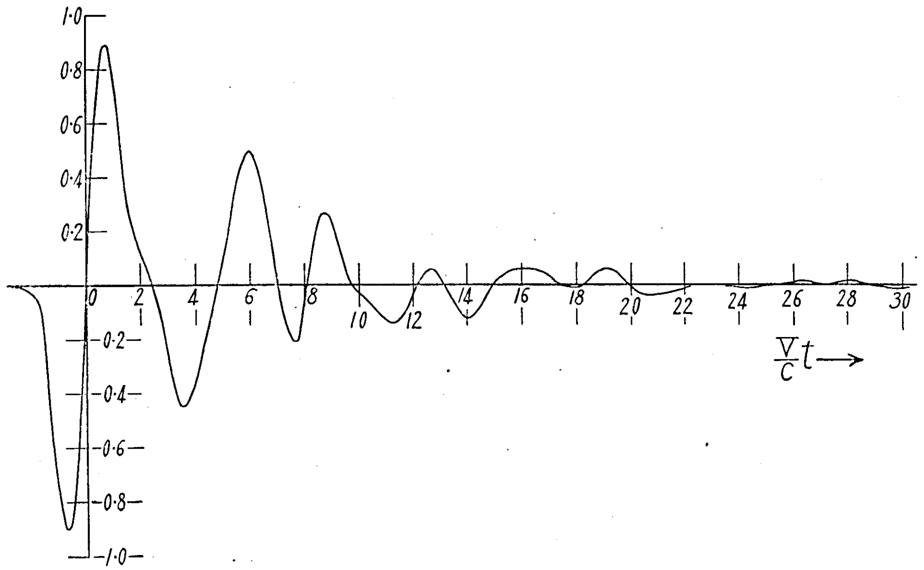

Fig. 2. The horizontal vibration on the bottom surface when $H/c=0{\cdot}48$.

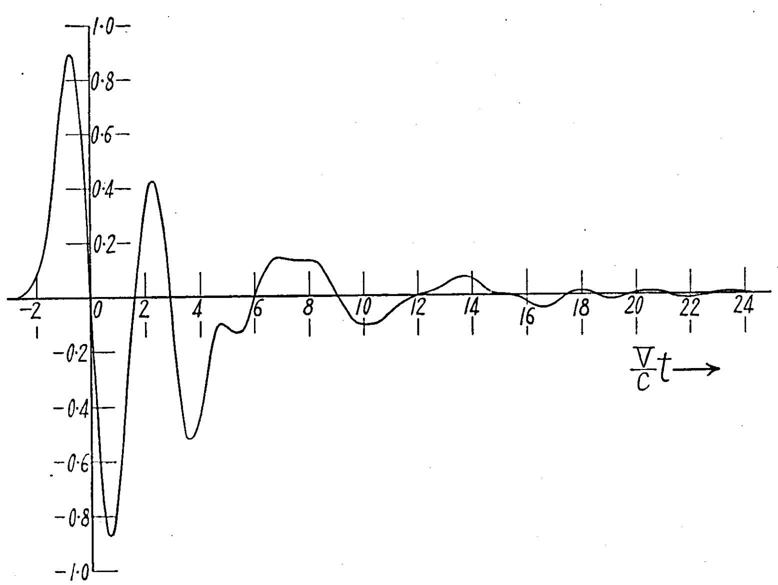

Fig. 3. The vertical vibration on the bottom surface when $H/c=0{\cdot}48$.

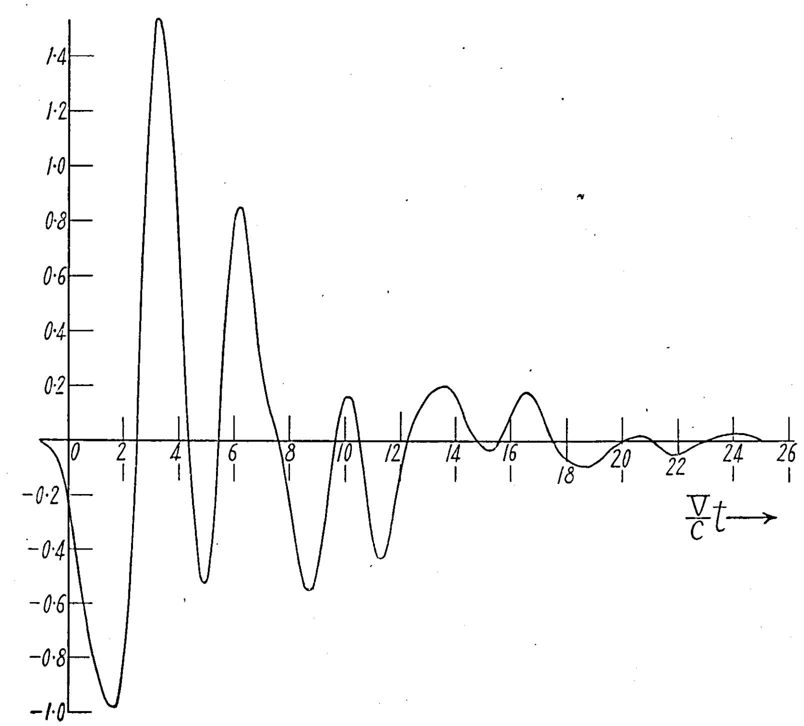

Fig. 4. The horizontal vibration on the top surface when $H/c=0{\cdot}48$.

