39. *On the Elastic Waves due to Pressure Variation on the Inner Surface of a Spherical Cavity in an Elastic Solid.*

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1. As is well known, when a great earthquake occurs, the epicentral region of the earth's crust is subjected to severe seismal vibrations due mainly to seismic wave-energy propagated from the earthquake origin, in addition to which permanent crustal deformations occur in the region, accompanied sometimes with considerable dislocation. This paper is one of two preliminary studies of the dynamical nature of earthquakes. The other preliminary study[1] (already published) discussed the statical deformations of the surface of a semi-infinite gravitating elastic solid when a force acts on the surface of its spherical cavity.

2. Using the solutions obtained by K. Sezawa[2] of equations of motion of homogeneous isotropic solid in the case of steady state of simple-harmonic motion, H. Kawasumi[3] and R. Yoshiyama obtained formulae of displacements expressing the diverging bodily waves (longitudinal and transverse) that emanate from a spherical cavity. The boundary-conditions of stresses assumed by them for the inner surface of a spherical cavity of radius a are

$$\left.\begin{aligned}
\widehat{rr}_{r=a}&=Nf(t)P_n^m(\cos\theta)\begin{matrix}\cos\\-\sin\end{matrix}\Big\}m\phi,\\
\widehat{r\theta}_{r=a}&=Tf(t)\frac{dP_n^m(\cos\theta)}{d\theta}\begin{matrix}\cos\\-\sin\end{matrix}\Big\}m\phi,\\
\widehat{r\phi}_{r=a}&=-mTf(t)\frac{P_n^m(\cos\theta)}{\sin\theta}\begin{matrix}\sin\\\cos\end{matrix}\Big\}m\phi.
\end{aligned}\right\}$$

Needless to say, the origin of the spherical polar co-ordinates $(r,\ \theta,\ \phi)$

1) G. NISHIMURA, *Bull. Earthq. Res. Inst.*, **12** (1934), 368~401.
2) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **10** (1932), 299~334.
3) H. KAWASUMI and R. YOSHIYAMA, *Disin*, **7** (1935), 367.

is the centre of the spherical cavity. K. Sezawa[4] and W. Inouye[5] have also studied the elastic waves radiated from a spherical cavity in an elastic solid for the case when a variable pressure acts on the inner surface of the cavity. The form $f(t)$ of the time-variation of pressure assumed by them are such that

$$\left.\begin{aligned} &f(t)=0, \quad t<0 \\ &f(t)=N(1-e^{-qt}), \quad t>0 \end{aligned}\right\} \tag{a}$$

$$\left.\begin{aligned} &f(t)=0, \quad t\lesseqgtr\tau \\ &f(t)=N, \quad -\tau<t<\tau \end{aligned}\right\} \tag{b}$$

where t indicates time. The form (a) is assumed by K. Sezawa and W. Inouye, while expession (b) corresponds to the form assumed by H. Kawasumi and R. Yoshiyama.

3. When the boundary-conditions on the inner surface of the spherical cavity are expressed by

$$\left.\begin{aligned} &\widehat{rr}_{r=a}=-f(t), \\ &\widehat{r\theta}_{r=a}=\widehat{r\phi}_{r=a}=0, \end{aligned}\right\} \tag{1}$$

the diverging wave[6] radiated from the cavity becomes merely a longitudinal wave, which causes only radial vibrations of particles in the medium, as

$$u=\frac{\partial\phi}{\partial r}, \tag{2}$$

where

$$\phi=-\frac{a}{2\pi\rho}\frac{1}{r}\int_{-\infty}^{\infty}\frac{\varphi(p)e^{-ip(t-\frac{r-a}{v})}}{(p-\alpha)(p-\beta)}dp, \tag{3}$$

$$\varphi(p)=\int_{-\infty}^{\infty}f(\lambda)e^{-ip\lambda}d\lambda, \tag{4}$$

$$\alpha=\frac{2V}{a}\sqrt{1-\left(\frac{V}{v}\right)^2}+i\frac{2V^2}{av}, \quad \beta=-\frac{2V}{a}\sqrt{1-\left(\frac{V}{v}\right)^2}+i\frac{2V^2}{av}, \tag{5}$$

4) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **13** (1935), 740.
K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **14** (1936), 10~17.
5) W. INOUYE, *Bull. Earthq. Res. Inst.*, **15** (1937), 90.
6) H. KAWASUMI and R. YOSHIYAMA, *loc. cit.*

and v and V are the respective velocities of the longitudinal and transverse waves in the medium, such as $\sqrt{\frac{\lambda+2\mu}{\rho}}$, $\sqrt{\frac{\mu}{\rho}}$.

Now let the normal stress $\widehat{rr}$ on the inner surface of the cavity be zero until the time becomes t_0, when we shall express it by

$$\widehat{rr}_{r=a}=-P\frac{t+t_0}{t_m}e^{1-\frac{t_0+t}{t_m}}, \tag{6}$$

where P denotes the maximum intensity of pressure when the time becomes t_m. Expression (3) then becomes

$$\phi=\frac{aP}{2\pi\rho}\frac{t_0}{rt_m}e^{1-\frac{t_0}{t_m}}K\left(t-\frac{r-a}{v}\right)+\frac{aP}{2\pi\rho r}\frac{e^{1-\frac{r-a}{t_m}}}{t_m}K'\left(t-\frac{r-a}{v}\right), \tag{7}$$

where

$$\left.\begin{aligned} K\left(t-\frac{r-a}{v}\right)&=\int_{-\infty}^{\infty}\frac{e^{ip\left(t-\frac{r-a}{v}\right)}}{(p-\alpha)(p-\beta)\left(p-i\frac{1}{t_m}\right)}dp,\\ K'\left(t-\frac{r-a}{v}\right)&=\int_{-\infty}^{\infty}\frac{e^{ip\left(t-\frac{r-a}{v}\right)}}{(p-\alpha)(p-\beta)\left(p-i\frac{1}{t_m}\right)^2}dp. \end{aligned}\right\} \tag{8}$$

Now using Cauchy's theorem of residues, we find that when $t\leq\frac{r-a}{v}$,

$$K\left(t-\frac{r-a}{v}\right)=0,\quad K'\left(t-\frac{r-a}{v}\right)=0, \tag{9}$$

and when $t\geq\frac{r-a}{v}$, the evaluated integrals become

$$\begin{aligned} K\left(t-\frac{r-a}{v}\right)=\frac{2\pi i}{(\alpha-\beta)\left\{\alpha\beta-\frac{1}{t_m^2}-\frac{i(\alpha+\beta)}{t_m}\right\}}\Bigg[&\left(\beta-i\frac{1}{t_m}\right)e^{i\alpha\left(t-\frac{r-a}{v}\right)}\\ &-\left(\alpha-i\frac{1}{t_m}\right)e^{i\beta\left(t-\frac{r-a}{v}\right)}+(\alpha-\beta)e^{-\frac{1}{t_m}\left(t-\frac{r-a}{v}\right)}\Bigg], \end{aligned} \tag{10}$$

$$
\begin{aligned}
K'\left(t-\frac{r-a}{v}\right)=&-\frac{2\pi i}{(\beta-\alpha)\left(\dfrac{1}{t_m^{\,2}}-\alpha^2+i\dfrac{2\alpha}{t_m}\right)}\Bigg[e^{i\alpha\left(t-\frac{r-a}{v}\right)}\\
&+i\left(\alpha-i\frac{1}{t_m}\right)\left(t-\frac{r-a}{v}\right)e^{-\frac{1}{t_m}\left(t-\frac{r-a}{v}\right)}-e^{-\frac{1}{t_m}\left(t-\frac{r-a}{v}\right)}\Bigg]\\
&+\frac{2\pi i}{(\beta-\alpha)\left(\dfrac{1}{t_m^{\,2}}-\beta^2+i\dfrac{2\beta}{t_m}\right)}\Bigg[e^{i\beta\left(t-\frac{r-a}{v}\right)}\\
&-i\left(\beta-i\frac{1}{t_m}\right)\left(t-\frac{r-a}{v}\right)e^{-\frac{1}{t_m}\left(t-\frac{r-a}{v}\right)}+e^{-\frac{1}{t_m}\left(t-\frac{r-a}{v}\right)}\Bigg]. \qquad (11)
\end{aligned}
$$

