Change of Volume and of Length in Iron, Steel, and Nickel Ovoids by Magnetization.

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With Plates VI. & VII.

1. In our former paper,[1] we described some effects of magnetization on the dimensions of nickel and iron, as well as those of hydrostatic pressure and longitudinal pull on the magnetization. We then showed that there is a reciprocal relation between the two, and that the Villari effect in iron is a natural consequence of the observed changes of dimensions. Unfortunately on that occasion the range of the magnetizing field was limited to a few huudred C.G.S. units, so that the investigation of the behaviour of these metals in high fields was reserved for further experiments. In addition to this, the ferromagnetics were not of a shape to be uniformly magnetized with the exception of the iron ovoids. It was therefore thought desirable to repeat

1) Nagaoka and Honda, Journal of the College of Science, **9,** 353, 1898; Phil. Mag. **46,** 262, 1898.

the experiment on ovoids of ferromagnetic metals, and so to extend the investigation into still stronger fields.

2. In his well-known researches on the changes of dimensions of iron and other metals by magnetization, Bidwell[1] pushed the field strength to 1500; in the present experiment, the field strength is greater than that of Bidwell by 700. In addition to ordinary soft iron and steel ovoids, wolfram steel from Bohler in Vienna was tested with a result which showed a remarkable difference from ordinary steel as regards the change of dimensions wrought by magnetization. As was generally supposed, the change of volume is very small in iron and nickel in weak fields, but with strong magnetizing force the effect becomes generally pronounced.

3. The apparatus already described was used in measuring the change of length and of volume. A small alteration was made in the arrangement of the magnetizing coil. Owing to the strong magnetizing current, special arrangements were made for keeping the interior of the coil at a constant temperature. A double walled tube of brass was inserted in the coil, and a constant stream of cold water was passed in the interspace for more than an hour before each experiment. As the resistance of the coil was only 0.56Ω, the rise of temperature was so small, that the ferromagnetics placed in its core were scarcely affected. The change of length was measured by an optical lever, as before described.[2] For measuring the change of volume, the ovoid was sealed in a glass tube with a capillary neck (internal diameter about 0.4 mm.) and so placed in the tube

1) Bidwell, Phil. Trans. **179,** 205, 1889.

2) Nagaoka, Phil. Mag. [5] **37,** 131, 1894; Wied. Ann. **53,** 487, 1894; Nagaoka and Honda loc. cit.

that it rested in the axial line, and never came in contact with the wall of the tube. The magnetizing coil and the tube were placed in a horizontal position. The motion of the meniscus was measured by a microscope provided with a micrometer ocular. For more minutely detailed particulars, we must refer the reader to the former paper.

3. The following are the dimensions of ovoids used in the present experiments:

Specimen No.	Metal	a (cm.)	c (cm.)	v (c.cm.)	ρ	N
1	Nickel	0.750	12.50	31.50	8.86	0.125
2	,,	0.500	10.00	10.48	8.86	0.0848
3	Soft iron	0.750	12.50	31.45	7.84	0.125
4	,,	0.500	10.00	10.53	7.83	0.0848
5	Ordinary steel	0.750	12.50	31.60	7.83	0.125
6	,,	0.500	10.00	10.57	7.81	0.0848
7	Wolfram steel	0.750	12.50	31.82	7.90	0.125
8	,,	0.500	10.00	10.53	7.95	0.0848

a gives the semi-minor axis, c the semi-major axis, v the volume, ρ the density, and N the demagnetizing factor of the ovoids. The volume of each specimen was measured by weighing the ovoids in water.

The elastic constants of the metals were measured by flexure and torsion experiments on rectangular prisms made from the same specimens as the ovoids. The prisms were 14.6 cm. long and 0.896 cm. square in cross-section.

Metal	E (C.G.S.)	K (C.G.S.)	θ
Nickel	2.07×10^{12}	0.771×10^{12}	1.082
Soft iron	2.10×10^{12}	0.800×10^{12}	0.844
Steel	2.04×10^{12}	0.838×10^{12}	0.384
Wolfram steel	2.02×10^{12}	0.849×10^{12}	0.306

E gives Young's modulus, K (= n. Thomson and Tait) the modulus of rigidity, and θ a constant defined by the equation

$$\frac{E}{2}\left(\frac{1+2\theta}{1+3\theta}\right)=K.$$

The magnetization of each of these ferromagnetics was determined by the magnetometric method, after the ovoids had been carefully annealed, with the following results:

