31. *Relation between the Normal-Tangential Viscosity Ratio and Poisson's Elasticity Ratio in Certain Soils.*

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1. Introduction.

In our previous papers[1] we discussed the elastic properties of certain kinds of soil exhibited by subjecting them to vibration. As already explained, the principal object of our experiments was to impart various vibration frequencies to the foot of the soil specimen where it touches the vibrating plate, and thus ascertain the fundamental resonance frequency, after which its elastic constants were computed. In order to determine the velocity of elastic waves in the soil specimen, a diagram was constructed, in which the fundamental resonance periods obtained from our experiments were plotted as ordinate, and the various heights of the same soil specimen as abscissa. Seeing that the curves showing these relations do not pass through the coordinate origin, we concluded that these results are due to the solid viscosity of the soil.

We therefore formulated equations of motion for the longitudinal and the torsional wave in the soil column with the assumption that the stresses in the soil specimen could be regarded as having been derived from the ordinary elastic stresses due to Hooke's law plus the viscous stresses, namely,

$$\rho\frac{\partial^2 w}{\partial t^2}=E\frac{\partial^2 w}{\partial z^2}+\gamma_l\frac{\partial^3 w}{\partial z^2\partial t}, \qquad (1)$$

$$\rho\frac{\partial^2 u}{\partial t^2}=\mu\frac{\partial^2 u}{\partial z^2}+\gamma_t\frac{\partial^3 u}{\partial z^2\partial t}, \qquad (2)$$

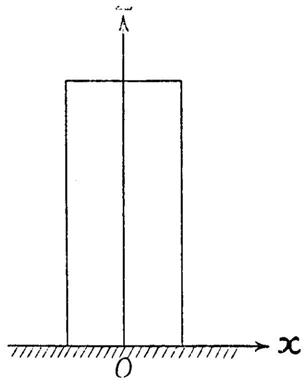

Fig. 1.

where w is the displacement in the direction of z, u the displacement in the direction of x (Fig. 1), ρ the density, t the time; E, μ Young's modulus and modulus of rigidity respectively, and γ_l, γ_t the normal and tan-

1) M. ISHIMOTO and K. IIDA, "Determination of Elastic Constants of Soils by means of Vibration Methods," *Bull. Earthq. Res. Inst.*, **14** (1936), 632; **15** (1937), 67.

gential solid viscosity coefficients respectively.

As already shown in our previous papers[2], the longitudinal and the torsional fundamental resonance periods, T_l and T_t, are given by

$$T_l=\frac{4h}{\sqrt{\frac{E}{\rho}-\frac{\pi^2\gamma_l^2}{16\rho^2h^2}}}, \tag{3}$$

$$T_t=\frac{4h}{\sqrt{\frac{\mu}{\rho}-\frac{\pi^2\gamma_t^2}{16\rho^2h^2}}}. \tag{4}$$

With these equations it is possible to determine the elastic constants E, μ and the solid viscosity coefficients γ_l, γ_t. Although we have already pointed out that the ratio of γ_l to γ_t thus obtained seems to diminish with increase in water content, we did not prove the accuracy of these quantities, so that it was found desirable to make further experiments in order to ascertain these relations. On the other hand, it is yet unknown whether or not the normal-tangential viscosity ratio, namely, γ_l/γ_t, might correspond to Poisson's elasticity ratio.

The objects of our experiments were to ascertain the nature of soil with reference to its elastic and viscous properties, for which purpose we studied not only the relations between the ratio γ_l to γ_t and the water content of the soil, but also the relations of the ratio γ_l to γ_t and Poisson's elasticity ratio.

2. Soil Specimens.

In these experiments we used the same kinds of soil as those previously used, namely, the loam at Hongô (Imperial University) and the silty-clay at Maru-no-uti in Tôkyô. The original conditions and the mechanical properties of these soils are similar to those described in previous papers[3]. The forms of the soil specimens were cylinders of initial heights of from 20 to 30 cm, with a diameter of about 5 cm as before. The soils were tested in their natural state, and sometimes in their altered state. The water contents were altered by leaving the original moist soil specimen to dry under ordinary room temperature. The physical properties in question were studied for the foregoing variety of conditions.

2) M. ISHIMOTO and K. IIDA, *loc. cit.*

3) *loc. cit.*

3. Experiments.

In the present experiments, both apparatuses and methods were the same as those employed previously. When a soil specimen becomes hard through loss of its water, it is difficult to observe the resonance frequencies of the higher orders within the limit of our scope of experiment, because of the increase in fundamental resonance frequency, for which reason it was not possible for us to investigate the properties of each soil for water contents other than those coming within the scope of this paper. The diagram showing the relations between the fundamental resonance period T_l or T_t and the height h of the soil specimens are shown in Figs. 2 and 3, in which the period T_l or T_t is taken as ordinate and the height h as abscissa.

In the present case, we determined both elastic constants E, μ, and solid viscosity coefficients γ_l, γ_t by means of the method of least squares with the aid of equations (3) and (4), into which we put the experimental values, such as T_l, T_t, corresponding to each height h. We then computed the longitudinal and the transverse wave-velocities V_l, V_t in the soil specimen by means of the following relations: $V_l=\sqrt{\dfrac{E}{\rho}}$, $V_t=\sqrt{\dfrac{\mu}{\rho}}$. All the values thus obtained are shown in Tables I, II, from which it will be seen that the probable errors in these values range from about 2 per cent to 20 per cent. The resonance periods, however, were determined with an accuracy of from 1 to 2 per cent.

