APPLICATION OF SEISMOGRAPHS TO THE MEASUREMENT OF THE VIBRATIONS OF RAILWAY BRIDGE PIERS. 3rd Paper.

By

F. OMORI, *Rigakuhakushi*,

Member of the Imperial Earthquake Investigation Committee.

With Plates VII–XVI.

PRINCIPAL CONTENTS.

1. *Introduction.* The study of the movements of railway bridge piers forms an interesting branch of applied seismology. My two previous reports on this subject are as follows:—

Note on the vibrations of railway bridge piers, the "Publications," No. 12;

Vibrations of a railway bridge pier, the "Bulletin," Vol. I, No. 3. These papers give an account of the measurements relating, firstly, to the Tone-*gawa* (Toride) and the Ibi-*gawa* (Kuwana) bridges, in which the heights of the piers were only from 19′ to 29.′4, although the well-sinkings reached to depths of 64.′9 to 95.′6; and, secondly, to the Kizu-*gawa* (Kasagi) and the Tone-*gawa* (Maebashi) bridges, whose piers have heights of 40′ to 60′, the river beds being hard.

The measurements in the present instance relate to the Takaya-*gawa* and the Shichimi-*gawa* bridges, both of which are near the town of Ayabe and are on the Shin-Maizuru section of the Government Railways, which connects Kyoto with the naval port of Maizuru; the experiments having been made on July 15th and 16th, 1910, previous to opening to traffic of this newly completed line.

A special feature in the construction of these bridges is in the employment of heavy trusses supported by very high masonry piers. Thus the tallest pier of the Takaya-*gawa* bridge has a height of 82′ 6″.5 and supports the fixed end of a 200′ deck truss, to which is attached one end of a 70′ plate girder. Again, the tallest pier of the Shichimi-*gawa* bridge is 82′ 1″.5 in height and supports the roller ends of two similar 200′ deck trusses, to each of whose other ends is attached a 40′ plate girder. The weights of the different trusses and girders are as follows:—

Takaya-*gawa* Bridge	200′ truss	155^t^	0^c^	0^q^	19^lbs.^
	70′ plate girder	27	12	3	21
Shichimi-*gawa* Bridge	200′ truss	152	19	1	12
	40′ plate girder	9	10	3	15

The depth of the 200′ trusses is 28′. The piers are each rectangular in section, the dimensions increasing toward the base. There is no well sinking, as the river beds are hard.

The locomotives employed in the different experiments were two similar tender engines, Nos. 8301 and 8302, each of which weighs, in the working order, 78.2 tons. The direction of the train was as follows :—

Up Train.............. *from* Ayabe, *toward* Kyoto;
Down Train .... *from* Kyoto, *toward* Ayabe.

The measurement of the pier motion was made with the portable horizontal vibration recorder* used on former occasions, consisting of a pair of horizontal pendulums, which register, with a magnification of 30, in ink on white paper driven by rollers. The two pointers of the instrument, which was set up at the centre of the top surface of a given pier, recorded the motion of the latter in two rectangular directions :—

(1) *Transverse Component*, or the vibration in direction normal to the pier plane, namely, parallel to the length of the bridge.

(2) *Longitudinal Component*, or the vibration in direction parallel to the pier plane, namely, normal to the length of the bridge.

The periods (complete) of the natural oscillation of the heavy masses of the vibration recorder were in the actual adjustment from 2 to 3 sec., as follows :—

	Transv. Comp. Pendulum.	Longit. Comp. Pendulum.
(Takaya-*gawa*)	2.3 sec.	2.8 sec.
(Shichimi-*gawa*)	2.0 „	2.5 „

As the natural periods of the horizontal pendulums were thus much longer than the proper periods of the pier, the instrument furnished nice and reliable records of the vibrations of the latter. Illustrative diagrams are given in Figs. 11 to 23.

A portion of the present series of experiments has been devoted to the observation of the effect on each of the piers of stopping the train on the bridge. The pier motion thus produced

* The picture of the instrument has been given in the Bulletin, Vol. I, No. 3.

will be, for the sake of brevity, be called the *break-effect.* In the case of the Takaya-*gawa* bridge, the vertical vibration of the 200′ truss was also measured.

In the analysis of the diagrams, $2a$ denotes the double amplitude or range, and T the complete period, of vibration.

EXPERIMENTS ON THE TAKAYA-GAWA BRIDGE TRUSS AND PIER.

2. ***Experiments.*** The single track Takaya-*gawa* bridge has a total span of 349′10″.5, and consists of one 200′ deck truss (at the Kyoto end) and three 70′ plate girders, supported by two abutments and three piers of masonry (Figs. 7 and 9) whose heights are, including in each case the thickness of the foundation, as follows :—

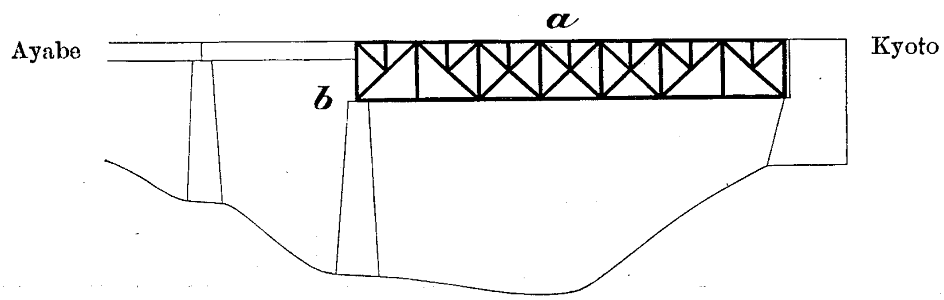
Kyoto end Abutment	46′ 3″¼
1st Pier..........................	82′ 6″½
2nd ,, ...	69′ 2″½
3rd ,, ...	51′ 3″½
Ayabe end Abutment	56′ 8″

The horizontal vibration recorder was set up on the 1st, or tallest, pier, whose height is 82′ 6″.5, and for which the vertical distance between the foundation and the rail level is 114′ 10″. (See Fig. 1.) The pier supports the fixed end of the 200′ truss, to the upper part of whose vertical end frame is attached one extremity of the neighbouring 70′ plate girder.

The top and base sections of the pier proper, or the part above the foundation (thickness$=$4′) are respectively 7′ 0″$\times$24′ 0″ and 17′ 7″$\times$30′ 5″.5. The two sides of the pier have a uniform batter of 1 : 24, while the two faces have three different batters according to height, as follows ;

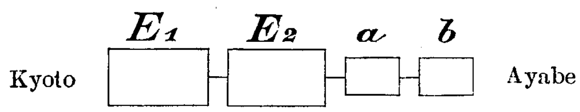
1st	35′ 0″	1 : 24
Middle	25′ 0″	1 : 12
Lowest	17′ 6″	1 : 10

Fig. 1.



The horiz. and vert. vibration recorders were set up at the top of the pier (b), and on the truss (a) respectively.

Fig. 2.



E_1 ..Engine No. 8301
E_2 .... ,, No. 8302
a b........Wagons.

The special experimental train, which passed over the truss with a velocity of $3\frac{1}{2}$ to 23 miles per hour, was composed, in the experiments Nos. 5 to 14, of two similar tender engines (Nos. 8301 and 8302) coupled in series and two open (unloaded) goods-wagons. (See Fig. 2.) In the experiments Nos. 1 to 4 and Nos. 15 to 20, the train was composed of the engine No. 8302 and the two wagons. The following table gives for the different experiments the transit velocity of the train, and the maximum motion (double amplitude$=2a$) and the period ($=T$) of the vertical vibration of the truss in question.

TABLE I. EXPERIMENTS ON TAKAYA-GAWA BRIDGE.

No. of Experiment.	Train.		Velocity of Train.
	Composition.	Direction.	
1	1 Engine and 2 Wagons.	Down	3.9m/h
2		Up	10.0
3		Down	11.0
4		Up	11.0
5	2 Engines and 2 Wagons.	Down	3.5
6		Up	4.0
7		Down	7.0
8		Up	9.0
9		Down	20.0
10		Up	23.0
11		Down	15.0
12		Up	13.7
13		Down	19.5
14		Up	21.0
15	1 Engine and 2 Wagons.	Down	4.5
16		Up	5.5
17		Down	(?)
18		Up	7.6
19		Down	6.8
20		Up	6.0

Vertical Vibration of the 200′ Truss.

3. *Vertical vibration of 200′ Truss: Analysis of diagrams.* The vertical vibration of the 200′ truss was measured by means of a vertical motion recorder with the pointer magnification of 2, set up at the middle of the up-stream side top-chord of the span; the observation being repeated 16 times, in Expts. Nos. 5 to 20.

EXPT. No. 5. $T=0.24$ sec., $2a=0.2$ mm.

EXPT. No. 6. In the preliminary portion, the motion was small :—

$T=0.24$ sec., $2a=0.15$ mm.

In the part immediately preceding the principal portion, for about 11.8 sec., the motion was larger and slower :—

$T=0.41$ sec., $2a=0.75$ mm; $T=0.32$ sec., $2a=0.50$ mm.

The vibrations were most active for the next 23.4 sec., as follows :—

during 1st 2.7	sec.,	motion maximum :	$T=0.39$	sec.,	$2a=1.3$	mm
,, next 4.3	,,	,, small :	$T=0.41$	,,	$2a=0.6$	,,
,, ,, 2.0	,,	,, max. :	$T=0.40$	,,	$2a=1.3$	,,
,, ,, 4.5	,,	,, small :	$T=0.20$	,,	$2a=0.3$	,,
,, ,, 10.0	,,	,, nearly uniform :	$T=0.28$	,,	$2a=0.9$	,,
			$T=0.36$	,,		

Thereafter the movements became suddenly quicker and were nearly uniform for 10.0 sec.: $T=0.23$ sec., $2a=0.4$ mm.
The subsequent motion was very small: $T=0.24$ sec.

EXPT. No. 7. Preliminary portion : $T=0.26$ sec., $2a=0.4$ mm; $T=0.36$ sec., $2a=0.65$ mm.

In the principal portion, vibrations of $T=0.40$ sec., $2a=0.8$ mm, were mixed with small movements of $T=0.18$ sec.

In the free end portion : $T=0.25$ sec., $2a=0.6$ mm.

EXPT. No. 8. Preliminary portion: $T=0.24$ sec., $2a=0.65$ mm.

Principal portion : $T=0.40$ sec., $2a=2.5$ mm.

EXPT. No. 9. (Record confused.)

EXPT. No. 10. The preliminary portion, which was well defined and lasted 1.9 sec., consisted of small quick vibrations mixed with slow ones. The principal portion, which lasted 4 sec., consisted of regular vibrations of $T=0.40$ sec., $2a=6.3$ mm.

EXPT. No. 11. Total duration of preliminary and principal portions$=11.4$ sec. In the principal portion, the motion was as follows :—

earlier part : $T=0.43$ sec., $2a=0.75$ mm.
later part : $T=0.26$ „ $2a=2.4$ „

The end portion did not exist.

EXPT. No. 12. Total duration$=8.8$ sec.

Preliminary portion. Small and irregular ; duration$=2.0$ sec.

Principal portion. During the 1st 5.0 sec., the motion was uniform :

$$T=0.40 \text{ sec., } 2a=4.8 \text{ mm.}$$

In the last 2.1 sec. : $T=0.25$ sec., $2a=2.0$ mm.

EXPT. No. 13. Preliminary and principal portions were continuous together :—

during 1st 1.3 sec. : $T=0.26$ sec., $2a=2.0$ mm.
„ next 1.1 „ $T=0.35$ „ $2a=1.8$ „
„ „ 4.0 „ $T=0.44$ „ $2a=2.3$ „
„ „ 1.1 „ motion small : $T=0.28$ sec., $2a=0.7$ mm.

In the end portion, the motion was uniform and extremely small :—

$$T=0.21 \text{ sec., } 2a=0.25 \text{ mm.}$$

EXPT. No. 14. At first the motion was very small: $T=0.25$ sec. The preliminary and principal portions were as follows :—

during 1st 1.6 sec., $T=$(irregular), $2a=2.5$ mm.
,, next 3.1 ,, motion active: $T=0.38$ sec., $2a=3.3$ mm.
,, ,, 1.5 ,, $T=0.21$ sec., $2a=1.5$ mm.

The free end portion did not exist.

EXPT. No. 15.* Motion very small: $T=0.33$ sec., $2a=0.3$ mm.

EXPT. No. 16.* Preliminary portion: motion very small, $T=0.31$ sec. Principal portion: during the 1st 5.6 sec., $T=0.27$ sec., $2a=0.6$ mm; during the last 2.8 sec., $T=0.34$ sec., $2a=0.5$ mm. The free end portion did not exist.

EXPT. No. 17.* During the 1st 9.0 sec., the motion was small: $T=0.18$ sec., $T=0.29$ sec., $2a=0.4$ mm. During the remaining 6.5 sec., the motion was regular: $T=0.36$ sec., $2a=0.75$ mm.

EXPT. No. 18.* Total duration$=11.9$ sec. In the well defined portion: $T=0.36$ sec., $2a=0.8$ mm.

EXPT. No. 19.* Principal portion (duration$=11.8$ sec.): $T=0.32$ sec., $2a=0.5$ mm. Toward the end: $T=0.37$ sec.

EXPT. No. 20.* Total duration$=11.5$ sec. In the well-defined portion (duration$=4.8$ sec.), $T=0.32$ sec., $2a=0.5$ mm.

4. *Remarks on vertical vibration of 200′ truss.* The period of vibration, which occurred in the principal, or most active, portion of the different diagrams, were, arranged according to order of magnitude, as follows :—

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* The engine passed on very slowly, stopping finally at the middle of the truss.