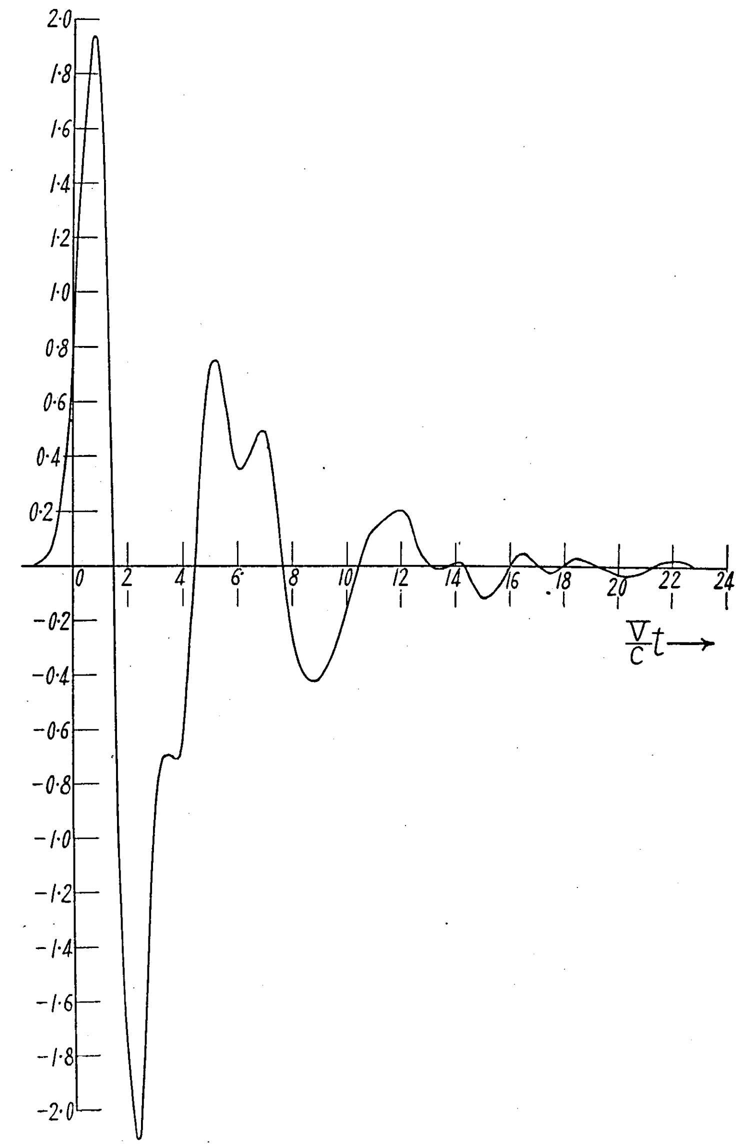

Fig. 5. The vertical vibration on the top surface when $H/c=0{\cdot}48$.

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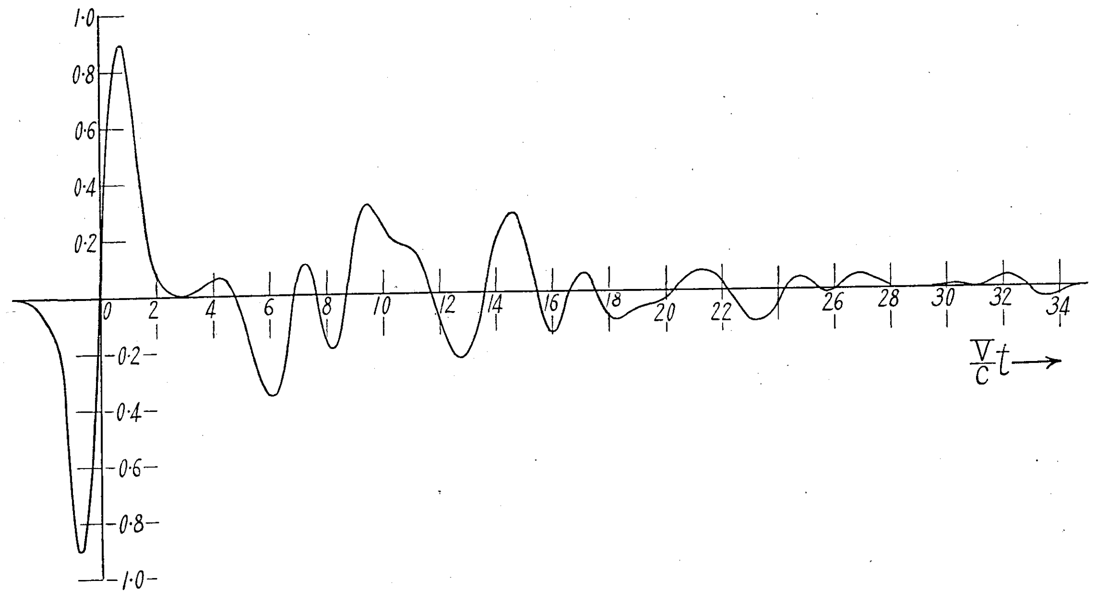

Fig. 6. The horizontal vibration on the bottom surface when $H/c=0{\cdot}80$.

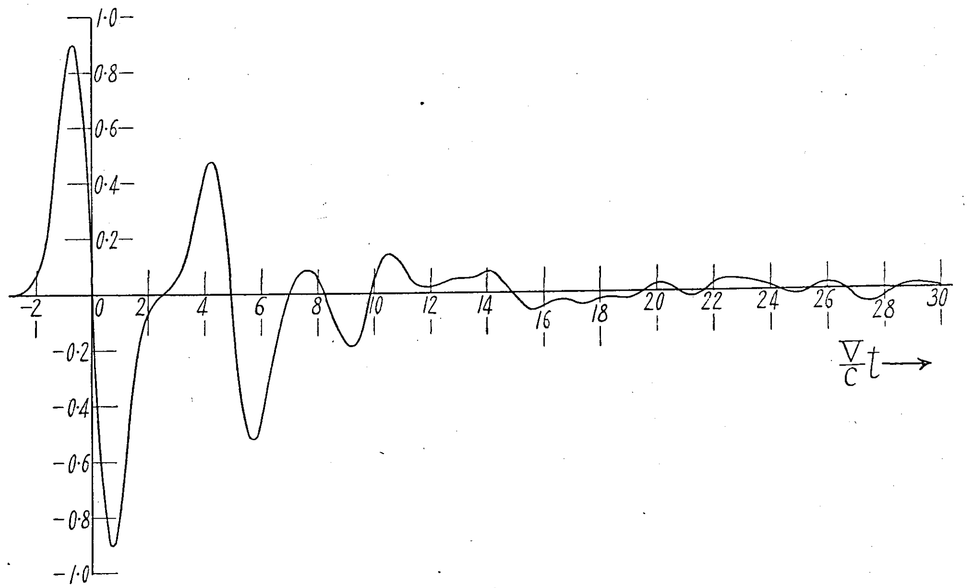

Fig. 7. The vertical vibration on the bottom surface when $H/c=0{\cdot}8$)