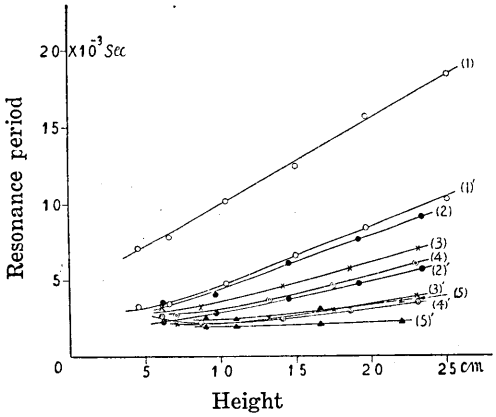

Fig. 2. Relation between the fundamental resonance period of vibration and the height of soil specimen (the silty-clay at Maru-no-uti).
(1)∼(5): torsional vibration.
(1)′∼(5)′: longitudinal vibration.
(1) Water content $w=48{\cdot}5\%$.
(2) $w=42{\cdot}3\%$. (3) $w=39{\cdot}5\%$.
(4) $w=37{\cdot}4\%$. (5) $w=31{\cdot}5\%$.

It is possible to get from Tables I, II several diagrams representing the relations between the elastic constants and the solid viscosity coefficients and the water contents.

These diagrams are shown in Figs. 4∼8, in which the abscissa al-

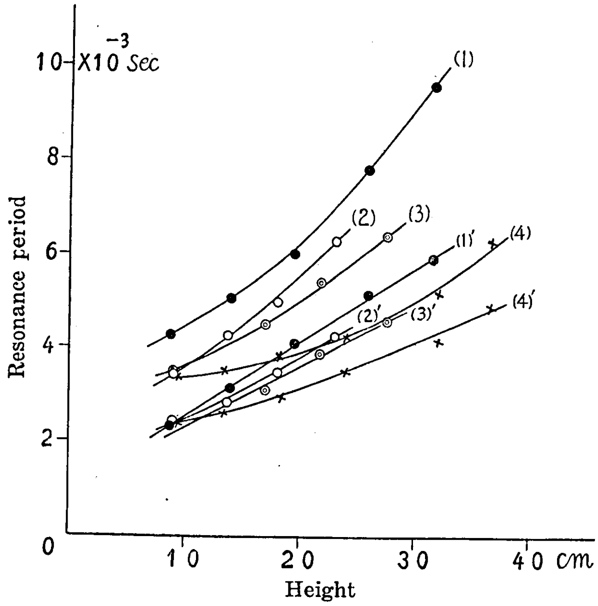

Fig. 3. Relation between the fundamental resonance period of vibration and the height of soil specimen (the loam at Hongô).
(1)~(4): torsional vibration.
(1)'~(4)': longitudinal vibration.
(1) Water content $w=48{\cdot}0\%$. (2) $w=46{\cdot}2\%$.
(3) $w=43{\cdot}6\%$. (4) $w=40{\cdot}0\%$.

Table I. Longitudinal and Torsional Wave-Velocities, Normal and Tangential viscosity Coefficients, Density, and Water Content of Soils.

Kind of soil	No.	Water cont. $w(\%)$	Density ρ	V_l (m/sec)	V_t (m/sec)	γ_l	γ_t	$\frac{V_l}{V_t}$
						$\times 10^5$	$\times 10^5$	
Silty-clay (Maru-no-uti)	1	51·0	1·46	63·6± 4·5	37·2± 3·0	0·359±0·010	0·212±0·009	1·709
	2	48·5	1·50	96·1± 6·1	56·0± 7·6	0·569±0·016	0·398±0·012	1·716
	3	42·3	1·56	167·6±23·1	103·8±22·5	1·576±0·045	0·921±0·044	1·615
	4	39·5	1·63	240·1±13·2	151·5± 8·7	3·182±0·053	1·760±0·035	1·585
	5	37·4	1·56	251·0±21·6	157·1± 8·6	3·065±0·633	1·368±0·025	1·597
	6	31·5	1·59	377·6±33·8	246·7± 6·1	6·600±0·891	2·890±0·325	1·531
	7*	47·9	1·49	34·5± 2·2	20·0± 1·5	0·396±0·015	0·212±0·011	1·729
Loam (Hongô)	1	53·5	1·31	185·3±10·1	110·0± 9·3	1·764±0·180	0·877±0·092	1·684
	2	48·0	1·24	217·0±19·6	134·2±13·6	3·525±0·450	1·481±0·104	1·617
	3	46·2	1·19	233·0± 9·1	149·0±21·0	3·750±0·256	1·585±0·121	1·564
	4	43·6	1·16	247·6± 8·0	162·0± 5·1	4·090±0·088	1·736±0·056	1·528
	5	40·0	1·09	299·8±20·9	198·5±19·5	4·760±0·490	2·155±0·362	1·510

Table II. Elastic Constants, γ_l/γ_t, and E/μ of Soils.

Kind of soil	No.	Young's modulus E (c. g. s.)	Rigidity μ (c. g. s.)	Poisson's ratio σ	λ (c.g.s.)	$\frac{\gamma_l}{\gamma_t}$	$\frac{E}{\mu}$
		$\times 10^8$	$\times 10^8$		$\times 10^8$		
Silty-clay (Maru-no-uti)	1	0·590±0·009	0·202±0·006	0·46±0·04	2·324	1·689±0·085	2·920±0·097
	2	1·385±0·012	0·470±0·015	0·47±0·02	7·369	1·708±0·065	2·944±0·037
	3	4·382±0·046	1·681±0·045	0·30±0·04	2·521	1·713±0·093	2·607±0·074
	4	9·400±0·264	3·741±0·174	0·26±0·06	4·053	1·808±0·047	2·511±0·136
	5	9·826±0·432	3·850±0·017	0·27±0·05	4·520	2·240±0·454	2·550±0·112
	6	22·670±0·676	9·677±0·121	0·17±0·04	4·984	2·280±0·400	2·344±0·075
	7*	0·176±0·004	0·055±0·003	0·49±0·03	2·911	1·866±0·121	2·991±0·051
Loam (Hongô)	1	4·498±0·202	1·585±0·018	0·42±0·06	8·321	2·012±0·294	2·836±0·131
	2	5·839±0·391	2·233±0·027	0·31±0·08	3·643	2·331±0·344	2·614±0·177
	3	6·460±0·184	2·642±0·042	0·22±0·04	2·076	2·366±0·242	2·445±0·079
	4	7·112±0·159	3·044±0·101	0·17±0·04	1·568	2·356±0·091	2·336±0·093
	5	9·800±0·419	4·295±0·389	0·14±0·02	1·670	2·209±0·434	2·281±0·228

