53. *Changes in Rigidity and Internal Friction of Amorphous Silica with Temperature.**

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1. Introduction. The changes in the form of seismic waves in the earth's crust and the decay of these waves as the result of internal friction of the materials composing it are familiar matters. These problems, first treated theoretically by Prof. H. Nagaoka,[1] were later studied by Prof. K. Sezawa[2] and other investigators,[3] and the relations of the velocities of seismic waves, to Young's modulus, rigidity, and density of the materials composing the earth's crust were obtained from observations of seismic waves by a number of investigators, whose studies however will not be described here. With the desire to attack such problems as the internal friction and the rigidity and elasticity of materials from the experimental side, the writer carried out the experiments described in this paper.

The form of the materials composing the earth's crust may either be amorphous or complicated crystals. In order to know the properties of such material, amorphous silica was selected as being readily accessible and easily worked. The properties of vitreous silica, its density, rigidity, Young's modulus, etc., have already been ascertained.[4] The modulus of torsional rigidity of quartz fibre was first investigated by Boys,[5] and later by Threlfall,[6] Barnett,[7] Guye and Fréedericksz,[8] and Horton.[9] The most precise measurements of rigidity of vitreous silica are those of F. Horton, who, by means of the method of oscillation of loaded fibre, measured its rigidity between temperatures 20° and

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* Communicated by M. Ishimoto.

1) H. NAGAOKA, *Proc. Phys.-Math. Soc., Japan,* **3** (1906), 17.

2) K. SEZAWA, *Bull. Earthq. Res. Inst.,* **3** (1927), 43; *ibid.,* **10** (1932), 19.

3) For example, B. GUTENBERG, *Phys. z. s.,* **30** (1929), 230.
N. M. HOSARI, *Proc. Roy. Soc.,* **104** (1923), 271.

4) R. B. SOSMAN, "*The Properties of Silica,*" U. S. A. (1927).

5) C. V. BOYS, *Phil. Trans. Roy. Soc., London,* **186** (1895), 65.

6) R. THRELFALL, *Phil. Mag.,* **30** (1890), 99.

7) S. J. BARNETT, *Phys. Rev.,* **6** (1898), 114.

8) C. E. GUYE and V. FRÉEDERICKSZ, *Arch. Sci. Phys. Nat.,* **29** (1910), 49

9) F. HORTON, *Phil. Trans. Roy. Soc., London,* **204** (1905), 407.

1058°C. He also observed the logarithmic decrements of the oscillations of quartz fibre. Similar measurements have also been made by Guye and Morein.[10]

The nature of internal friction in metals has received the attention of a number of workers, such as Honda and Konno,[11] Iokibe and Sakai,[12] Sezawa and Kubo,[13] Ishimoto,[14] Barus,[15] Voigt,[16] Cady,[17] Horton,[18] Higuchi,[19] and Itihara.[20] The values of the coefficients of internal friction (the normal viscosity) of metals obtained by Honda and Konno are of the order of 10^8 in C. G. S. units at room temperature, while those obtained by Sezawa and Kubo are so small as to be of the order of 10^6 in the same units.

The objects of the experiments described here were to gain some knowledge with respect to the geophysical problems just mentioned, to test whether damping is proportional to the velocity of the deformation of the substance, and also whether or not the internal friction of quartz fibre is similar to that of metals.

2. Methods of Experiment. The quartz fibre experimented on were drawn from a bar of quartz, the chemical analysis of which, in bulk composition, was

$$SiO_2 \quad 99{\cdot}95 \text{ wt. } \%, \qquad Al_2O_3 \quad \text{trace},$$

which shows that the article was fairly pure. Great care was taken in making them in order to get them free of air bubbles and to obtain a true circular cross-section. The dimensions of the tested quartz fibre are shown in Table I. The diameters were measured by a micrometer with an accuracy of 1/100 mm, and their lengths by a calliper of an accu-

Table I. Dimension of Quartz Fibres.

No.	Radius, r. (cm)	Length, l. (cm)
1	$18{\cdot}52 \times 10^{-3}$	10·15
2	28·05 ″	10·00
3	42·45 ″	11·93
4	51·17 ″	12·36
5	65·40 ″	9·74

10) C. E. GUYE and A. MOREIN, *Arch. Sci. Phys. Nat.*, [v], **2** (1920), 350.
11) K. HONDA and S. KONNO, *Phil. Mag.*, **42** (1921), 115.
12) K. IOKIBE and S. SAKAI, *Phil. Mag.*, **42** (1921), 397.
13) K. SEZAWA and K. KUBO, *Rep. Aeron. Res. Inst., Tokyo*, **7** (1932), 198.
14) M. ISHIMOTO, *Proc. Phys.-Math. Soc., Japan*, [iii], **1** (1919), 267.
15) C. BARUS, *Phil. Mag.*, **29** (1890), 337.
16) W. VOIGT, *Wied. Ann.*, **47** (1892), 671.
17) W. G. CADY, *Phys. Rev.*, **15** (1920), 146; **19** (1922), 1.
18) F. HORTON, *Phil. Trans. Roy. Soc., London*, [A], **204** (1905), 1~55.
19) S. HIGUCHI, *Tech. Rep. Tohoku Imp. Univ.*, **10** (1931), 506.
20) M. ITIHARA, *Trans. Soc. Mech. Eng.*, **1** (1935).

racy of 1/10 mm.

