35. *Growth and Decay of Seiches in an Epicontinental Sea.*[1]

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1. The fact that a kind of seiches, which however decays rapidly, appears very often even in an open coast free from the effect of bays or inlets, was discovered by Honda, Terada, Yoshida, and Isitani as the result of a series of observations made at a number of points along the coasts of the Japanese islands.[2] Nomitsu,[3] judging from a recent paper of his, seems to conclude that seiches of that kind are due partly to the free oscillation of water in an epicontinental sea. From the mathematical point of view, the problem whether such seiches that depend on the particular form of sea bottom or other boundary conditions, are likely to persist or not, is rather an important one.[4]

2. We shall first discuss the forced oscillation of the epicontinental sea. Supposing for simplicity that the epicontinental sea and the outer open sea are of uniform depths, their values being ξ', ξ, and we take the axis of x directed outwards with its origin at the common boundary of the two seas under consideration, the elementary solutions for incident and reflected waves in the outer open sea as well as in the epicontinental sea are expressed by

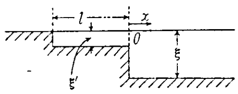

Fig. 1.

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1) Preliminary report published in the *Proc. Imp. Acad.*, **11** (1935), 177.

2) K. HONDA, T. TERADA, Y, YOSHIDA, and D. ISITANI, *Publ. Earthq. Inv. Comm.*, No. 26 (1908); *Journ. Coll. Sci., Tokyo Imp. Univ.*, **24** (1908).

3) T. NOMITSU's paper read at the General Meeting of Math.-Phys. Soc., Japan, held April 1935, in Oosaka.

4) The paper of M. NAKANO, J. NAKAGAWA, and K. TAKEDA on the experimental study of the accumulation and dissipation of energy of secondary undulations in a bay seems to be more in line than others with the phenomena that I have been studying; *Geophys. Mag.*, **8** (1934), 121~124.

These authors used Lord Rayleigh's semi-empirical equation in the analysis of dissipation, which however is not a satisfactory one from the point of view of dynamical theory, although it may be helpful in relieving the difficulty of the analysis to some extent.

$$\left.\begin{array}{ll} u_1=-ie^{i(\sigma t+fx)}\ , & w_1=f\xi e^{i(\sigma t+fx)} \\ u_2=iAe^{i(\sigma t-fx)}\ , & w_2=f\xi Ae^{i(\sigma t-fx)} \\ u_1'=-iBe^{i(\sigma t+f'x)}\ , & w_1'=f'\xi' Be^{i(\sigma t+f'x)} \\ u_2'=iCe^{i(\sigma t-f'x)}\ , & w_2'=f'\xi' Ce^{i(\sigma t-f'x)}\ , \end{array}\right\} \tag{1}$$

where u_1, w_1, u_2, w_2, u_1', w_1', u_2', w_2' are horizontal displacements and surface elevations of the incident and reflected waves in the outer sea and of similar ones in the epicontinental sea respectively, while $2\pi/f$, $2\pi/f'$ are the lengths of the waves in the two seas corresponding to the period $2\pi/\sigma$, so that $\sigma/f=\sqrt{g\xi}$, $\sigma/f'=\sqrt{g\xi'}$. The boundary conditions at $x=0$ may be taken as

$$\xi(u_1+u_2)=\xi'(u_1'+u_2'),\quad w_1+w_2=w_1'+w_2', \tag{2}$$

while that at the end of the epicontinental sea, $x=-l$, is

$$u_1'+u_2'=0. \tag{3}$$

Substituting (1) in (2) and (3), we get

$$\left.\begin{array}{l} A=\dfrac{-(\alpha^2-\alpha)+(\alpha^2+\alpha)e^{-2if'l}}{(\alpha^2+\alpha)-(\alpha^2-\alpha)e^{-2if'l}}\ , \\[2ex] B=\dfrac{2}{(\alpha^2+\alpha)-(\alpha^2-\alpha)e^{-2if'l}}\ , \\[2ex] C=\dfrac{2e^{-2if'l}}{(\alpha^2+\alpha)-(\alpha^2-\alpha)e^{-2if'l}}\ , \end{array}\right\} \tag{4}$$

in which $\alpha=\sqrt{\xi'/\xi}=f'\xi'/f\xi$.