* The original moist soil (No. 2) was tested in its altered state of packing.

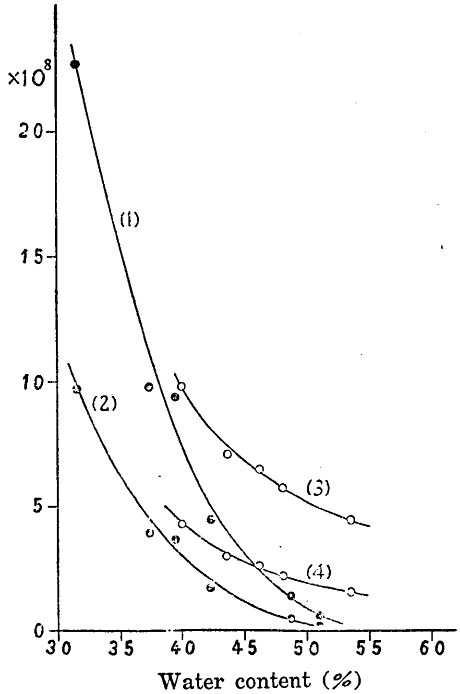

Fig. 4. Showing that the elastic constants diminish with increase in water content.
(1), (3): Young's modulus E.
(2), (4): modulus of rigidity μ.
Ordinate: E or μ.

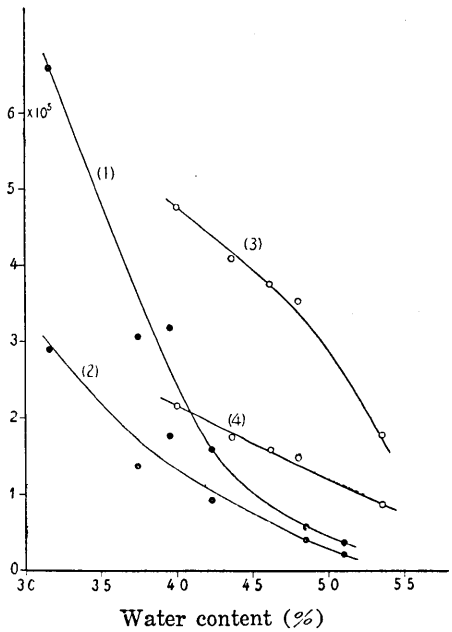

Fig. 5. Showing that the solid viscosity coefficients diminish with increase in water content.
(1), (3): normal viscosity γ_l.
(2), (4): tangential viscosity γ_t.
Ordinate: γ_l or γ_t.

• The silty-clay at Maru-no-uti. ∘ The loam at Hongô.

ways represents the water content and the ordinate one kind of the following values, namely, the wave-velocities; the elastic constants E and μ; σ; the normal and tangential viscosity coefficients γ_l and γ_t; the ratio of γ_l to γ_t; and the ratio of E to μ.

The relations between any one of V_l, V_t, E, μ, σ, γ_l, γ_t and the water content are similar to those previously obtained, that is, the terms V_l, V_t, E, μ, γ_l, γ_t all somewhat rapidly diminish with increase in water content, σ or E/μ increasing with increase in water content, while γ_l/γ_t seems to become maximum when the water content is about 45 per cent, especially in the case of the loam at Hongô (Imperial University). The variations in the ratio of γ_l/γ_t to water in the case of the loam at

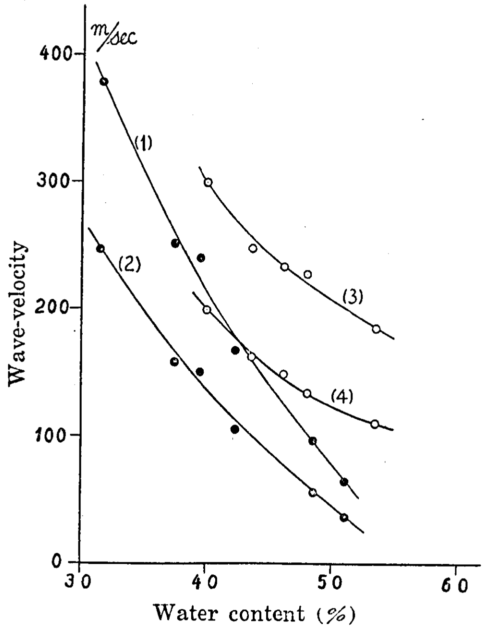

Fig. 6. Relation between the longitudinal wave-velocity as well as the transverse wave-velocity and the water content.
• The silty-clay at Maru-no-uti.
° The loam at Hongô.
(1), (3) Longitudinal wave-velocity.
(2), (4) Transverse wave-velocity.