(*a*) *The First Experiments.* The apparatus used is shown in Fig. 1. The quartz fibre Q was firmly welded at both its ends to the two quartz rods B and C, about 8 mm dia. and 20 cm long. The upper end of B was firmly wedged in between the two brass clamps D, which itself could be firmly clamped by means of other clamps. The lower end of C was affixed by means of screws to the brass rod E, which, with the soft iron plate S, 5·8 cm × 1·4 cm × 0·4 cm, forms the vibrator. To increase the moment of inertia of the vibrator, weights can be attached to the brass rod E by means of the screw K.

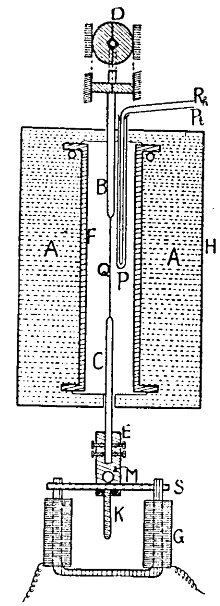


Fig. 1. Sketch of the apparatus.

A, asbestos; B, C, quartz rods; Q, quartz fibre; F, electric furnace; M, lens mirror; E, brass rod; H, tin-plate box; S, soft iron plate; G, electromagnet; P, porcelain tube; K, screw; D, brass clamp, which is divisible into two parts; P_t and R_h are wires of the thermo-couple.

F is an electric furnace, 5 cm internal diameter and 30 cm long, capable of being separated into two parts. Nichrome wires are wound along the longitudinal axis of the inner side of the furnace, which latter is heated by means of an alternating current, 100 volts and 9 amperes, its temperature being measured by means of a thermo-couple consisting of wires of platinum and rhodium. The lower end of the porcelain tube P in which these two wires are placed, is situated near the middle of the quartz fibre Q to be tested. The temperatures were read off from a millivoltmeter, which was standardised by taking the melting points of such substances as Cu, Ni, Al, Pb, etc., and the boiling point of water. The furnace is placed in a tin-plate box H, 35 cm × 17 cm × 17 cm. To prevent heat loss from the furnace, asbestos was packed in the space between the furnace and the box. The temperature, after being raised slowly from that of the room to about 1100°C, was then allowed to cool. The torsional oscillations of the specimen were observed during both rise and fall of the temperature.

The vibrator was set in oscillation by the following device: A momentary current passing through the electromagnet G, turns the soft iron plate S and sets the specimen in torsional oscillation. The position of the electromagnet was carefully adjusted so as not to cause any lateral vibration of the specimen. The oscillations of the soft iron plate S over the magnet may be the cause of the damping, but since the residual magnetism is very small, its action may be ignored.

A small lens mirror M attached to the brass rod E enables observation of the torsional oscillations. The motion of the mirror is photographically recorded on a rapidly rotating drum (40 cm circumference, rev. 13 sec) on which a bromide paper is wound, and which is placed about one meter distant from the lens mirror M. An electric lamp (motor-car head light) of 10 v. 3·3 amp. serves as the light source. The device for obtaining the period of the oscillations causes the light from a slit to be cut off every 0·5 seconds. A chronometer and a Post Office standard relay was used, and this connected to a battery, formed circuit (A). We made another circuit (B) by means of the relay, another battery, and the solenoidal coil L in Fig. 2. N is a piece of soft iron magnetised by the solenoid L; R the bar of soft steel that is drawn to the magnet N when the current flows through the circuit (B). When R is drawn to the magnet N, the light from slit X is interrupted by R. In the record produced by the motion of the mirror M, many broken points were consequently observed. From the intervals of these points the length of the period of the oscillations was calculated.

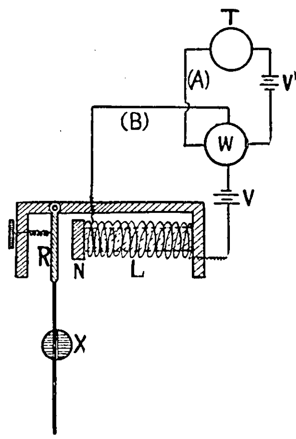


Fig. 2. Arrangement of the timing apparatus.

N, piece of soft iron; L, solenoid coil; R, bar of soft iron; T, chronometer; W, Post Office standard relay; V, battery; X, slit; (A),(B), circuits.

(*b*) *The Second Experiments.* As the result of both internal friction and air resistance, the amplitude of oscillations gradually diminishes. Since we wished to know the internal friction of the specimen, we had to observe the effects of air resistance. To achieve this purpose the second experiment was made with the above described oscillating system enclosed in a bell-jar of thick glass resting tightly on a plane disk. The bell-jar was evacuated by a Gaede rotary pump, by means of which the pressure in the bell-jar was reduced, the lowest value of which was about 1 mm. The pressure is measured with a mercury manometer connected to the bell-jar. The experiments were conducted under different pressures and with different sizes of fibre. We obtained the relation between the logarithmic decrement of the oscillation and the pressure in the bell-jar. Such experiments as those of the bell-jar should, strictly speaking, be made also at temperature ranges of 20°∼1100°C, but to simplify matters they were performed at the room temperature of 20°C. In this way the modulus of rigidity and the coefficient of internal friction were measured.