Nickel (2)		Soft iron (4)		Steel (6)		Wolfram steel (8)	
H	I	H	I	H	I	H	I
0.7	24.2	1.0	62	1.9	23	2.7	18
1.4	49.8	2.5	160	2.4	44	6.8	65
3.0	138.6	4.3	291	6.9	183	12.6	193
5.4	238.0	9.5	587	9.7	279	20.2	498
10.9	336.8	12.7	750	13.1	385	25.8	748
37.8	395.7	19.9	948	23.3	651	44.5	992
74.1	420.0	37.2	1111	32.3	815	83.6	1116
125.3	434.5	99.6	1255	50.2	984	118.0	1170
171.6	438.7	155.5	1309	116.3	1196	191.0	1224
240.3	440.7	270.3	1400	174.4	1260	344.6	1301
481.4	443.4	433.6	1479	345.0	1379	512.3	1348
674.2	444.5	584.6	1520	520.2	1440	666.6	1373
914.0	446.8	792.8	1546	873.7	1489	940.3	1400
1233.0	447.7	992.6	1562	1149.8	1512	1213.3	1423
1747.0	448.7	1585.8	1607	1822.6	1549	1674.9	1452

Change of Length.

4. *Iron* (Fig. 1).—The change of length experienced by soft iron is too well-known to need any description. The ovoid elongates in weak fields till it attains a maximum, being longer by about 3- to 4-millionths of its initial length; it then decreases in length and becomes shorter than in the unmagnetized state. The contraction goes on gradually increasing, and, in the present experiment, it does not seem to reach an asymptotic value, even in fields of 2200 C.G.S. units, where the contraction amounts to about $\frac{1}{100,000}$. The present result agrees qualitatively with Bidwell's experiment, but the contraction is much greater. The discrepancy is perhaps to be chiefly accounted for by the difference of shape.

5. *Steel* (Fig. 1).—Ordinary steel behaves just like iron, the difference being the smallness of elongation and contraction, while the field at which the elongation vanishes lies in the stronger. The field of maximum elongation in wolfram steel is greater than in ordinary steel or iron, that of no-elongation in the unannealed state being several times greater than in iron or ordinary steel. Such a field lies in H=1200. When the wolfram steel is annealed, the retraction after reaching the maximum takes place very slowly and the characteristic as regards the field of no elongation becomes exceedingly pronounced. From the curve of length change, it does not appear that it will ever cut the line of no-elongation even in intense fields.

6. The curve of elongation (in dots) plotted against the intensity of magnetization is given in Fig. 1. The change of length

at first takes place very slowly, but on reaching saturation, the rate of decrease becomes very rapid. So far as the present experiment goes, the rate does not diminish except in annealed wolfram steel, in which we notice a slight flattening.

7. *Nickel* (Fig. 1).—The behaviour of annealed nickel ovoid as regards the length change is nearly the same as that already observed by one of us. With an increasing field, the contraction reaches an asymptotic value, which in the present case is greater than that obtained by Bidwell from experiments on a nickel wire. The explanation of this discrepancy is to be sought for partly in the difference of shape, and partly in the difference of treatment, as will be clearly illustrated by experiments on the change of volume. We have also reason to believe that repeated annealing alters the elastic behaviour of ferromagnetics as regards the strain wrought by magnetization. Plotting the curve of length change against the intensity of magnetization, we find a slight bend when the magnetization becomes saturated and the contraction approaches its asymptotic value.

Change of Volume.

8. Experiments by several physicists prove that magnetization produces change of volume in ferromagnetics, in contradiction to the popular belief which is based on Joule's experiment. The alteration of volume accompanying the magnetization of ferromagnetics is generally very small in weak fields, but as will be seen from the present experiment, the phenomenon becomes more marked as the field is made stronger. As we have already remarked, the change of volume as measured by Cantone[1] in an iron ovoid must

1) Cantone, Mem. della R. Accad. dei Lincei **6**, 487, 1891.

have been exceedingly minute as the magnetizing field was very small. Dr. Knott[1] has published several papers on the change of internal volume of ferromagnetic tubes, showing that iron, nickel, and cobalt are subject to the change by magnetization. As our former result regarding the same question was somewhat different, especially in the case of nickel, we have thought it advisable to settle the discrepancy by fresh experiments.

9. *Iron and Steel* (Fig. 2).—Preliminary experiments on soft iron and steel ovoid showed that considerable increase in the volume change takes place as the ovoids are annealed. The increase becomes more significant as the field is made stronger. In steel, the effect of annealing is greater than in iron. In strong fields, the volume change of the annealed steel ovoid is nearly twice as great as in the unannealed state. Wolfram steel is very little affected by annealing as regards the volume change, but the change itself is much greater than in nickel or iron. The motion of the capillary meniscus in the dilatometer can be easily followed by the naked eye. The curves in Fig. 2 have been plotted from measurements made on annealed ovoids.

10 *Nickel* (Fig. 2).—As specimens of nickel almost always contain traces of iron, the change of volume will probably depend on the chemical nature. In addition to this, the mechanical process which the metal had to undergo before it could be brought to a form suitable for experiment, must have substantially altered its elastic behaviour.