PERIODS IN THE PRINCIPAL PORTION

Expts. Nos. 5-12. (2 Engines and 2 Wagons).	Expts. Nos. 13-20. (1 Engine and 2 Wagons).
0.18 sec.	0.21 sec.
0.20	0.26 ($2a=2.0$ mm.)
mean .... **0.19** $=t_1'$	0.27
	0.27
0.24	*mean* .... **0.25** $=t_2$
0.24	
0.26 ($2a=2.4$ mm.)	0.32
0.28	0.32
mean .... **0.26** $=t_2'$	0.33
	0.34
0.36	0.35
0.39	0.36
0.40 ($2a=6.3$ mm.)	0.36
0.40 ($2a=4.8$ mm.)	0.37
0.40 ($2a=2.5$ mm.)	0.38 ($2a=3.3$ mm.)
0.40	0.44
0.40	*mean* .... **0.36** $=t_3$
0.41	
0.43	
mean .... **0.40** $=t_3'$	

Thus, in the cases of the passage of the train with two engines, there were essentially three kinds of periods, namely:

$$t_1'=0.19 \text{ sec.} \qquad t_2'=0.26 \text{ sec.} \qquad t_3'=\mathbf{0.40} \text{ sec.}$$

Of these t_3' was of a very frequent occurrence, while t_1' happened only rarely. In the passage of the train with one engine the periods were as follows:

$$t_2=0.25 \text{ sec.} \qquad t_3=0.36 \text{ sec.}$$

t_2 and t_3 are evidently equivalent respectively to t_2' and t_3', the only difference being the lengthening of the third period in consequence of the greater weight of the moving load in the cases of the train with two engines. The period t_1' may be regarded as being equal to half of the period t_3' or t_3. The period in the preliminary portion of the motion varied between 0.18 sec. and 0.41 sec.

Of the periods in the principal portion, the 2nd one, namely, t_2 or t_2', probably represents the period of the natural oscillation of the truss. Thus the periods occurring in the very earliest part of the preliminary portion and in the free end portion give a mean value of 0.24 sec., as follows :

	sec.	
Period of the free vibrations. (Expts. Nos. 1–16)	0.21	
	0.23	
	0.24	
	0.24	···· *mean* **0.24** sec. $=t_2$
	0.25	
	0.25	
	0.25	

The vertical vibration caused by two men running over the truss from end to end also indicated the same period (t_2), as follows :

$$\text{max. } 2a=0.3 \text{ mm}, \quad T=0.25 \text{ sec.}$$

With regards to the relation between the velocity of the train and the $2a$ of the vertical motion, Expts. Nos. 5–12 (Table I) give the following results :

Velocity of Train (2 Engines).		Max. Motion.	
23.0 m/h		$2a=6.3$ mm.	
15.0		2.4	
13.7	····· *mean* 12.6	4.8	······ *mean* 3.2
9.0		2.5	

m/h		mm	
7.0		0.8	
4.0	····· *mean* 4.8	1.3	····· *mean* 0.8
3.5		0.2	

As far as these figures indicate, the $2a$ of the vertical vibration seems to increase approximately in a linear relation to the velocity of the train. Such, however, would not be the case had the train been composed of a great number of passenger carriages or goods wagons. A long goods train running at a moderate speed over a truss or girder of 200′ span often causes, by reason of the accumulation of motion, the vertical vibration larger than that due to the passage of two heavy locomotives, coupled in series, running at a high speed. Again, a comparison of the results in Expts. Nos. 13 and 14 with those in Expt. No. 10, the motion caused by the passage of the two engines Nos. 8301 and 8302 was nearly double that in the case of one engine, No. 8302.

Transverse and Longitudinal Vibrations of Pier No. I.

5. *Measurements of vibrations of pier.* The measurements of the transverse and the longitudinal component vibrations of Pier No. 1 of the Takaya-*gawa* bridge was repeated 20 times, in Expts. Nos. 1 to 20, in the last six of which the break-effect was *measured.*

During the experiments, it was accidentally noted that the vibration recorder indicated some small vibrations when people walked on the 200′ truss whose one end is supported by the pier. I describe in the next § the effect on the pier due to this artificial cause of disturbance.

6. *Vibrations of pier, caused by one man or two running over 200′ truss.* The measurement of the movements of the tall pier of the Takaya-*gawa* bridge caused by one man or two running over the 200′ truss was repeated four times, as follows.

EXPT. No. 1′. One man ran over Truss.

Transverse component. $T=0.30$ sec., $2a=0.03$ mm.

Longitudinal ,, Zero.

EXPT. No. 2′. Same as before.

Transverse component. Motion was distinct for 8.8 sec.: $T=0.33$ sec., $2a=0.02$ mm.

EXPT. No. 3′. Two men ran together over Truss.

Transverse component. Distinct motion lasted 16.5 sec., and consisted of a series of groups, each made up of 12 or 13 vibrations: $T=0.25$ sec., $2a=0.063$ mm.

Longitudinal component. Motion lasted about 30 sec., although the amplitude was much smaller than in the transverse component. It consisted also of a series of groups of 15 to 20 vibrations: $T=0.25$ sec., $2a=0.027$ mm.

EXPT. No. 4′. Same as before.

Transverse component. Duration$=20.0$ sec. The motion gradually reached the maximum and then again gradually decreased: $T=0.31$ sec., $2a=0.04$ mm.

Longitudinal component. Very slight: $T=0.31$ sec.

7. *Vibrations of pier caused by passage of train: Analysis of diagrams.* I give next the result of the analysis of the diagrams obtained in Expts. Nos. 1 to 20, in each of which the tall pier of the Takaya-*gawa* bridge was caused to vibrate by the passage of the train.

EXPT. No. 1. Down Train; 1 Engine. Velocity$=3.9^{\text{m}}/_{\text{h}}$.

The transverse and longitudinal vibrations began nearly at the same moment, lasting respectively 96 and 111 sec.

Transverse component. The vibrations during the successive stages of motion were as follows :—

(i) For 11.6 sec., motion small : $T=0.29$ sec., $2a=0.05$ mm, mixed with small slow movements of $T=1.15$ sec.

(ii) For 6.4 sec., motion distinct : $T=0.29$ sec., $2a=0.08$ mm.

(iii) ,, 10.3 ,, motion larger and regular : $T=0.38$ sec., $2a=0.13$ mm.

(iv) For 8.0 sec., motion very large and uniform : $T=0.40$ sec., $2a=0.23$ mm.

(v) For 8.8 sec., motion smaller but regular :

at first.......... $T=0.35$ sec., $2a=0.22$ mm ;
toward the end .. $T=0.32$,, $2a=0.13$,,

During the rest of the motion, the vibrations were on the whole uniform, the end being quite abrupt :—

at first.......... $T=0.33$ sec., $3a=0.05$ mm ;
toward the end .. $T=0.24$,, $2a=0.08$,,

there being also traces of slow movements of $T=0.83$ sec.

Longitudinal component. The motion was as follows :—

(i) For 7.7 sec., motion small : $T=0.28$ sec., $2a=0.027$ mm.

(ii) ,, 4.0 ,, ,, ,, : $T=0.28$,, $2a=0.053$,,

(iii) ,, 11.9 ,, : $T=0.51$ sec., $2a=0.07$ mm ; $T=0.20$ sec., $2a=$ (small).

(iv) For 5.8 sec., $T=0.19$ sec., $2a=0.06$ mm : $T=0.26$ sec., $2a=0.04$ mm.

(v) For 24.0 sec., motion most active and nearly uniform :—

$T=0.49$ sec., $2a=0.08$ mm. ; $T=0.18$ sec., $2a=0.07$ mm.
$T=0.23$,, $2a=0.08$,, $T=1.13$,, $2a=$ (small).

(vi) For 11.6 sec., motion small and uniform : $T=0.39$ sec., $2a=0.07$ mm. ; $T=0.18$ sec., $2a=0.05$ mm.

(vii) For 12.5 sec., motion consisted almost entirely of quick vibrations of $T=0.19$ sec., $2a=0.03$ mm.
Thereafter the movements became slower and perfectly regular : $T=0.26$ sec., $2a=0.03$ mm.

EXPT. No. 2. Up Train ; 1 Engine. Velocity $=10.0$ $^m/_h$.

At the commencement, the transverse component was much larger than the longitudinal. The latter component, however, lasted longer.

Transverse component. Duration $=44$ sec.

(i) For 4.5 sec., motion small : $T=0.27$ sec., $2a=0.08$ mm.

(ii) ,, 12.5 ,, ,, regular : $T=0.30$,, $2a=0.23$,,

(iii) ,, 10.0 ,, ,, irregular : $\begin{cases} T=0.35 \text{ ,, } 2a=0.16 \text{ ,,} \\ T=0.30 \text{ ,, } 2a=0.26 \text{ ,,} \\ T=0.45 \text{ ,, } 2a=0.17 \text{ ,,} \\ T=0.092 \text{ ,, } 2a=\text{(small).} \end{cases}$

(iv) ,, 7.5 ,, ,, largest and regular : $T=0.37$ sec., $2a=0.39$ mm.

(v) For 3.5 sec., motion small : $\begin{cases} T=0.29 \text{ sec , } 2a=0.19 \text{ mm ;} \\ T=0.079 \text{ ,, } 2a=\text{(small).} \end{cases}$

In the remaining 6.5 sec., the motion was rapidly reduced to rest ; $T=0.27$ sec., $2a=0.05$ mm ; $T=0.08$ sec., $2a=$(small).

Longitudinal component. Duration $=56$ sec. During the first 9.6 sec., the motion was very small : $T=0.24$ sec., $2a=0.01$ mm. For the next 2.4 sec., the motion was larger : $T=0.24$ sec., $2a=0.063$ mm. Then for 5.3 sec., the motion was most active and consisted of quick well-defined vibrations of $T=0.18$ sec., $2a=0.23$ mm, more or less divided into movements of $T=0.44$ sec. For the next 5.7 sec., there appeared more marked traces of slow vibrations of $T=0.44$ sec., $2a=0.24$ mm, mixed with quick movements of $T=0.16$ sec. For the next 4.9 sec., the vibrations of $T=0.21$ sec., $2a=0.02$ mm., were more or less distinctly grouped into oscillations of $T=0.47$ sec. For the next 9.3 sec., which corresponded to the epoch of maximum motion in the other component, the vibrations were smaller: $T=0.15$ sec., $2a=$(small); $T=0.19$ sec., $2a=0.13$ mm; $T=0.49$ sec., $2a=0.13$ mm. In the remaining portion, the motion was smaller: $T=0.18$ sec., $2a=0.07$; $T=0.22$ sec., $2a=$(small).

EXPT. No. 3. Down Train; 1 Engine. Velocity$=11.0$ m/h.

Transverse component. The motion began very gradually.

(i) For 4.9 sec., motion gradually increased up to $2a=0.04$ mm, $T=0.28$ sec.

(ii) For 1.05 sec.: $T=0.26$ sec., $2a=0.10$ mm.

(iii) ,, 4.5 ,, : $T=0.41$,, $2a=0.32$,,

(iv) ,, 5.7 ,, : motion largest and regular, being arranged in two maximum groups: $T=0.32$ sec., $2a=0.92$ mm.

(v) For 10.9 sec., motion was much smaller and on the whole uniform: $T=0.29$ sec., $2a=0.33$ mm, (mixed at first with quick tremors of $T=0.09$ sec.)

In the next 5.7 sec., the motion was quickly reduced to zero: $T=0.29$ sec., $2a=0.047$ mm.

Longitudinal component.

(i) For 6.9 sec., motion was practically zero.

(ii) ,, 1.4 ,, , motion became suddenly distinct: $T=0.19$ sec., $2a=0.10$ mm.

(iii) For 2.6 sec., there appeared slow vibrations of $T=0.51$ sec., $2a=0.19$ mm., mixed with quick vibrations of $T=0.18$ sec.

(iv) For an interval of 10.5 sec., whose commencement coincided with that of the maximum portion in the other component, the motion was on the whole uniform: $T=0.18$ sec., $2a=0.19$ mm; $T=0.17$ sec., $2a=0.27$ mm.

(v) For 5.8 sec.: $T=0.20$ sec., the end of this epoch coinciding with that of the well-marked portion in the transverse component.

(vi) For 26.0 sec. (free end portion), motion was regular: $T=0.23$ sec., $2a=0.05$ mm.

EXPT. No. 4. Up Train; 1 Engine. Velocity$=11^{m}/_{h}$.

Transverse component: $2a=0.28$ mm.

Longitudinal ,, : $2a=0.20$,,

EXPT. No. 5. Down Train; 2 Engines. Velocity$=3.5^{m}/_{h}$.

Transverse component.

(i) For 8.7 sec., motion small: $T=0.30$ sec., $2a=0.04$ mm; $T=1.45$ sec.

(ii) For 4.9 sec.: $T=0.35$ sec., $2a=0.07$ mm.

(iii) ,, 11.8 ,, , motion constant: $T=0.37$ sec., $2a=0.13$ mm.

(iv) ,, 9.6 ,, ,, regular and most active: $T=0.38$ sec., $2a=0.30$ mm.

(v) For 21.8 sec., motion much smaller; $T=0.33$ sec., $2a=0.13$ mm; $T=1.55$ sec.

(vi) For 17.1 sec.: $T=0.29$ sec., $2a=0.05$ mm.

(vii) ,, 22.0 ,, , motion nearly constant: $T=0.26$ sec., $2a=0.05$ mm.

Then the motion came abruptly to end.

Longitudinal component.

(i) For 3.1 sec., motion was small.