3. Results of the Experiments. Some examples of actual photographs of torsional oscillations obtained by the apparatus are shown in Fig. 3. It will be seen from these records that the oscillations of this system are of the regular sine wave form with diminishing amplitude. It will also be noticed that since the amplitudes of the oscillations rapidly diminish at high temperature, the logarithmic decrements are great. At over 1000°C the internal friction of the fibre was so great that the oscillations were nearly dead-beat. The internal friction of the fibre is calculated from the logarithmic decrement of the oscillations as follows:

If v be the velocity of a fluid moving in the direction of x, and z an axis perpendicular to x, then the tangential (ordinary) viscosity and the normal viscosity per unit area are given by

$$\left.\begin{aligned} f_t &= \eta\frac{dv}{dz}, \\ f_n &= \xi\frac{dv}{dx}, \end{aligned}\right\} \tag{1}$$

respectively, where η and ξ are the coefficients of tangential and normal viscosity respectively. If resistance be represented by ordinary formula, the equation of motion of this torsional system at any time t is given by

$$I\frac{d^2\theta}{dt^2}+\varepsilon\frac{d\theta}{dt}+k\theta=0, \tag{2}$$

where θ is the angle of twist, I the moment of inertia of the attached body, and ε, k are two constants depending on the dimensions and the nature of the suspended substance.

If l and r be the length and radius of the specimen respectively, and n its rigidity modulus, we would have the relations[21)]

$$k=\frac{\pi n r^4}{2l}, \qquad \varepsilon=\frac{\pi\eta r^4}{2l}. \tag{3}$$

The solution of equation (2) is given by

$$\theta=\theta_0 e^{-\frac{\varepsilon t}{2I}}\cos\left(\frac{2\pi t}{T}+\alpha\right), \tag{4}$$

21) K. HONDA and S. KONNO, *loc. cit.*

where θ_0 is the initial amplitude of oscillation, T the period of oscillation, and α a constant.

If λ be the logarithmic decrement of the oscillation, we would have

$$\lambda = \frac{\varepsilon T}{4I} = \frac{\pi \eta r^4 T}{8Il},$$

$$\eta = \frac{8Il\lambda}{\pi r^4 T}. \tag{5}$$

Hence, by observing λ and T, η can be determined. For the modulus of torsional rigidity of the specimen, we have

$$n = \frac{8\pi Il}{T^2 r^4}, \tag{6}$$

whence

$$\frac{n}{\eta} = \frac{\pi^2}{T\lambda}. \tag{7}$$

By observing the period of the torsional oscillations the modulus of rigidity n can be calculated if the values of I, l, and r are all constant. The values of n and the observed logarithmic decrement λ, calculated from the observed periods and amplitudes of torsional oscillations at each temperature, are shown in Tables II∼IV, and some

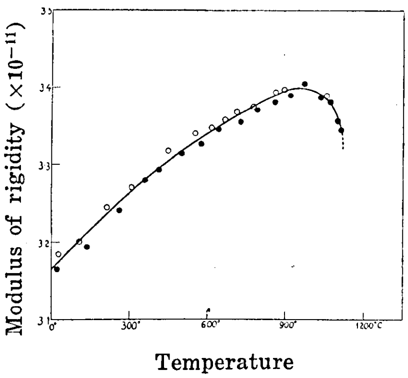


Fig. 4. Curve showing the change in rigidity of amorphous silica fibre with temperature.
Radius of fibre $=18{\cdot}52\times10^{-3}$ cm, length 10·15 cm.
●, heating; ○, cooling.

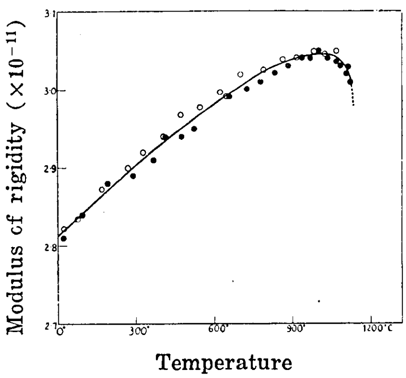


Fig. 5. Curve showing the change in rigidity of amorphous silica fibre with temperature.
Radius of fibre $=65{\cdot}4\times10^{-3}$ cm, length 9·74 cm.
●, heating; ○, cooling.

examples in Figs. 4∼7. In these figures, n and λ are taken as ordinates and the temperature as abscissa.

Since up to a temperature of about 200°C, the rigidity of the fibre increased with temperature as a linear function of it, we may assume

Table II. Modulus of Rigidity and Coefficient of Internal Friction of Amorphous Silica Fibre.