The nickel rod, which we used in the former experiment, was hammered from a nickel plate to a prism of square cross-section. It contained 1.75 % of iron, besides traces of manganese and carbon. The ovoids used in the present experiment

1) Knott, Trans. Roy. Soc. Edinb. **38**, 527, 1896; **39**, 457, 1898.

were prepared from a thick plate, and were nearly pure nickel, the quantity of iron present as an impurity being inmeasurably small. As the material is likely to become homogeneous by repeated annealing, the ovoids were carefully annealed for about 50 hours. The ovoid was wrapped in asbestus and placed in a thick metal tube, the interspace between the ovoid and the wall of the tube being filled with fine charcoal powder. The tube was then placed in charcoal fire. When the ovoid was annealed in this way, there were some traces of surface oxidation. The change of volume after each annealing was examined with the result that it became evident that the process of annealing increases the effect. It therefore appears that the previous history of the specimen exercises an important effect on the magnetization and on the dimensions of ferromagnetics as affected by magnetization. The anomaly in the length change noticed by Bidwell in two specimens of nickel wire is probably not the effect of temperature, but is perhaps to be ascribed to the cause above stated. In contradiction to our former result with a square prism, the ovoid showed increase of volume. The amount of increase was small compared with the decrease noticed in the previous experiment. Cantone[1] obtained a tolerably large increase of volume in nickel ovoids; our former result was nearly half as large, while in the present experiment, there is a slight increase. The discrepancy is probably due to the difference of treatment before the specimen can be converted into a proper shape for experimenting, and also to its chemical composition.

11. The volume change of ferromagnetics considered as a function of the magnetizing field takes place very slowly in weak fields; it then increases in a more rapid ratio till it reaches the

1) Cantone, Atti della R. Accad. die Lincei, **6**, (1), 257, 1891.

'wendepunkt'; after that the change becomes slower, but still goes on increasing nearly in a straight line. Up to H=2000, the rate of change shows no tendency to decrease. With a still stronger field, the increase of volume will probably become more considerable.

12. In our former experiment, the range of the magnetizing force was confined to a few hundred C. G. S. units. In the present experiment, the increased field strength unveiled the character of the change of the volume considered as function of the intensity of magnetization. As will be seen from the curves (Fig. 2.) in dotted lines, the increase in nickel and steel takes place quite slowly before the magnetization reaches saturation. As soon as the magnetization reaches this state, the increase becomes very rapid, so that the branch of the curve ascends nearly parallel to the axis of volume increase. There we find that a slight increase in magnetization is attended with a large increase of volume. As the rate of increase appears to be nearly constant, it would be very interesting, if we could push the field strength still farther to see whether the volume change ultimately attains an asymptotic value.

The observed changes of volume and of length are exhibited in the following table :—

1) Cantone, Atti della R. Accad. dei Lincei, **6**, (1), 257, 1891.

Nickel (1)		Soft iron (3)		Steel (5)		Wolfram steel (8)	
H	$\frac{\delta v}{v}$	H	$\frac{\delta v}{v}$	H	$\frac{\delta v}{v}$	H	$\frac{\delta v}{v}$
13	0.09×10^{-7}	8	0.10×10^{-7}	7	0.08×10^{-7}	19	0.30×10^{-7}
30	0.29	11	0.52	12	0.47	42	1.52
90	0.65	18	1.56	33	1.95	93	3.03
218	0.82	167	3.12	192	3.13	216	5.01
282	0.97	443	3.85	376	4.69	442	8.04
517	1.38	691	4.58	586	6.22	692	11.68
877	2.06	958	5.88	792	8.01	1001	16.68
1141	2.44	1115	7.18	1044	10.16	1117	18.96
1547	3.24	1342	9.47	1376	14.07	1296	22.75
1740	3.53	1563	11.45	1646	17.20	1704	28.82
2253	4.12	2089	14.68	2171	22.20	2153	32.62

Nickel (2)		Soft iron (4)		Steel (6)		Wolfram steel (8)	
H	$\frac{\delta l}{l}$	H	$\frac{\delta l}{l}$	H	$\frac{\delta l}{l}$	H	$\frac{\delta l}{l}$
4	-14.1×10^{-7}	6	2.5×10^{-7}	13	3.1×10^{-7}	18	4.1×10^{-7}
6	−64.0	15	19.0	19	7.1	25	12.4
10	−118.2	51	31.6	28	15.1	39	21.7
15	−163.6	127	23.8	54	22.2	62	28.9
33	−217.5	224	3.1	96	23.1	106	32.1
59	−264.3	354	−17.7	160	17.8	170	32.3
124	−317.6	575	−52.6	225	8.2	235	31.7
302	−343.6	698	−62.6	374	−11.5	349	30.2
561	−353.8	883	−73.5	589	−36.0	592	23.1
839	−356.0	1077	−78.9	844	−49.2	781	18.7
1145	−360.0	1180	−82.2	1061	−55.5	1052	17.0
1289	−360.9	1324	−86.6	1177	−59.5	1188	15.4
1483	−362.2	1447	−89.9	1361	−64.5	1345	13.9
1849	−362.7	1538	−91.6	1729	−69.9	1697	12.4
2322	−365.3	2180	−102.0	2172	−78.1	2235	10.9

Kirchhoff's Constants k' and k''.