(ii) ,, 9.1 ,, : $\begin{cases} T=0.54 \text{ sec.,} & 2a=0.063 \text{ mm.} \\ T=0.20 \text{ ,,} & 2a=0.033 \text{ ,,} \\ T=0.30 \text{ ,,} & 2a=0.060 \text{ ,,} \end{cases}$

(iii) For 11.5 sec. : $\begin{cases} T=0.25 \text{ sec., } 2a=0.07 \text{ mm.} \\ T=0.50 \text{ ,, } 2a=\text{(small).} \end{cases}$

(iv) For 12.5 sec., which interval approximately coincided with the active part of the transverse component : $T=0.21$ sec., $2a=0.073$ mm ; $T=1.14$ sec., $2a=0.07$ mm.

(v) For 17.3 sec., motion constant : $T=0.51$ sec., $2a=0.093$ mm ; $T=0.17$ sec., $2a=0.073$ mm.

(vi) For 4.0 sec. : $T=0.36$ sec., $2a=0.07$ mm.

(vii) ,, 10.8 ,, : $T=0.32$,, , $2a=0.03$,,

For the remaining 17.7 sec., the motion was quicker and regular : $T=0.21$ sec., $2a=0.03$ mm.

EXPT. No. 6. Up Train ; 2 Engines. Velocity $=4.0^{m}/_{h}$.

Transverse component.

(i) During the first 29.2 sec., the motion was small, there being occasional groups, each composed of about 5 vibrations : $T=0.28$ sec., $2a=0.04$ mm. For the next 14.0 sec., the motion was nearly constant, except that in the latter half were more distinct the slow vibration of $T=1.25$ sec., $2a=0.053$ mm ; the superposed vibration being uniform : $T=0.29$ sec., $2a=0.07$ mm. The first engine passed over the pier at the middle instant of this epoch.

(ii) For 11.1 sec., at the end of which interval the 2nd engine passed over the pier, the motion was regular and much larger : $T=0.33$ sec., $2a=0.18$ mm.

(iii) For 12.0 sec., the motion was largest and slower in period : $T=0.40$ sec., $2a=0.22$ mm.

(iv) For 13.7 sec. : $\begin{cases} T=0.38 \text{ sec., } 2a=0.16 \text{ mm (during 1st half) ;} \\ T=0.29 \text{ ,, } 2a=0.12 \text{ ,, (,, last ,,) ;} \end{cases}$

there being, at first, also quick tremors of $T=0.08$ sec., $2a=0.02$ mm.

During the remaining 7.9 sec., the motion became uniform and suddenly smaller: $T=0.29$ sec., $2a=0.04$ mm. Thereafter the motion was reduced abruptly to zero.

Longitudinal component. For the first 16.5 sec., the motion was zero. For the next 14.3 sec., the motion consisted only of small vibrations of $2a=0.03$ mm, whose T was 0.25 sec. at first, and 0.18 sec. at the end. During the next 14.3 sec., there appeared distinct slower vibrations:

for the 1st 7.7 sec. . . . . $T=0.39$ sec., $2a=0.12$ mm;
,, ,, last 6.6 ,, ... $T=0.51$,, $2a=0.16$,, ;

the T of the superposed quick vibration being 0.17 sec. The next 20.3 sec. was characterized by the appearance of slow oscillations of $T=1.08$ sec., $2a=0.073$ mm; mixed with movements of $T=0.50$ sec.; $T=0.18$ sec., $2a=0.08$ mm. During the next 16.7 sec., the motion was marked by the existence of slow vibration of $T=0.50$ sec., $2a=0.06$ mm., mixed with quicker movements of $T=0.27$ sec., $2a=0.07$ mm. For the remaining 15 sec., the motion became abruptly very small.

EXPT. No. 7. Down Train; 2 Engines. Velocity$=7^{m}/_{h}$.

Transverse component.

(i) For 2.5 sec., motion small and regular: $T=0.28$ sec., $2a=0.063$ mm.

(ii) For 1.9 sec., motion larger: $T=0.27$ sec., $2a=0.16$ mm, being mixed with micro-tremors of $T=0.08$ sec.

(iii) For 2.5 sec., motion regular and largest: $T=0.26$ sec., $2a=0.5$ mm.

(iv) For 2.8 sec., motion uniform but distinctly slower: $T=0.47$ sec., $2a=0.42$ mm, being mixed with micro-tremors of $T=0.08$

sec. The 1st engine passed over the pier at the end of this portion.

(v) For 2.4 sec., motion equally large but a little quicker; $T=$ 0.40 sec., $2a=0.55$ mm, mixed with micro-tremors. The 2nd engine passed over the pier at the commencement of this portion.

(vi) For 8.2 sec., motion nearly constant: $T=0.28$ sec., $2a=$ 0.43 mm.

Thereafter the motion became abruptly small and free: $T=0.27$ sec., $2a=0.037$ mm.

Longitudinal component.

(i) For 3.3 sec., motion zero.

(ii) ,, 5.0 ,, ,, large and regular: $T=0.21$ sec., $2a=$ 0.28 mm.

(iii) For 9.3 sec., motion most active: $T=0.18$ sec., $2a=0.51$ mm.

(iv) ,, 10.0 ,, ,, smaller and regular: $T=0.24$ sec., $2a=$ 0.13 mm.

EXPT. No. 8. Up Train; 2 Engines. Velocity$=9.0^{m}/_{h}$.

Transverse component.

(i) For 5.5 sec., motion very small: $T=0.28$ sec., $2a=0.03$ mm.

(ii) ,, 3.5 ,, ,, became suddenly large and regular: $T=$ 0.27 sec., $2a=0.41$ mm.

(iii) For 3.5 sec., motion smaller: $T=0.29$ sec., $2a=0.27$ mm.

(iv) ,, 2.6 ,, ,, larger and slower: $T=0.32$ sec., $2a=$ 0.50 mm.

(v) For 6.3 sec., motion very great, consisting of 17.5 vibrations, as follows:—

(1) $T=0.45$ sec., $2a=1.02$ mm (5 vibrations),
(2) $T=0.39$,, $2a=1.07$,, (3 ,,),
(3) $T=0.31$,, $2a=1.03$,, ($9\frac{1}{2}$,,).

The subsequent portion became abruptly small, remaining nearly constant: $T=0.28$ sec., $2a=0.12$ mm.

Longitudinal component. The motion was zero for the first 7.5 sec. For the next 3.5 sec., the motion gradually increased: $T=0.19$ sec., $2a=0.17$ mm. The next 5.5 sec. was characterized by the appearance of slow vibration of $T=0.52$ sec., $2a=0.33$ mm; mixed with conspicuous quick vibration of $T=0.18$ sec., $2a=0.42$ mm. For the next 6.7 sec., the motion was constant: $T=0.24$ sec., $2a=0.24$ mm. For the next 5.2 sec.: $T=0.29$ sec., $2a=0.1$ mm; $T=0.18$ sec., $2a=0.06$ mm. In the subsequent portion the vibrations were perfectly regular: $T=0.26$ sec., $2a=0.057$ mm.

EXPT. No. 9. Down Train; 2 Engines. Velocity$=20^{m}/_{h}$.

The diagram is very similar to that in Expt. No. 7, only the motion being here slightly larger.

(i) For 2.7 sec., motion small: $T=0.32$ sec.

(ii) „ 1.7 „ „ regular: $T=0.30$ sec., $2a=0.15$ mm.

(iii) „ 1.05 „ „ irregular, mixed with micro-tremors of $T=0.07$ sec.

(iv) For 4.03 sec., motion was maximum and regular, consisting of 14 vibrations of $T=0.31$ sec., $2a=0.83$ mm.

(v) For 3.35 sec., motion smaller: $T=0.40$ sec., $2a=0.47$ mm.

(vi) „ 6.3 „ „ again active and regular: $T=0.28$ sec., $2a=0.62$ mm.

(vii) For 2.25 sec., motion irregular and small.

(viii) „ 5.35 „ „ active and regular: $T=0.28$ sec., $2a=0.33$ mm.

Thereafter the vibrations became abruptly very small: $T=0.30$ sec.

Longitudinal component. The motion, which was zero for the first 4.9 sec., became active for 16.1 sec., consisting principally of vibration of $T=0.20$ sec., $2a=0.44$ mm. This maximum occurred about 14.3 sec. after the commencement, and simultaneously with the 2nd maximum group in the transverse component. There were also movements of $T=0.51$ sec. The subsequent portion which was much smaller, consisted of perfectly regular vibrations of $T=0.25$ sec., $2a=0.10$ mm.

EXPT. No. 10. Up Train; 2 Engines. Velocity$=23.0^{m}/_{h}$.

The diagram is perfectly similar to that in Expt. No. 8.

Transverse component. The motion was small for the first 5.5 sec.: $T=0.28$ sec. For the next 9.9 sec., the motion was regular: $T=0.29$ sec., $2a=0.52$ mm. Then for 7.8 sec., the vibrations were largest and regular :—

1st	2.4 sec. :	$T=0.40$ sec.,	$2a=1.03$ mm	(6 vibrations),
next	2.0 ,, :	$T=0.33$,,	$2a=0.93$,,	(6 ,,),
last	3.3 ,, :	$T=0.30$,,	$2a=0.71$,,	(11 ,,).

The subsequent motion was abruptly small but regular : $T=0.28$ sec., $2a=0.07$ mm.

Longitudinal component.

(i) For 7.6 sec., motion zero.

(ii) ,, 2.5 ,, ,, gradually increased to $2a=0.1$ mm, $T=0.21$ sec.

(iii) For 15.2 sec., motion active :—

$T=0.20$ sec., $2a=0.45$ mm ; $T=0.24$ sec., $2a=0.25$ mm ;
$T=0.27$,, $2a=0.33$,, ; $T=0.43$,, $2a=0.37$,,

The vibrations in the free end portion were as follows : $T=0.25$ sec., $2a=0.07$ mm.

EXPT. No. 11. Down Train; 2 Engines. Velocity$=15^{m}/_{h}$.

Transverse component.

(i) For 2.2 sec., motion small.
(ii) ,, 2.3 ,, ,, irregular: $T=0.27$ sec., $2a=0.13$ mm.
(iii) ,, 1.3 ,, there were 4 equal vibrations of $T=0.31$ sec., $2a=0.28$ mm.
(iv) For 2.8 sec., there were $6\frac{1}{2}$ vibrations of $T=0.43$ sec., $2a=0.58$ mm.
(v) For 1.8 sec., there were $4\frac{1}{2}$ vibrations of $T=0.42$ sec., $2a=0.38$ mm.

The subsequent motion was smaller: $T=0.32$ sec., $2a=0.3$ mm.
In the free end portion: $T=0.29$ sec., $2a=0.03$ mm.

Longitudinal component. For the first 2.3 sec., the motion was zero. Then the motion gradually increased during tne next 2.9 sec. up to $2a=0.15$ mm. For the next 7.1 sec., the vibrations were large: $T=0.18$ sec., $2a=0.3$ mm; $T=0.25$ sec., $2a=0.53$ mm. The maximum movement occurred 10.3 sec. after the commencement. In the subsequent portion, the motion was smaller: $T=0.22$ sec., $2a=0.17$ mm.

EXPT. No. 12. Up Train; 2 Engines. Velocity$=13.7^{m}/_{h}$.

Transverse component. For the first 8.2 sec.: $T=0.30$ sec., $2a=0.22$ mm. For the next 5.7 sec., the vibrations were regular and largest: $T=0.37$ sec., $2a=0.61$ mm. In the remaining 7.3 sec., the motion was as follows: $T=0.28$ sec., $2a=0.18$ mm.

Longitudinal component.

In earlier part: $T=0.24$ sec., $2a=0.05$ mm.
In the most active part: $T=0.22$ sec., $2a=0.37$ mm; $T=0.21$ sec., $2a=0.23$ mm.
In the free end portion: $T=0.20$ sec., $2a=0.07$ mm.

EXPT. No. 13. Down Train; 2 Engines. Velocity$=19.5^{m}/_{h}$.

Transverse component.

(i) For 3.6 sec. : motion small.

(ii) ,, 8.8 ,, : T=0.26 sec., $2a$=0.11 mm ; T=0.30 sec., $2a$=0.13 mm.

(iii) For 11.5 sec. : motion regular and large : T=0.39 sec., $2a$= 0.28 mm.

(iv) For 10.7 sec. : motion very regular and quicker : T=0.30 sec., $2a$=0.21 mm.

(v) For 18.9 sec., motion nearly constant except for grouping in occasional maxima : T=0.29 sec., $2a$=0.11 mm ; T=0.33 sec., $2a$=0.12 mm ; T=0.25 sec., $2a$=0.04 mm.

Then the motion abruptly ended. In (ii) and (iii), there were also some micro-tremors.

Longitudinal component.

(i) For 4.0 sec., motion very small.

(ii) ,, 4.6 ,, ,, gradually increased to $2a$=0.07 mm.

(iii) ,, 15.2 ,, ,, nearly uniform : T=0.19 sec., $2a$= 0.15 mm ; T=0.30 sec., $2a$=0.17 mm ; T=0.53 sec., $2a$= 0.11 mm.

(iv) For 8.2 sec., motion most active : T=0.31 sec., $2a$= 0.21 mm ; T=0.18 sec., $2a$=0.13 mm.

(v) For 8.6 sec., motion smaller : T=0.39 sec., $2a$=0.07 mm ; T=0.17 sec., $2a$=0.11 mm.

In the free end portion : T=0.24 sec., $2a$=0.06 mm.

EXPT. No. 14 Up Train ; 2 Engines. Velocity=21.0^{m}/h.

Transverse component.