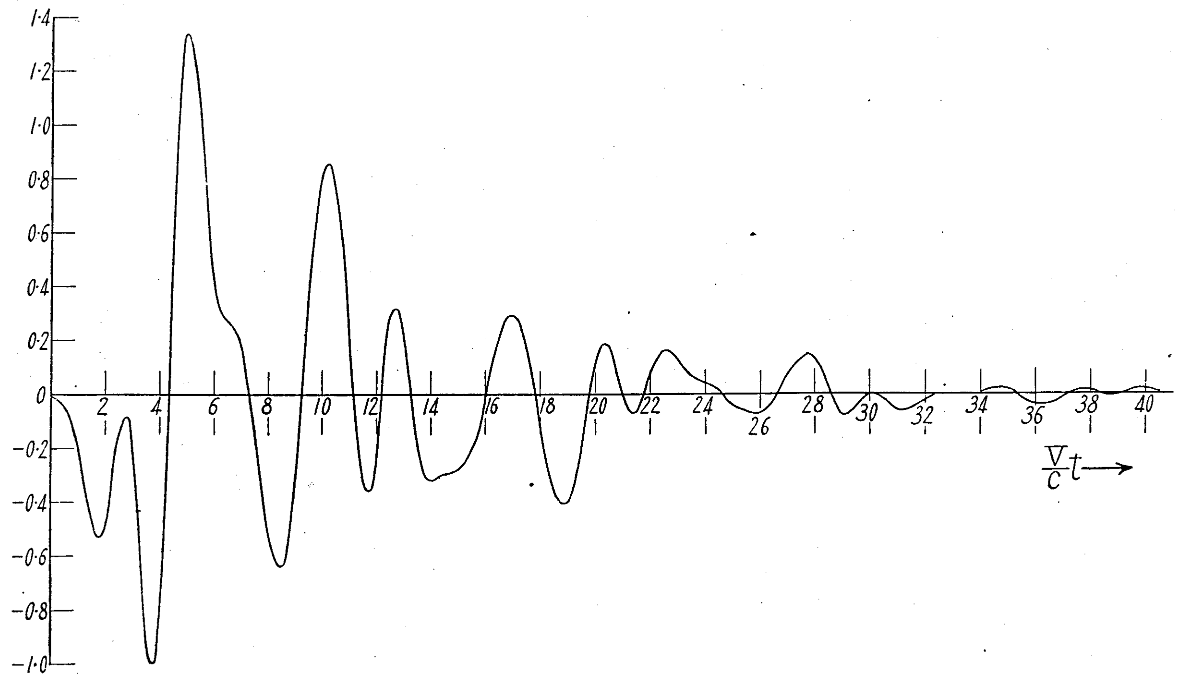

Fig. 8. The horizontal vibration on the top surface when $H/c=0{\cdot}80$.

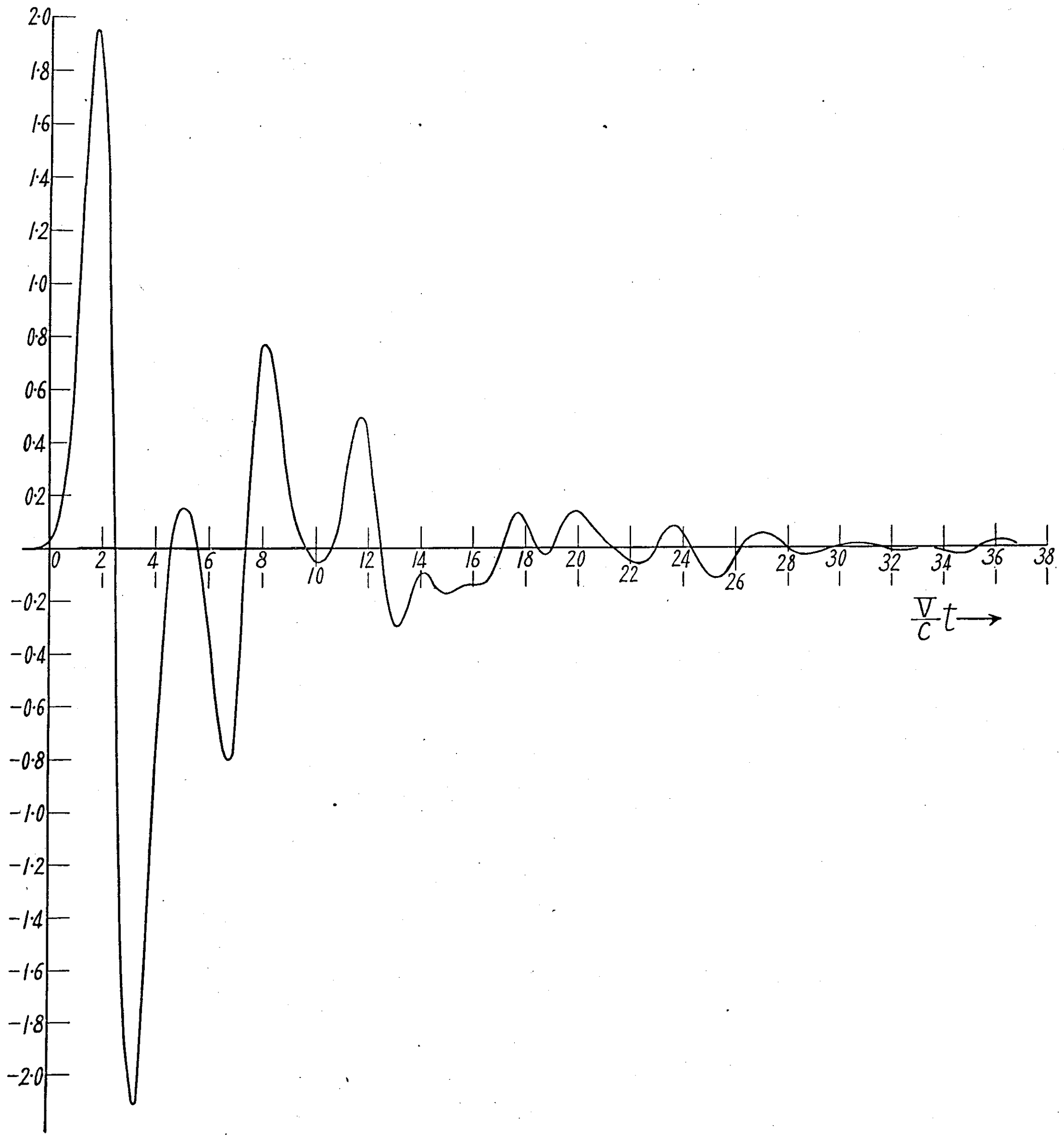

Fig. 9. The vertical vibration on the top surface when $H/c=0$ 80.