13. Starting from the formulæ

$$\lambda=\frac{\delta l}{l}=\left\{\frac{4\pi k^2}{3}\left(\frac{1+\theta}{1+2\theta}\right)+\frac{k-k'}{2(1+2\theta)}-\frac{k''}{2}\right\}\frac{H^2}{E},$$

and $$\sigma=\frac{\delta v}{v}=\left\{\pi k^2+\frac{3(k-k')}{4}-\frac{k''}{4}\right\}\frac{H^2}{K(1+3\theta)},$$

which give the change of volume and of length of ferromagnetic ovoids in terms of Kirchhoff's constants k' and k'', we obtain the following expressions for these two constants:

$$k'=\frac{p(1+2\theta)-q}{2(1+3\theta)},$$

and $$k''=\frac{3q-p}{2(1+3\theta)},$$

where $$p=-\frac{4K(1+3\theta)}{H^2}\sigma+4\pi k^2+3k,$$

and $$q=-\frac{2E(1+2\theta)}{H^2}\lambda+\frac{8\pi k^2}{3}(1+\theta)+k.$$

These constants, as calculated from the change of dimensions of ovoids, are given in the following table, and graphically drawn in Fig. 3:

H	Nickel		Soft iron		Steel		Wolfram steel	
	k'	k''	k'	k''	k'	k''	k'	k''
5	−229100	712800	21900	−22610	1017	−1865	348	−1252
10	−188900	578900	23520	−28450	3840	−3322	986	−1512
20	− 71000	216700	13280	−16420	4248	−4615	3600	−3983
30	− 36370	111200	7302	− 8650	4048	−5080	4881	−5440
60	− 8163	34540	2139	− 2222	1738	−1864	1946	−2217
80	− 6906	20960	1207	− 1102	1069	−1004	1198	−1385
100	− 4653	14120	750	− 550	701	− 546	794	− 880
120	− 3373	10260	500	− 255	477	− 279	557	− 595
160	− 1297	3968	239	18	240	− 16	315	− 317
250	− 843	2553	55	175	69	117	128	− 109
300	− 591	1790	25	175	38	124	88	− 66
500	− 216	655	−9	130	−1	99	28	− 8
800	− 86	259	−9	70	−6	57	8	5
1200	− 39	117	−6	37	−4	31	3	5
1600	− 22	66	−4	23	−3	19	1	4
2000	− 14	42	−3	16	−2	13	0	3

14. The curves for k' and k'' present the same general feature in iron and steel. k' increases in low fields; and there attaining the maximum value, it rapidly diminishes till it becomes less than zero; it then reaches a minimum, after which it again gradually increases. The exact position of the minimum is very vague; the curve for k' ultimately coincides with the axis of H. k'' is at first negative, and attaining the minimum value, goes on gradually increasing till it becomes greater than zero, and then reaches a maximum. With the farther increase of the field, the value of k'' decreases very slowly. The position of maximum for k' and that of minimum for k'' lie nearly in the same field, which is greater for wolfram steel than for soft iron, while that for ordinary steel occupies an intermediate position. The absolute value of k' and k'' is greater in iron than in steel. In nickel, the values of k' and k'' are far greater than those for iron and steel, and moreover are of *opposite* signs. The maximum of k'', or the minimum of k', seems to lie in a weak field; the rate of decrease or increase is quite rapid and the curves for k' and k'' soon approach the axis of H. Compared with the results of former experiments, the absolute values of k' and k'' are generally small for iron,—far greater for nickel. This difference arises from the fact that for iron, the change of length in weak fields is less in this case than in the former experiment, and that for nickel the contrary is the case. As regards the sign, these two experiments show fair agreement.

Consequences of the theory.

15. *Effect of longitudinal pull.*—The change of magnetization produced by the elongation of a wire can be easily calculated from the formula

$$\delta I = H\left\{k'\frac{E}{K} - 3\left(k' + \frac{k''}{3}\right)\right\}\lambda.$$

Putting $\lambda = 4.67 \times 10^{-6}$, 4.80×10^{-6}, and 4.85×10^{-6} for soft iron, ordinary steel, and wolfram steel respectively, each corresponding to a pull of 0.1 Kilog. per sq. mm., we get the following results:

H	Soft iron.	Steel.	Wolfram steel.
	δI	δI	δI
10	0.919	0.055	0.044
20	1.074	0.212	0.170
30	0.831	0.402	0.350
60	0.399	0.254	0.294
80	0.244	0.153	0.249
100	0.127	0.072	0.188
120	0.039	0.005	0.145
160	−0.080	−0.092	0.094
200	−0.164	−0.146	0.061
300	−0.258	−0.210	0.017
500	−0.311	−0.236	−0.002
800	−0.275	−0.205	−0.038
1200	−0.219	−0.163	−0.037
1600	−0.183	−0.134	−0.033
2000	−0.157	−0.115	−0.028