(Radius of the fibre $r=18{\cdot}52\times10^{-3}$ cm, length $l=10{\cdot}15$ cm)

Temp. (°C)	Frequency	Logarithmic decrement $\lambda\times10^3$	Modulus of rigidity n(c. g. s.)$\times10^{-11}$	Coeff. of internal friction η(c. g. s.)$\times10^{-9}$
21·9	1·152	0·60	3·16	4·18
138	1·157	0·72	3·19	4·30
225	1·177	0·68	3·24	4·75
360	1·179	0·69	3·28	4·83
419	1·176	0·75	3·30	5·34
505	1·180	0·78	3·32	5·51
579	1·182	1·10	3·33	7·74
649	1·185	1·56	3·35	10·99
722	1·187	2·62	3·36	18·45
789	1·189	4·50	3·37	31·74
848	1·189	7·13	3·38	50·32
922	1·192	12·77	3·39	90·41
973	1·196	17·94	3·41	127·42
1032	1·192	27·24	3·39	192·89
1073	1·190	39·64	3·37	280·18
1095	1·187	52·04	3·36	366·89
1112	1·185	64·32	3·35	452·72
895	1·194	6·77	3·40	48·01
840	1·196	2·77	3·39	19·65
777	1·193	1·75	3·38	12·34
710	1·192	1·07	3·37	7·60
664	1·193	0·89	3·36	6·32
611	1·190	0·86	3·35	6·10
544	1·185	0·79	3·34	5·54
445	1·183	0·61	3·32	4·29
313	1·170	0·63	3·27	4·37
214	1·167	0·62	3·24	4·33
110	1·159	0·63	3·20	4·58
30	1·155	0·62	3·18	4·30

Table III. Modulus of Rigidity and Coefficient of Internal Friction of Amorphous Silica Fibre.

(Radius of the fibre $r=51.17\times10^{-3}$ cm, length $l=12.36$ cm)

Temp. (°C)	Frequency	Logarithmic decrement $\lambda\times10^3$	Modulus of rigidity n(c. g. s.)$\times10^{-11}$	Coeff. of internal friction η(c. g. s.)$\times10^{-5}$
21	7·625	0·50	2·79	4·72
105	7·620	0·49	2·79	4·61
176	7·691	0·33	2·84	3·19
232	7·733	0·35	2·87	3·39
293	7·765	0·28	2·89	2·66
370	7·769	0·29	2·90	2·76
435	7·800	0·33	2·92	3·23
494	7·800	0·40	2·93	3·89
547	7·826	0·40	2·94	3·90
592	7·826	0·38	2·95	2·92
639	7·846	0·49	2·95	4·75
692	7·857	0·66	2·96	6·41
725	7·875	0·92	2·98	8·97
764	7·882	1·32	2·98	12·91
805	7·888	2·04	2·99	20·00
855	7·889	3·58	3·00	35·09
906	7·894	5·74	3·01	56·24
942	7·924	8·35	3·02	82·11
966	7·940	10·74	3·02	105·77
990	7·954	13·82	3·04	136·41
1009	8·000	16·22	3·03	161·03
1029	7·961	17·38	3·04	171·71
1053	7·950	24·15	3·03	238·25
1075	7·939	29·01	3·02	284·71
1090	7·925	34·26	3·01	336·56
1100	7·927	36·36	3·01	357·67
1116	7·916	42·88	3·00	421·28
1119	7·905	42·66	3·00	418·48
1062	7·960	27·25	3·04	269·18
984	7·960	10·79	3·04	109·07
900	7·944	4·58	3·02	45·15
822	7·920	2·01	3·01	19·76
762	7·900	1·08	2·99	10·59
701	7·881	0·87	2·98	8·55
638	7·860	0·65	2·96	6·33
570	7·840	0·51	2·95	4·39
400	7·794	0·36	2·91	3·50
338	7·800	0·40	2·91	3·95
262	7·769	0·32	2·89	2·10
181	7·739	0·35	2·87	2·44
125	7·714	0·34	2·86	3·22
63	7·690	0·25	2·84	3·00
19	7·625	0·24	2·79	2·33

Table IV. Modulus of Rigidity and Coefficient of Internal Friction of Amorphous Silica Fibre.

($r=65{\cdot}40\times10^{-3}$ cm, $l=9{\cdot}74$ cm)

Temp. (°C)	Frequency	Logarithmic decrement $\lambda\times10^3$	Mod. of rigidity n(c. g. s.)$\times10^{-11}$	Coeff. of internal friction η(c. g. s.)$\times10^{-5}$
20·8	13·833	0·16	2·81	0·82
94	13·900	0·18	2·84	0·89
192	14·000	0·20	2·88	1·03
291	14·000	0·18	2·89	0·93
370	14·081	0·18	2·91	0·95
413	14·130	0·20	2·94	1·05
477	14·133	0·26	2·94	1·37
525	14·176	0·25	2·95	1·28
605	14·095	0·31	2·96	1·62
661	14·266	0·44	2·99	2·29
719	14·307	0·74	3·01	3·89
780	14·316	1·28	3·01	6·70
817	14·350	2·24	3·03	11.76
886	14·363	3·55	3·03	18·66
940	14·355	5·53	3·03	29·05
972	14·378	6·79	3·04	35·73
1004	14·527	8·71	3·06	40·71
1039	14·356	10·53	3·05	55·50
1067	14·372	13·12	3·04	69·01
1082	14·394	15·37	3·04	80·99
1098	14·404	17·59	3·03	92·66
1108	14·392	19·22	3·02	101·31
1117	14·370	22·56	3·01	118·65
1116	14·382	22·18	3·04	116·77
1070	14·413	14·73	3·05	77·70
1025	14·371	9·04	3·05	47·55
984	14·419	5·84	3·06	30·84
920	14·382	3·08	3·04	16·20
845	14·400	1·47	3·04	7·73
772	14·353	0·72	3·03	3·79
707	14·380	0·42	3·02	2·21
653	14·250	0·32	2·99	1·65
626	14·353	0·19	3·00	1·00
544	14·285	0·23	2·98	1·22
471	14·266	0·19	2·97	0·98
410	14·100	0·19	2·94	0·99
327	14·087	0·18	2·93	0·90
270	14·091	0·19	2·92	0·99
167	14·000	0·20	2·88	1·06
81	13·882	0·21	2·83	1·07
21	13·857	0·18	2·82	0·92