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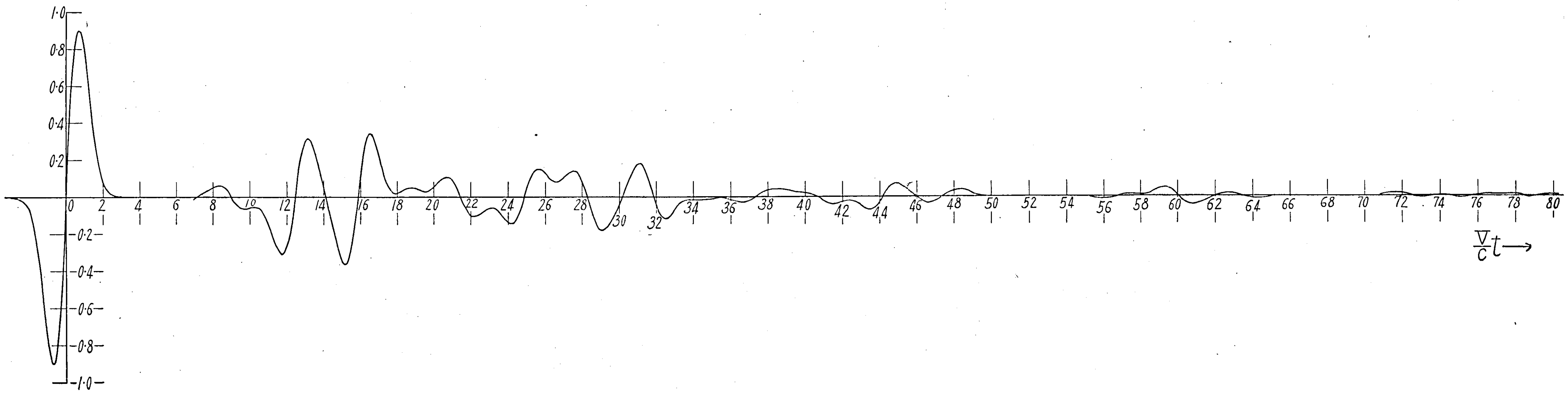

Fig. 10. The horizontal vibration on the bottom surface when $H/c=1{\cdot}46$.

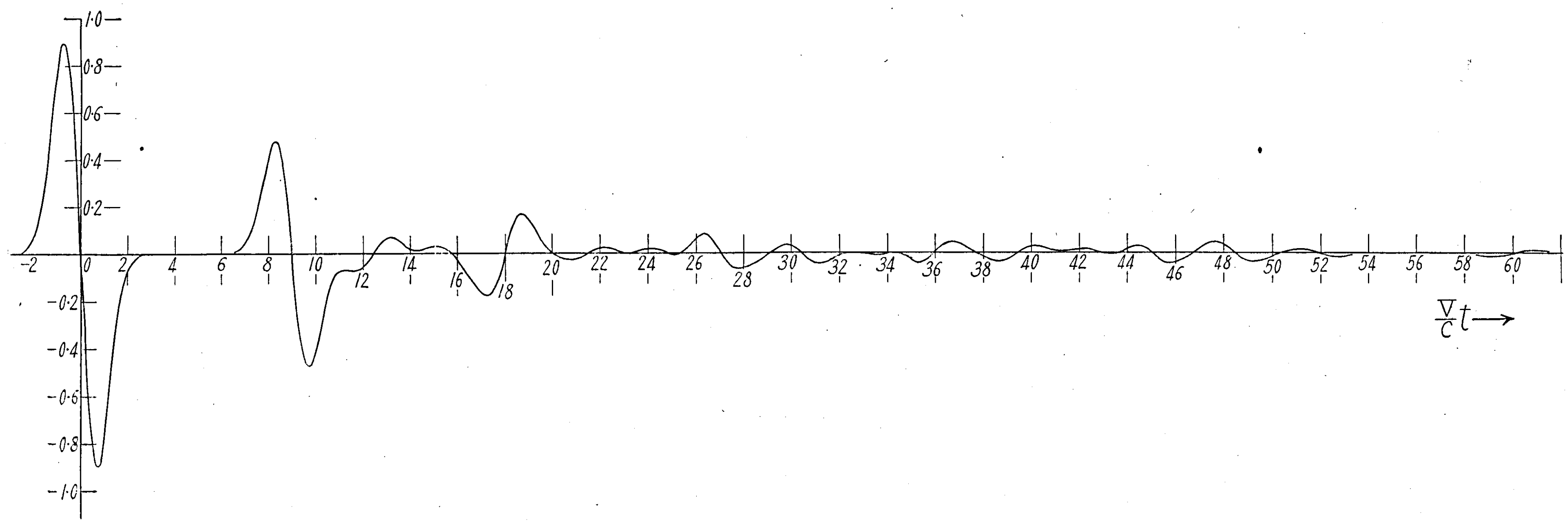

Fig. 11. The vertical vibration on the bottom surface when $H/c=1{\cdot}46$.