It will be seen from the above table that there is an increase of magnetization in low fields, till it reaches a maximum, after which it gradually decreases. The decrease does not proceed continuously, but reaches a maximum, whence the magnetization begins to recover. Although the former result here arrived at is the well-known Villari effect, we do not know whether the maximum decrease due to longitudinal stress has as yet been experimentally ascertained. With nickel, we obtain the following values for the change of magnetization due to elongation, $\lambda =$

4.74×10^{-6}, which corresponds to a pull of 0.1 Kilog. per sq. mm.:

H	δI
10	−24.58
20	−18.87
30	−14.16
60	− 9.09
80	− 7.12
100	− 5.99
120	− 5.22
160	− 2.70
300	− 2.28
500	− 1.39
800	− 0.88
1200	− 0.60
1600	− 0.45
2000	− 0.36

There is nothing remarkable in nickel. Longitudinal pull produces decrease of magnetization, which becomes gradually less as the field strength is increased. This is such a well established experimental fact that we need not enter into further discussion of the subject.

16. *Effect of hydrostatic pressure.*—We can easily see that the change of magnetization δI due to change of volume σ by hydrostatic pressure is given by

$$\delta I = -\left(k' + \frac{k''}{3}\right)H\sigma .$$

If we calculate the change of magnetization due to contractions 4.68×10^{-6}, $5{,}38 \times 10^{-6}$, $8{,}42 \times 10^{-6}$ and $9{,}33 \times 10^{-6}$ for nickel, soft iron, steel, and wolfram steel respectively, each corresponding to a pressure of 10 atm., we obtain the following values:

H	Nickel	Soft iron	Steel	Wolfram steel
	δI	δI	δI	δI
0	0.000	0.000	0.000	0.000
10	0.190	0.757	0.230	0.045
20	0.119	0.840	0.457	0.237
30	0.080	0.713	0.595	0.858
60	0.037	0.398	0.565	0.675
80	0.030	0.362	0.495	0.550
100	0.024	0.305	0.437	0.467
200	0.012	0.178	0.268	0.259
300	0.008	0.135	0.200	0.185
500	0.005	0.093	0.134	0.118
800	0.002	0.062	0.088	0.073
1200	0.001	0.042	0.061	0.048
1600	0.000	0.031	0.043	0.034
2000	0.000	0.024	0.034	0.026

It thus appears that in nickel the effect of hydrostatic pressure is very small compared to that of longitudinal pull. There is increase of magnetization with the volume contraction of the magnet. Such an increase reaches a maximum in low fields, whence the effect gradually diminishes. Similar changes are also noticed in the case of iron and steel. In our former experiment, we found that hydrostatic pressure increases the magnetization in nickel, while it decreases it in iron. The agreement between theory and experiment is very close in nickel, but there is a wide discrepancy in iron and steel, as we have already noticed.

17. *Effect of torsion on longitudinally or circularly magnetized wire.* There are other important consequences to be drawn from the constant k'' with regard to the effect of torsion on

longitudinally magnetized wire and on ferromagnetic wire traversed by an electric current. The strain caused by twisting a circular wire can be resolved in elongation and contraction in directions perpendicular to each other and inclined to the axis of the wire at 45°. Taking these two principal axes of the strain for those of x and y, we have for the strain.

$$\frac{\partial u}{\partial x} = \tfrac{1}{2}\omega r\,,$$

$$\frac{\partial v}{\partial y} = -\tfrac{1}{2}\omega r\,,$$

$$\frac{\partial w}{\partial z} = 0\,,$$

where ω denotes the amount of torsion and r the distance from the axis. Resolving the magnetizing force which is in the direction of the axis of the cylinder, along the axis of elongation and of contraction, we find that the circular magnetization which will be called into play is equal to $-\frac{1}{2}\omega r k''H$ at a distance r from the axis, the mean circular magnetization being $-\omega k''HR$, where R is the radius of the wire.

The transient current which will be thus induced in the wire by suddenly twisting it is proportional to $-k''H$.

Next suppose that the wire is traversed by an electric current of intensity C. Then the circular magnetizing force at a distance r from the axis is

$$H = \frac{2Cr}{R^2}\,.$$

By applying similar reasoning, we find that the mean longitudinal magnetization is equal to $-\omega k''C$. We therefore conclude that twisting the wire carrying the electric current gives rise to longitudinal magnetization proportional to $-k''C$. Thus the circular magnetization produced by twisting a longitudinally magnetized wire has a reciprocal relation to the longitudinal magnetization caused by twisting a circularly magnetized wire.[1]

The view propounded by Prof. Ewing[2] to account for the existence of transient current by means of æolotropic susceptibility is similar to what would follow from Kirchhoff's theory, but it fails to give the amount of the current or of the magnetization which would be produced by twisting.