that change of rigidity with temperature obeys some such linear law as

$$n_t = n_0(1+\alpha t).$$

It is possible therefore to determine the temperature coefficient α of the modulus, n_0 and n_t being the values of the modulus of the rigidity at 0°C and t°C respectively. From the straight lines drawn in these figures, the values of coefficient α of the modulus are as in the subjoined table.

No.	Radius $r\times10^3$cm	Temp. coeff. α	Mean value of α
1	18·52	0·000109	
2	28·05	0·000110	
3	42·45	0·000113	0·000111
4	51·17	0·000108	
5	65·40	0·000113	

Observer	Value of α
Boys	——
Threlfall	0·000128
Barnett	0·000115
Horton	0·0001235

The mean values of temperature coefficient α between 15°C and 100°C as obtained by other observers have been brought together in the preceding table, from which it will be seen that there are considerable differences in the values of α obtained by different observers. The mean value of α obtained from the present experiments is nearest the result obtained by Barnett. The fibre used by us were much thicker than those used by either Horton or Threlfall.

With rise of temperature the rate of increase gradually diminished, maximum rigidity being attained at about 1000°C. After passing this temperature, rigidity diminished with increase of temperature. The temperature at which the rigidity of the fibre reaches maximum differs slightly with different fibres. Reduction in the diameter of the fibre seems to demand a lower temperature for attaining maximum rigidity. The smaller the diameter of the fibre the larger is their modulus of rigidity. Generally speaking, the modulus of rigidity of the fibres after annealing is larger than that before annealing.

The logarithmic decrement of the torsional oscillations increased with temperature at a rate, which although at first was constant, became much more rapid after about 600°C was reached (Figs. 6, 7). In order to ascertain what part of the observed logarithmic decrement was due to the viscosity of the fibres, another experiment, similar to that just described in the preceding section, was resorted to by using different fibres in the bell-jar. An example of the relation between the logarithmic decrement of the oscillations, without the effect of the air-damping and the pressure in the bell-jar, is shown in Fig. 8. We take the common logarithm of $\sqrt{\lambda_0-\lambda_a}$ and λ_0 as ordinate and the pressure p as

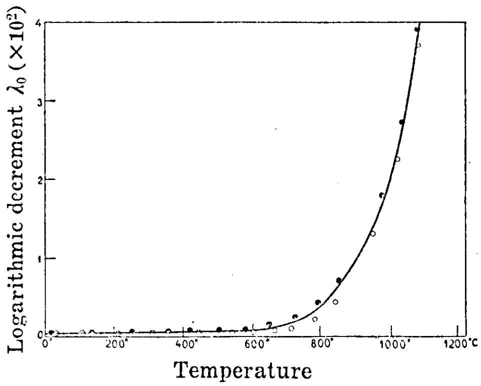


Fig. 6. Relation between observed logarithmic decrement λ_0 of the oscillation and temperature.

The values of λ_0 in the case of cooling are smaller than that in the case of heating.

Radius of the fibre $r=18{\cdot}52\times10^{-3}$ cm.

●, heating; ○, cooling.

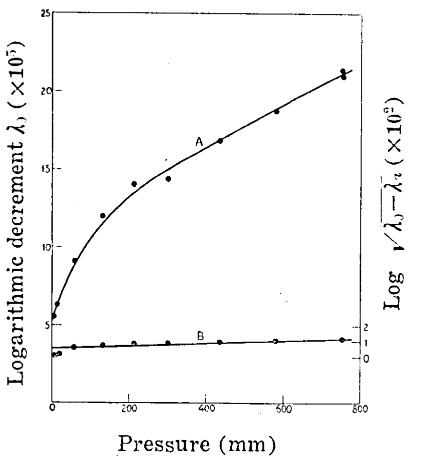


Fig. 8. A. Relation between logarithmic decrement λ_0 and pressure in the bell-jar.