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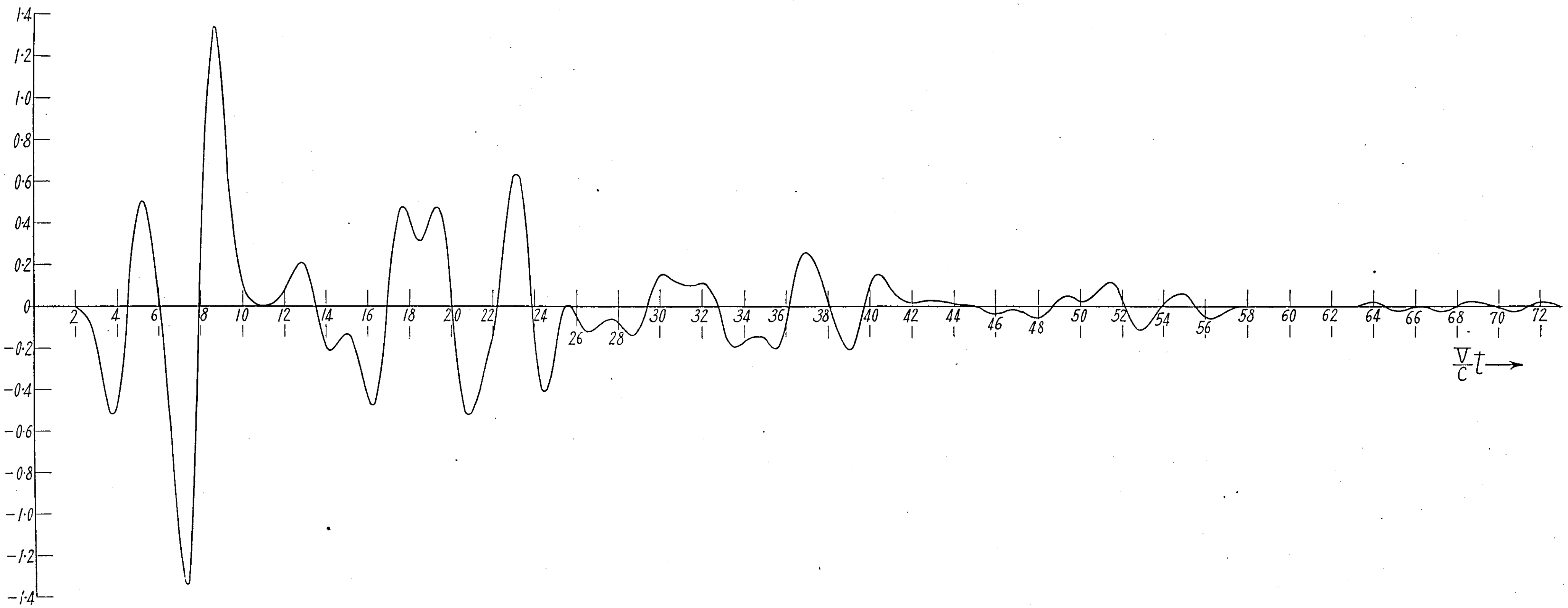

Fig. 12. The horizontal vibration on the top surface when $H/c=1{\cdot}46$.

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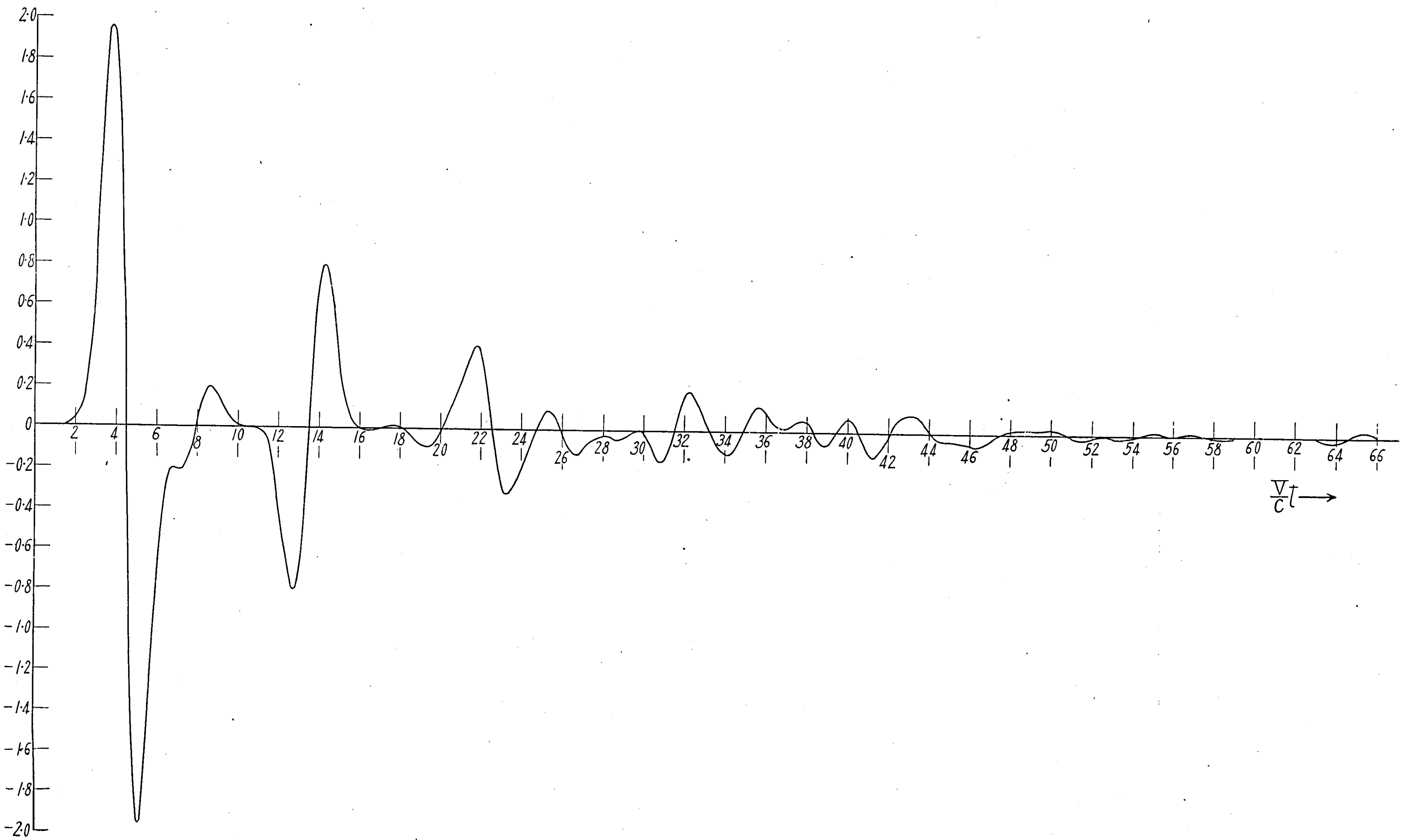

Fig. 13. The vertical vibration on the top surface when $H/c=1{\cdot}46$.