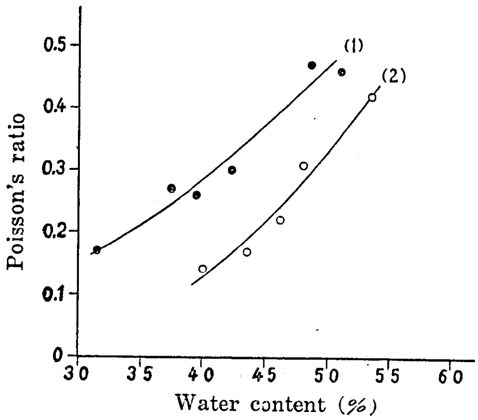

Fig. 7. Showing that Poisson's ratios increase with increase in water content. Ordinate: Poisson's ratio.

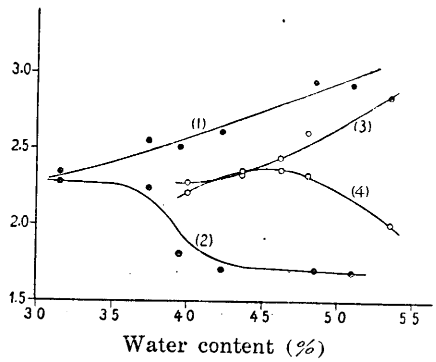

Fig. 8. Relation between the water content and E/μ as well as γ_l/γ_t. Ordinate: E/μ or γ_l/γ_t.
(1), (3): E/μ. (2), (4): γ_l/γ_t.

• The silty-clay at Maru-no-uti. ° The loam at Hongô.

Hongô are smaller than in the silty-clay at Maru-no-uti, so far as the present experiments are concerned.

In order to know the possible relation between E/μ and γ_l/γ_t, we plotted E/μ against γ_l/γ_t for every case of water content as shown in Fig. 9, from which we found the relation in question within a certain range of water content. As will be seen from Fig. 9, the white cireles 3, 4, 5 and the black cirele 6 fall on a part of such a straight line as represented by $\gamma_l/\gamma_t = E/\mu$, while the white circle 2 and the black cirele 5 fall on a part of the curve represented by $\gamma_l/\gamma_t = 1/3 \cdot (E/\mu)^2$,

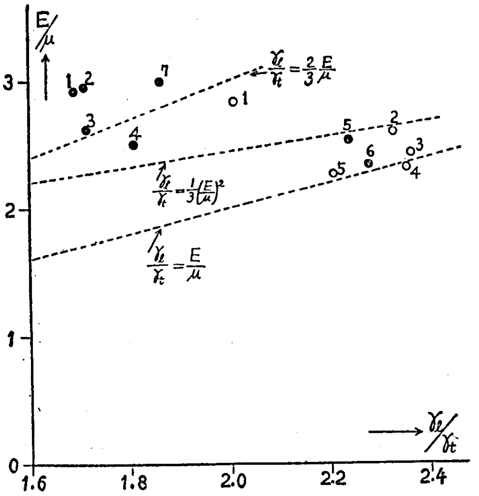

Fig. 9. Relation between E/μ and γ_l/γ_t. The numbers in this figure correspond to those in Table I (showing different water content).

• The silty-clay at Maru-no-uti.

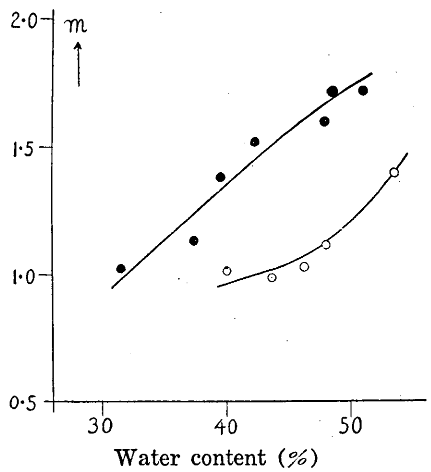

Fig. 10. Relation between m and the water content. m assumes a decreasingly smaller value with decrease in water content.

∘ The loam at Hongô.

the remaining circles lying similarly on other curves. We shall discuss these relations later. If, experimentally, there were always such a linear relation between γ_l/γ_t and E/μ for certain water content, it would be possible to assume that the relation is of the form

$$\frac{E}{\mu} = m\frac{\gamma_l}{\gamma_t},$$

where m is a constant depending on the water content.

The values of m may be determined from the experimental result as shown in Table I. The values of m thus determined (Table III) are plotted against the water content (Fig. 10), which shows that the greater the increase of water in the soil the larger the value of m.

We are also going to determine viscous coefficients λ' and μ'[4] analogous to Lamé constants λ and μ in the case of elasticity. For this purpose we use equations of motion having the same form as those of (1) and (2) from the generalized stress-strain relations in a visco-elastic body.

Table II. The Value of m and Water Content of Soils.

Kind of soil	No.	Water content w (%)	m
Silty-clay (Maru-no-uti)	1	51·0	1·729
	2	48·5	1·723
	3	42·3	1·521
	4	39·5	1·388
	5	37·4	1·138
	6	31·5	1·028
	7*	47·9	1·602
Loam (Hongô)	1	53·5	1·409
	2	48·0	1·121
	3	46·2	1·033
	4	43·6	0·991
	5	40·0	1·032

4. Longitudinal and Torsional Vibration of a Visco-elastic Rod.

The problem of longitudinal vibrations in a visco-elastic rod has been theoretically treated by W. G. Cady[5] and S. L. Quimby[6]. Cady used an equation of motion similar to our equation (1), adding that the expressions derived will afford a means for determining the coefficient of viscosity of a solid when in a state of rapid vibration. Quimby solved this problem under the assumption that the ratio of the lateral to longitudinal strain is $-\sigma$, σ being Poisson's ratio. Further, his theoretical treatments are based on the assumption that the stress in the rod due to viscosity is proportional to the time rate of shearing strain. Recently Thompson[4], who worked out the analysis of this problem, obtained an approximate solution of the equation of motion derived from the general stress-strain relations.