During the first 19.3 sec., the motion was irregular : T=0.30 sec., $2a$=0.15 mm ; T=0.22 sec., $2a$=0.10 mm.

The vibrations were largest for the next 18.3 sec., as follows :—

(1) 6 vibrations, for 1.6 sec. ; $T=0.27$ sec., $2a=0.26$ mm.
(2) 10 „ , „ 3.0 „ ; $T=0.30$ „ $2a=0.42$ „
(3) 6 „ , „ 1.55 „ ; $T=0.26$ „ $2a=0.14$ „
(4) 6 „ , „ 2.1 „ ; $T=0.35$ „ $2a=0.23$ „
(5) 2 „ , „ 0.50 „ ; $T=0.25$ „ $2a=0.20$ „
(6) 8½ „ , „ 3.5 „ ; $T=0.41$ „ $2a=0.44$ „
(7) 12 „ , „ 4.0 „ ; $T=0.33$ „ $2a=0.47$ „
(8) 7 „ , „ 1.95 „ ; $T=0.28$ „ $2a=0.20$ „

For the next 3.6 sec., the motion was small: $T=0.31$ sec., $2a=$ 0.15 mm. The next 5.3 sec. forms the free end portion: $T=0.29$ sec., $2a=0.03$ mm.

Longitudinal component.

(i) For 4.1 sec., motion zero.
(ii) „ 6.3 „ , „ small: $T=0.28$ sec., $2a=0.03$ mm; $T=0.20$ sec., $2a=0.02$ mm.
(iii) For 1.6 sec., motion large and regular (6½ vibrations): $T=0.25$ sec., $2a=0.17$ mm.
(iv) For 2.2 sec., motion small: $T=0.17$ sec., $2a=0.08$ mm.
(v) „ 6.0 „ „ large: $T=0.20$ „ $2a=0.21$ „
(vi) „ 6.0 „ „ largest: $T=0.50$ „ $2a=0.32$ „
(vii) „ 5.0 „ „ regular: $T=0.19$ „ $2a=0.22$ „ ; $T=0.50$ sec., $2a=$(small).
(viii) For 9.9 sec.; $T=0.51$ sec., $2a=0.18$ mm; $T=0.22$ sec., $2a=0.19$ mm.

8. *Observation of effect on pier caused by stopping train on bridge; Analysis of diagrams.* In each of the Expts. Nos. 15 to 20 was measured the effect on the pier produced by causing the train, composed of Engine No. 8302 and two goods wagons, to stop on the bridge. These experiments were carried on with great precaution, and the velocity of the train, whose initial value did not, in any case, exceed 7.6 miles per hour, was gradually reduced, such that the engine was finally brought to rest at the middle of the 200′ truss whose one end is supported by the pier in question. The initial velocity and the time interval between the first application of the break and the stopping of the engine were, in the different experiments, as follows :—

No. of Expt.	Initial Velocity.	Time Interval between application of Break and stopping of Engine.
15	4.5 miles/hour	10 seconds.
16	5.5	7
17	(?)	9
18	7.6	6
19	6.8	—
20	6.0	—

It may be noted that, as the 200′ truss is the first Kyoto end span of the bridge, the up train engine comes on it after passing over the pier, which was not the case with the down train engine. Illustrative diagrams are reproduced on Pls. XIV and XV.

EXPT. No. 15. Down Train ; 1 Engine. Initial Velocity $=4.5^{m}/_{h}$.

Transverse component.

(i) For 10.2 sec., motion small : $T=0.30$ sec., $2a=0.08$ mm.

(ii) ,, 5.0 ,, ,, regular and formed a maximum group : $T=0.30$ sec., $2a=0.16$ mm.

(ii) For 10.3 sec., motion smaller : $T=0.31$ sec., $2a=0.10$ mm. Toward the end of this latter part, the motion was small ($2a=0.03$ mm), when took place the effect due to stoppage of the engine :

1st motion.... $a=0.39$ mm. toward Kyoto, followed by
2nd ,, ...$2a=0.46$,, ,, Ayabe.

The 2nd complete vibration was nearly equal to the 1st vibration: $2a=0.4$ mm. The 3rd vibration was smaller: $2a=0.19$ mm. The period of these vibrations was $T=0.34$ sec. The subsequent motion was small: $T=0.34$ sec., $2a=0.093$ mm. Later on, when the engine went out gradually: $T=0.30$ sec., $2a=0.093$ mm; $T=1.30$ sec., 0.041 sec.

Longitudinal component. No break-effect was indicated. In the earlier part: $T=0.52$ sec., $2a=0.087$ mm. $T=0.27$ sec., $2a=0.10$ mm. Later on (after the re-start of the engine): $T=0.47$ sec., $2a=0.07$ mm, mixed with quicker vibration. In the free end portion: $T=0.24$ sec.

EXPT. No. 16. Up Train; 1 Engine. Initial Velocity$=5.5^{m}/_{h}$.

Transverse component.

(i) For 6.0 sec., motion small: $2a=0.07$ mm.

(ii) ,, 7.0 ,, ,, perfectly regular and forms a gradually increasing maximum group: $T=0.31$ sec., $2a=0.18$ mm.

(iii) For 3.0 sec., motion small: $T=0.37$ sec., $2a=0.12$ mm.

(iv) ,, 7.2 ,, ,, regular and large, forming a maximum group: $T=0.31$ sec., $2a=0.38$ mm.

(v) For 7.7 sec., motion smaller, there being 2 maximum groups: $T=0.34$ sec., $2a=0.25$ mm; $T=0.076$ sec., $2a=$(small).

(vi) For 2.4 sec., motion uniform: $T=0.35$ sec., $2a=0.07$ mm.

When the engine completely stopped, there took place a sudden displacement of 0.31 mm ($=a$) toward Ayabe, followed by the counter motion of 0.45 mm ($=2a$) toward Kyoto. The next two vibrations were respectively of $2a=0.46$ mm and $2a=0.28$ mm, the period being $T=0.33$ sec. After the re-start of the engine: $T=0.31$ sec, $2a=0.19$ mm. The small free end vibrations were: $T=0,30$ sec., $2a=0.05$ mm.

Longitudinal component. Different vibrations in the principal portion were: $T=0.22$ sec., $2a=0.08$ mm; $T=0.19$ sec., $2a=0.16$ mm; $T=0.31$ sec., $2a=0.13$ mm. After the re-start of the engine: $T=0.75$ sec., $T=0.50$ sec., $2a=0.04$ mm.

EXPT. No. 17. Down Train; 1 Engine. Initial Velocity=(?)

Transverse component. For the first 7.4 sec., the motion was small: $T=0.27$ sec., $2a=0.06$ mm. For the next 15.6 sec., the vibrations were regular, being maximum at the middle of this interval: $T=0.35$ sec., $2a=0.20$ mm. Just before the engine came to rest: $2a=0.07$ mm. The break effect was as follows:

1st displacement	: 0.32	mm	$(=a)$,	toward	Kyoto;
2nd ,,	: 0.43	,,	$(=2a)$,	,,	Ayabe;
3rd ,,	: 0.35	,,	(,,)	,,	Kyoto;
4th ,,	: 0.13	,,	(,,)	,,	Ayabe, etc.

These formed vibrations of $T=0.33$ sec. The very small subsequent movements were: $T=0.29$ sec., $2a=0.03$ mm. After the re-start of the engine: $T=0.33$ sec., $2a=0.12$ mm. Toward the very end: $T=1.14$ sec.; $T=0.28$ sec., $2a=0.03$ mm.

Longitudinal component. In the principal portion: $T=0.30$ sec., $2a=0.08$ mm; $T=0.55$ sec., $2a=0.11$ mm; $T=0.22$ sec., $2a=0.02$ mm. After the re-start of the engine: $T=0.48$ sec., $2a=0.15$ mm; $T=0.20$ sec., $2a$=(small).

EXPT. No. 18. Up Train; 1 Engine. Initial Velocity=$7.6^{m}/_{h}$.

Transverse component.

(i) For 3.3 sec., motion small; $T=0.33$ sec.

(ii) „ 11.1 „ : $T=0.29$ sec., $2a=0.10$ mm; $T=0.42$ sec., $2a=0.03$ mm.

(iii) For 17.7 sec., motion regular, increasing on the whole: $T=0.39$ sec., $2a=0.07$ mm; $T=0.35$ sec., $2a=0.22$ mm.

(iv) For 9.4 sec., there were alternations of maximum and minimum groups: $T=0.35$ sec., $2a=0.18$ mm.

Then the motion was practically reduced to zero, when took place the break effect, as follows :—

1st Vibration { 1st displacement : 0.23 mm ($=a$) toward Ayabe,
2nd „ : 0.35 „ ($=2a$), „ Kyoto.

2nd „ : $2a=0.37$ mm.

3rd „ : $2a=0.23$ „

The period of these vibrations was $T=0.33$ sec. After the re-start of the engine: $T=0.34$ sec., $2a=0.19$ mm. The free end vibrations were: $T=0.28$ sec., $2a=0.07$ mm. The engine, which had entered on the neighbouring 70′ plate girder on the Ayabe side 15.3 sec. after the commencement of the motion, passed over the pier in question 9.4 sec. later on. It came to rest in further 16.3 sec.

Longitudinal component. In the earlier part: $T=0.23$ sec., $2a=0.02$ mm; $T=0.17$ sec., $2a=0.08$ mm. In the maximum portion: $T=0.33$ sec., $2a=0.15$ mm; $T=0.70$ sec. In the end portion: $T=0.80$ sec.

EXPT. No. 19. Down Train; 1 Engine. Initial Velocity$=6.8^{m}/_{h}$.

Transverse component.

(i) For 8.5 sec., motion small: $T=0.30$ sec., $2a=0.02$ mm.
(ii) ,, 3.8 ,, , $T=0.35$ sec., $2a=0.12$ mm.
(iii) ,, 7.5 ,, motion largest and regular: $T=0.33$ sec., $2a=$ 0.57 mm.
(iv) For 5.6 sec., motion small: $T=0.36$ sec., $2a=0.20$ mm.

The motion was then reduced to $2a=0.07$ mm, when took place the break effect as follows :—

1st Vibration	1st displacement	: 0.36 mm ($=a$), toward Kyoto;
	2nd ,,	: 0.42 ,, ($=2a$), ,, Ayabe.
2nd ,,	$2a=0.38$ mm.	
3rd ,,	$2a=0.12$,,	

The period of these vibrations was $T=0.30$ sec. After the re-start of the engine: $T=0.31$,, $2a=0.10$ mm.

Longitudinal component.

(i) For 5.0 sec., motion was zero.
(ii) ,, 3.0 ,, ,, very small.
(iii) ,, 3.5 ,, : $T=0.26$ sec., $2a=0.05$ mm.
(iv) ,, 7.9 ,, motion most active: $T=0.29$ sec., $2a=0.17$ mm.

The subsequent motion was small.

EXPT. No. 20. Up Train; 1 Engine. Initial Velocity$=6.0^{m}/_{h}$.

Transverse component. The commencement was quite sharp. For the first 13.6 sec., the motion was small: $T=0.30$ sec., $2a=$ 0.07 mm. For the next 21.2 sec., the motion was regular and gradually increased: $T=0.34$ sec., $2a=0.12$ mm; $T=0.34$ sec., $2a=0.17$ mm. Then for 4.0 sec., the motion was much smaller: $T=0.30$ sec., $2a=0.06$ mm. The motion just before the stopping of the engine was $2a=0.03$ mm. The break effect was as follows :—

1st Vibration	1st displacement : 0.29 mm, toward Ayabe ;	
	2nd ,, : 0.41 ,, ,, Kyoto.	
2nd ,,	$2a=0.43$ mm.	
3rd ,,	$2a=0.28$,,	

The period of these vibrations was $T=0.33$ sec.

Longitudinal component. In the earlier part: $T=0.18$ sec., $2a=0.11$ mm. In the most active part: $T=0.25$ sec., $2a=0.08$ mm; $T=0.31$ sec., $2a=0.12$ mm.

Summary of Results of Experiments on Takaya-gawa Bridge Pier.

9. *Amplitude of vibration.* Table II gives, for the different experiments, the maximum transverse and longitudinal vibrations of the pier motion as well as the maximum vertical motion of the 200′ truss. The greatest transverse pier movements of over 1 mm occurred in the up-train Expts. Nos. 8 and 10, as follows :—

Expt. No. 8....	$2a=1.07$ mm, $T=0.39$ sec.
	$2a=1.03$,, , $T=0.31$,,
Expt. No. 10....	$2a=1.03$,, , $T=0.40$,,
	$2a=0.93$,, , $T=0.33$,,

The velocities of the train in these two cases were respectively 9 and 23 miles per hour. The greatest longitudinal vibrations of about half a millimeter occurred in the two down-train Expts. Nos. 7 and 11, as follows :—

Expt. No. 7....$2a=0.51$ mm, $T=0.18$ sec. ;
,, No. 11....$2a=0.53$,, $T=0.25$,, ;

the velocities of the train being respectively 7 and 15 miles per hour. The ratio of the maximum transverse to the maximum longitudinal motion varied in the different experiments, from 1.08

to 4.11, giving the mean value of 2.1. The pier in question may therefore be regarded as moving twice more in direction normal, than in direction parallel, to its plane.

TABLE II. COMPARISON OF TRANSVERSE AND LONGITUDINAL VIBRATIONS OF PIER WITH VERTICAL VIBRATION OF 200′ TRUSS.