The theoretical inferences which we can draw at a glance from the curves of $-k'' H$ (Fig. 3) are as follows:

1. The transient current as well as the longitudinal magnetization produced by twisting an iron or steel wire is opposite to that produced by twisting one of nickel, up to moderate fields.
2. The transient current as well as the longitudinal magnetization produced by twisting an iron, steel, or nickel wire reaches a maximum in low fields.
3. In strong fields the direction of the current as well as the longitudinal magnetization is the same in iron, steel, and nickel.

It has been established by G. Wiedemann[3] that the longitudinal magnetization produced by twisting an iron wire carrying an electric current is opposite to that produced in a nickel one. The opposite character of the transient current in these two metals has also been observed by Zehnder[4] and independently by one of us[5]. The existence of a maximum transient current in

1) Voigt, *Kompendium der theoretischen Physik*, **2**, 203, 1896, Leipzig; Drude, Wied. Ann. **63**, 8, 1897.

2) Ewing, *Proc. Roy. Soc.* **36**, 1884.

3) Wiedemann, *Electricität*, **3**.

4) Zehnder, Wied. Ann., **38**, 68, 1889.

5) Nagaoka, Phil. Mag. [5] **29**, 123,1890; Journal of the College of Science, Tōkyō, **3**, 335, 1890.

these two metals has been clearly established, although there is some difference in the field strength between iron and nickel. It appears from the experiments of Dr. Knott[1] that the area of the hysteresis curve in the longitudinal magnetization produced by twisting circularly magnetized wire reaches a maximum as the field strength is increased; but on account of the feebleness of the current, the existence of the maximum in the longitudinal magnetization is not well established. To judge from the course of the curve given by the same experimenter, it seems highly probable that the maximum would be reached if we could push the circularly magnetizing force a little further. The conclusion (3) is still an open question, although some experiments of Matteucci[2] seem to corroborate the view just stated.[3]

18. Looking at the curves of $k''H$, we cannot but be struck with the close resemblance of the curves representing the amount of torsion produced by the combined action of the circular and the longitudinal magnetizing forces on a ferromagnetic wire. We can no doubt co-ordinate the effect of torsion on a magnetized wire with the Wiedemann effect. The discussion of the last mentioned effect we hope to lay before the public in the near future.

In spite of the qualitative explanations which Kirchhoff's theory affords with regard to the effect of longitudinal pull, of the hydrostatic pressure, and of torsion, there are instances in which the theory apparently fails in several quantitative details that it necessarily calls for modification. We may remark that k'

1) Knott, Trans. Roy. Soc. Edinb., **36**, 485, 1891.

2) Matteucci, Annales de Chimie et de Physique, 1858.

3) While this paper was passing through the press, we found that the direction of the transient current produced by twisting a magnetized iron wire is reversed in strong magnetizing fields.

and k'' are physically functions of the strain, as is borne out by the numerous experiments on the effect of stress on magnetization. The present state of the theory of magnetostriction may perhaps be compared with that stage in the history of the theory of magnetization when the intensity of magnetization was supposed to be simply proportional to the magnetizing force. In fact, the theory is still in its infancy, so that there are ample grounds for expecting further developments on further researches.