In order to express γ_l or γ_t by λ' and μ', we also used the general stress-strain relations of a visco-elastic substance.

The directions of the axes are denoted by x, y, z (Fig. 1), and the components of displacement by u, v, w. The equation of motion then becomes

$$\rho\frac{\partial^2 w}{\partial t^2}=\frac{\partial \widehat{xz}}{\partial x}+\frac{\partial \widehat{yz}}{\partial y}+\frac{\partial \widehat{zz}}{\partial z}. \tag{5}$$

As is generally known, the stress-components are given by the equ-

4) K. SEZAWA had already used these coefficients in his study on waves in visco-elastic solid bodies. *Bull. Earthq. Res. Inst.*, **3** (1927), 43.

5) W. G. CADY, *Phys. Rev.*, **15** (1920), 146; **19** (1922), 1.

6) S. L. QUIMBY, *Phys. Rev.*, **25** (1925), 558.

7) J. H. C. THOMPSON, *Phil. Tran. Roy. Soc.*, **231** (1933), 339.

ations[8)]

$$\widehat{xx}=\left(\lambda+\lambda'\frac{\partial}{\partial t}\right)\Delta+2\left(\mu+\mu'\frac{\partial}{\partial t}\right)e_{xx}\,, \tag{6}$$

$$\widehat{yy}=\left(\lambda+\lambda'\frac{\partial}{\partial t}\right)\Delta+2\left(\mu+\mu'\frac{\partial}{\partial t}\right)e_{yy}\,, \tag{7}$$

$$\widehat{zz}=\left(\lambda+\lambda'\frac{\partial}{\partial t}\right)\Delta+2\left(\mu+\mu'\frac{\partial}{\partial t}\right)e_{zz}\,, \tag{8}$$

$$\widehat{yz}=2\left(\mu+\mu'\frac{\partial}{\partial t}\right)e_{yz}\,, \quad \text{etc.,} \tag{9}$$

$$\Delta=e_{xx}+e_{yy}+e_{zz}\,, \tag{10}$$

when the strain-components become

$$e_{xx}=\frac{\partial u}{\partial x}\,, \quad \text{etc.,} \qquad e_{yz}=\frac{1}{2}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right), \quad \text{etc.}$$

If a rod, uniform over its plane ends, and free from traction on its lateral surfaces, is subjected to $\widehat{zz}$, it will be in a state of stress, such that

$$\widehat{xx}=\widehat{yy}=\widehat{xy}=\widehat{zx}=\widehat{yz}=0. \tag{11}$$

The rod will then be in a state of strain, such that

$$\left(\lambda+\lambda'\frac{\partial}{\partial t}\right)\Delta+2\left(\mu+\mu'\frac{\partial}{\partial t}\right)e_{xx}=0\,, \tag{12}$$

$$\left(\lambda+\lambda'\frac{\partial}{\partial t}\right)\Delta+2\left(\mu+\mu'\frac{\partial}{\partial t}\right)e_{yy}=0\,, \tag{13}$$

$$\left(\lambda+\lambda'\frac{\partial}{\partial t}\right)\Delta+2\left(\mu+\mu'\frac{\partial}{\partial t}\right)e_{zz}=\widehat{zz}\,. \tag{14}$$

Substituting (12) in (13), we obtain

$$\left(\mu+\mu'\frac{\partial}{\partial t}\right)e_{xx}=\left(\mu+\mu'\frac{\partial}{\partial t}\right)e_{yy}\,. \tag{15}$$

Adding (12), (13), to (14), we obtain

$$\widehat{zz}=\left\{(3\lambda+2\mu)+(3\lambda'+2\mu')\frac{\partial}{\partial t}\right\}\Delta\,, \tag{16}$$

whence

8) K. SEZAWA, *loc. cit.*, 4)

$$\left(\lambda+\lambda'\frac{\partial}{\partial t}\right)\widehat{zz}=\left\{(3\lambda+2\mu)+(3\lambda'+2\mu')\frac{\partial}{\partial t}\right\}\left(\lambda+\lambda'\frac{\partial}{\partial t}\right)\Delta. \tag{17}$$

Substituting (12), (13) in (17), we obtain

$$\left(\lambda+\lambda'\frac{\partial}{\partial t}\right)\widehat{zz}=-2\left(\mu+\mu'\frac{\partial}{\partial t}\right)\left\{(3\lambda+2\mu)+(3\lambda'+2\mu')\frac{\partial}{\partial t}\right\}e_{xx}, \tag{18}$$

$$\left(\lambda+\lambda'\frac{\partial}{\partial t}\right)\widehat{zz}=-2\left(\mu+\mu'\frac{\partial}{\partial t}\right)\left\{(3\lambda+2\mu)+(3\lambda'+2\mu')\frac{\partial}{\partial t}\right\}e_{yy}. \tag{19}$$

By means of the same operation as (17), we get

$$2\left(\mu+\mu'\frac{\partial}{\partial t}\right)\widehat{zz}=2\left(\mu+\mu'\frac{\partial}{\partial t}\right)\left\{(3\lambda+2\mu)+(3\lambda'+2\mu')\frac{\partial}{\partial t}\right\}\Delta. \tag{20}$$

Adding (18), (19) to (20), we obtain

$$\left\{\lambda+\mu+(\lambda'+\mu')\frac{\partial}{\partial t}\right\}\widehat{zz}=\left(\mu+\mu'\frac{\partial}{\partial t}\right)\left\{(3\lambda+2\mu)+(3\lambda'+2\mu')\frac{\partial}{\partial t}\right\}e_{zz}. \tag{21}$$