No. of Experiment.	Velocity.	200′ Truss. Vertical Vibrations. 2a	T	Pier. Transverse Vibration. 2a	T	Longitudinal Vibration. 2a	T
	m/h	mm	sec.	mm	sec.	mm	sec.
1	3.9			0.23 0.22	0.40 0.35	0.08 0.08	0.49 0.23
2	10.0			0.39	0.37	0.23 0.24	0.18 0.44
3	11.0			0.92	0.32	0.27	0.17
4	11.0			0.28	—	0.20	—
5	3.5	0.2	0.24	0.30	0.38	0.073 0.073	0.17 0.21
6	4.0	1.3	0.40	0.22	0.40	0.12 0.08	0.39 0.18
7	7.0	0.8	0.40	0.55 0.50 0.42	0.40 0.26 0.47	0.51	0 18
8	9.0	2.5	0.40	1.07 1.03 1.02	0.39 0.31 0.45	0.42	0.18
9	20.0	—	—	0.83	0.31	0.44	0.20
10	23.0	6.25	0.40	1.03 0.93 0.71	0.40 0.33 0.30	0.45 0.33	0.20 0.27
11	15.0	2.4 0.75	0.26 0.43	0.58	0.43	0.53	0.25
12	13.7	4.75	0.40	0.61	0.37	0.37	0.22

No. of Experiment.	Velocity.	200′ Truss. Vertical Vibration. 2a	200′ Truss. Vertical Vibration. T	Pier. Transverse Vibration. 2a	Pier. Transverse Vibration. T	Pier. Longitudinal Vibration. 2a	Pier. Longitudinal Vibration. T
	m/h	mm	sec.	mm	sec.	mm	sec.
13	19.5	{2.0 2.25	0.26 0.44	0.28	0.39	{0.15 0.21 0.17	0.19 0.31 0.30
14	21.0	3.25	0.38	{0.44 0.47	0.41 0.33	{0.22 0.21 0.17	0.19 0.20 0.25
15	4.5*	0.3	0.33	0.16	0.30	0.10	0.27
16	5.5*	{0.6 0.5	0.27 0.34	0.38	0.31	{0.16 0.13	0.19 0.31
17	(?*)	0.75	0.36	0.20	0.35	0.08	0.30
18	7.6*	0.8	0.36	0.22	0.35	{0.15 0.08	0.33 0.17
19	6.8*	0.5	0.32	0.57	0.33	0.17	0.29
20	6.0*	0.5	0.32	0.17	0.34	0.12	0.31

* The velocity is that with which the train started, the engine having stopped in each of the Expts. Nos. 15 to 20, at the middle of the 200′ truss.

10. ***Periods of the transverse vibration.*** The principal period of vibration in the preliminary and the free end portion of the transverse component of the pier motion was, according to Expts. Nos. 1 to 20, as follows :—

Preliminary Portion.		Free End Portion.	
sec.	sec.	sec.	sec.
0.27	0.29	0.24	0.29
0.27	0.30	0.26	0.29
0.28	0.30	0.27	0.29
0.28	0.30	0.27	0.29
0.28	0.30	0.28	0.30
0.28	0.32	0.28	0.30
0.28	0.35	0.28	0.33
		0.28	
(*mean*)	**0.29**		$t_1=$ **0.28** (*mean*)

The period t_1, whose average length is 0.28 sec., (taking the value in the free end portion), may be regarded as the proper period of the transverse oscillation of the pier in question.

Again, the period occurring in the principal portion of the transverse component caused by the passage of the train with two engines (Expts. Nos. 5 to 12) was as follows :—

(*A*) Most active part of the principal portion.	(*B*) Principal Portion, excepting (*A*).	
sec.	sec.	sec.
0.26 ($2a$=0.50 mm)	0.27	0.37
0.28 ($2a$=0.62 ,,)	0.27	0.38
0.29 ($2a$=0.52 ,,)	0.27	0.42
0.30 ($2a$=0.71 ,,)	0.28	t_3=**0.39** sec.
0.31 ($2a$=1.03 ,,)	0.28	
0.31 ($2a$=0.83 ,,)	0.28	
0.33 ($2a$=0.93 ,,)	0.29	
0.30 sec.=t_2	0.29	
	0.29	
0.37 ($2a$=0.61 mm)	0.30	
0.38	0.30	
0.39 ($2a$=1.09 mm)	0.31	
0.40 ($2a$=0.47 ,,)	0.32	
0.40	0.32	
0.40 ($2a$=0.53 mm)	0.33	
0.40 ($2a$=1.03 ,,)	0.33	
0.43 ($2a$=0.58 ,,)	t_2=**0.30** sec.	
0.45 ($2a$=1.02 ,,)		
0.47		
t_3=**0.41** sec.		

Thus the periods in the principal portion, and especially in its most active part, are two-fold, namely, t_2=**0.30** sec., and t_3=**0.41** sec. Large movements of over 1.0 mm occurred with the t_2 as well as with the t_3 period.

In the case of the transverse motion caused by the passage of the train with 1 engine, the periods corresponding to t_2 and t_3 were respectively 0.29 sec. and 0.36 sec.

Besides the principal periods considered above there were also in some cases three others whose mean values were respectively

0.081 sec.; 1.08 sec.; 1.43 sec..

Of these the first occurred chiefly in the principal portion alone.

11. *Periods of the longitudinal vibration.* The period in the free end portion of the longitudinal component was, according to Expts. Nos. 1 to 20, as follows :—

sec.	sec.	sec.	sec.
0.20	0.22	0.24	0.26
0.21	0.23	0.25	0.26
0.22	0.24	0.25	(*mean*) **0.42**=l_1

The period l_1 (=0.24 sec.) may be regarded as the period of the proper longitudinal oscillation of the pier.

The periods of the most active vibrations in the principal portion were, taking together the results in Expts. Nos. 5 to 20 as follows :—

Most active Vibrations in the Principal Portion.

sec.	sec.
0.17	0.19
0.18 ($2a$=0.51 mm)	0.20 ($2a$=0.44 mm)
0.18 ($2a$=0.42 ,,)	0.20 ($2a$=0.45 ,,)
0.18 ($2a$=0.30 ,,)	0.20
0.18	0.21
0.18	0.22 ($2a$=0.37 mm)
0.19	(*mean*) **0.19**=l_3

sec.		sec.
0.22		0.29
0.24		0.31
0.25 (2a=0.53 mm)		0.31
0.25		0.31
0.27 (2a=0.33 mm)		0.33
	(*mean*)	**0.28**=l_2

Thus there were two different sets of the most active vibrations, whose periods had the mean values respectively of l_2=0.19 sec., and l_3=0.28 sec. The periods occurring generally in the principal portion were three-fold as follows :

(*A*) Train with 2 Engines.		(*B*) Train with 1 Engine.
0.27 sec.	..........l_2............	0.27 sec.
0.19 ,,	..........l_3............	0.18 ,,
0.48 ,,	..........l_4............	0.48 ,,

These three sets of periods were of nearly equal frequency, the l_4 movements being also often well pronounced.

In a few cases there were slow oscillations of periods of 0.75 and 1.12 sec.

12. ***Comparison of periods of transverse and longitudinal components of pier motion with those of vertical vibration of truss.*** The mean values of the different periods occurring in the transverse and the longitudinal components of the pier motion, and those in the vertical movement of the 200′ truss are, according to §§4, 10, and 11, as shown in the following table :—

TABLE III. COMPARISON OF PERIODS.

Nature of Vibration.	Pier.		Truss.
	Transverse Component.	Longitudinal Component.	Vertical Vibration.
Free end vibration.	$t=0.28$ sec.	$l=0.42$ sec.	$u=0.24$ sec.
Most active vibration (2 engines).	$t_1=$ **0.30** (max. $2a=1.03$ mm) $t_1=$ **0.41** (max. $2a=1.09$ mm)	$l_1=$ **0.28** (max. $2a=0.53$ mm) $l_2=$ **0.19** (max. $2a=0.51$ mm)	$u^1=0.19$ $u_1=0.26$ (max. $2a=2.4$ mm) $u_2=$ **0.40** (max. $2a=6.3$ mm)
Motion in Principal Portion, in general, (2 engines).	$t_1=0.30$ $t_2=0.39$	$l_1=0.27$ $l_2=0.19$ $l_3=0.48$ (max. $2a=0.37$ mm)	
Do. (1 engine).	$t_1=0.29$ $t_2=0.36$	$l_1=0.27$ $l_2=0.18$	$u_1=0.25$ $u_2=0.36$
Slow oscillation, (1 engine).	— 1.08 1.43	0.75 1.12 —	
Quick vibration.	0.08		

One of the periods of the strongest transverse vibrations, namely, t_1 ($=0.30$ sec.) is probably equivalent, with a proportional lengthening due to the weight of the train, to the period t ($=0.28$ sec.) which characterized the free proper movements of the pier. The other period t_2 ($=0.41$ sec.) is, however, to be regarded as being identical with u_2 ($=0.40$ sec.) the period of the most active vertical vibration of the truss.

Again, one of the periods of the strongest longitudinal vibrations, namely, l_1 (=0.28 sec.) is probably equivalent in nature to the period l (=0.24 sec.) of the free pier motion. The other period l_2 (=0.19 sec.) is to be regarded as being equal to half of that of the vertical motion, u_2 (=0.40 sec.). Similar relations also hold among the periods t_1, t_2, l_1, l_2, and u_2, of the movements in the principal portion in general.

Thus the principal motion in question are two-fold in origin : (1) the vibrations of periods t_1 and l_1 are due to the proper movement of the pier itself; (2) the vibrations of the periods t_2 add l_2 are, on the other hand, caused by the vertical movement of the 200′ truss whose one end rests on the pier.

The longitudinal vibration with the period l_3 (=0.48 sec.) may possibly be due to the horizontal motion of the truss in direction at right angles to the span.

The vibration measurement made on the pier between the 7th and 8th 200′ trusses of the Tone-*gawa* bridge indicated exactly similar characteristics in the periods of the transverse and the longitudinal movements.* Thus in the case of the passage of a long goods train at a low speed, the principal elements of motion was as follows :

Transverse component : T=0.39 sec., $2a$=0.63 mm ;
Longitudinal „ : T=0.22 „ $2a$=0.33 „

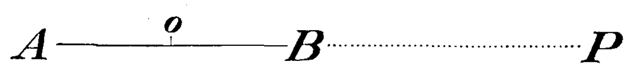
the lengths of the periods of the transverse and the longitudinal vibrations were thus nearly 2 to 1, the amplitudes of the two components being also in nearly the same ratio. The period of the vertical vibration of the 200′ Double Warren Girders of the Tone-*gawa* bridge was on the average 0.43 sec.

The relation between the periods of the transverse and the

* The "Bulletin", Vol. I, p. 155–157.

longitudinal components of the pier motion, which depends on the vertical vibrations of the bridge truss, may be explained as follows. Let *o* be a source of disturbance in an isotropic medium, which makes periodic vibrations in a fixed direction *A B*. Then a particle *P* situated in the direction of *AB* prolonged must execute two sets of movements: Firstly, *P* moves forward and backard in resonance to the motion of the disturbing origin, *o*, that is to say, the period of the radial vibration of *P* is equal to that of *o* itself. Secondly, *P* moves in a direction at right angles to the line *o p*, such that the forward as well as backward motion of the origin causes each a complete vibratory movement of the point *P*, whose period is thus equal to half that of the disturbance at *o*. These suppositions have been verified in the measurement made recently in Tokyo of the movements of the ground caused by a gas engine. Now in the case of a railway bridge, the vertical vibration of a truss or girder, which is maximum at the middle of the span and vanishes towards the extremities, is communicated to the adjacent piers as horizontal movement in direction normal to the pier faces. By exactly similar reasonings as stated above, the periods of the transverse and the longitudinal component vibrations of the pier motion ought to be respectively equal to, and half of, the period of the vertical motion of the truss or girder.

Fig. 3.



13. ***Break effect.*** According to Expts. Nos. 15 to 20, the stopping of the train on the bridge truss was, irrespective of the low initial velocity and its gradual reduction, accompanied by quite remarkable results. Its effect on the pier consisted in the production, at the moment when, the engine was brought to rest, of a sudden displacement in the transverse component, which was often much glarer than the vibration during the running of the train. As may be expected, there was no corresponding effect in the longitudinal component. The following table embodies the results of the experiments under consideration.

TABLE IV. LIST OF BREAK EFFECTS ON TRANSVERSE VIBRATION OF PIER OCCURRING AT MOMENT WHEN ENGINE WAS GRADUALLY BROUGHT TO FINAL REST.

No. of Expt.	Initial Velocity.	Break Effect.				Max. motion during the previous running of the train.
		Direction of 1st Displacement.	1st Vibration (Transverse).			
			1st Displacement $=a$	2nd Displacement T' $=2a$		
		DOWN TRAIN.				
	m/h		mm	mm	sec.	mm
15	4.5	Toward Kyoto	0.39	0.46	0.34	0.16
17	(?)	,,	0.32	0.43	0.33	0.20
19	6.8	,,	0.36	0.42	0.30	0.57
Mean	**5.7**	....	**0.36**	**0.44**	**0.32**	**0.31**
		UP TRAIN.				
16	5.5	Toward Ayabe	0.31	0.45	0.33	0.38
18	7.6	,,	0.23	0.35	0.33	0.22
20	6.0	,,	0.29	0 41	0.33	0.17
Mean	**6.4**	....	**0.28**	**0.40**	**0.33**	**0.26**

According to Table IV, the mean values of the first and second displacements of the first break-effect vibration were, taking together the cases of the up and down trains, as follows :—

1st displacement (a) : 0.32 mm.
2nd ,, ($2a$) : 0.42 ,,

Thus the mean values of the 1st and 2nd displacements were respective by 0.32 and 0.42 mm ; these being not less than the mean maximum vibration during the running of the train in the different experiments. The significance of the break-effect may be seen from the consideration that the 1st displacement is a single, and not the double, amplitude, and consequently equivalent to a vibration double in magnitude. Had a heavy train running with a high velocity over a bridge been suddenly stopped on the latter, the effect on the pier would be enormous and might prove dangerous for the stability of the piers.