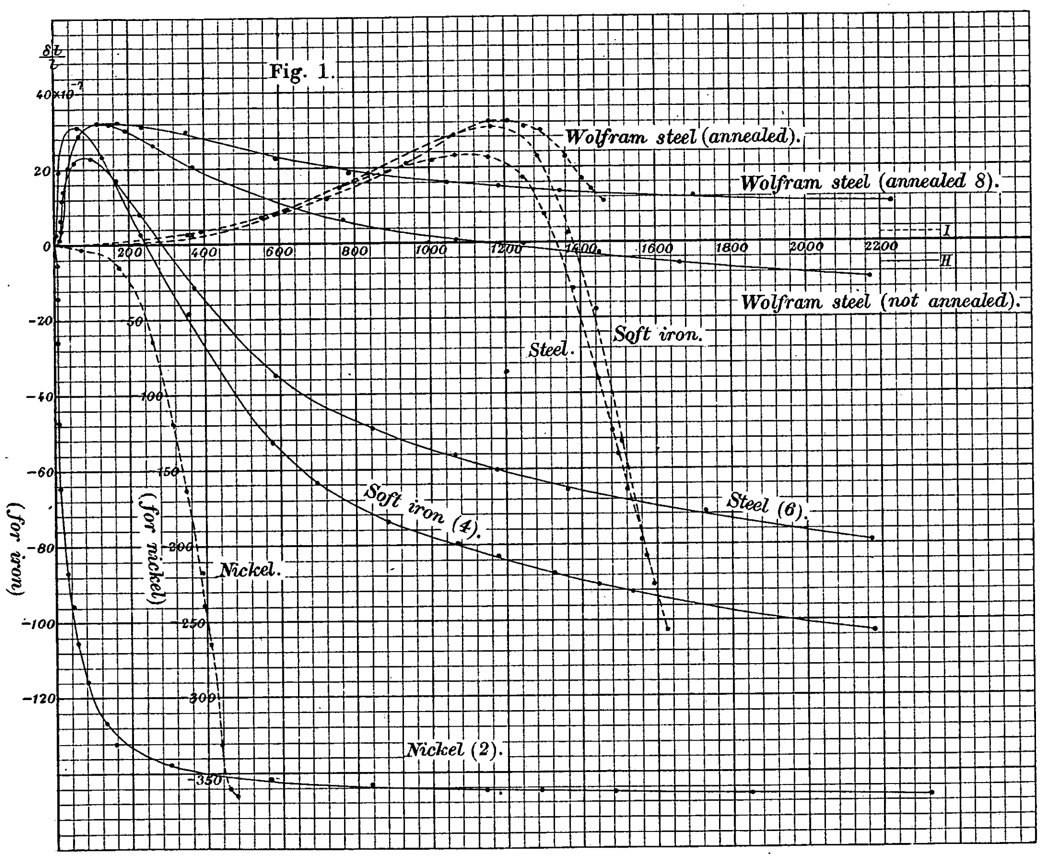


Fig. 1.
Wolfram steel (annealed).
Wolfram steel (annealed 8).
Wolfram steel (not annealed).
Steel.
Soft iron.
Steel (6).
Soft iron (4).
Nickel.
Nickel (2).
(for iron)
(for nickel)
I
H