Thus the equation of motion, namely, equation (5), becomes

$$\rho\left\{(\lambda+\mu)+(\lambda'+\mu')\frac{\partial}{\partial t}\right\}\frac{\partial^2 w}{\partial t^2}=\left(\mu+\mu'\frac{\partial}{\partial t}\right)\left\{(3\lambda+2\mu)+(3\lambda'+2\mu')\frac{\partial}{\partial t}\right\}\frac{\partial^2 w}{\partial z^2}. \tag{22}$$ [9]

The solution of the equation (22) is of the form

$$w=Ae^{ifz}e^{pt},$$

or

$$w=(A\cos fz+B\sin fz)e^{pt}, \tag{23}$$

where p is the root of the equation,

$$\rho\left\{(\lambda+\mu)+(\lambda'+\mu')p\right\}p^2+f^2(\mu+\mu'p)\left\{(3\lambda+2\mu)+(3\lambda'+2\mu')p\right\}=0. \tag{24}$$

We shall determine the constants A and B in (23) under the condition, such that

$$\left.\begin{aligned} w&=0, &&\text{at}\quad z=0,\\ \frac{\partial w}{\partial z}&=0, &&\text{at}\quad z=h, \end{aligned}\right\} \tag{25}$$

where h is the height of the rod. We then obtain

$$A=0, \qquad f=\frac{\pi}{2h}(2r-1), \qquad r=1, 2, 3, \ldots\ldots. \tag{26}$$

Thus we have the solution of (22) for $r=1$, namely,

9) J. H. C. THOMPSON got the same expression in a like manner. *loc. cit.*, 7)

$$w=\sin\frac{\pi}{2h}z\sum_{i=1}^{3}A_i e^{p_i t},$$

where p_i is the root of (24) when $r=1$.

We can solve mathematically the equation (24), but owing to the complexity of its solution, it is difficult to discuss the nature of the vibration of a rod so that we have to resort to the method of physics. Thompson shows that one of the roots is always large, provided that μ/μ' and λ/λ' are large, while the other two roots may be small if the height of the rod is large.

Since the value of μ/μ' of soil is of the order of 10^4, and the height of soil specimen is of the order of 30 cm, we may suppose, according to Thompson's results, that one of the roots is large and the other small. We may then put (24) approximately in a form such that

$$\rho p^2+f^2\frac{\mu(3\lambda+2\mu)}{\lambda+\mu}(1+\nu p)=0\,, \tag{27}$$

where,

$$\nu=\frac{\lambda\mu\left(\frac{\lambda'}{\lambda}-\frac{\mu'}{\mu}\right)+\frac{\mu'}{\mu}(\lambda+\mu)\,(3\lambda+2\mu)}{(\lambda+\mu)\,(3\lambda+2\mu)}, \tag{28}$$

whence the equation of motion becomes approximately

$$\left.\begin{aligned}\rho\frac{\partial^2 w}{\partial t^2}&=E\left(1+\nu\frac{\partial}{\partial t}\right)\frac{\partial^2 w}{\partial z^2},\\ \text{where,}\qquad E&=\frac{\mu(3\lambda+2\mu)}{\lambda+\mu}.\end{aligned}\right\} \tag{29}$$

This equation is of the same form as equation (1), by comparing which with (1), we obtain

$$\begin{aligned}\gamma_l&=E\nu\\ &=\mu\left\{\lambda\mu\left(\frac{\lambda'}{\lambda}-\frac{\mu'}{\mu}\right)+\frac{\mu'}{\mu}(\lambda+\mu)\,(3\lambda+2\mu)\right\}\frac{1}{(\lambda+\mu)^2}.\end{aligned} \tag{30}$$

In the case of the torsional vibration of a visco-elastic rod, the stress-components are given by

$$\left.\begin{aligned}&\widehat{xx}=\widehat{yy}=\widehat{zz}=\widehat{xy}=0\,,\\ &\widehat{yz}=x\left(\mu+\mu'\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial z},\quad \widehat{zx}=-y\left(\mu+\mu'\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial z},\end{aligned}\right\} \tag{31}$$

whence the equation of motion is given by

$$\rho\frac{\partial^2 u}{\partial t^2}=\left(\mu+\mu'\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial z^2}. \tag{32}$$

Thus we obtain the relation between γ_t and μ' by comparing (32) with (2), that is

$$\lambda_t=\mu'. \tag{33}$$

5. The Ratio of γ_l to γ_t.

We can obtain the ratio of γ_l to γ_t from equation (30) and (31).

$$\frac{\gamma_l}{\gamma_t}=\frac{\lambda\mu\left(\frac{\lambda'}{\lambda}-\frac{\mu'}{\mu}\right)+\frac{\mu'}{\mu}(\lambda+\mu)(3\lambda+2\mu)}{\frac{\mu'}{\mu}(\lambda+\mu)^2}. \tag{34}$$

Table IV. λ'/μ', n, and Water Content of Soils.