The direction of the 1st displacement of the break-effect vibration was, always contrary to that of the progress of the train, namely, it was directed, in the cases of the down train, toward Kyoto, and, in the cases of the up train, toward Ayabe. It thus seems that a train passing over a bridge pushes along with it the different spans and the piers forward through a small range, causing the piers to bound back when the engine is brought to complete stoppage.

The mean period of the vibrations due to the break-effect was 0.32 or 0.33 sec., being equal to the t_1, which was supposed to characterize the most active vibrations due to the proper oscillation of the pier in question (§ 12).

14. *Inclination of pier.* An inspection of the diagrams of the motion of the pier shows that the pointer of the transverse component suffered during the passage of the train over the 200′ truss a change of its zero-position, which was toward Kyoto end and which attained its maximum when the engine was at the middle of the span. (See figures 11-16 and 18-21.) Now this pointer displacement was due to nothing else than the inclination of the top surface of the pier, on which the vibration recorder was set up, in consequence of the deflection of the truss under the weight of the engine. The longitudinal component diagrams indicated no such effect.

In Expts. Nos. 15 to 20, the amount (r) of the pointer displacement on the diagrams was from 5.4 to 7.5 mm, giving the average of 6.0 mm. The corresponding inclination (θ) of the pier top can be estimated by the formula :

$$\theta = \frac{r\,T_0^2}{n\,L\,T^2}$$

in which n, L, T_0 and T are the instrumental constants of the horizontal tremor recorder, as follows :

n = Magnification ratio of the pointer = 30 times.

L = Length of the horizontal pendulum, or the distance between its axis and the junction to the magnifying pointer = 186 mm.

T = Oscillation period of the horizontal pendulum as actually set up = 2.3 sec.

T_0 = Oscillation period of the same, when suspended as an ordinary pendulum = 0.52 sec.

The inclination θ is found by calculation to be about 11″.5, this being the effect due to the passage of the train with a single engine No. 8302.

EXPERIMENTS ON THE SHICHIMI-GAWA BRIDGE PIER.

15. *Bridge and experiments.* The single track Shichimi-*gawa* bridge has a total span of 626′ 8″$\frac{3}{8}$, and consists of four 40′ plate girders (Kyoto end), two 200′ deck trusses, and one 40′ plate girder (Ayabe end), supported by two abutments and six piers of masonry, (Figs. 8 and 10), whose heights are, including in each case the thickness of the foundation, as follows :—

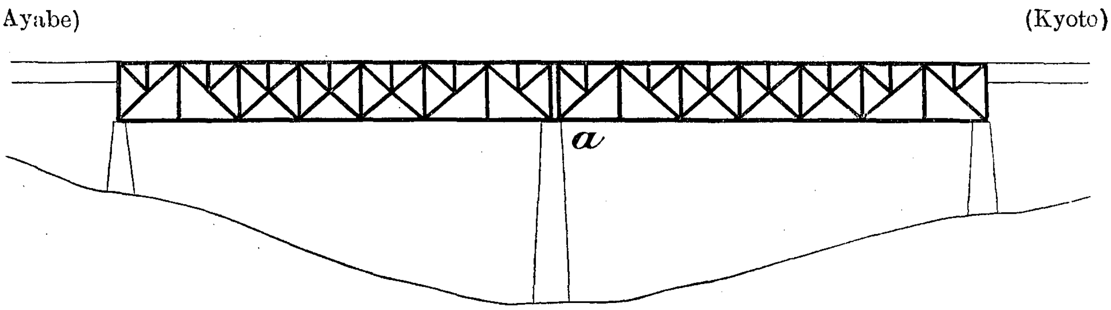
Kyoto and Abutment	23′ 6″
1st Pier	26′ 1″
2nd ,,	50′ 0″$\frac{1}{2}$
3rd ,,	64′ 2″$\frac{1}{2}$
4th ,,	39′ 10″
5th ,,	82′ 1″$\frac{1}{2}$
6th ,,	26′ 10″
Ayabe end Abutment	18′ 6″

The horizontal vibration recorder was set up on the top surface of the 5th, or tallest, pier, whose height is 82′ 1″$\frac{1}{2}$, and for which the vertical distance between the foundation and the rail level is 114′ 4″. (See Fig. 4.) The pier supports the free roller ends of the two 200′ trusses. The down end of the 4th and the up end of the Ayabe side 40′ plate girder were each attached to the upper part of the fixed-end side frame of the 200′ trusses. The top and the base sections of the pier proper or the part above the foundation (thickness=4′) are respectively 8′ 6″×24′ 0″ and 18′ 5″×30′ 5″. The sides of the pier have a uniform batter of 1 : 24, while the faces have three different batters according to height, as follows :—

First	40′ 0″	....	1 : 24
Middle	25′ 0″	....	1 : 12
Lowest	12′ 1″	....	1 : 10

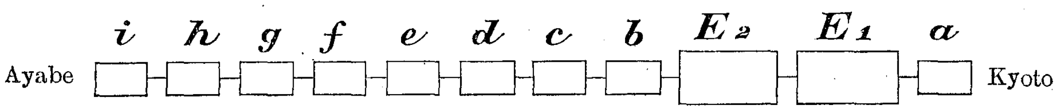
The measurement was repeated 15 times, the different experiments being numbered 1 to 15. In all these cases, with the exception of Expt. No. 15, the train was composed of two engines Nos. 8301 and 8302, and one break-van and eight open goods wagons (Fig. 5), of which one was empty and the others were each loaded with about 20 tons of rails ; the weight of the break-van was about 10.4 tons. In Expt. No. 15, the train consisted only of Engine No. 8301 and a wagon. The direction and the velocity of the train in each of the different experiments are given in Table V.

Fig. 4.



The vibration recorder was set up on the top of the pier (*a*)

Fig. 5.



E_1 ....Engine No. 8301　　　*b*........Break-van

E_2 .... „ No. 8302　　　*a*, *c*, *d*,....*i*....goods wagons..

Transverse and Longitudinal Vibrations of Pier No. 5, Shichimi-gawa Bridge.

16. *Vibrations of pier caused by passage of train: Analysis of diagrams.*

EXPT. No. 1. Up Train. Velocity$=3^{m}/_{h}$.

Transverse component.

(i) For 17 $\frac{1}{4}$ sec., motion very small.
(ii) „ 4.2 „ : $T=0.29$ sec., $2a=0.015$ mm.
(iii) „ 38.6 „ : $T=0.35$–0.36 sec.; $T=0.41$ sec., $2a=0.07$ mm.
(iv) „ 12.6 „ : $T=0.35$ sec., $2a=0.04$ mm; $T=0.31$ sec.
(v) „ 11.1 „ : motion small: $T=0.30$ sec., $2a=0.03$ mm.
(vi) „ 12.3 „ : „ again slow: $T=0.42$ sec., $2a=0.05$ mm.

The motion, which thereafter became small, was continued for 80 sec. more: $T=0.32$ sec., $2a=0.03$ mm.

Longitudinal component. For the first 9.9 sec., the motion was small, but much more distinct than in the transverse component: $T=0.40$ sec., $2a=0.02$ mm, being mixed with some traces of smaller quick vibrations. The subsequent motion was made up of the different vibrations as follows:—

$T=0.18$ sec., $2a=0.03$ mm;	$T=0.73$ sec., $2a=0.13$ mm.
$T=0.19$ „ , ———— ;	$T=0.69$ „ , $2a=0.15$ „
$T=0.20$ „ , $2a=0.07$ mm;	$T=0.68$ „
$T=0.36$ „ , $2a=0.09$ „ ;	$T=0.67$ „ , $2a=0.07$ mm.
$T=0.32$ „ , $2a=0.06$ „ ;	$T=0.61$ „

EXPT. No. 2. Down Train. Velocity$=4^{m}/_{h}$.

Transverse component.

(i) For 56.6 sec., motion very small: $T=0.30$ sec., $2a=0.02$ mm.
(ii) „ 10.7 „ „ more pronounced: $T=0.34$ sec., $2a=$ 0.04 mm.

(iii) For 15.1 sec., there was a maximum group; $T=0.41$ sec., $2a=0.07$ mm.

(iv) For 15.9 sec., motion quicker and small except for a single maximum group composed of about 7 vibrations: $T=0.31$ sec., $2a=0.053$ mm.

(v) For 18.1 sec., motion again larger and slower: $T=0.38$ sec., $2a=0.11$ mm.

For the remaining 22 sec., the motion became suddenly smaller and quicker: $T=0.33$ sec., $2a=0.03$ mm.

Longitudinal component.

(i) For 15.2 sec., motion small and regular: $T=0.20$ sec., $2a=0.03$ mm.

(ii) For 34.8 sec., amplitude was nearly constant:
$T=0.66$ sec., $2a=0.07$ mm; $T=0.18$ sec., $2a=0.05$ mm.

(iii) For 21.5 sec., the vibrations were largest and regular:
$T=0.65$ sec., $2a=0.17$ mm; $T=0.31$ sec., $2a=0.13$ mm.

(iv) For 9.0 sec.. motion small, this epoch corresponding to a maximum portion in the transverse component.

(v) For 22.6 sec.; $T=0.34$ sec., $2a=0.12$ mm.

Then the motion gradually decreased, continuing for further 36 sec.

EXPT. No. 3. Up Train. Velocity$=17^{m}/_{h}$.

Transverse component.

(i) For 5.5 sec., motion small: $T=0.28$ sec., $2a=0.03$ mm.

(ii) ,, 2.6 ,, ,, larger: $T=0.29$ sec., $2a=0.08$ mm.

(iii) ,, 4.8 ,, ,, large and slow: $T=0.53$ sec., $2a=0.26$ mm.

(iv) For 6.2 sec., ,, regular: $T=0.33$ sec., $2a=0.20$ mm.

The subsequent motion was small and regular: $T=0.32$ sec., $2a=0.05$ mm.

Longitudinal component.

(i) For 2.5 sec., motion practically zero.

(ii) „ 3.5 „ , „ regular; $T=0.20$ sec., $2a=0.08$ mm.

(iii) „ 7.4 „ , „ larger, mixed with slower movements.

(iv) For 14.0 „ , motion largest:

$T=0.78$ sec., $2a=0.60$ mm; $T=0.26$ sec., $2a=0.17$ mm.

(v) For 8.0 sec., motion smaller; $T=0.62$ sec., $2a=0.17$ mm at first, thence gradually decreasing. There were also small movements of $T=0.19$ sec.

(vi) For the last 21.5 sec., the motion was regular: $T=0.61$ sec., $2a=0.05$ mm, mixed with some slight traces of quicker vibrations.

EXPT. No. 4. Down Train. Velocity$=9^{m}/_{h}$.

Transverse component.

(i) For 10.8 sec., motion very small.

(ii) „ 6.9 „ , „ regular: $T=0.29$ sec., $2a=0.02$ mm.

(iii) „ 15.7 „ , „ larger and uniform: $T=0.32$ sec., $2a=0.04$ mm.

(iv) For 15.7 sec., „ much larger and uniform: $T=0.32$ sec., $2a=0.16$ mm.

(v) For 6.8 sec., motion largest and slower: $T=0.40$ sec., $2a=0.21$ mm.

(vi) For 5.3 sec., motion very small: $T=0.35$ sec., $2a=0.08$ mm. The subsequent motion was very small: $T=0.30$ sec.

Longitudinal component.

(i) For 12.7 sec., motion gradually increased from zero up to $2a=0.03$ mm, $T=0.26$ sec.

(ii) For 16.1 sec., motion much larger and regular: $T=0.24$ sec., $2a=0.12$ mm.

(iii) For 19,5 sec., motion most active and neary uniform:

$T=0.24$ sec., $2a=0.27$ mm; $T=0.21$ sec., $2a=0.20$ mm.

(iv) For next 14.0 sec., the vibrations were smaller, but there appeared the pendulum oscillations of $T=2.7$ sec., $2a=0.67$ mm, mixed with vibration of $T=0.19$ sec., $2a=0.18$ mm.

(v) For the last 16.0 sec., motion small: $T=0.60$ sec.

EXPT. No. 5. Up Train. Velocity$=22^{m}/_{h}$.

Transverse component.

(i) For 2.0 sec., motion practically zero.

(ii) „ 2.0 „ , „ small but distinct: $T=0.31$ sec.

(iii) „ 14.5 „ , „ most active and regular: $T=0.35$ sec., $2a=0.32$ mm, mixed with pendulum oscillations of $T=3.2$ sec., $2a=0.67$ mm.

(iv) In the subsequent portion, motion small: $T=0.33$ sec., $2a=0.07$ mm, grouped into vibrations of $T=0.64$ sec.

Longitudinal component.

(i) For 2.1 sec., motion was zero.

(ii) „ 1.6 „ $T=0.21$ sec., $2a=0.08$ mm.

(iii) „ 3.9 „ motion well-defined and uniform: $T=0.19$ sec., $2a=0.27$ mm.

(iv) „ 8.7 sec., motion largest: $T=0.21$ sec., $2a=0.40$ mm, being mixed with slower vibration.

(v) For 13.8 sec., motion constant: $T=0.63$ sec., $2a=0.29$ mm; $T=0.32$ sec., $2a=0.13$ mm.

(vi) During the last 20 sec., motion very small: $T=0.26$ sec., $2a=0.05$ mm.