Therefore the diverging wave radiated from the cavity is expressed by

$$
\begin{aligned}
u=&\frac{aP}{\rho}\frac{t_m}{v}e^{1-\frac{t_0}{t_m}}\frac{1}{r}\Bigg[\frac{e^{-(1+\eta)\tau}}{\zeta(\eta^2+\zeta^2)}\left\{(\zeta^2+\eta^2+\eta)\left(\frac{t_0}{t_m}\right)+\frac{\zeta^2(1-\eta)-\eta^2(1+\eta)}{(\eta^2+\zeta^2)}\right\}\sin\zeta\tau\\
&+\frac{e^{-(1+\eta)\tau}}{\zeta(\eta^2+\zeta^2)}\left\{\zeta\frac{t_0}{t_m}-\frac{\zeta(\eta^2+\zeta^2+2\eta)}{(\eta^2+\zeta^2)}\right\}\cos\zeta\tau\\
&+\left\{\frac{1-\dfrac{t_0}{t_m}}{({}^2\eta+\zeta^2)}+\frac{2\eta}{(\eta^2+\zeta^2)^2}\right\}e^{-\tau}-\frac{1}{(\eta^2+\zeta^2)}\tau e^{-\tau}\Bigg]\\
&-\frac{aP}{\rho}\frac{t_m^2}{r^2}e^{1-\frac{t_0}{t_m}}\Bigg[\frac{e^{-(1+\eta)\tau}}{\zeta(\eta^2+\zeta^2)}\left\{\eta\left(\frac{t_0}{t_m}\right)+\frac{\zeta^2-\eta^2}{\eta^2+\zeta^2}\right\}\sin\zeta\tau\\
&+\frac{e^{-(1+\eta)\tau}}{\zeta(\eta^2+\zeta^2)}\left\{\zeta\left(\frac{t_0}{t_m}\right)-\frac{2\eta\zeta}{(\eta^2+\zeta^2)}\right\}\cos\zeta\tau\\
&+\left\{\frac{2\eta}{(\eta^2+\zeta^2)^2}-\frac{\dfrac{t_0}{t_m}}{(\eta^2+\zeta^2)}\right\}e^{-\tau}-\frac{1}{(\eta^2+\zeta^2)}\tau e^{-\tau}\Bigg], \qquad (12)
\end{aligned}
$$

where

$$
\tau=\frac{1}{t_m}\left(t-\frac{r-a}{v}\right),\quad \eta=\frac{2t_mV^2}{av}-1,\quad \zeta=2\frac{t_mV}{a}\sqrt{1-\frac{V^2}{v^2}}. \qquad (13)
$$

It will be seen from expression (12) that there are two kinds of radial vibrations, one of which has its amplitude proportional to the inverse ratio of the distance of the particle from the center of the cavity and the other proportional to the square of the inverse ratio of the distance; whence it follows that the vibrations of a particle near the cavity are more complicated than those some distance away from it. The longitudinal vibrations of a particle in the medium far from the cavity are expressed approximately by

$$u \approx \frac{aP_0}{\mu}\left(\frac{a}{r}\right)e^{1-\frac{t_0}{t_m}}\left[\frac{e^{-(1+\eta)\tau}}{\zeta(\eta^2+\zeta^2)}\left\{(\zeta^2+\eta^2+\eta)\frac{t_0}{t_m}+\frac{\zeta^2(1-\eta)-\eta^2(1+\eta)}{(\eta^2+\zeta^2)}\right\}\sin\zeta\tau\right.$$

$$+\frac{e^{-(1+\eta)\tau}}{\zeta(\eta^2+\zeta^2)}\left\{\zeta\left(\frac{t_0}{t_m}\right)-\frac{\zeta({}^2+\eta^2+2\eta)}{(\eta^2+\zeta^2)}\right\}\cos\zeta\tau$$

$$\left.+\left\{\frac{1-\frac{t_0}{t_m}}{(\eta^2+\zeta^2)}+\frac{2\eta}{(\eta^2+\zeta^2)^2}\right\}e^{-\tau}-\frac{1}{(\eta^2+\zeta^2)}\tau e^{-\tau}\right], \quad (14)$$

which are derived from expression (12).

To ascertain the nature of the radial vibrations of a particle, we obtained Figs. 1∼3 from expression (14) when the Poisson ratio of the medium is 1/4, $a/t_m v$ being tentatively put as 1. Fig. 4 shows the time-variations in the pressure on the surface of the cavity corresponding to $\frac{t_0}{t_m}=0$, 0·50, and 1·50 respectively. It will be seen from Figs. 1∼4 that the particles in the medium that is far from the spherical cavity become damped radial vibrations and their amplitudes reach maximum when $\frac{t_0}{t_m}=0$, in which case their apparent periods of vibrations in the initial state become also the longest. The apparent

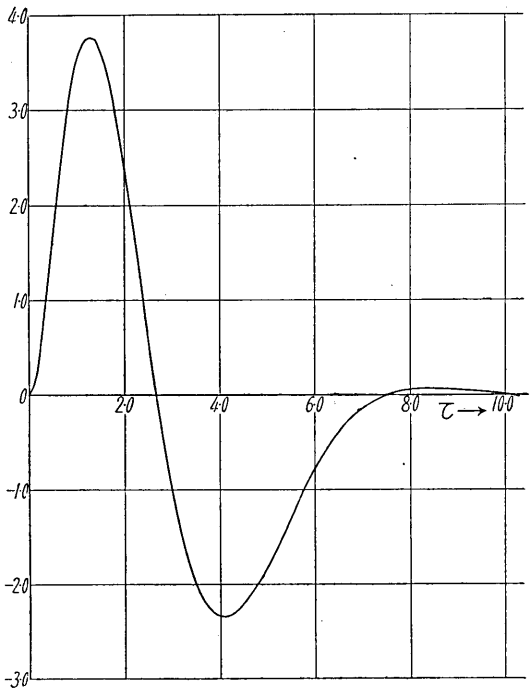

Fig. 1. The radial vibration of a particle in the medium of Poisson's ratio $\frac{1}{4}$, when $\frac{a}{t_m v}=1$ and $\frac{t_0}{t_m}=0$. Unit of ordinate$=\frac{Pa}{27\mu}\left(\frac{a}{r}\right)$, abscissa $\tau=\frac{v}{a}\left\{t-\frac{1}{v}(r-a)\right\}$.

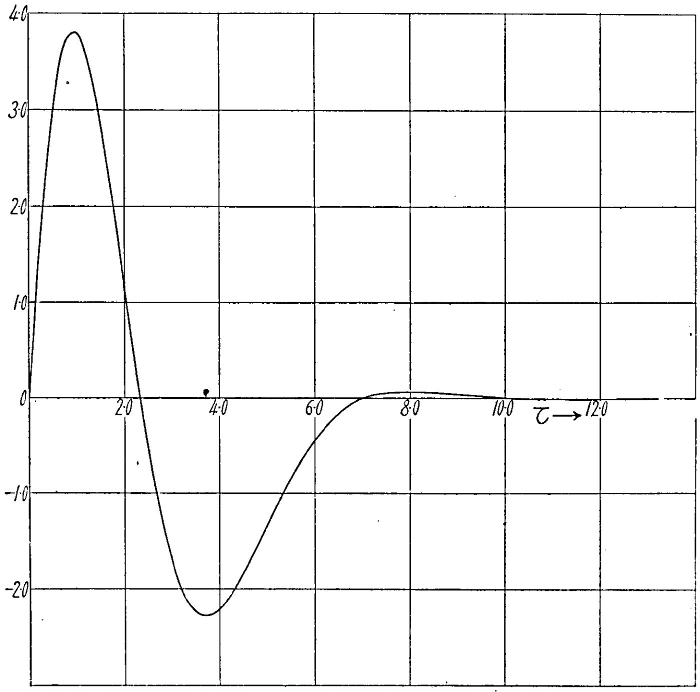

Fig. 2. The radial vibration of a particle when Poisson's ratio$=\frac{1}{4}$, $\frac{a}{t_m v}=1$ and $\frac{t_0}{t_n}=0{\cdot}5$. Unit of ordinate$=\frac{Pa}{27\mu}\left(\frac{a}{r}\right)$, abscissa $\tau=\frac{v}{a}\left\{t-\frac{1}{v}(r-a)\right\}$.