Kind of soil	No.	Water content w(%)	$\frac{\lambda'}{\mu'}$	n
Silty-clay (Maru-no-uti)	1	51·0	−180·78±20·12	−15·73
	2	48·5	−327·77±30·56	−20·90
	3	42·3	−4·08± 0·31	−2·72
	4	39·5	−2·59± 0·23	−2·39
	5	37·4	−0·29± 0·09	−0·25
	6	31·5	0·37± 0·12	0·72
	7*	47·9	−2762·5±293·1	−56·4
Loam (Hongô)	1	53·5	−26·95± 3·86	−5·15
	2	48·0	−0·33± 0·18	−0.20
	3	46·2	0·53± 0·21	0·68
	4	43·6	0·56± 0·18	1·09
	5	40·0	0·25± 0·10	0.64

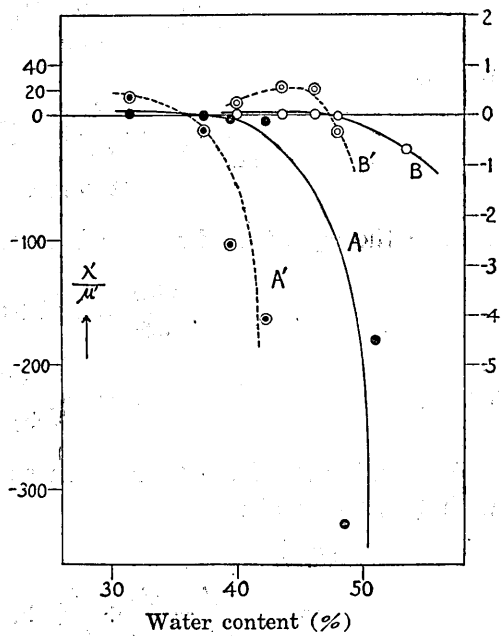

Fig. 11. Relation between λ'/μ' and the water content. λ'/μ' tends to assume a negatively greater value with increase in water content. The ordinates of curves A, B are shown in the left-hand side scale; those of A′ and B′ in the right-hand side scale.
A: The silty-clay at Maru-no-uti.
B: The loam at Hongô.

Since the values of γ_l/γ_t are found experimentally, we can determine, by means of equation (34), the viscous coefficient λ' and also the value of λ'/μ' for various water contents, the results being shown in Table IV. The relations between λ'/μ' and the water content thus found are shown in Fig. 11, λ'/μ', the water content being taken as ordinate and abscissa. As will be seen from this figure, λ'/μ' assumes an increasingly larger negative value with increase in water content. In the case of the silty-clay at Maru-no-uti, if its water content w is less than about 36 per cent, $\lambda'/\mu' > 0$, and if its water content is about 36 per cent, $\lambda'/\mu' = 0$. In the case of the loam at Hongô, if w is larger than about 47 per cent, $\lambda'/\mu' < 0$, and if w is less than about 47 per cent, $\lambda'/\mu' > 0$.

Hence the stress behaviour in the soil due to viscosity obeys the law of compressible or incompressible viscous fluid, in agreement with water content in the soils.

It would, moreover, be interesting to consider special cases of the equation (34).

(a) If $\lambda'/\mu' = \lambda/\mu$, from (34), we obtain

$$\frac{\lambda_l}{\gamma_t} = \frac{E}{\mu}. \tag{35}$$

This relation corresponds to $m=1$ in the empirical formula shown in Section 3, such as $\frac{E}{\mu} = m\frac{\gamma_l}{\gamma_t}$.

(b) If $\lambda' + \frac{2}{3}\mu' = 0$, as in hydrodynamics, then (34) becomes

$$\frac{\gamma_l}{\gamma_t} = \frac{1}{3}\left(\frac{E}{\mu}\right)^2. \tag{36}$$

(c) If Quimby's equations are satisfied, the following relation can be obtained. Since Quimby's equation[10] of motion is

$$\rho\frac{\partial^2 w}{\partial t^2} = E\frac{\partial^2 w}{\partial z^2} + \frac{4}{3}(1+\sigma)\mu'\frac{\partial^3 w}{\partial z^2 \partial t},$$

γ_l can be obtained. Therefore the ratio of γ_l to γ_t becomes

$$\frac{\gamma_l}{\gamma_t} = \frac{2}{3}\frac{E}{\mu}. \tag{37}$$

(d) If $\frac{\lambda'}{\lambda} = n\frac{\mu'}{\mu}$,

then (34) becomes

$$\frac{\gamma_l}{\gamma_t} = 2\left\{\sigma(1-2\sigma)(n-1) + (1+\sigma)\right\}. \tag{38}$$

10) S. L. Quimby, *loc. cit.*, 6)

These relations we have investigated experimentally, with results as shown in Figs. 11～13. As already described in Section 3, soil seems to be in a state represented by relations, such as (a), (b), (c) respectively for certain ranges of water content. In other words, soil may be regarded as changing with decrease in water content from the state expressed by (c) to that of (b), and even to that of (a). It will thus be noticed from these relations that, with increase in water content, the soil will approach the condition of a viscous fluid.

Since the cases (a), (b), (c), may be included in the cases of (d), we specially studied the nature of n obtained from the relation of (d) by introducing our experimental results. We plotted the values of n (Table IV) against the water content as shown in Fig. 12, from which we found that the value of n assumes an increasingly larger negative value with increase in water content. In the case of the silty-clay at Maru-no-uti, if the water content w is less than 36 per cent, $n > 0$, whereas if the water content w is larger than 36 per cent, $n<0$. Such a condition also held in the case of the loam at Hongô. If w is smaller than 47 per cent, $n>0$, whereas if w is larger than about 47 per cent, $n<0$. Thus there must be such a case as $n=0$. The water content in question differs with the kind of soil, namely, the content is about 47～48 per cent for loam, and about 36～38 per cent for silty-clay.

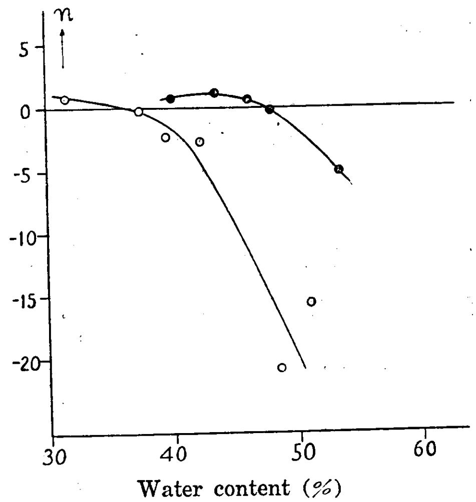

Fig. 12. Relation between n and the water content. n tends to assume a negatively greater value with increase in water content.
∘ The silty-clay at Maru-no-uti.
• The loam at Hongô.