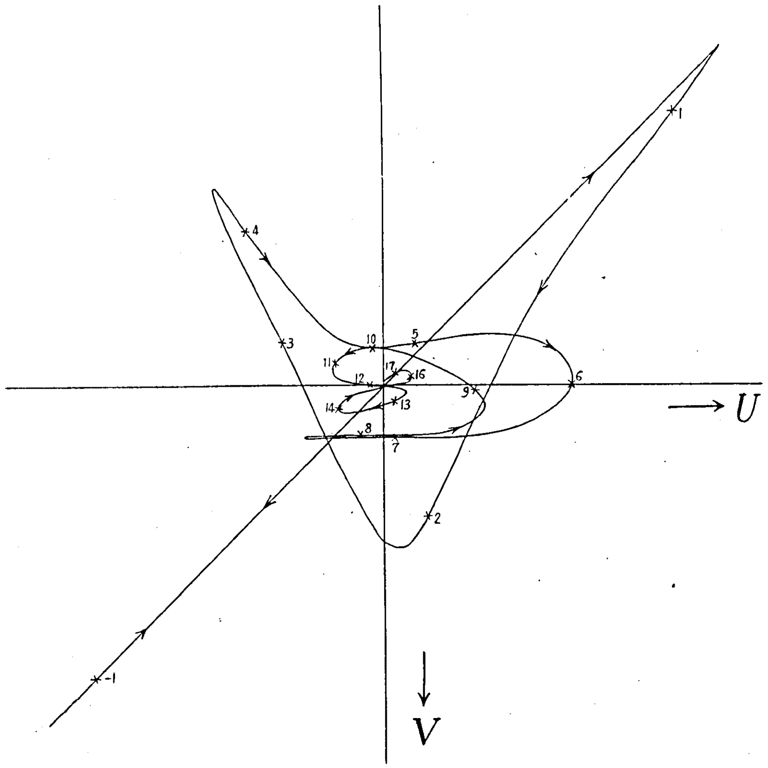

Fig. 14. The orbital motion on the bottom surface when $H/c=0{\cdot}48$. The numerals on the curve show the values of $V_1\,t/c$.

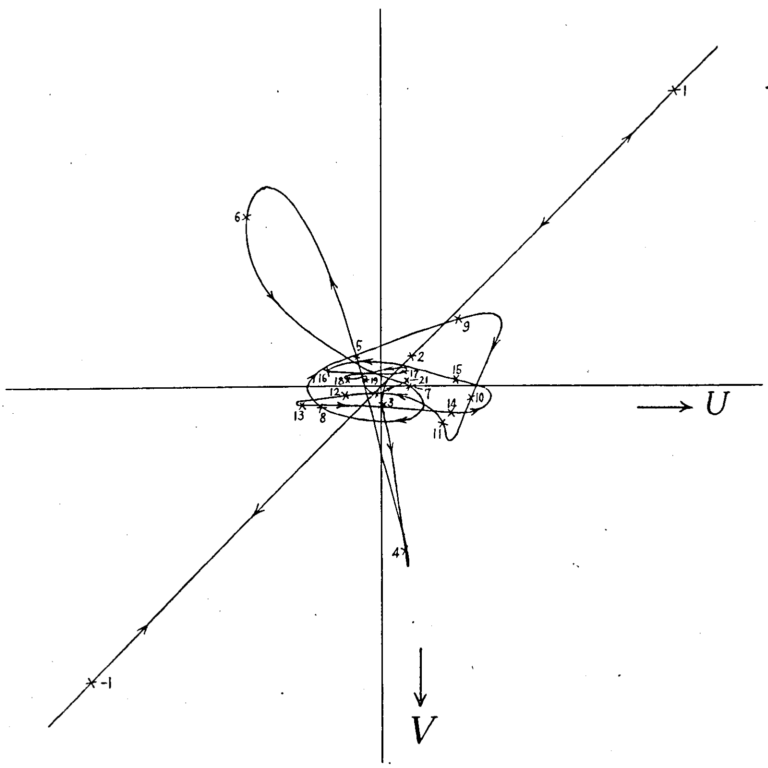

Fig. 16. The orbital motion on the botto msurface when $H/c=0{\cdot}80$. The numerals on the curve show the values of $V_1\,t/c$.