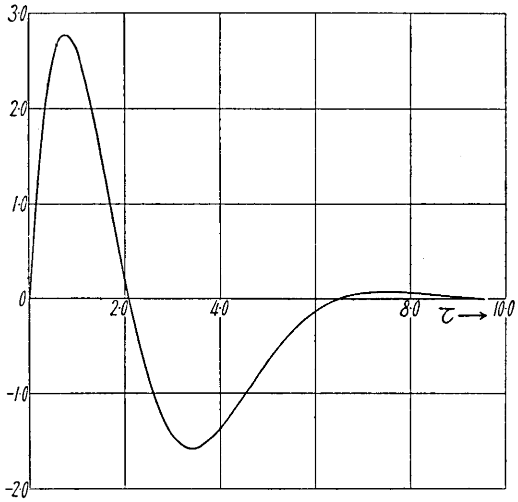

Fig. 3. The radial vibration of a particle when Poisson's ratio$=\frac{1}{4}$, $\frac{a}{t_m v}=1$ and $\frac{t_0}{t_m}=1{\cdot}5$. Unit of ordinate$=\frac{Pa}{27\mu}\left(\frac{a}{r}\right)$, abscissa $\tau=\frac{v}{a}\left\{t-\frac{1}{v}\left(r-a\right)\right\}$.

periods, of radial vibrations in the state of initial motion in the case of sudden commencement of pressure generally become shorter than those in the case of gradual commencement, and the periods of radial vibrations, however, of the tail end do not depend on the form of the time-variations in pressure on the surface of the cavity, but on the dimensions of the cavity. The periods of vibrations in the tail are generally

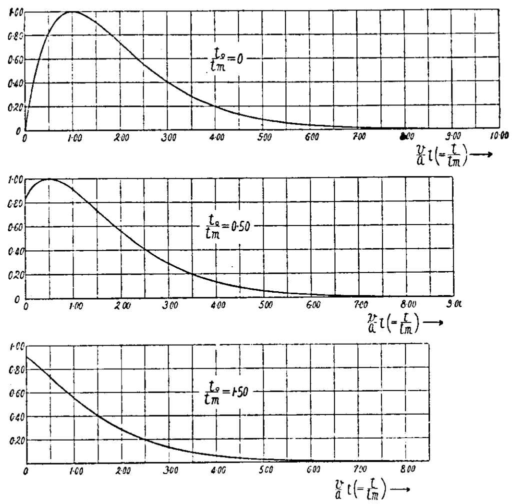

Fig. 4. Time-variations in pressure on the surface of the spherical cavity when $\frac{t_0}{t_m}=0$, $\frac{t_0}{t_m}=0{\cdot}50$ and $\frac{t_0}{t_m}=1{\cdot}50$. Unit of ordinate$=-P$.

longer than those of the initial motion. Moreover, it may be seen that Shida's law of push-pull of the initial motion holds also in this case.

4. When the normal stress on the surface of the spherical cavity has co-latitudinal and azimuthal distributions[7], like

$$\widehat{rr}_{r=a}=-Pf(t)\sin 2\theta\cos\phi, \tag{15}$$

and the shear stresses $\widehat{r\theta}_{r=a}$, $\widehat{r\phi}_{r=a}$ are zero on this surface, the diverging waves radiated from the spherical cavity then become two bodily waves, like the longitudinal and transversal waves, and the particles in the medium as the effect of these two waves assume the following vibrational motions of radial, co-latitudinal, and azimuthal displacements:

7) Honda has already studied the wave-motions when the pressure distribution on the surface of a spherical cavity has the same distribution as in Section 4, but the time-factor in his paper is simple-harmonic, like e^{ipt}.

H. HONDA, *Geophy. Mag.*, 8 (1934), 153~164.

$$\left.\begin{aligned}
u_1&=\frac{1}{2\pi}\frac{a^3}{r^4}\int_{-\infty}^{\infty}K(p)\frac{P(p)e^{ip\left\{t-\frac{1}{v}(r-a)\right\}}}{p^2\psi(p)}\sin 2\theta\cos\phi dp,\\
v_1&=\frac{1}{\pi}\frac{a^3}{r^4}\int_{-\infty}^{\infty}K(p)\frac{Q(p)e^{ip\left\{t-\frac{1}{v}(r-a)\right\}}}{p^2\psi(p)}\cos 2\theta\cos\phi dp,\\
w_1&=-\frac{1}{\pi}\frac{a^3}{r^4}\int_{-\infty}^{\infty}K(p)\frac{Q(p)e^{ip\left\{t-\frac{1}{v}(r-a)\right\}}}{p^2\psi(p)}\cos\theta\cos\phi dp,
\end{aligned}\right\}\qquad(16)$$

$$\left.\begin{aligned}
u_3&=-\frac{3}{\pi}\frac{a^3}{r^4}\int_{-\infty}^{\infty}K(p)\frac{S(p)e^{ip\left\{t-\frac{1}{V}(r-a)\right\}}}{p^2\psi(p)}\sin 2\theta\cos\phi dp,\\
v_3&=-\frac{1}{\pi}\frac{a^3}{r^4}\int_{-\infty}^{\infty}K(p)\frac{T(p)e^{ip\left\{t-\frac{1}{V}(r-a)\right\}}}{p^2\psi(p)}\cos 2\theta\cos\phi dp,\\
w_3&=\frac{1}{\pi}\frac{a^3}{r^4}\int_{-\infty}^{\infty}K(p)\frac{T(p)e^{ip\left\{t-\frac{1}{V}(r-a)\right\}}}{p^2\psi(p)}\cos\theta\sin\phi dp,
\end{aligned}\right\}\qquad(17)$$

where

$$K(p)=\int_{-\infty}^{\infty}f(\lambda)e^{-ip\lambda}d\lambda.\qquad(18)$$

In these expressions, u_1, v_1, w_1 are the radial, co-latitudinal, and azimuthal components of displacement due to the longitudinal wave, and u_3, v_3, w_3 are those due to the transverse wave. The functions of p, such as $\psi(p)$, $P(p)$, $Q(p)$, $S(p)$, and $T(p)$ in expressions (16) and (17) are

$$\begin{aligned}
\psi(p)=\Bigg[&\left(\frac{ap}{v}\right)^6-i\left\{\frac{3\lambda+10\mu}{(\lambda+2\mu)}+5\frac{V}{v}\right\}\left(\frac{ap}{v}\right)^5\\
&-\left\{\frac{(15\lambda+26\mu)}{(\lambda+2\mu)}\frac{V}{v}+21\left(\frac{V}{v}\right)^2+\frac{(3\lambda+34\mu)}{(\lambda+2\mu)}\right\}\left(\frac{ap}{v}\right)^4\\
&+i\left\{72\frac{\mu}{\lambda+2\mu}+9\frac{(7\lambda+10\mu)}{(\lambda+2\mu)}\left(\frac{V}{v}\right)^2+5\frac{(3\lambda+10\mu)}{(\lambda+2\mu)}\frac{V}{v}+48\left(\frac{V}{v}\right)^3\right\}\left(\frac{ap}{v}\right)^3\\
&+\left\{72\frac{\mu}{(\lambda+2\mu)}\frac{V}{v}+48\frac{(3\lambda+4\mu)}{(\lambda+2\mu)}\left(\frac{V}{v}\right)^3+3\frac{(3\lambda+10\mu)}{(\lambda+2\mu)}\left(\frac{V}{v}\right)^2\right.\\
&\left.+48\left(\frac{V}{v}\right)^4+72\frac{\mu}{(\lambda+2\mu)}\right\}\left(\frac{ap}{v}\right)^2
\end{aligned}$$

$$-i\left\{72\frac{\mu}{(\lambda+2\mu)}\left(\frac{V}{v}\right)^2+48\frac{(3\lambda+4\mu)}{(\lambda+2\mu)}\left(\frac{V}{v}\right)^4+72\frac{\mu}{(\lambda+2\mu)}\frac{V}{v}\right.$$

$$\left.+48\frac{(3\lambda+4\mu)}{(\lambda+2\mu)}\left(\frac{V}{v}\right)^3\right\}\left(\frac{ap}{v}\right)$$

$$\left.-\left\{72\frac{\mu}{(\lambda+2\mu)}\left(\frac{V}{v}\right)^2+48\frac{(3\lambda+4\mu)}{(\lambda+2\mu)}\left(\frac{V}{v}\right)^4\right\}\right], \tag{19}$$

$$P(p)=-i\left(\frac{1}{V}\right)^4\left(\frac{1}{v}\right)^3a^4r^3p^7-\left\{5\left(\frac{1}{V}\right)^3\left(\frac{1}{v}\right)^3(ar)^3+4\left(\frac{1}{V}\right)^4\left(\frac{1}{v}\right)^2a^4r^5\right\}p^6$$

$$+i\left\{21\left(\frac{1}{V}\right)^2\left(\frac{1}{v}\right)^3a^2r^3+20\left(\frac{1}{V}\right)^3\left(\frac{1}{v}\right)^2a^3r^2+9\left(\frac{1}{V}\right)^4\left(\frac{1}{v}\right)a^4r\right\}p^5$$

$$+\left\{48\left(\frac{1}{V}\right)\left(\frac{1}{v}\right)^3ar^3+84\left(\frac{1}{V}\right)^2\left(\frac{1}{v}\right)^2a^2r^2\right.$$

$$\left.+45\left(\frac{1}{V}\right)^3\left(\frac{1}{v}\right)a^3r+9\left(\frac{1}{V}\right)^4a^4\right\}p^4$$