Hence when the water content increases, the soil behaves as a viscous fluid.

6. Summary and Conclusion.

In this paper, the writer discussed the experimental results in connection with the elastic properties and the solid viscosity of certain kinds of soil.

The relations between their properties and the water content are investigated. The elastic constants and solid viscosity coefficients diminish somewhat rapidly with increase in water content, as before.

The Poisson's ratios of these soils increase with increase in water content; the variation in which is the same as that previously described.

The ratio of γ_l to γ_t seems to be maximum at a certain water content. In the case of the loam at Hongô, the variation in the ratio of γ_l/γ_t to water, so far as the present experiments are concerned, is smaller than that of the silty-clay at Maru-no-uti.

The relations between the ratio of γ_l to γ_t and that of E to μ are investigated.

The value of m obtained under the assumption $\frac{E}{\mu}=m\frac{\gamma_l}{\gamma_t}$, varies with the water content, the greater the increase of water in the soil the larger the value of m.

The viscous coefficients λ' and μ' similar to both Lamé elastic constants λ, μ are determined by means of an approximate solution derived from the generalized stress-strain relations in a visco-elastic body.

The relations between λ'/μ' and the water content are investigated. λ'/μ' assumes an increasingly larger negative value with increase in water content. In the case of the silty-clay at Maru-no-uti, if its water content is about 36 per cent, $\lambda'/\mu'=0$. In the case of the loam at Hongô, if its water content is about 47 per cent, $\lambda'/\mu'=0$. It was ascertained that in the case of water contents exceeding these values, the soils behave like a viscous fluid. It is supposed that λ'/μ' may assume an increasingly larger positive value with decrease in water content below the values in question, and the state of the soils may then approach to that of a solid.

The relations between the ratios of γ_l/γ_t and E/μ obtained in our experiments are investigated in the following three cases.

(a) If $\frac{\lambda'}{\mu'}=n\frac{\lambda}{\mu}$, then $\frac{\gamma_l}{\gamma_t}=2\left\{\sigma(1-2\sigma)(n-1)+(1+\sigma)\right\}$.

(b) If $\lambda'+\frac{2}{3}\mu'=0$, then $\frac{\gamma_l}{\gamma_t}=\frac{1}{3}\left(\frac{E}{\mu}\right)^2$.

(c) If Quimby's equation holds,

then
$$\frac{\gamma_l}{\gamma_t}=\frac{2}{3}\frac{E}{\mu}.$$

We observed that, from our experimental results, these relations seem to be approximately satisfied with different conditions of water contents. In the case of the loam at Hongô, we obtained $\frac{\gamma_l}{\gamma_t}\approx\frac{E}{\mu}$ for a water content ranging from about 40 per cent to 46 per cent, with

$\frac{\gamma_l}{\gamma_t} \approx \frac{2}{3} \frac{E}{\mu}$ for a water content of 54 per cent, and $\frac{\gamma_l}{\gamma_t} \approx \frac{1}{3} \left(\frac{E}{\mu}\right)^2$ for a water content of 48 per cent. In the case of the silty-clay at Maru-no-uti, $\frac{\gamma_l}{\gamma_t} \approx \frac{E}{\mu}$ at about 32 per cent, $\frac{\gamma_l}{\gamma_t} \approx \frac{1}{3} \left(\frac{E}{\mu}\right)^2$ for a water content ranging from about 37 per cent to 40 per cent, $\frac{\gamma_l}{\gamma_t} \approx \frac{2}{3} \frac{E}{\mu}$ at about 42 per cent. Since the water content exceeded nearly 42 per cent, these simple relations do not hold.

The relations between n in the conditions of (a) and the water content are investigated. In the case of the silty-clay at Maru-no-uti, if the water content w is less than 36 per cent, $n > 0$, while if w is greater than about 36 per cent, $n < 0$. In the case of the loam at Hongô, if w is smaller than 47 per cent, $n > 0$, while if w is greater than about 47 per cent, $n < 0$. It was thus ascertained that these water relations differ with the kind of soil.

From the fact that n assumes an increasingly larger negative value with increase in water content, it was concluded that the condition of the soil approaches that of a viscous fluid.

It was ascertained that the stress behaviour in the soil due to viscosity obeys the law of compressible or incompressible viscous fluid, in agreement with different water content in the soils.

In conclusion, the writer wishes to express his sincerest thanks to Professor Katsutada Sezawa for his kind advices and critiques, and to Drs. C. Tsuboi, H. Kawasumi, and G. Nishimura for their valuable discussions.

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31. 二三の土に於ける2種の粘性比とポアソン彈性比との關係

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本論文に於いては土の物理的性質特に彈性及び粘性に關係せる性質の實驗的研究の結果が述べてある.

土の彈性がそれが含有する水の量によつて大いに變化する事を已に屢々述べたが，今回は更に粘性に就いても研究し，且つ縦振動から求めた固體粘度と捩振動から求めた固體粘度との比に對する彈性 Poisson 比の關係を實驗的に檢べた. 又 λ, μ に相當する固體粘度 λ', μ' 等を求めた.

以上の諸量の含水量に對する變化を檢べて見ると本郷の赤土は含水率約 47～48% 以上，丸の內の沈泥質粘土は含水率約 36% 以上に於て夫々粘性液體の樣な性質を示す事が判明した.