B. Relation between log $\sqrt{\lambda_0-\lambda_a}$ and pressure in the bell-jar.

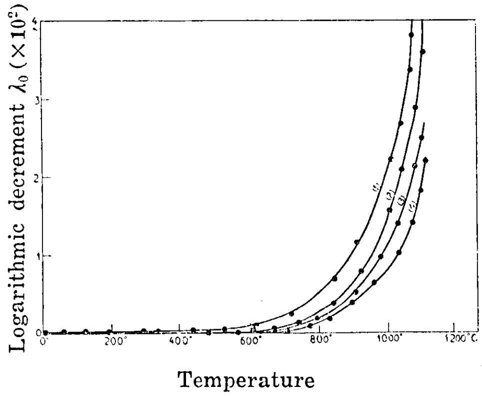


Fig. 7. Curves showing that the logarithmic decrements of the oscillations differ with different fibres at high temperature.

(1) Radius of the fibre $r=18{\cdot}52\times10^{-3}$cm.

(2) $r=42{\cdot}45\times10^{-3}$ cm.

(3) $r=51{\cdot}17\times10^{-3}$ cm.

(4) $r=65{\cdot}40\times10^{-3}$ cm.

abscissa, where λ_0 is the observed logarithmic decrement and λ_a is the logarithmic decrement due to the air-damping.

The logarithmic decrement of the oscillations, $\sqrt{\lambda_0-\lambda_a}$, is proportional to the square of the pressure, p^2, except in the case of very small pressure; the value of λ_0 in the open air being about 4 times that in vacuum. We can therefore deduct the effect of the logarithmic decrement due to air resistance from the observed logarithmic decrement.

Since the temperature of the air surrounding the vibrator was constant during the experiments, the logarithmic decrement due to air-damping was assumed to have the same value throughout the experiments. We shall now apply the relation between the pressure and the logarithmic decrement at room temperature to the case of high temperature. Strictly speaking, this would not be possible, seeing that for the viscosity of the air surrounding the vibrator, the temperattre of which is raised, would increase with temperature. The effect of this increase, however, is probably very small compared with the whole damping effect due to the air, since most of the torsional oscillations would doubtless be caused by the brass rod and the soft iron plate (E and S in Fig. 1). It will also be seen from Tables II$\sim$IV that the logarithmic decrement λ_a due to air-damping is itself a small fraction of the actually observed logarithmic decrement λ_0 at high temperature, while from Fig. 6 it is obvious that annealing reduces the coefficient of internal friction. The mean values of the observed logarithmic decrement λ_0 of the different fibres obtained by both heating and cooling against the corresponding temperatures are shown in Fig. 7. It will be observed that the value of λ_0 increases with reduction in the diameter of the fibre.

We take the logarithmic decrement λ_a due to air-damping from the observed logarithmic decrement λ_0 and calculate the coefficient of internal friction of the fibre by substituting this residual logarithmic decrement λ_0-λ_a for λ in equation (5). The values of the coefficient of internal friction η of the fibres against the corresponding temperature are shown in Tables II$\sim$IV and Fig. 9. The relation between the coefficient of internal friction η at a certain temperature and the frequency of the oscillating fibres is shown in Fig. 10, in which the frequency is taken as abscissa and the common logarithm of the coefficient of internal friction η as ordinate.

From the last-named two figures we can see that the coefficient of internal friction of the fibres differs not only with different fibres, but also with the frequency of the oscillations. At constant temperature, the smaller the diameter of the fibres, the greater their coefficient of internal

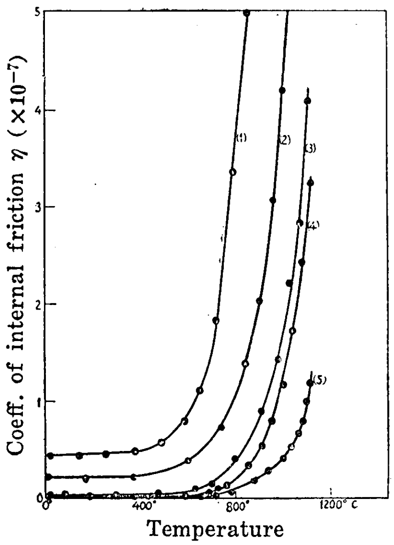


Fig. 9. Curves showing the relation between the coefficients of internal friction and temperature.
(1) Radius of the fibre $r=18{\cdot}52\times10^{-3}$ cm.
(2) $r=28{\cdot}05\times10^{-3}$ cm.
(3) $r=42.45\times10^{-3}$ cm.
(4) $r=51{\cdot}17\times10^{-3}$ cm.
(5) $r=65{\cdot}40\times19^{-3}$ cm.

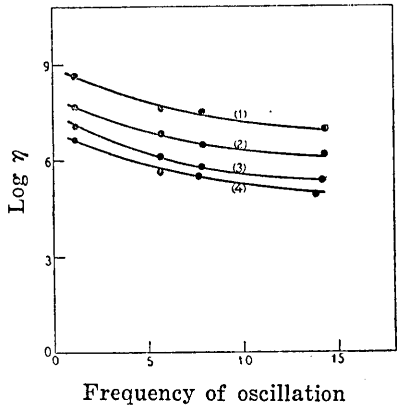


Fig. 10. Relation between log η and the frequency of oscillation at constant temperature θ.
(1) $\theta=1100°$C.
(2) $\theta=850°$C.
(3) $\theta=650°$C.
(4) $\theta=250°$C.

friction values; and the longer the period of the oscillations, the larger the value of the coefficient of internal friction.