$$-i\left\{48\left(\frac{1}{v}\right)^3r^3+192\left(\frac{1}{V}\right)\left(\frac{1}{v}\right)^2ar^2+189\left(\frac{1}{V}\right)^2\left(\frac{1}{v}\right)a^2r+45\left(\frac{1}{V}\right)^3a^3\right\}p^3$$

$$-\left\{192\left(\frac{1}{v}\right)^2r^2+432\left(\frac{1}{v}\right)\left(\frac{1}{V}\right)ar+189\left(\frac{1}{V}\right)^2a^2\right\}p^2$$

$$+i\left\{432\left(\frac{1}{v}\right)r+432\left(\frac{1}{V}\right)a\right\}p+432, \tag{20}$$

$$Q(p)=\left(\frac{1}{V}\right)^4\left(\frac{1}{v}\right)^2a^4r^2p^6-i\left\{5\left(\frac{1}{V}\right)^3\left(\frac{1}{v}\right)^2a^3r^2+3\left(\frac{1}{V}\right)^4\left(\frac{1}{v}\right)a^4r\right\}p^5$$

$$-\left\{21\left(\frac{1}{V}\right)^2\left(\frac{1}{v}\right)^2a^2r^2+15\left(\frac{1}{V}\right)^3\left(\frac{1}{v}\right)a^3r+3\left(\frac{1}{V}\right)^4a^4\right\}p^4$$

$$+i\left\{48\left(\frac{1}{V}\right)\left(\frac{1}{v}\right)^2ar^2+63\left(\frac{1}{V}\right)^2\left(\frac{1}{v}\right)a^2r+15\left(\frac{1}{V}\right)^3a^3\right\}p^3$$

$$+\left\{48\left(\frac{1}{v}\right)^2r^2+144\left(\frac{1}{V}\right)\left(\frac{1}{v}\right)ar+63\left(\frac{1}{V}\right)^2a^2\right\}p^2$$

$$-i\left\{144\left(\frac{1}{v}\right)r+144\left(\frac{1}{V}\right)a\right\}p-144, \tag{21}$$

$$S(p)=i\left\{2\left(\frac{1}{v}\right)^3\left(\frac{1}{V}\right)^2a^3r^2\right\}p^5$$

$$+\left\{10\left(\frac{1}{v}\right)^2\left(\frac{1}{V}\right)^2a^2r^2+6\left(\frac{1}{v}\right)^3\left(\frac{1}{V}\right)a^3r\right\}p^4$$

$$-i\left\{24\left(\frac{1}{v}\right)\left(\frac{1}{V}\right)^2ar^2+30\left(\frac{1}{v}\right)^2\left(\frac{1}{V}\right)a^2r+6\left(\frac{1}{v}\right)^3a^3\right\}p^3$$

$$-\left\{24\left(\frac{1}{V}\right)^2r^2+72\left(\frac{1}{v}\right)\left(\frac{1}{V}\right)ar+30\left(\frac{1}{v}\right)^2a^2\right\}p^2$$

$$+i\left\{72\left(\frac{1}{V}\right)r+72\left(\frac{1}{v}\right)a\right\}p+72, \tag{22}$$

$$T(p)=2\left(\frac{1}{v}\right)^3\left(\frac{1}{V}\right)^3a^3r^3p^6-i\left\{10\left(\frac{1}{v}\right)^2\left(\frac{1}{V}\right)^3a^2r^3+6\left(\frac{1}{v}\right)^3\left(\frac{1}{V}\right)^2a^3r^2\right\}p^5$$

$$-\left\{24\left(\frac{1}{v}\right)\left(\frac{1}{V}\right)^3ar^3+30\left(\frac{1}{v}\right)^2\left(\frac{1}{V}\right)^2a^2r^2+12\left(\frac{1}{v}\right)^3\left(\frac{1}{V}\right)a^3r\right\}p^4$$

$$+i\left\{24\left(\frac{1}{V}\right)^3r^3+72\left(\frac{1}{v}\right)\left(\frac{1}{V}\right)^2ar^2+60\left(\frac{1}{v}\right)^2\left(\frac{1}{V}\right)a^2r+12\left(\frac{1}{v}\right)^3a^3\right\}p^3$$

$$+\left\{72\left(\frac{1}{V}\right)^2r^2+144\left(\frac{1}{v}\right)\left(\frac{1}{V}\right)ar+60\left(\frac{1}{v}\right)^2a^2\right\}p^2$$

$$-i\left\{144\left(\frac{1}{V}\right)r+144\left(\frac{1}{v}\right)a\right\}p-144. \tag{23}$$

Now let the time-factor $f(t)$ in expression (15) be

$$f(t)=\frac{t+t_0}{t_m}e^{1-\frac{t_0+t}{t_m}}, \tag{24}$$

when $t\geq 0$, and

$$f(t)=0, \tag{25}$$

when $t\leq 0$ as in the case of Section 3. But since the integral expressions in (16) and (17) have singular points of higher order, evaluation of the contour-integrals becomes impossible, with the result that it may be impossible to study the wave-motions of particles on and near the surface of the spherical cavity with the present method[8]. We shall therefore study the vibrational motions of particles far from the cavity when the pressure distributions on the surface of it are expressed by (15). To simplify the mathematical treatment, let the Poisson ratio

8) If we calculate afresh the displacement-potentials consistent with the boundary-conditions of stresses on the inner surface of the spherical cavity, it is possible to investigate the vibrational motions of the particles on and near the surface of the spherical cavity.

be 1/4, as in the preceding section, when the radial vibration u_1 due to the longitudinal wave, and the co-latitudinal and azimuthal vibrations v_3, w_3 due to the transverse wave become

$$u_1 \approx \frac{V^2 v}{6\pi\mu a^3}\frac{1}{r}\int_{-\infty}^{\infty} K(p)\frac{P'(p)e^{ip\left\{t-\frac{1}{V}(r-a)\right\}}}{\psi'(p)}\sin 2\theta\cos\phi dp, \tag{26}$$

$$v_3 \approx -\frac{v^4}{3\pi\mu V a^3}\frac{1}{r}\int_{-\infty}^{\infty} K(p)\frac{T'(p)e^{ip\left\{t-\frac{1}{v}(r-a)\right\}}}{\psi'(p)}\cos 2\theta\cos\phi dp, \tag{27}$$

$$w_3 \approx \frac{v^4}{3\pi\mu V a^3}\frac{1}{r}\int_{-\infty}^{\infty} K(p)\frac{T'(p)e^{ip\left\{t-\frac{1}{v}(r-a)\right\}}}{\psi'(p)}\cos\theta\sin\phi dp, \tag{28}$$

where

$$P'(p) = -i\left(\frac{1}{V}\right)^4 a^4p^5 - 5\left(\frac{1}{V}\right)^3 a^3p^4 - i21\left(\frac{1}{V}\right)a^2p^3 + 48\left(\frac{1}{V}\right)ap^2 - i48p, \tag{29}$$

$$T'(p) = 2\left(\frac{1}{v}\right)^3 a^3p^4 - i10\left(\frac{1}{v}\right)^2 a^2p^3 - 24\left(\frac{1}{v}\right)ap^2 + i24p, \tag{30}$$

$$\psi'(p) = (p+a_1)(p+a_2)(p+a_3)(p+a_4)(p+a_5)(p+a_6), \tag{31}$$

and

$$\left.\begin{array}{l} a_1 = (\alpha_1 - i\beta_1)\frac{v}{a}, \quad a_2 = -(\alpha_1 + i\beta_1)\frac{v}{a}, \quad a_3 = (\alpha_2 - i\beta_2)\frac{v}{a}, \\ a_4 = -(\alpha_2 + i\beta_2)\frac{v}{a}, \quad a_5 = (\alpha_3 - i\beta_3)\frac{v}{a}, \quad a_6 = -(\alpha_3 + i\beta_3)\frac{v}{a}, \\ \left.\begin{array}{l}\alpha_1 = 0{\cdot}83199, \\ \beta_1 = 2{\cdot}10618,\end{array}\right\} \left.\begin{array}{l}\alpha_2 = 2{\cdot}416332, \\ \beta_2 = 1{\cdot}040118,\end{array}\right\} \left.\begin{array}{l}\alpha_3 = 0{\cdot}600844, \\ \beta_3 = 0{\cdot}463744.\end{array}\right\} \end{array}\right\} \tag{32}$$

$\psi'(p)$ expressed by (31) may be obtained from expression (19), and is due to H. Kawasumi[9]. In this case, the respective amplitudes of the co-latitudinal and the azimuthal components of displacements of the longitudinal wave are less than the radial displacement of the wave, and those of the radial component of displacement due to the transverse wave are also less than those of the co-latitudinal and the azimuthal components of displacement. We therefore omit these displacements in studying vibrations far from the cavity.