Using (1) and (4) and taking the real parts only, we obtain, corresponding to the incident long waves of the stationary type,

$$u_1=\sin f(\sqrt{g\xi}t+x),\quad w_1=f\xi\cos f(\sqrt{g\xi}t+x), \tag{5}$$

the seiches excited in the epicontinental sea of the form

$$u'=\frac{2\sin f'(x+l)\cos\{f\sqrt{g\xi'}t-\tan^{-1}(\alpha\tan f'l)\}}{\alpha\sqrt{\cos^2 f'l+\alpha^2\sin^2 f'l}}\ , \tag{6a}$$

$$w'=\frac{2f\xi\cos f'(x+l)\cos\{f'\sqrt{g\xi'}t-\tan^{-1}(\alpha\tan f'l)\}}{\sqrt{\cos^2 f'l+\alpha^2\sin^2 f'l}}\ , \tag{6b}$$

where $u'=u_1'+u_2'$, $w'=w_1'+w_2'$, $\alpha=\sqrt{\xi'/\xi}$.

These equations show that the maximum horizontal or vertical displacement of the seiches under resonance condition, $\cos f'l=0$, in the usual sense, is $1/\alpha$ times that of the case in which the incident waves are exceedingly long. This is due to the energy of the seiches having dissipated outward from the common boundary of both seas as waves propagated in the direction opposite to that of the incident waves; otherwise, under resonance conditions, the displacement under consideration would become infinitely large. It is also remarkable that in such resonance condition the vertical displacement at $x=0$ is zero, which is evident from the nature of the problem.

3. In the second place the decaying nature of free seiches will be dealt with. The generalisation of (1) by means of Fourier's integral gives the expressions of incident waves and of the free vibrations of the epicontinental sea as follows:

$$w_1=\frac{1}{2\pi}\int_{-\infty}^{\infty}df\int_{-\infty}^{\infty}\varphi(\tau)e^{if(\sqrt{g\xi}t+x-\tau)}d\tau\,, \tag{7a}$$

$$w'=\frac{\alpha}{\pi}\int_{-\infty}^{\infty}\frac{df}{(\alpha^2+\alpha)-(\alpha^2-\alpha)e^{-if/\alpha}}\int_{-\infty}^{\infty}\varphi(\tau)e^{\frac{if}{\alpha}(\sqrt{g\xi'}t+x-\alpha\tau)}d\tau$$

$$+\frac{\alpha}{\pi}\int_{-\infty}^{\infty}\frac{e^{-2ifl/\alpha}df}{(\alpha^2+\alpha)-(\alpha^2-\alpha)e^{-ifl/\alpha}}\int_{-\infty}^{\infty}\varphi(\tau)e^{\frac{if}{\alpha}(\sqrt{g\xi'}t-x-\alpha\tau)}d\tau\,, \tag{7b}$$

provided $\varphi(x)$, which is the form of the incident waves at $t=0$, is so sharp that it does not much disturb the subsequently excited free oscillation of the epicontinental sea. The above equations generally reduce to

$$w_1=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{if(\sqrt{g\xi}t+x)}df\int_{-\infty}^{\infty}\varphi(\tau)e^{-if\tau}d\tau\,, \tag{8a}$$

$$w'=\frac{1}{(1+\alpha)\pi}\sum_{m=0}^{\infty}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^m\left\{\int_{-\infty}^{\infty}e^{\frac{if}{\alpha}(\sqrt{g\xi'}t+x-2ml)}df\int_{-\infty}^{\infty}\varphi(\tau)e^{-if\tau}d\tau\right.$$

$$\left.+\int_{-\infty}^{\infty}e^{\frac{if}{\alpha}\{\sqrt{g\xi'}t-x-2(m+1)l\}}df\int_{-\infty}^{\infty}\varphi(\tau)e^{-if\tau}d\tau\right\}. \tag{8b}$$