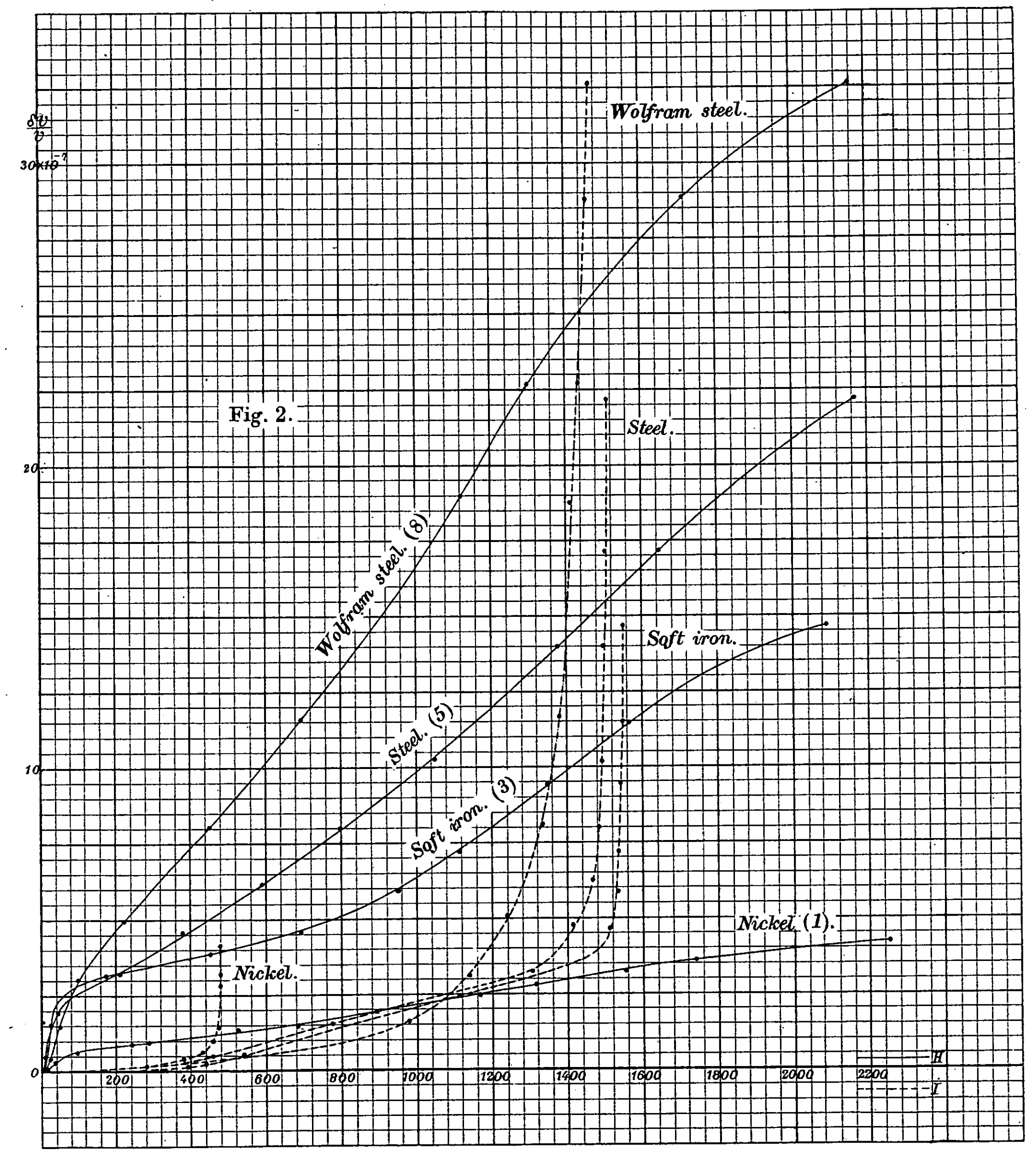
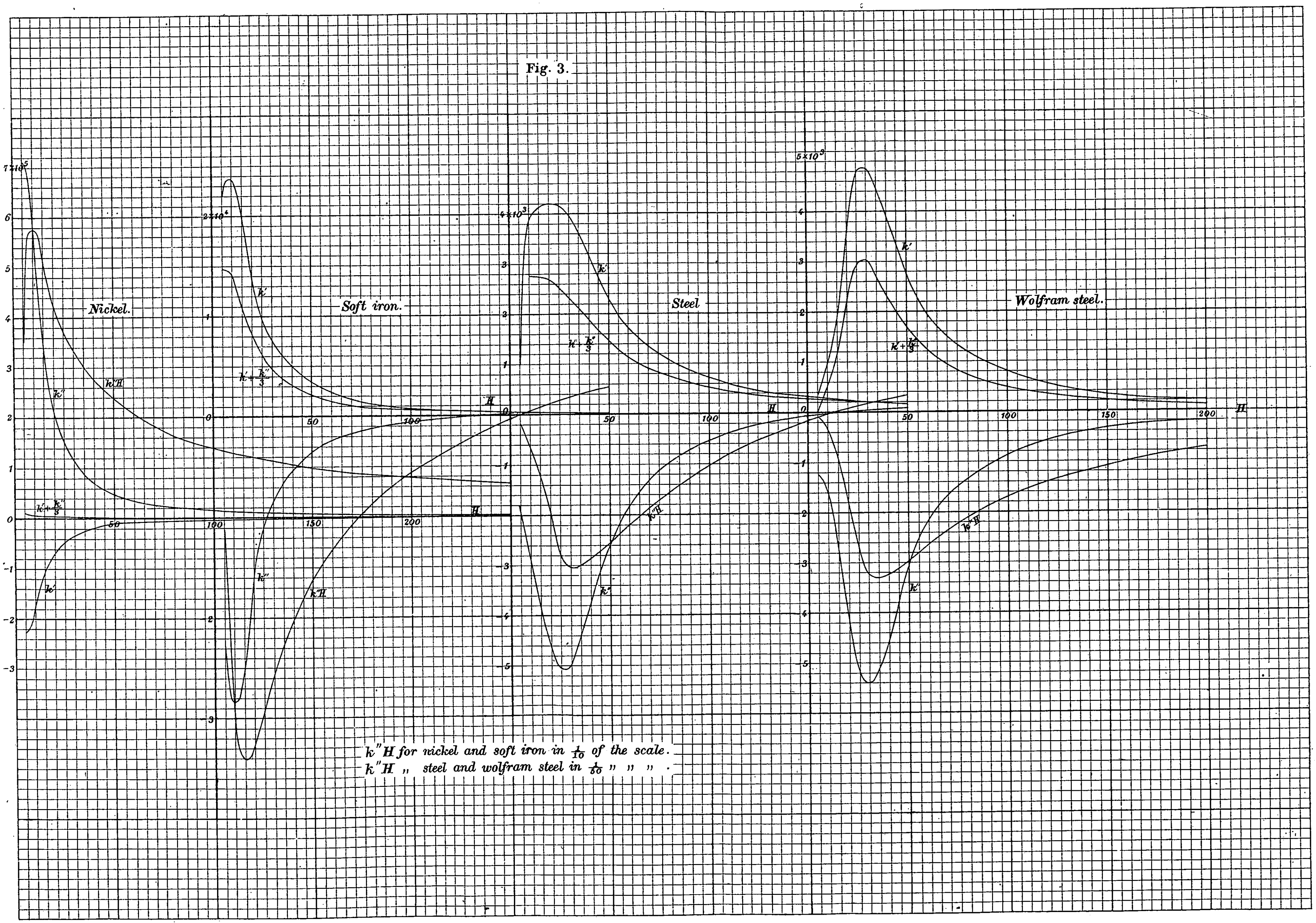


Fig. 2.
Wolfram steel.
Wolfram steel (8).
Steel.
Steel (5).
Soft iron.
Soft iron (3).
Nickel.
Nickel (1).
H
I

Fig. 3.



k''H for nickel and soft iron in $\frac{1}{10}$ of the scale.
k''H „ steel and wolfram steel in $\frac{1}{60}$ „ „ „ .