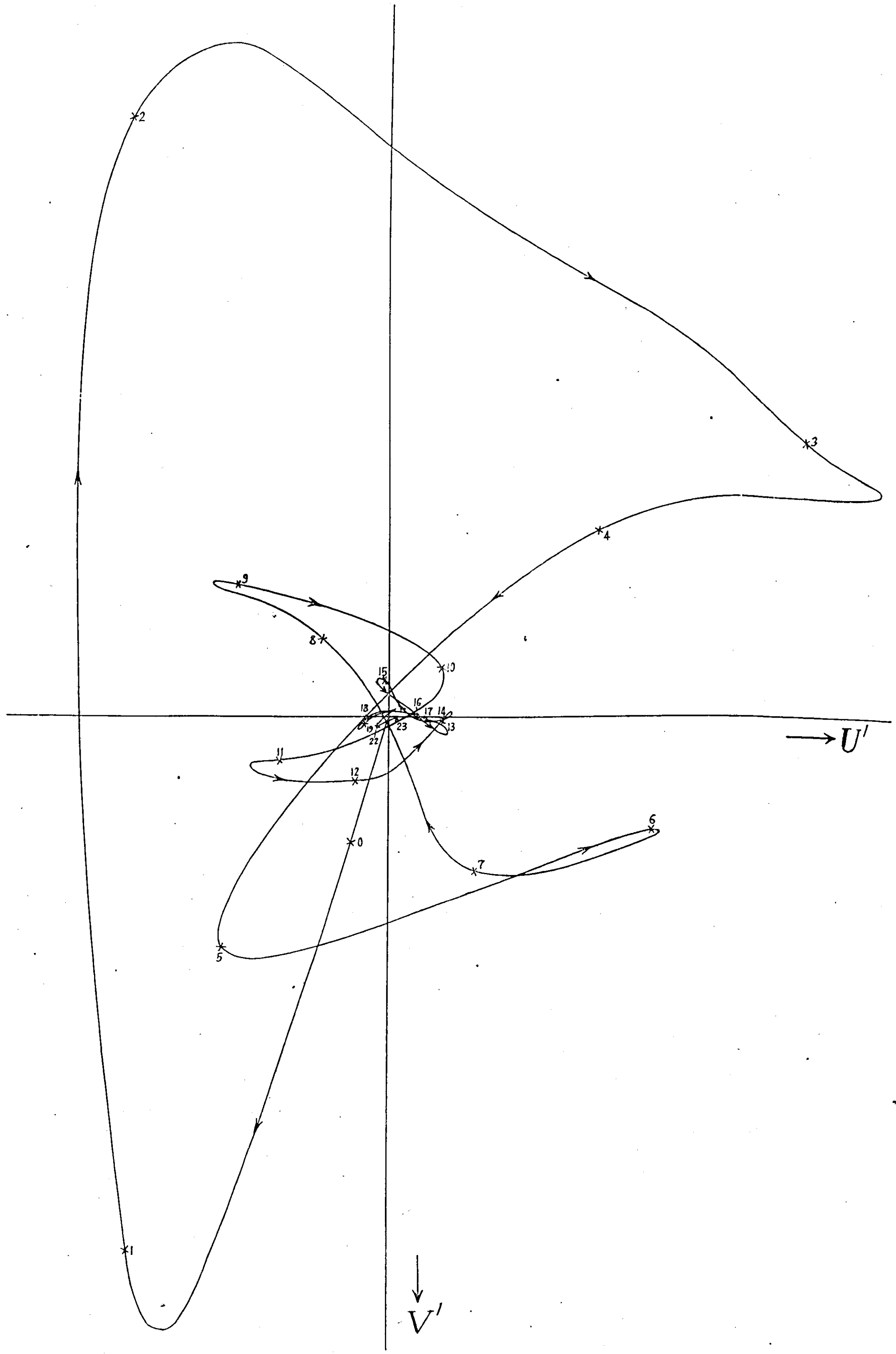

Fig. 15. The orbital motion on the top surface when $H/c=0{\cdot}48$. The numerals on the curve show the values of $V_1\,t/c$.

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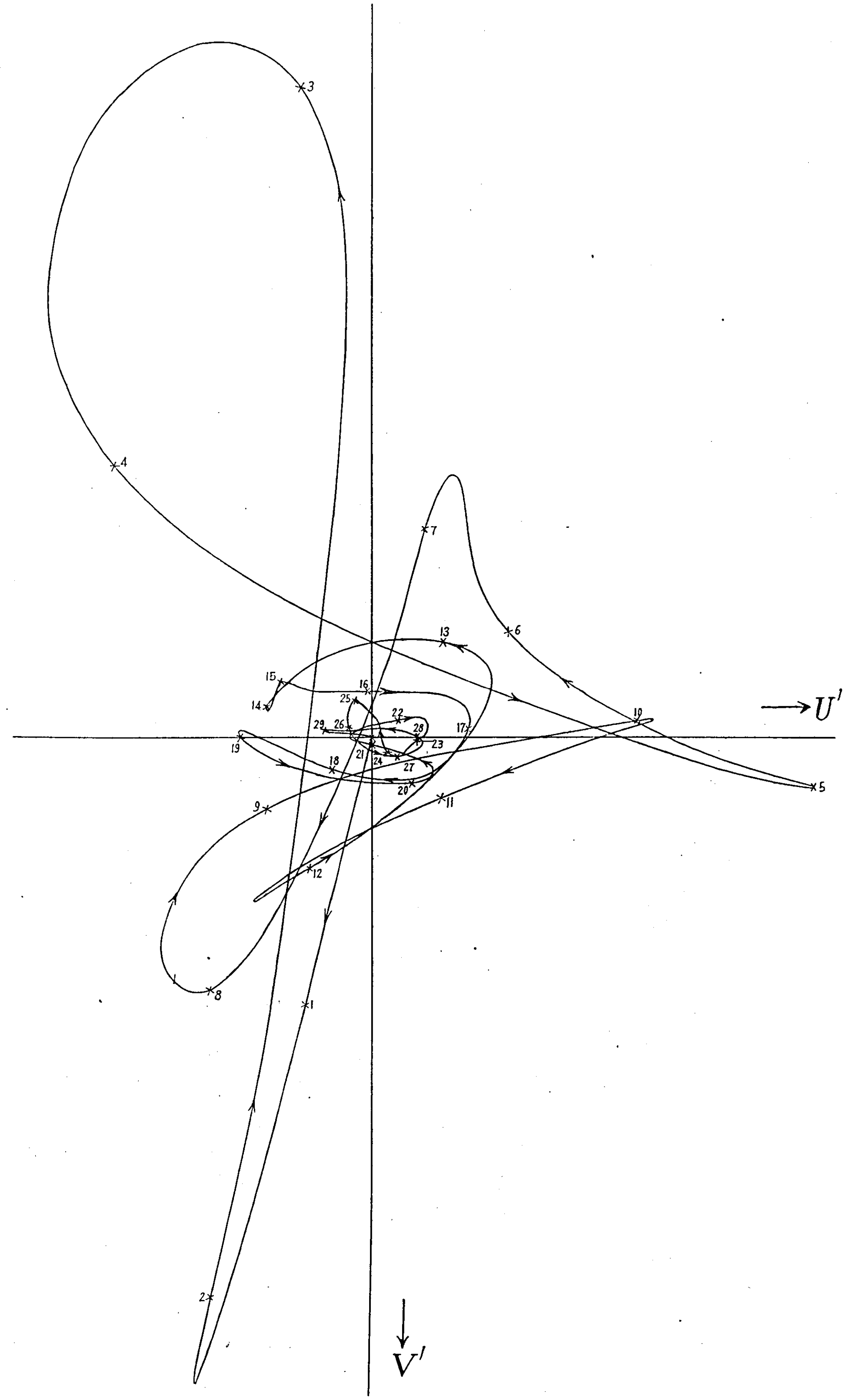

Fig. 17. The orbital motion on the top surface when $H/c=0{\cdot}80$.
The numerals on the curve show the values of $V_1\, t/c$.