From these equations it follows that, if the initial impulse involved in the incident waves were of the form

$$w_1=F(\sqrt{g\xi}t+x), \tag{9}$$

the free seiches excited in the epicontinental sea would be

$$w'=\frac{2}{1+\alpha}\sum_{m=0}^{\infty}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^m\left\{F\left(\frac{\sqrt{g\xi'}\,t+x-2ml}{\alpha}\right)\right.$$
$$\left.+F\left(\frac{\sqrt{g\xi'}\,t-x-2(m+1)l}{\alpha}\right)\right\}. \tag{10}$$

Such an oscillation is the exponentially damping type with logarithmic decrement

$$\frac{\sqrt{g\xi'}}{2l}\log e\left|\frac{1+\alpha}{1-\alpha}\right|. \tag{11}$$

The greater the difference in the depths of both the seas, the more slowly do the free seiches decay in the epicontinental sea. Since

$$\log e\left|\frac{1+\alpha}{1-\alpha}\right|=\log e\left|\frac{1+\dfrac{1}{\alpha}}{1-\dfrac{1}{\alpha}}\right|,$$

the logarithmic decrement is the same whether $\sqrt{\xi'/\xi}$ is equal to a constant α or equal to the reciprocal $1/\alpha$ of the same constant. In other words, the decrement of the case in which the ratio of depths is specified, is the same as that of the case in which the ratio of the depths is the reciprocal of the former case.

4. Further, equation (7) or (10) may be used in the solution of the growth of the seiches without any alteration in the forms of those equations. By investigating the oscillation indicated by such equations over a range of period in which the disturbance $F(\sqrt{g\xi}t+x)$ is still going on, it is possible to determine the behaviour in the growth of seiches, which is evident from the meaning implied in the equations under consideration.

If the form of the incident waves transmitted in the outer open sea be

$$\left.\begin{aligned} w_1&=\sin f(\sqrt{g\xi}t+x), && [\sqrt{g\xi}t+x>0] \\ w_1&=0, && [\sqrt{g\xi}t+x<0] \end{aligned}\right\} \tag{12}$$

we obtain from (10)

$$w'=\frac{2}{1+\alpha}\sum_{m=0}^{\infty}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^m\Big\{F_1(\sqrt{g\xi'}t+x-2ml)$$

$$+F_2(\sqrt{g\xi'}\,t-x-2\overline{m+1}l)\Big\}, \tag{13}$$

where

$$F_1(\sqrt{g\xi'}t+x-2ml)=\sin f'(\sqrt{g\xi'}t+x-2ml),$$

$$[\sqrt{g\xi'}t+x-2ml>0] \tag{14a}$$

$$=0, \qquad [\sqrt{g\xi'}t+x-2ml<0] \tag{14b}$$

$$F_2(\sqrt{g\xi'}t-x-2\overline{m+1}l)=\sin f'(\sqrt{g\xi'}t-x-2\overline{m+1}l),$$

$$[\sqrt{g\xi'}t-x-2\overline{m+1}l>0] \tag{14c}$$

$$=0. \qquad [\sqrt{g\xi'}t-x-2\overline{m+1}l<0] \tag{14d}$$

If $(2n+1)l>\sqrt{g\xi'}t>2nl$, we have

$$w'=\frac{2}{1+\alpha}\Big\{\sum_{m=0}^{n}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^m F_1(\sqrt{g\xi'}t+x-2ml)$$

$$+\sum_{m=0}^{n-1}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^m F_2(\sqrt{g\xi'}t-x-2\overline{m+1}l)$$

$$=\frac{4\cos f'(l+x)}{1+\alpha}\sum_{m=0}^{n-1}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^m\sin f'(\sqrt{g\xi'}t-2ml)$$

$$+\frac{2a_n}{1+\alpha}(-1)^n\left(\frac{1-\alpha}{1+\alpha}\right)^n\sin(\sqrt{g\xi'}t+x-2nl), \tag{15}$$

in which $a_n=1$ or $=0$ according as $\sqrt{g\xi'}t>$ or $<2nl-x$.