Consequently, in the present experiments, the same coefficient of internal friction could not be obtained for all the different fibres.

4. Remarks. Internal resistance does not depend wholly on the rate of strain, but also on stress-amplitude.[22] Honda and Konno[23] observed that the logarithmic decrement of annealed specimens is greatly affected by the amplitude of oscillations. Sezawa and Kubo[24] also found that the larger the amplitude, the greater becomes the coefficient of solid viscosity. The change in the logarithmic decrement of the oscillations with temperature was observed by both Horton,[25] Ishimoto,[26] and Iokibe and Sakai.[27] Internal friction therefore does

22) Kimball and Lovell, *Phys. Rev.*, **30** (1927), 948.
23) K. Honda and S. Konno, *loc. cit.*
24) K. Sezawa and K. Kubo, *loc. cit.*
25) F. Horton, *Phil. Trans. Roy. Soc., London*, [A], **204** (1905), 1.
26) M. Ishimoto, *loc. cit.*
27) K. Iokibe and S. Sakai, *loc. cit.*

not seem to be constant irrespective of the conditions of oscillation.

As to the coefficient of internal friction, we think that all quartz fibres should give the same value, which however the present experiments did not. The solid viscosity coefficients differed with thickness of fibre; the smaller the diameter of the fibre, the larger being the values of their internal friction coefficient. We shall now examine the causes of this, the chief of which as introduced in the present experiments and computation are believed to be the following:

(1) Unsatisfactory character of the ordinary equation (2) in section 3.

(2) Effect of the surface of the substance.

(3) Errors in reading the period and amplitude of the oscillations.

(4) The fact that the dissipation of energy from these vibrational bodies differs with the various diameters of the fibre.

In these, cause (2) is believed to amount to only a small fraction. In order to see the surface effect of the substance, we assume that the internal friction is divisible into two parts, the one being proportional to the surface of the fibre and the other to the volume of the fibre, but from calculation we found that the former was very small compared with the latter. This problem therefore will demand much further study. In order to avoid the errors in (3), the records were repeatedly read and their values corrected whenever possible. As to cause (4), although the diameter of the fibre was changed, the moment of inertia of the oscillating system remained constant throughout the experiments, its value being 109·878 in C.G.S. units. The smaller the diameter of the fibres, the longer became the period of the oscillations. During oscillation, a small fraction of energy is lost through the point of suspension outwards, and this energy loss may, in the case of short period oscillation, be more than in the case of long period.

Another form of equation (2) in section 3, namely, an equation involving the normal viscosity coefficient ξ, in place of η, in the damping term, was treated by Honda and Konno[28] and found correct. The effect of tension moreover may be considered. According to Boys,[29] the rigidity of vitreous silica in the form of a fibre is influenced by the tension on the fibre.

Until, therefore, further experiments have been made it is not possible to say which of the foregoing causes is the most important.

28) K. HONDA and S. KONNO, *loc. cit.*
29) C. V. BOYS, *loc. cit.*

5. Summary and Conclusion. We may now briefly summarise the results of the present study.

1) In contrast to metals, the modulus of rigidity of quartz fibre increases with increasing temperature, the maximum value being attained at about 1000°C. As the diameter of the fibre is reduced, the maximum temperature seems to be lowered. As to the geophysical problems, the fact that the modulus of rigidity of quartz fibre increases with increasing temperature seems interesting.

2) The smaller the diameter of the fibres, the larger become their moduli of rigidity. The modulus of rigidity after annealing is larger than that before annealing.

3) Assuming that the change in rigidity with temperature obeyed a linear law, the mean value obtained for the temperature coefficient of the rigidity modulus was 0·000111. The mean value of the modulus of rigidity at room temperature was found to be $2{\cdot}93 \times 10^{11}$ dynes per sq. cm.

4) The coefficient of internal friction of the fibre increased with increasing temperature at a rate which, although at first was constant, became much more rapid after about 600°C.

5) The coefficient of internal friction differs with different fibres. The smaller the diameter of the fibre, the larger the coefficient of internal friction, the coefficient of internal friction after annealing being smaller than that before annealing.

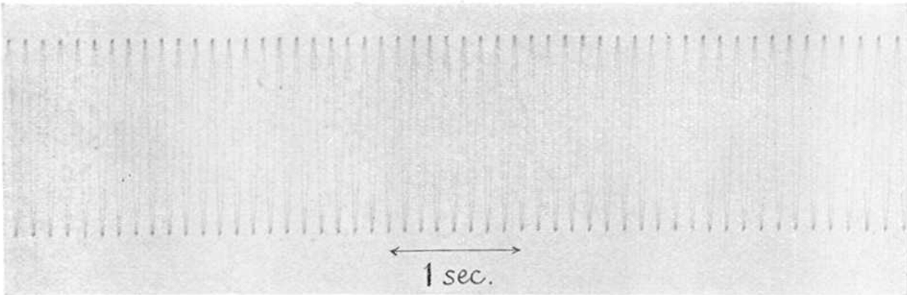
6) The coefficient of internal friction of the fibre at room temperature is of the order of 10^5 in C.G.S. units.