By letting the time-factor $f(t)$ be (24) and (25), the expressions (26), (27), (28) become

9) H. KAWASUMI and R. YOSHIYAMA, *loc. cit.*

$$u_1 \approx -\frac{V^2 v P}{6\mu\pi a^3}\frac{1}{r}e^{1-\frac{t_0}{t_m}}\sin 2\theta\cos\phi\, X\left(t-\frac{1}{v}(r-a)\right), \tag{33}$$

$$v_3 \approx \frac{1}{3\pi}\frac{v^4 P}{\mu V a^3}\frac{1}{r}e^{1-\frac{t_0}{t_m}}\cos 2\theta\cos\phi\, Y\left(t-\frac{1}{V}(r-a)\right), \tag{34}$$

$$w_3 \approx -\frac{1}{3\pi}\frac{v^4 P}{\mu V a^3}\frac{1}{r}e^{1-\frac{t_0}{t_m}}\cos\theta\sin\phi\, Y\left(t-\frac{1}{V}(r-a)\right), \tag{35}$$

where

$$X\left(t-\frac{1}{v}(r-a)\right)=\int_{-\infty}^{\infty}\frac{P'(p)}{\psi'(p)}\left\{\frac{t_0}{t_m}\frac{i}{\left(p-i\frac{1}{t_m}\right)}+\frac{1}{t_m}\frac{1}{\left(p-i\frac{1}{t_m}\right)^2}\right\}e^{ip\left\{t-\frac{1}{v}(r-a)\right\}}dp, \tag{36}$$

$$Y\left(t-\frac{1}{V}(r-a)\right)=\int_{-\infty}^{\infty}\frac{T'(p)}{\psi'(p)}\left\{\frac{t_0}{t_m}\frac{i}{\left(p-i\frac{1}{t_m}\right)}+\frac{1}{t_m}\frac{1}{\left(p-i\frac{1}{t_m}\right)^2}\right\}e^{ip\left\{t-\frac{1}{V}(r-a)\right\}}dp. \tag{37}$$

For evaluating the integrals of expressions (36) and (37), we found, after laborious mathematical work, the following relations when $\frac{a}{t_m v}=1$[10]:

$$\begin{aligned}\frac{P'(p)}{\psi'(p)\left(p-i\frac{1}{t_m}\right)^2}=&\frac{(\gamma_1-i\delta)\frac{a^6}{v^6}}{\left\{p-(-\alpha_1+i\beta_1)\frac{v}{a}\right\}}+\frac{(-\gamma_1-i\delta)\frac{a^6}{v^6}}{\left\{p-(\alpha_1+i\beta_1)\frac{v}{a}\right\}}\\&+\frac{(-\gamma_2+i\delta_2)\frac{a^6}{v^6}}{\left\{p-(-\alpha_2+i\beta_2)\frac{v}{a}\right\}}+\frac{(\gamma_2+i\delta_2)\frac{a^6}{v^6}}{\left\{p-(\alpha_2+i\beta_2)\frac{v}{a}\right\}}\\&+\frac{(-\gamma_3+i\delta_3)\frac{a^6}{v^6}}{\left\{p-(-\alpha_3+i\beta_3)\frac{v}{a}\right\}}+\frac{(\gamma_3+i\delta_3)\frac{a^6}{v^6}}{\left\{p-(\alpha_3+i\beta_3)\frac{v}{a}\right\}}\\&-\frac{\gamma_4\frac{a^5}{v^5}}{\left(p-i\frac{v}{a}\right)^2}+\frac{i\delta_4\frac{a^6}{v^6}}{\left(p-i\frac{v}{a}\right)},\end{aligned} \tag{38}$$

---

10) The vibrations of particles for the cases when $\frac{a}{t_m v}>1$ and <1 will be the next object of our study.

$$\frac{P'(p)}{\psi'(p)\left(p-i\frac{1}{t_m}\right)}=\frac{(-\xi_1+i\eta_1)\frac{a^5}{v^5}}{\left\{p-(-\alpha_1+i\beta_1)\frac{v}{v^5}\right\}}+\frac{(-\xi_1-i\eta_1)\frac{a^5}{v^5}}{\left\{p-(\alpha_1+i\beta_1)\frac{v}{a}\right\}}$$

$$+\frac{(\xi_2-i\eta_1)\frac{a^5}{v^5}}{\left\{p-(-\alpha_2+i\beta_2)\frac{v}{a}\right\}}+\frac{(\xi_3+i\eta_3)\frac{a^5}{v^5}}{\left\{p-(\alpha_2+i\beta_2)\frac{v}{a}\right\}}$$

$$+\frac{(\xi_3-i\eta_3)\frac{a^5}{v^5}}{\left\{p-(-\alpha_3+i\beta_3)\frac{v}{a}\right\}}+\frac{(\xi_3+i\eta_3)\frac{a^5}{v_5}}{\left\{p-(\alpha_3+i\beta_3)\frac{v}{a}\right\}}$$

$$+\frac{\xi_4\frac{a^5}{v^5}}{\left(p-i\frac{v}{a}\right)}, \tag{39}$$

$$\frac{T'(p)}{\psi'(p)\left(p-i\frac{1}{t_m}\right)^2}=\frac{(\gamma_1'-i\delta_1')\frac{a^6}{v^6}}{\left\{p-(-\alpha_1+i\beta_1)\frac{v}{a}\right\}}+\frac{(-\gamma_1'-i\delta_1')\frac{a^6}{v^6}}{\left\{p-(\alpha_1+i\beta_1)\frac{v}{a}\right\}}$$

$$+\frac{(-\gamma_2'-i\delta_2')\frac{a^6}{v^6}}{\left\{p-(-\alpha_2+i\beta_2)\frac{v}{a}\right\}}+\frac{(\gamma_2'-i\delta_2')\frac{a^6}{v^6}}{\left\{p-(\alpha_2+i\beta_2)\frac{v}{a}\right\}}$$

$$+\frac{(\gamma_3'-i\delta_3')\frac{a^6}{v^6}}{\left\{p-(-\alpha_3+i\beta_3)\frac{v}{a}\right\}}+\frac{(-\gamma_3'-i\delta_3')\frac{a^6}{v^6}}{\left\{p-(\alpha_3+i\beta_3)\frac{v}{a}\right\}}$$

$$+\frac{\gamma_4'\frac{a^5}{v^5}}{\left(p-i\frac{v}{a}\right)^2}+\frac{i\delta_4'\frac{a^6}{v^6}}{\left(p-i\frac{v}{a}\right)}, \tag{40}$$

$$\frac{T'(p)}{\psi'(p)\left(p-i\frac{1}{t_m}\right)}=\frac{(-\xi_1'+i\eta_1')\frac{a^5}{v^5}}{\left\{p-(-a_1+i\beta_1)\frac{v}{a}\right\}}+\frac{(-\xi_1'-i\eta_1')\frac{a^5}{v^5}}{\left\{p-(a_1+i\beta_1)\frac{v}{a}\right\}}$$

$$+\frac{(\xi_2'+i\eta_2')\frac{a^5}{v^5}}{\left\{p-(-\alpha_2+i\beta_2)\frac{v}{a}\right\}}+\frac{(\xi_2'-i\eta_2')\frac{a^5}{v^5}}{\left\{p-(\alpha_2+i\beta_2)\frac{v}{a}\right\}}$$

$$+\frac{(-\xi_3'-i\eta_3')\dfrac{a^5}{v^5}}{\left\{p-(-\alpha_3+i\beta_3)\dfrac{v}{a}\right\}}+\frac{(-\xi_3'+i\eta_3')\dfrac{a^5}{v^5}}{\left\{p-(\alpha_3+i\beta_3)\dfrac{v}{a}\right\}}$$

$$+\frac{\xi_4'\dfrac{a^5}{v^5}}{\left(p-i\dfrac{v}{a}\right)},$$

where

$$\left.\begin{matrix}\gamma_1=3{\cdot}144601, & \gamma_2=0{\cdot}403808, & \gamma_3=0{\cdot}513517, & \gamma_3=1{\cdot}49546, \\ \delta_1=1{\cdot}453949, & \delta_2=0{\cdot}120422, & \delta_3=0{\cdot}888087, & \delta_4=0{\cdot}89088.\end{matrix}\right\} \quad (42)$$

$$\left.\begin{matrix}\xi_1=3{\cdot}9083103, & \xi_2=2{\cdot}029293, & \xi_3=1{\cdot}427375, & \xi_4=0{\cdot}903283, \\ \eta_1=0{\cdot}468814, & \eta_2=0{\cdot}307162, & \eta_3=0{\cdot}258230, & \end{matrix}\right\} \quad (43)$$

$$\left.\begin{matrix}\gamma_1'=0{\cdot}171372, & \gamma_2'=0{\cdot}033059, & \gamma_3'=0{\cdot}620073. & \gamma_4'=1{\cdot}102391, \\ \delta_1'=0{\cdot}055933, & \delta_2'=0{\cdot}016417, & \delta_3'=0{\cdot}332791, & \delta_4'=0{\cdot}810283,\end{matrix}\right\} \quad (44)$$

$$\left.\begin{matrix}\xi_1'=0{\cdot}080683, & \xi_2'=0{\cdot}080538, & \xi_3'=0{\cdot}551045, & \xi_4'=1{\cdot}102381, \\ \eta_1'=0{\cdot}236124, & \eta_2'=0{\cdot}038354, & \eta_3'=0{\cdot}132560, & \end{matrix}\right\} \quad (45)$$