If $2(n+1)l>\sqrt{g\xi'}t>(2n+1)l$, we get similarly

$$w'=\frac{4\cos f'(l+x)}{1+\alpha}\sum_{m=0}^{n-1}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^m\sin f'(\sqrt{g\xi'}t-2ml)$$

$$+\frac{2}{1+\alpha}(-1)^n\left(\frac{1-\alpha}{1+\alpha}\right)^n\sin f'(\sqrt{g\xi'}t+x-2nl)$$

$$+\frac{2b_n}{1+\alpha}(-1)^n\left(\frac{1-\alpha}{1+\alpha}\right)^n\sin f'(\sqrt{g\xi'}t-x-2\overline{n+1}l), \tag{16}$$

in which $b_n=1$ or $=0$ according as $\sqrt{g\xi'}t>$ or $<(2n+1)l+(l+x)$.

Now, we are familiar with the expression

$$\frac{1+(-1)^n r^{n+1}\{\cos(n+1)\phi+i\sin(n+1)\phi\}}{1+r(\cos\phi+i\mathrm{son}\,\phi)}$$

$$=1-r(\cos\phi+i\sin\phi)+\cdots\cdot+(-1)^n r^n(\cos n\phi+i\sin n\phi),$$

so that

$$\sum_{m=0}^{n-1}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^m\cos 2f'lm$$

$$=\frac{1+(-1)^{n-1}\left\{\left(\frac{1-\alpha}{1+\alpha}\right)^{n+1}\cos 2\overline{n-1}f'l+\left(\frac{1-\alpha}{1+\alpha}\right)^n\cos 2nf'l\right\}+\frac{1-\alpha}{1+\alpha}\cos 2f'l}{\left\{1+\frac{1-\alpha}{1+\alpha}\cos 2f'l\right\}^2+\left\{\frac{1-\alpha}{1+\alpha}\sin 2f'l\right\}^2}, \tag{17a}$$

$$\sum_{m=0}^{n-1}(-1)^m\left(\frac{1-\alpha}{1+\alpha}\right)^m\sin 2f'lm$$

$$=\frac{-\frac{1-\alpha}{1+\alpha}\sin 2f'l+(-1)^{n-1}\left\{\left(\frac{1-\alpha}{1+\alpha}\right)^{n+1}\sin 2\overline{n-1}f'l+\left(\frac{1-\alpha}{1+\alpha}\right)^n\sin 2nf'l\right\}}{\left\{1+\frac{1-\alpha}{1+\alpha}\cos 2f'l\right\}^2+\left\{\frac{1-\alpha}{1+\alpha}\sin 2f'l\right\}^2}. \tag{17b}$$

Substituting from these equations in (15) and (16), we obtain

$$w'=\frac{2\cos f'(x+l)}{\sqrt{\cos^2 f'l+\alpha^2\sin^2 f'l}}\Big[\sin\{f'\sqrt{g\xi'}t-\tan^{-1}(\alpha\tan f'l)\}$$

$$+(-1)^{n-1}\left(\frac{1-\alpha}{1+\alpha}\right)^n\sin\{f'\sqrt{g\xi'}t-2f'nl-\tan^{-1}(\alpha\tan f'l)\}\Big]$$

$$+\frac{2A}{1+\alpha}(-1)^n\left(\frac{1-\alpha}{1+\alpha}\right)^n, \tag{18}$$

where (i) $A=a_n\sin f'(\sqrt{g\xi'}t+x-2nl)$, $\begin{cases}a_n=1, & \sqrt{g\xi'}t>2nl-x,\\ a_n=0, & \sqrt{g\xi'}t<2nl-x,\end{cases}$

or (ii) $A=\sin f'(\sqrt{g\xi'}t+x-2nl)$,

$+b_n\sin f'(\sqrt{g\xi'}t-x-\overline{2n+1}l)$, $\begin{cases}b_n=1, & \sqrt{g\xi'}t>(2n+1)l+(l+x),\\ b_n=0, & \sqrt{g\xi'}t<(2n+1)l+(l+x),\end{cases}$

according as (i) $(2n+1)l>\sqrt{g\xi'}t>2nl$, or (ii) $2(n+1)l>\sqrt{g\xi'}t>(2n+1)l$. These oscillations correspond to the incident waves indicated in (12).