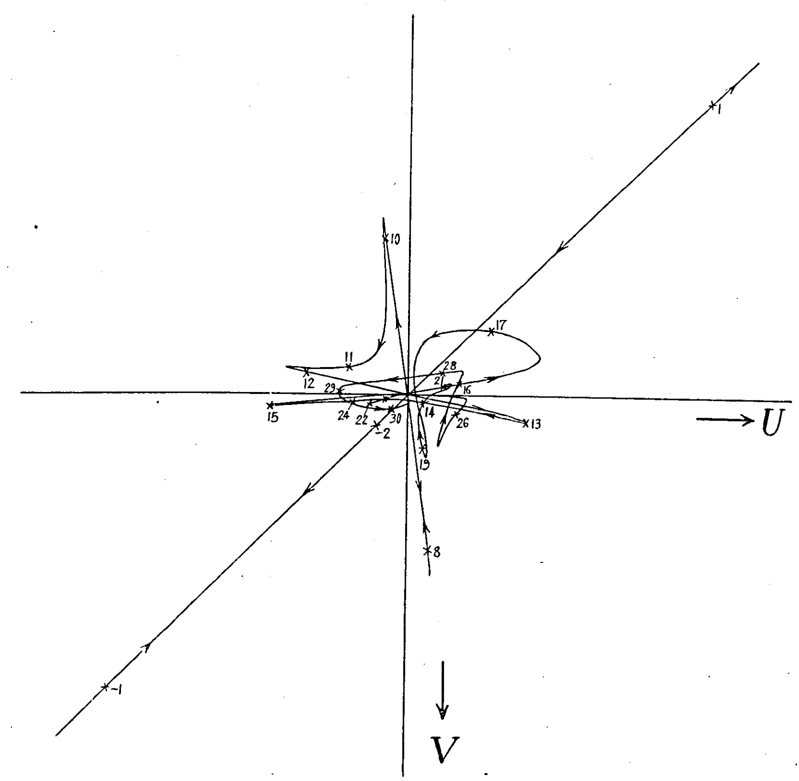

Fig. 18. The orbital motion on the bottom surface when $H/c=1{\cdot}46$.
The numerals on the curve show the values of $V_1\, t/c$.

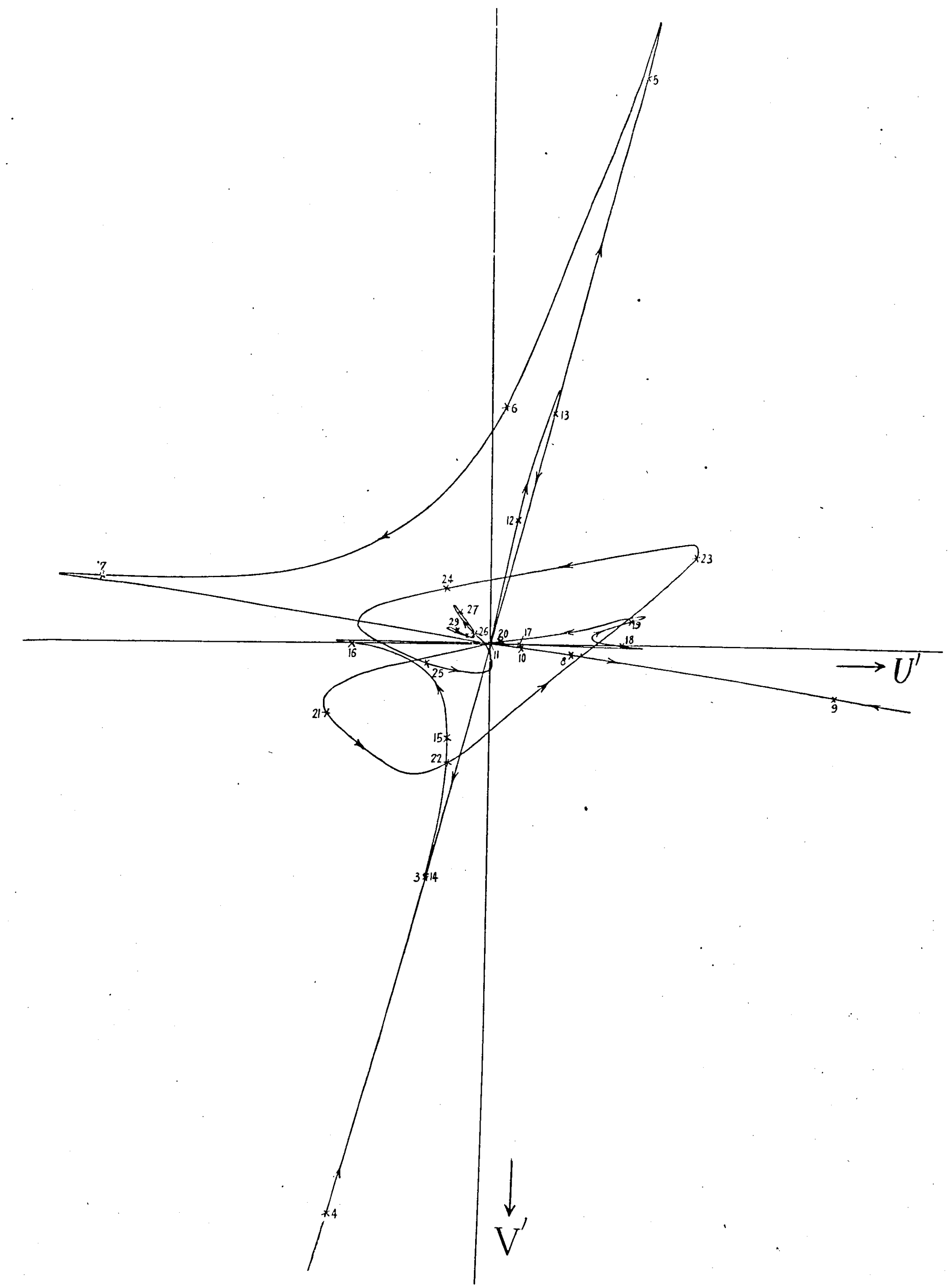

Fig. 19. The orbital motion on the top surface when $H/c=1{\cdot}46$.
The numerals on the curve show the values of $V_1\, t/c$.