By using these relations, we may be able to evaluate the integrals in (36) and (37) by Cauchy's residue methods, the diverging waves far from the cavity being determined as follows:

$$u_1 \approx -\frac{Pa}{27\mu}\left(\frac{a}{r}\right)e^{1-\frac{t_0}{t_m}}\sin 2\theta\cos\phi\cdot$$

$$\cdot\Bigg[2\sqrt{\left(-\gamma_1+\frac{t_0}{t_m}\eta_1\right)^2+\left(-\delta_1-\frac{t_0}{t_m}\xi_1\right)^2}\,e^{-\beta_1\tau}\sin\left\{\alpha_1\tau+\tan^{-1}\left(\frac{-\delta_1-\dfrac{t_0}{t_m}\xi_1}{-\gamma_1+\dfrac{t_0}{t_m}\eta_1}\right)\right\}$$

$$+2\sqrt{\left(\gamma_2-\frac{t_0}{t_m}\eta_2\right)^2+\left(\delta_2+\frac{t_0}{t_m}\xi_2\right)^2}\,e^{-\beta_2\tau}\sin\left\{\alpha_2\tau+\tan^{-1}\left(\frac{\delta_2+\dfrac{t_0}{t_m}\xi_2}{\gamma_2-\dfrac{t_0}{t_m}\eta_2}\right)\right\}$$

$$+2\sqrt{\left(\gamma_3-\frac{t_0}{t_m}\eta_3\right)^2+\left(\delta_3+\frac{t_0}{t_m}\xi_3\right)^2}\,e^{-\beta_3\tau}\sin\left\{\alpha_3\tau+\tan^{-1}\left(\frac{\delta_3+\dfrac{t_0}{t_m}\xi_3}{\gamma_3-\dfrac{t^0}{t_m}\eta_3}\right)\right\}$$

$$-\gamma_4\tau e^{-\tau}+\left(\delta_4+\frac{t_0}{t_m}\xi_4\right)e^{-\tau}\Bigg], \quad (46)$$

$$v_3 \approx \frac{Pa}{27\mu}\left(\frac{a}{r}\right)e^{1-\frac{t_0}{t_m}}\cos 2\theta\cos\phi\left(\frac{18}{1\cdot 7325}\right)\cdot$$

$$\cdot\left[2\sqrt{\left(-\gamma_1'+\frac{t_0}{t_m}\eta_1'\right)^2+\left(-\delta_1'-\frac{t_0}{t_m}\xi_1'\right)^2}\,e^{-\beta_1\tau'}\sin\left\{\alpha_1\tau'+\tan^{-1}\left(\frac{-\delta_1'-\frac{t_0}{t_m}\xi_1'}{-\gamma_1'+\frac{t_0}{t_m}\eta_1'}\right)\right\}\right.$$

$$+2\sqrt{\left(\gamma_2'+\frac{t_0}{t_m}\eta_2'\right)^2+\left(-\delta_2'+\frac{t_0}{t_m}\xi_2'\right)^2}\,e^{-\beta_2\tau'}\sin\left\{\alpha_2\tau'+\tan^{-1}\left(\frac{-\delta_2'+\frac{t_0}{t_m}\xi_2'}{\gamma_2'+\frac{t_0}{t_m}\eta_2'}\right)\right\}$$

$$+2\sqrt{\left(-\delta_3'-\frac{t_0}{t_m}\xi_3'\right)^2+\left(-\gamma_3'-\frac{t_0}{t_m}\eta_3'\right)^2}\,e^{-\beta_3\tau'}\sin\left\{\alpha_3\tau'+\tan^{-1}\left(\frac{-\delta_3'-\frac{t_0}{t_m}\xi_3'}{-\gamma_3'-\frac{t_0}{t_m}\eta_3'}\right)\right\}$$

$$\left.+\gamma_4'\tau' e^{-\tau'}+\left(\delta_4'+\frac{t_0}{t_m}\xi_4'\right)e^{-\tau'}\right], \tag{47}$$

$$w_3 \approx -\frac{Pa}{27\mu}\left(\frac{a}{r}\right)e^{1-\frac{t_0}{t_m}}\cos\theta\sin\phi\left(\frac{18}{1\cdot 7325}\right)\cdot$$

$$\cdot\left[2\sqrt{\left(-\gamma_1'+\frac{t_0}{t_m}\eta_1'\right)^2+\left(-\delta_1'-\frac{t_0}{t_m}\xi_1'\right)^2}\,e^{-\beta_1\tau'}\sin\left\{\alpha_1\tau'+\tan^{-1}\left(\frac{-\delta_1'-\frac{t_0}{t_m}\xi_1'}{-\gamma_1'+\frac{t_0}{t_m}\eta_1'}\right)\right\}\right.$$

$$+2\sqrt{\left(\gamma_2'+\frac{t_0}{t_m}\xi_2'\right)^2+\left(-\delta_2'+\frac{t_0}{t_m}\xi_2'\right)^2}\,e^{-\beta_2\tau'}\sin\left\{\alpha_2\tau'+\tan^{-1}\left(\frac{-\delta_2'+\frac{t_0}{t_m}\xi_2'}{\gamma_2'+\frac{t_0}{t_m}\eta_2'}\right)\right\}$$

$$+2\sqrt{\left(-\gamma_3'-\frac{t_0}{t_m}\eta_3'\right)^2+\left(-\delta_3'-\frac{t_0}{t_m}\xi_3'\right)^2}\,e^{-\beta_3\tau'}\sin\left\{\alpha_3\tau'+\tan^{-1}\left(\frac{-\delta_3'-\frac{t_0}{t_m}\xi_3'}{-\gamma_3'-\frac{t_0}{t_m}\eta_3'}\right)\right\}$$

$$\left.+\gamma_4'\tau' e^{-\tau'}+\left(\delta_4'+\frac{t_0}{t_m}\xi_4'\right)e^{-\tau'}\right], \tag{48}$$

where

$$\tau=\frac{v}{a}\left\{t-\frac{1}{v}(r-a)\right\}, \quad \tau'=\frac{v}{a}\left\{t-\frac{1}{V}(r-a)\right\}. \tag{49}$$

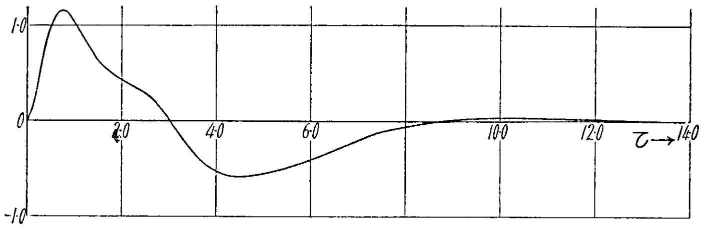

Fig. 5a. The radial vibration of a particle due to the longitudinal wave when $\frac{a}{t_m v}=1$, $\frac{t_0}{t_m}=0$. Unit of ordinate $=\frac{Pa}{27\mu}\left(\frac{a}{r}\right)\sin 2\theta\cos\phi$, abscissa $\tau=\frac{v}{a}\left\{t-\frac{1}{v}(r-a)\right\}$.