Closer examination of the above expressions shows that the seiches grow relatively quickly to the state of stationary oscillation, particularly in the resonance condition, $f'l=\pi/2$. The vertical movement at the end, $x=-l$, begins at the instant $f'\sqrt{g\xi'}t=\pi/2$, and the successive amplitudes at $f'\sqrt{g\xi'}t=\pi,\ 2\pi, \cdots n\pi, \cdots$ are

$$\frac{2}{a}\left\{1-\frac{1-a}{1-a}\right\}, \frac{2}{a}\left\{1-\left(\frac{1-a}{1+a}\right)^2\right\}, \cdots \cdot \frac{2}{a}\left\{1-\left(\frac{1-a}{1+a}\right)^n\right\}, \cdots \cdot \qquad (19)$$

respectively. From these expressions it follows that, in the resonance condition, at any rate, the increment of amplitudes of successive oscillations relative to that of the initial one in the case of the growth of seiches is proportional to the amplitudes of successive oscillations in the case of the decay of the seiches. In non-resonance conditions, however, the seiches are not likely to grow at such a rapid rate.

5. With a view to compare the present calculation with actual data, I studied the result obtained by Honda, Terada, etc..[5] The places shown in the following table may be taken as good examples of open coasts.

Place	l (km)	Mean depth (m)		Period=T (min)		Decay const. $=\left(\frac{1-a}{1+a}\right)^2$	
		ξ	ξ'	Calc.	Obs.	Calc.	Obs.
Kanaiwa	30	1000	70	76	59·0	0·34	0·30
Wazima	40	1100	100	85	81·5	0·30	0·40
Kasiwazaki	18	900	90	43	43·3	0·27	0·40
Iwasaki	9	1000	70	23	15·8~17·3	0·35	0·30
Inubô	34	1500	80	81	66·0	0·81	0·70
Onmaizaki	10	500	55	29	27·7	0·64	0·60
Tei	25	500	60	69	73·9~77·5	0·62	0·60

I took the longest possible observed periods, excepting however that at Iwasaki, where the observed period is the one with conspicuous amplitudes. The observed decaying constants naturally correspond to the oscillations of these periods.

My sincere thanks are due to Professor Terada for his kind advices during the course of this investigation.

5) K. HONDA, T. TERADA, Y. YOSHIDA, D. ISITANI, *loc. cit.*

35. 陸棚に於けるセイシの成長及老衰

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本多博士，寺田博士其他の方々が昔試みられた研究によれば，灣内は勿論のこと，陸棚と思はれる所にも潮汐の副振動があるやうである．それを數理的に研究して見た所が，陸棚の深さと外海の深さの割合によつて種々の性質のあることがわかつた．

外海に週期的の波動があるときに，陸棚に起るセイシは波動の週期によつて振幅が違ふけれども，共振に當る所の振幅は，極めて長週期の波動の場合の振幅に外海と陸棚の深さの割合の平方根を乘じたものに等しくなり，且つその場合に陸棚の外海と連る所の上下の振幅が零になるものである．之等は波動勢力の外海への逸散を考へて始めて得られる結果である．

次に外海の波動が止つた場合に，陸棚に於けるセイシの老衰狀態は同樣な數理で解き得るが之は勿論勢力の外海への逸散といふことが原因をなすものである．セイシは指數函數的に老衰し，其對數的減率は

$$(\sqrt{g\xi'}/2l)\{\log e(1+\alpha)-\log e(1-\alpha)\}$$

で示される．$\alpha=\sqrt{\xi'/\xi}$ であり，ξ', ξ, l は夫々陸棚，外海の深さ及び陸棚の奥行である．

セイシが外海の波動の爲に成長する機構も數理的に一定の式で表される．但し共振に當るやうな波動では老衰の場合よりも寧ろ速にセイシが成長することがわかるのである．

數理が實際とあふかを見る爲に，本多博士，寺田博士等の觀測結果を借用して灣に無關係な著しい陸棚と思はれる場所の結果と比較して見た所が，セイシの週期は勿論，自由セイシの減衰の割合も可なりよく一致することがわかつたのである．