EXPT. No. 6. Down Train. Velocity$=27^{m}/_{h}$.

Transverse component. The motion increased from zero, in the course of the 1st 15.0 sec., up to $2a=0.12$ mm, $T=0.30$ sec. For the next 11.5 sec., the motion was regular and largest: $T=0.35$ sec., $2a=0.45$ mm. The subsequent motion became abruptly much smaller: $T=0.61$ sec., $2a=0.02$ mm; $T=0.30$ sec.

Longitudinal component.

(i) For 5.8 sec., motion zero.

(ii) „ 6.8 „ „ gradually increased:

$T=0.19$ sec., $2a=0.13$ mm; $T=0.31$ sec., $2a=0.12$ mm.

(iii) For 14.8 sec., motion most active:

$T=0.20$ sec., $2a=0.38$ mm; $T=0.29$ sec., $2a=0.27$ mm.

Thereafter the motion gradually decreased: $T=0.21$ sec., $2a=0.09$ mm; the motion continuing for further 25 sec., and being as usual much greater than in the transverse component.

EXPT. No. 7. Up Train. Velocity$=25^{m}/_{h}$.

Transverse component. In the most active portion, which lasted 11.8 sec., the vibration were as follows: $T=0.33$ sec., $2a=0.47$ mm. The subsequent portion was much smaller: $T=0.62$ sec., $2a=0.03$ mm; $T=0.32$ sec., $2a=0.07$ mm.

EXPT. No. 8. Down Train. Velocity$=27^{m}/_{h}$.

Transverse component. Preliminary portion: $T=0.29$ sec., $2a=0.02$ mm. Principal portion: $T=0.30$ sec., $2a=0.58$ mm. End portion: $T=0.32$ sec., $T=0.64$ sec.

Longitudinal component. In the most active portion: $T=0.21$ sec., $2a=0.41$ mm; $T=0.29$ sec., $2a=0.45$ mm; $T=0.32$ sec., $2a=0.32$ mm. In the end portion: $T=0.20$ sec., $2a=0.05$ mm.

17. *Observation of effect on pier caused by stopping train on bridge; Analysis of diagrams.*

EXPT. No. 9. Up Train, the engines (initial velocity$=6.5^{m}/_{h}$) stopping at the middle of the 2nd 200′ truss.

Transverse component. Preliminary portion: $T=0.32$ sec. The most active part: $T=0.36$ sec., $2a=0.23$ mm. In the subsequent portion, the period was $T=0.30$ sec. The break effect was only about 0.08 mm$=(2a)$.

Longitudinal component. Preliminary portion: $T=0.20$ sec. In the earlier part of the principal portion: $T=0.64$ sec., $2a=0.087$ mm; $T=0.23$ sec., $2a=0.12$ mm. In the most active portion: $T=0.38$ sec., $2a=0.22$ mm; $T=0.85$ sec., $2a=0.25$ mm, there being also other smaller vibrations.

EXPT. No. 10. Down Train, the engines (initial velocity$=6.8^{m}/_{h}$) stopping on the 1st 200′ Truss.

Transverse component. Maximum portion: $T=0.32$ sec., $2a=0.17$ mm. Just before the engine came to rest: $T=0.39$ sec., $2a=0.093$ mm. The break effect was 0.073 mm$(=2a)$, toward Ayabe.

Longitudinal component. In the most active portion, motion consisted principally of vibration $T=0.20$ sec., $2a=0.27$ mm.

EXPT. No. 11. Up Train, the engines (initial velocity$=10.2^{m}/_{h}$). stopping on the 1st 200′ Truss.

Transverse component. In the preliminary portion: $T=0.27$ sec., $2a=0.03$ mm. In the maximum portion, which preceded immediately the stoppage of the engine: $T=0.38$ sec., $2a=0.083$ mm. The break effect was 0.10 mm$(=2a)$. The subsequent motion was: $T=0.38$ sec., $2a=0.063$ mm.

Longitudinal component. In the most active portion: $T=0.20$ sec., $2a=0.12$ mm. In the portion preceding immediately the stoppage of the engine: $T=0.46$ sec., $2a=0.083$ mm.

EXPT. No. 12. Up Train, the engines (initial velocity$=12^{m}/_{h}$). stopping on the pier.

Transverse component. In the preliminary portion; $T=0.29$ sec., $2a=0.02$ mm. In the most active portion: $T=0.36$ sec., $2a=0.38$ mm. Immediately before the stoppage of the engine: $T=0.30$ sec., $2a=0.03$ mm. The break effect was:

1st displacement: 0.043 mm ($=a$) toward Ayabe,
2nd ,, : 0.11 ,, ($=2a$) ,, Kyoto;

the period being $T=0.31$ sec.

Longitudinal component. In the most active portion: $T=0.21$ sec., $2a=0.23$ mm. Immediately before, and during, the stoppage of the engine: $T=0.50$ sec., $T=0.20$ sec.

The motion after the re-start of the engine was as follows.

Transverse component: $T=0.32$ sec., $2a=0.11$ mm;

Longitudinal ,, : $T=0.21$,, $2a=0.14$,, ; $T=0.31$ sec, $2a=0.28$ mm; $T=0.80$ sec., $2a=0.25$ mm; $T=1.33$ sec., $2a=0.08$ mm.

EXPT. No. 13. Down Train, the engines (initial velocity$=12.8^{m}/_{h}$) stopping at the middle of the 1st 200′ Truss.

Transverse component. In the earlier part of the principal portion: $T=0.33$ sec., $2a=0.14$ mm. In the most active part: $T=0.36$ sec., $2a=0.19$ mm. The vibrations which occurred immediately before the stoppage of the engine were: $T=0.40$ sec., $2a=0.05$ mm. The break effect was as follows:—

1st displacement: 0.053 mm ($=a$). toward Kyoto.
2nd ,, : 0.10 ,, ($=2a$), ,, Ayabe.

Longitudinal component. In the preliminary portion: $T=0.50$ sec., $2a=0.14$ mm, mixed with quick vibrations. In the most active portion: $T=0.20$ sec., $2a=0.27$ mm.

EXPT. No. 14. Up Train, the engines (initial velocity$=16.4^{m}/_{h}$) stopping at the middle of the 2nd 200′ Truss.

Transverse component. In the most active portion : $T=0.33$ sec., $2a=0.20$ mm. No break effect was produced.

Longitudinal component. $T=0.20$ sec., $2a=0.18$ mm.

The motion after the re-start of the engine was as follows.

Transverse component. In the most active portion : $T=0.33$ sec., $2a=0.10$ mm. In the end portion : $T=0.60$ sec., $2a=0.03$ mm ; $T=0.30$ sec.

Longitudinal component : $T=0.28$ sec., $2a=0.33$ mm ; $T=0.20$ sec., $2a=0.20$ mm ; $T=0.88$ sec., $2a=0.26$ mm.

EXPT. No 14′. Down Train, the engines not stopping on the bridge.

Transverse component : $T=0.34$ sec., $2a=0.10$ mm.

Longitudinal ,, : $\begin{cases} T=0.29 \text{ ,, } 2a=0.38 \text{ ,,} \\ T=0.19 \text{ ,, } 2a=0.18 \text{ ,,} \end{cases}$

EXPT. No. 15. Down Train ; 1 Engine (initial velocity$=10.2^{m}/_{h}$) and a goods wagon.

Transverse component : $T=0.34$ sec., $2a=0.13$ mm.

There was almost no break effect.

Longitudinal component : $\begin{cases} T=0.31 \text{ sec., } 2a=0.16 \text{ mm.} \\ T=0.19 \text{ ,, } 2a=0,13 \text{ ,,} \end{cases}$

Summary of Results of Experiments on Shichimi-gawa Bridge Pier.

18. *Amplitude of vibration.* Table V gives, for the different experiments, the maximum transverse and longitudinal vibrations of the pier. The greatest transverse motion of 0.5 mm ($T=0.30$

sec.) occurred with the velocity of 27 miles per hour in Expt. No. 8. the greatest quick-period longitudinal motion of 0.45 mm (T=0.29 sec.) having taken place also in the same case. In fact, the transverse vibration increased with the velocity. The longitudinal vibration of the shorter period also indicates a similar relation.

The ratio of the maximum transverse to the maximum longitudinal motion varied, in the different experiments, from 0.47 to 1.36, giving the mean value of 0.81. Thus the pier, of which the width is twice of the thickness, is very abnormal in its behaviour and moves about 1.4 times more in the longitudinal than in the transverse component.

TABLE V. VIBRATIONS OF SHICHIMI-GAWA BRIDGE PIER.

No. of Experiment.	Direction.	Velocity	Transverse Vibration.		Longitudinal Vibration.		RATIO: Transv. $2a$ / Longit. $2a$
			$2a$	T	$2a$	T	
		m/h	mm.	sec.	mm	sec.	
1	Up	3	0.07	0.41	{0.15 0.09	0.69 0.36	0.47
2	Down	4	0.11	0.38	{0.17 0.13	0.65 0.31	0.65
3	Up	17	{0.26 0.20	0.53 0.33	{0.60 0.17	0.78 0.26	0.43
4	Down	9	{0.21 0.16	0.40 0.32	{0.27 0.18	0.24 0.19	0.78
5	Up	22	0.32	0.35	{0.40 0.29	0.21 0.63	0.80
6	Down	27	0.45	0.35	{0.38 0.27	0.20 0.29	1.18
7	Up	25	0 47	0.33	—	—	—
8	Down	27	0.58	0.30	{0.45 0.41	{0.29 0.21	1.29

No. of Experiment.	Direction.	Velocity.	Transverse Vibration.		Longitudinal Vibration.		RATIO : Transv. 2a / Longit. 2a
			2a	T	2a	T	
		m/h	mm.	sec.	mm.	sec.	
9*	Up	6.5	0.23	0.36	{0.22 0.25	0.38 0.85	0.92
10*	Down	6.8	0.17	0.32	0.27	0.20	0.63
11*	Up	10.2	0.083	0.38	0.12	0.20	0.69
12*	Up	12.0	0.38	0.36	{0.23 0.28 0.25	0.21 0.31 0.80	1.36
13*	Down	12.8	0.19	0.36	0.27	0.20	0.70
14*	Up	16.4	0.20	0.33	{0.33 0.20 0.26	0.28 0.20 0.88	0.61
15*	Down	10.2	0.13	0.34	{0.16 0.13	0.31 0.19	0.81
mean	....	.....	....	....	....	....	**0.81**

19. ***Periods of transverse vibration.*** The period of vibration in the preliminary and end portions varied between 0.27 and 0.33 sec., giving the mean value of 0.30 sec. ($=t$). This is probably the period of the proper transverse motion of the pier, that is to say, the pier supporting the heavy truss weight. According to Expts. Nos. 1–15, the period of the most active vibrations which varied between 0.30 and 0.40 sec., was as follows :—

sec.	sec.
0.30 ($2a=0.58$ mm)	0.35 ($2a=0.45$ mm).
0.32	0.35 ($2a=0.32$ „)
0.32	0.36 ($2a=0.38$ „)
0.33 ($2a=0.47$ mm)	0.36
0.33	0.36
0.33	0.38
0.33	0.40
0.34	*mean* **0.34** sec. $=t'$

Thus the mean value of the period of the most active vibrations was 0.34 sec. ($=t'$). The period of the vibrations in the principal portion, in general, varied, in the different experiments, between 0.29 and 0.42 sec., giving the average value also of 0.34 sec. ($=t'$). The t' may be regarded as being equal to the t with lengthening due to the weight of the train. In a few cases there were some slow vibrations of $T=0.61$ sec.

20. *Periods of longitudinal vibration.* The vibrations in the principal portion of the longitudinal component motion were essentially of three different periods, which, according to Expts. Nos. 1–15, varied respectively between 0.18 and 0.24 sec., between 0.28 and 0.38 sec., and between 0.46 and 0.88 sec., as follows :—

0.18 sec.	0.20 sec.
0.18	0.20
0.19 ($2a=0.27$ mm)	0.20
0.19	0.20
0.19	0.21 ($2a=0.41$ mm)
0.19	0.21 ($2a=0.40$ „)
0.19	0.21
0.19	0.21
0.19	0.21
0.20 ($2a=0.38$ mm)	0.21
0.20 ($2a=0.27$ „)	0.21
0.20 ($2a=0.27$ „)	0.23
0.20	0.24
0.20	0.24 ($2a=0.27$ mm)
0.20	0.26
(*mean*)	**0.20** sec. $=t_2$

sec.	sec.
0.28 ($2a$=0.33 mm)	0.46 ($2a$=0.83 mm)
0.29 ($2a$=0.29 ,,)	0.50
0.29 ($2a$=0.38 ,,)	0.60
0.29	0.61
0.31 ($2a$=0.28 mm)	0.62
0.31	0.63 ($2a$=0.29 mm)
0.31	0.64
0.31	0.65
0.32 ($2a$=0.32 mm)	0.66
0.32	0.67
0.32	0.68
0.33	0.69
0.34	0.73
0.36	0.78 ($2a$=0.60 mm)
0.83	0.80 ($2a$=0.25 ,,)
0.32 sec.$=l_1$	0.85 ($2a$=0.25 ,,)
	0.88 ($2a$=0.26 ,,)
	0.76 sec.$=l_3$

The amplitude of motion was greatest in the l_3 vibration, while it was nearly equal in the l_1 and l_2 vibrations. The l_2 vibration occurred most frequently.

The period l_1 (=0.32 sec.) is probably due to the proper oscillation of the pier, while the l_2 (=0.20 sec.) must, as in the case of the Takaya-*gawa* bridge, be equal to half of the period of the vertical movement of the 200′ trusses supported by the pier. The period l_3 is likely to be due to the horizontal vibration of the same trusses in direction normal to their length.