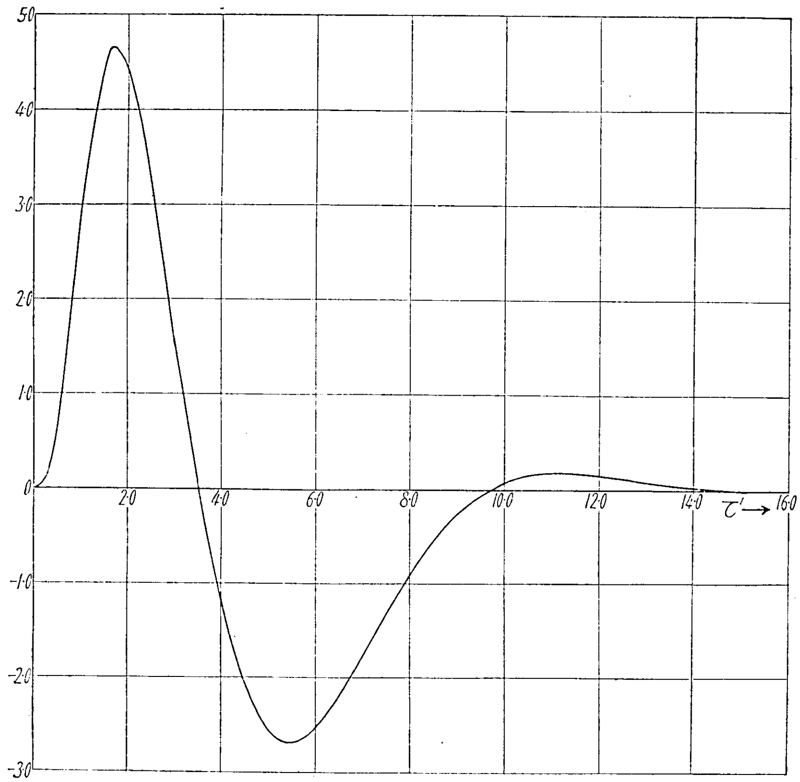

Fig. 5b. The azimuthal or the co-latitudinal vibrations of a particle due to the transverse wave when $\frac{a}{t_m v}=1$, $\frac{t_0}{t_m}=0$. Units of ordinate are $\frac{Pa}{27\mu}\left(\frac{a}{r}\right)\cdot\cos 2\theta\cos\phi$ and $\frac{Pa}{27\mu}\left(\frac{a}{r}\right)\cos\theta\sin\phi$ for the azimuthal and the co-latitudinal vibrations respectively. Abscissa $\tau'=\frac{v}{a}\left\{t-\frac{1}{V}(r-a)\right\}$.

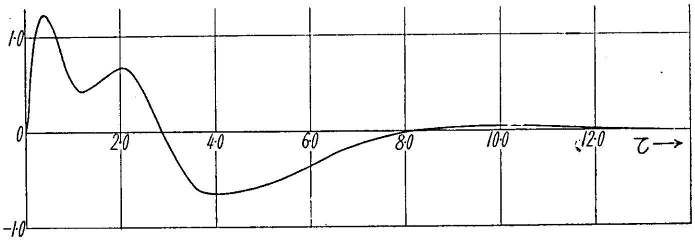

Fig. 6a. The radial vibration of a particle due to the longitudinal wave when $\frac{a}{t_m v}=1$, $\frac{t_0}{t_m}=0{\cdot}50$. Unit of ordinate $=\frac{Pa}{27\mu}\left(\frac{a}{r}\right)\sin 2\theta\cos\phi$, abscissa $\tau=\frac{v}{a}\left\{t-\frac{1}{v}(r-a)\right\}$.

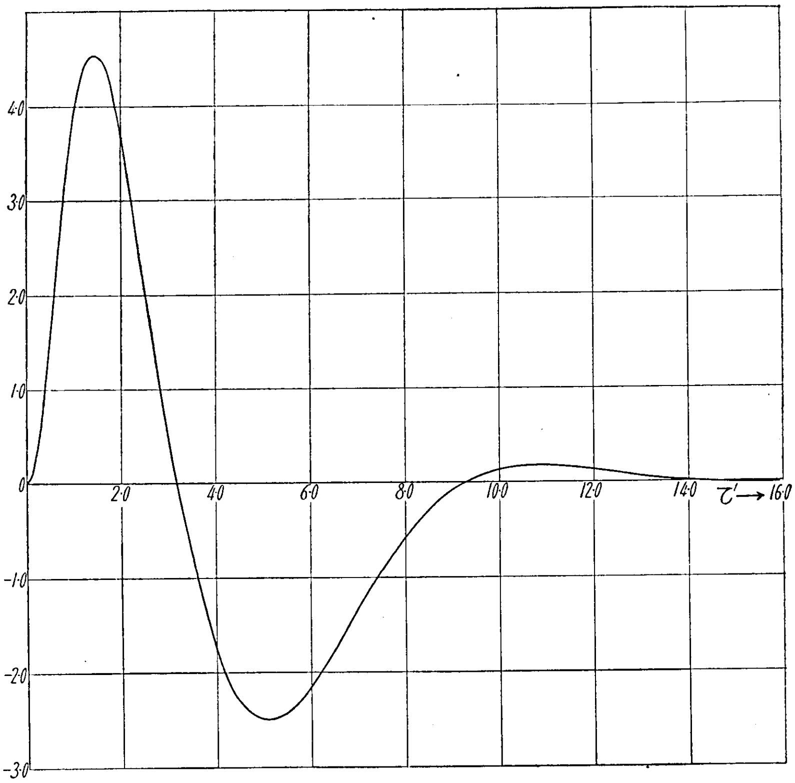

Fig. 6b. The azimuthal or the co-latitudinal vibrations of a particle due to the transverse wave when $\frac{a}{t_m v}=1$, $\frac{t_0}{t_m}=0{\cdot}50$. Unit of ordinate are $\frac{Pa}{27\mu}\left(\frac{a}{r}\right)$. $\cdot\cos 2\theta\cos\phi$ and $\frac{Pa}{27\mu}\left(\frac{a}{r}\right)\cos\theta\sin\phi$ for the azimuthal and the co-latitudinal vibrations respectively. Abscissa $\tau'=\frac{v}{a}\left\{t-\frac{1}{V}(r-a)\right\}$.

To get the vibrations of particles which are some distance away from the origin of disturbance, we calculate numerically the expressions (46), (47), and (48) for the cases when $\frac{t_0}{t_m}=0$, 0·50, and 1·50, the results being shown in Figs. 5a, 5b, 6a, 6b, and 7a, 7b respectively. The time-

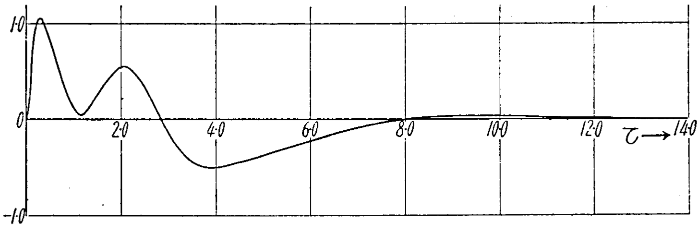

Fig. 7a. The radial vibration of a particle due to the longitudinal wave when $\frac{a}{t_m v}=1$, $\frac{t_0}{t_m}=1\cdot 50$. Unit of ordinate $=\frac{Pa}{27\mu}\left(\frac{a}{r}\right)\sin 2\theta\cos\phi$, abscissa $\tau=\frac{v}{a}\left\{t-\frac{1}{v}(r-a)\right\}$.

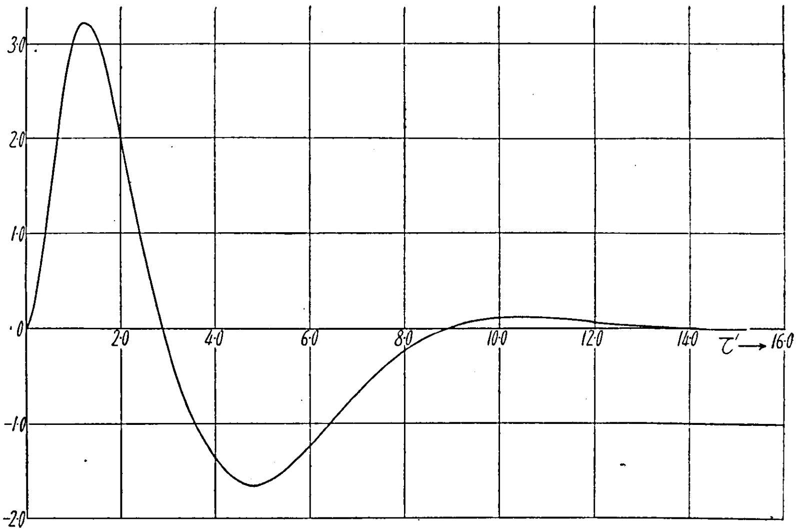

Fig. 7b. The azimuthal or the co-latitudinal vibration of a particle due to the transverse wave when $\frac{a}{t_m v}=1$, $\frac{t_0}{t_m}=1\cdot 50$. Units of ordinate are $\frac{Pa}{27\mu}\cdot\left(\frac{a}{r}\right)\cos 2\theta\cos\phi$, and $\frac{Pa}{27\mu}\left(\frac{a}{r}\right)\cos\theta\sin\phi$ for the azimuthal and the co-latitudinal vibrations respectively. Abscissa $\tau'=\frac{v}{a}\left\{t-\frac{1}{V}(r-a)\right\}$.

variations in the normal stresses on the surface of the spherical cavity for the cases when $\frac{t_0}{t_m}=0$, 0·50, and 1·50 are shown in Fig. 4. In Figs.