In conclusion, the writer wishes to express his sincere thanks to Professor Mishio Ishimoto for his kind guidance and valuable advices throughout the course of these experiments, and to Mr. S. Tanaka for his chemical analysis of the specimens.

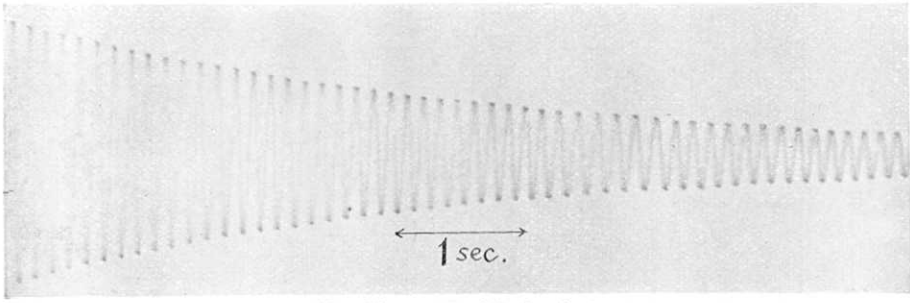
53. シリカの剛性率及び固體粘度の溫度による變化

飯 田 汲 事

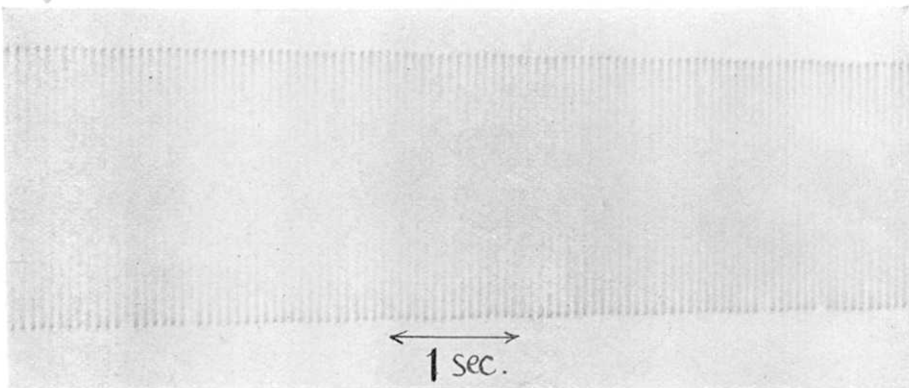
地震波が地殼内に於て老衰し，又變形すると云ふ事は既によく知られてゐる事實であつて，その理論的方面の研究に於ては，長岡博士を始め妹澤博士その他の人々によつて取扱はれてゐる．地殼物質の剛性，ヤング率，及び密度等に關係せる地震波速度については，地震記象の觀察其の他から多くの人々によつて求められてゐる．これらは何れもその地殼物質の性質が根本問題となつてゐる．筆者は以上の問題に關係せる地殼物質の性質を，實驗的に研究して見たいと考へてその研究に着手したのであつて，その第1報として前號にてピッチ，パラフイン等の性質が岩石に類似せる點を報告した次第である．本論文も地殼物質がこの研究の對照となつてゐる．地殼物質は色々な結晶形のものから成り立つてゐるであらうから，その性質を知るには結晶形構造の性質をも調べなければならぬ．先づ之に至る段階として容易に得られ且細工のたやすい無定形シリカを選びその粘性，剛性の2性質を實驗的に調べて見た．實驗から得られた結果によれば，シリカの剛性率は多くの金屬と異なり約 1000° 迄は徐々に增加する．又粘性は 600° 位から急激に增加する．常溫に於けるシリカの粘性係數は 10^5 (c.g.s.) の程度である．かくの如くシリカの性質を考へると，地殼物質の中には，地殼表層に於けるものよりかへつて大きな剛性率と大きな粘性を持つた物質も，地球の内部に存在する事が可能のやうに思はれる．



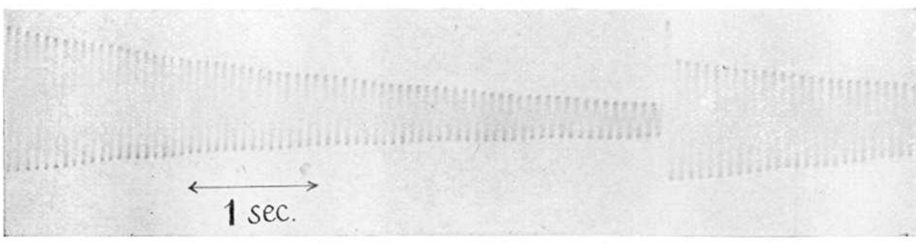
(A) Temp. 232°C (heating).



(B) Temp. 1090°C (heating).



(C) Temp. 845°C (cooling).



(D) Temp. 1098°C (heating).

Fig. 3. Actual records.

(A) and (B), radius of the fibre $r=51{\cdot}17\times10^{-3}$ cm.
length $l=12{\cdot}36$ cm.
(C) and (D), $r=65{\cdot}40\times10^{-3}$ cm.
$l=9{\cdot}74$ cm.

(震研彙報 第十三號 圖版 飯田)