5a, 6a, 7a are shown the radial vibrations of particles due to the longitudinal wave for the cases when $\frac{t_0}{t_m}=0$, 0·50, and 1·50 respectively. Figs. 5b, 6b, 7b show the co-latitudinal or the azimuthal vibrations due to the transverse wave for the respective cases when $\frac{t_0}{t_m}=0$, 0·50 and 1·50. It will be seen from these figures that the vibrations of particles, which are some distance away from the origin of disturbance, are due to longitudinal and transverse waves radiated from the origin of disturbance and that they are damped oscillations. When the maximum intensities of pressure on the surface of the cavity are constant, the amplitudes of vibrations of the initial motions in the case of sudden commencement of pressure, do not necessarily become larger than those in the case of gradual commencement. It will also be seen from these figures that the apparent periods of vibrations in the state of initial motion in the case of sudden commencement of pressure generally become shorter than those in the case of gradual commencement. Obviously, Shida's push-pull law holds also in this case. It will be seen, moreover, that the amplitudes of the transverse wave become considerably larger than those of the longitudinal wave, and that the apparent periods of the former wave in the state of initial motion are also longer than those of the latter wave.[11] Needless to say, these theoretical results concerning the amplitude and period of the bodily waves conform with seismometrical results. It will also be seen from these figures that the apparent periods of waves in the state of initial motion are generally shorter than those of the tail of the waves. It will be seen, comparing Figs. 5a, 6a, 7a with respective Figs. 1, 2, 3 in section 3, that the apparent periods of radial vibrations due to the longitudinal wave in the state of initial motion in the case of pressure variation (15) are comparatively longer than those in the case of (1).

5. When pressure is suddenly applied to the surface of the cavity in the elastic medium, and its intensity exceeds the breaking stress of the material, cracks may suddenly appear in the material near the cavity, when the intensity of pressure in the cavity may vary discontinuously. Using agar-agar as the material, S. Yamaguti[12] has

11) Concerning the longitudinal and transverse waves of shock wave-type, W. Inouye has also shown that the duration of time of the former wave becomes comparatively shorter than that of the latter wave.

W. INOUYE, *Bull. Earthq. Res. Inst.*, *loc. cit.*

12) S. YAMAGUTI, *Bull. Earthq. Res. Inst.*, **13** (1935), 772.

shown by his model experiment on the mechanism of the occurrence of earthquakes that the pressure in the cavity varies discontinuously at the moment the agar-agar begins to break. We shall now consider the wave-motions when the pressure in the cavity suddenly increases and a discontinuity appears in the timevariation in pressure.

(a) When the normal stress acts uniformly on the surface of the spherical cavity in the material, and it shows discontinuous time-variations like A, B, C, D in Fig. 8, the diverging waves radiated from the spherical cavity are respectively obtained by suitably superposing the waves shown in Figs. 1∼3 in Section 3. The results are shown in Fig. 9, in which the respective waves are the diverging longitudinal waves radiated from the cavity. The waves with letters A, B, C, D correspond of course to the respective normal stresses A, B, C, D, the time-variations of which are shown in Fig. 8.

(b) When the pressure acting on the surface of the spherical cavity has the distribution $\sin 2\theta \cos\phi$, and it shows discontinuous time-variations, such as A, B, C, D in Fig. 8, the diverging longitudinal and transverse waves radiated from the

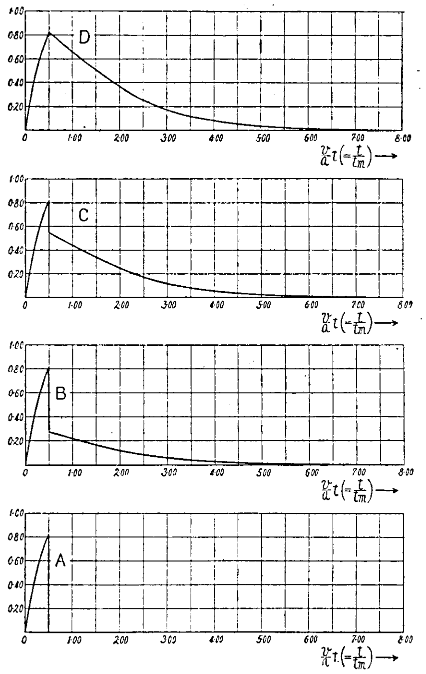

Fig. 8. The time-variations in pressure on the surface of a spherical cavity. Unit of ordinate $= -P$.

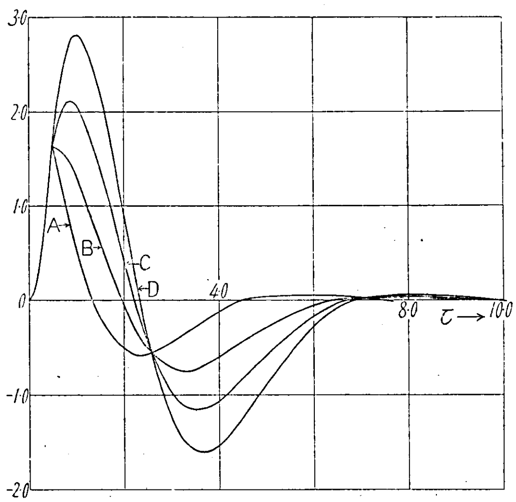

Fig. 9. The radial vibrations of particle due to the longitudinal wave. Unit of ordinate $= \frac{Pa}{27\mu}\left(\frac{a}{r}\right)$, abscissa $\tau = \frac{v}{a}\left\{t - \frac{1}{v}(r-a)\right\}$.

cavity are respectively shown in Figs. 10a and 10b. Of course we obtained these figures by suitably superposing the waves shown in Fgs. 5a, 6a, 7a, 5b, 6b, and 7b.

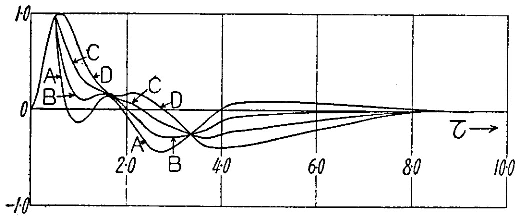

Fig. 10a. The radial vibrations of particle due to the longitudinal waves. Unit of ordinate$=\frac{Pa}{27\mu}\left(\frac{a}{r}\right)\sin 2\theta\cos\phi$, abscissa $\tau=\frac{v}{a}\left\{t-\frac{1}{v}(r-a)\right\}$.

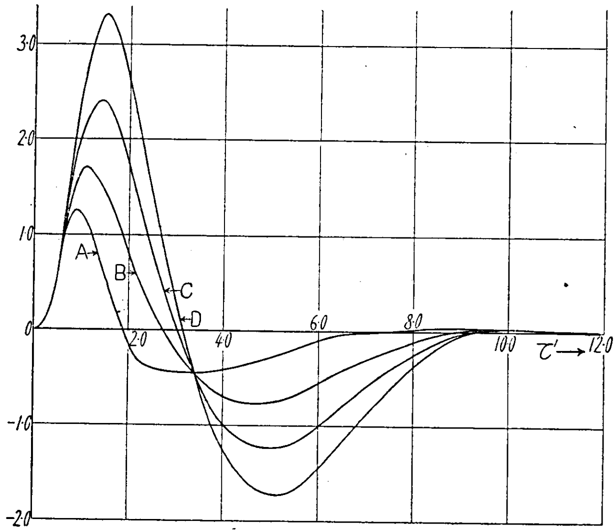

Fig. 10b. The azimuthal or the co-latitudinal vibrations of particle due to the transverse waves. Unit of ordinate are $\frac{Pa}{27\mu}\left(\frac{a}{r}\right)\cos 2\theta\cos\phi$ and $\frac{Pa}{27\mu}\cdot$ $\cdot\left(\frac{r}{a}\right)\cos\theta\sin\phi$ for the azimuthal and the co-latitudinal vibrations respectively. Abscissa $\tau'=\frac{v}{a}\left\{t-\frac{1}{V}(r-a)\right\}$.

It will be seen from Figs. 9～10b that when the pressure acting on the inner surface of the cavity shows discontinuous time-variations as shown in Fig. 8, the apparent periods of the waves generally become short.

In conclusion the author expresses his sincere thanks to Professor

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39. 彈性體內に在る球狀空窩の內面に作用する壓力變化によつて生ずる彈性波

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大地震の際，震央附近は主として震源より來た波動勢力による烈しい地震動に見舞はれる．それと同時に，そこには永久變位性の地殼變形が見られる．時には斷層が出現する事もある．

本論文は此れ等の事實に撞着しない樣な震源を假想してその力學的性質を研究する爲めの準備研究の一つである．本論文では表題の樣に彈性體內に球狀空窩を假想して，その內面に作用する壓力の分布及びその時間的變化を種々與へた場合に生ずる彈性波の性質を明かにした．