21. ***Rigidity of pier and vibration periods.*** From Tables II and V, it will be seen that the transverse vibration of the

Shichimi-*gawa* bridge pier was considerably less than that of the Takaya-*gawa* bridge pier, while the longitudinal vibration did not much differ in the two cases. Thus confining our attention to the cases of the trains with two engines, we have :—

TABLE VI. COMPARISON OF MAX. 2a'S OF TAKAYA-GAWA AND SHICHIMI-GAWA BRIDGE PIERS.

Velocity.	Ratio of Max. $2a$'s : Takaya/Shichimi.	
	Transverse Component.	Longitudinal Component.
m/h 3- 4	2.9	0.69
7- 9	4.3	2.04
14–17	2.3	1.15
20–25	2.3	1.15
Mean	**3.0**	**1.3**

The transverse and the longitudinal vibrations of the Takaya-*gawa* pier are therefore greater than those of the Shichimi-*gawa* pier respectively in the ratio of 3.0 : 1 and of 1.3 : 1. In other words, the Shichimi-*gawa* bridge pier is, from some reason, very rigid in its transverse direction. Consequently the pier seems to oscillate only with its own natural period t or t', not being affected by the vertical motion of the trusses it supports. The longitudinal vibrations were essentially similar to those in the case of the Takaya-*gawa* bridge.

22. ***Break effect.*** From the consideration in the preceding §, it is evident that the break effect on the motion of the Shichimi-*gawa* bridge pier must be very slight. Thus, the transverse vibration ($=2a$) occurring at the moment when the two engines coupled in series were gradually brought to rest was, in the different experiments, less than 0.11 mm, as follows :—

Expt. No.	9	$2a=0.08$ mm.
,, ,,	10	$2a=0.073$,,
,, ,,	11	$2a=0.10$,,
,, ,,	12	$2a=0.11$,,
,, ,,	13	$2a=0.10$,,
,, ,,	14	$2a=0.00$,,
,, ,,	15	$2a=0.00$,,

The initial velocity of the train in these experiments varied between 6.5 and 16.4 miles per hour. The pier also indicated no inclination effect.

CONCLUDING REMARKS.

23. The movements of the Takaya-*gawa* bridge pier may be supposed to be typical also of those cases in which the river bed is made up of a soft muddy formation of great thickness, and in which the piers are always very shaky notwithstanding the considerable depths to which the wells are sunk. Such a pier moves with its own natural period, as well as with the period of the vertical motion of the trusses it supports. On the other hand, the Shichimi-*gawa* bridge is an example of the case in which the transverse pier motion is exceptionally small and not sensibly affected by the vertical vibration of the trusses. The break-effect vibration is markedly shown only by those piers whose transverse strength is small.

The period of the most active transverse and longitudinal vibrations of the Takaya-*gawa* and the Shichimi-*gawa* bridge piers varied from about 0.2 sec. to about 0.4 sec., being in any way much shorter than the period of a destructive earthquake motion, which would be generally about 1.0 to 1.5 seconds. Hence the tall piers in question are to be regarded as " short columns " and would be fractured, in case of a great earthquake, at their bases, differing materially in these respects from high brick chimneys. These latter are " tall columns " and have natural oscillation periods longer than the period of the destructive seismic motion, being broken by an earthquake shock at two-thirds of their hights, and not at the base. Again, as the longitudinal component motion of each of the piers under consideration was by no means insignificant, it is to be expected that the latter would be fructured in a destructive earthquake both in direction parallel, and in direction at right angles, to the bridge length.

Seismological Institute. April 1911.

List of Plates.

Fig. 19. Expt. No. 17.

Pl. XV. Vibration of Pier No. 1 of the Takaya-*gawa* Bridge: Effect caused by stopping the Train on the 200′ Truss.

Up Train; 1 Engine. Magnification = 30.

Fig. 20. Expt. No. 16. Initial Velocity = 5.5 miles per hour.

Fig. 21. ,, ,, 18. ,, ,, = 7.6 ,, ,, ,,

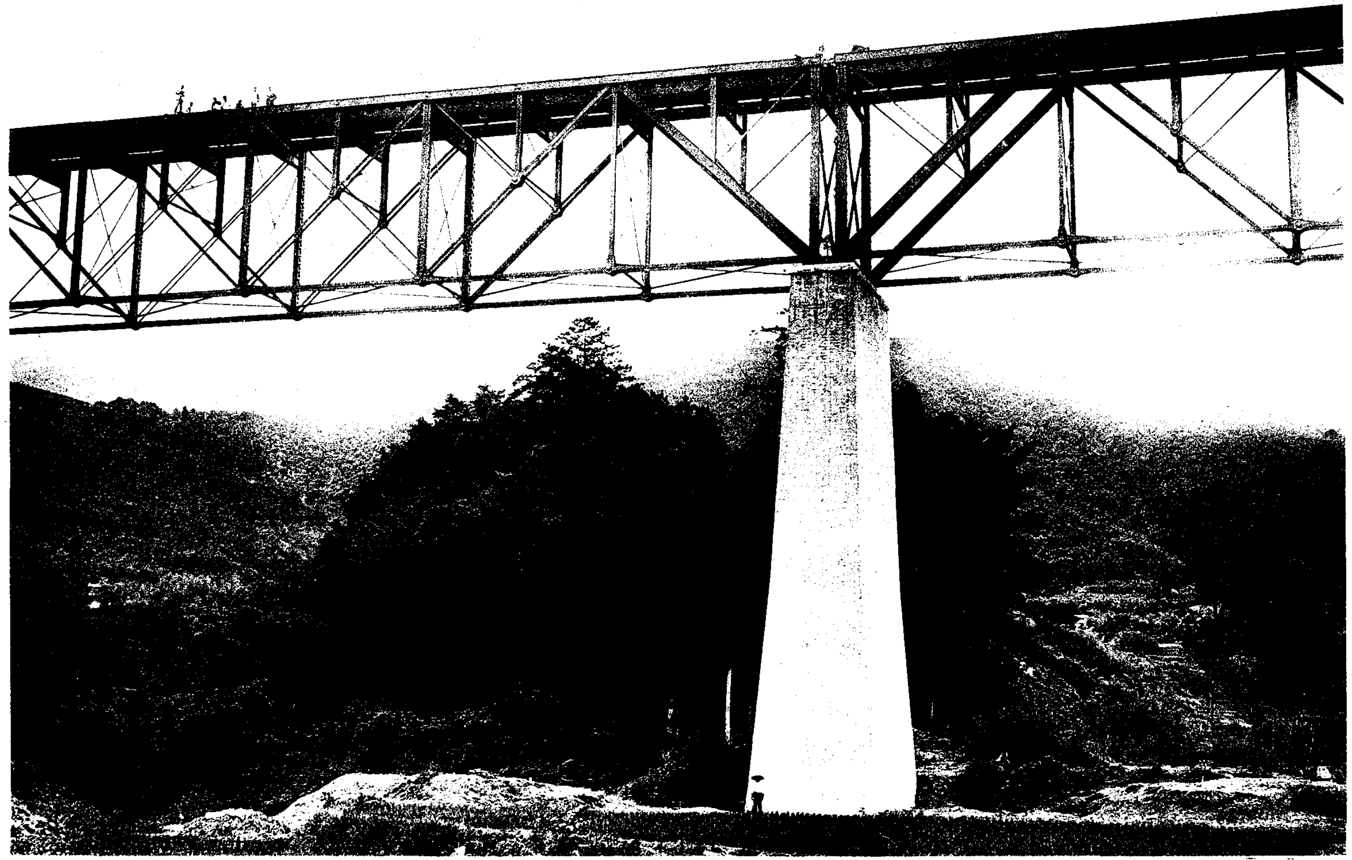
Pl. XVI. Vibration of Pier No. 5 of the Shichimi-*gawa* Bridge.

Magnification = 30.

Fig. 22. Expt. No. 8. Down Train: 2 Engines. Velocity = $27^{m}/_{h}$.

Fig. 23. Expt. No. 12. Up Train: 2 Engines. (Motion after the restart of the train.)

Fig. 6. Pier No. 5 and the two 200′ Trusses, Shichimi-*gawa* Bridge.



The horizontal vibration recorder was set up on the top of the pier.

Fig. 7. The Takaya-*gawa* Bridge. Elevation.

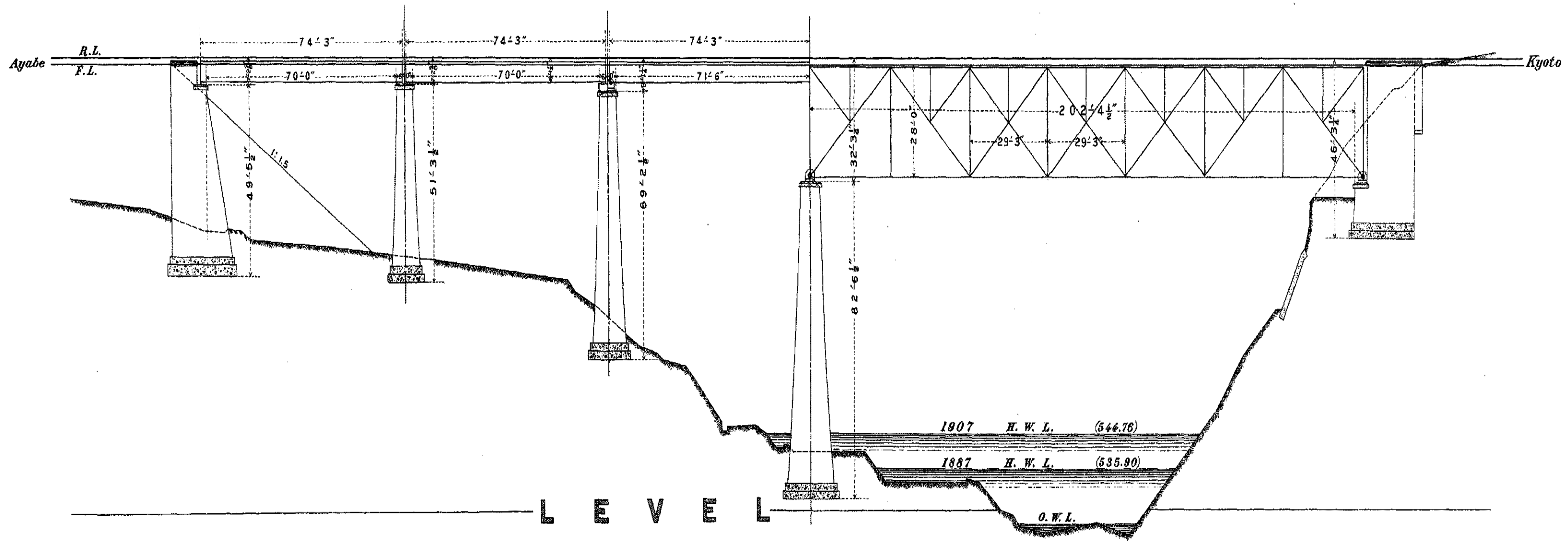


Fig. 8. The Shichimi-*gawa* Bridge. Elevation.

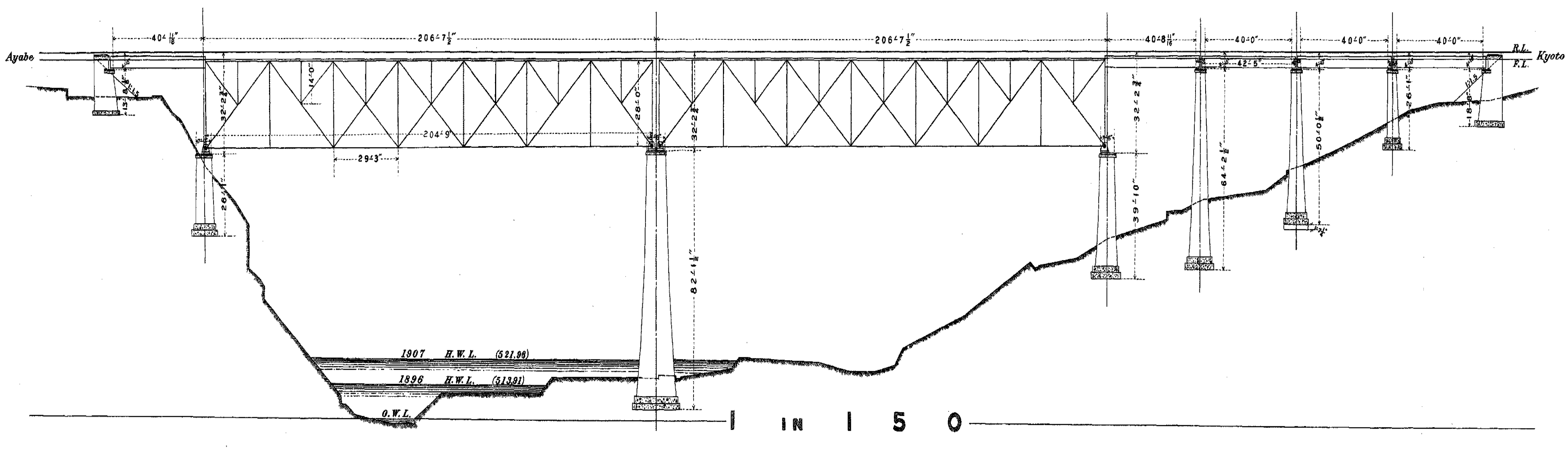


Fig. 9. Elevations and Sections of Pier No. 1 of the Takaya-*gawa* Bridge.

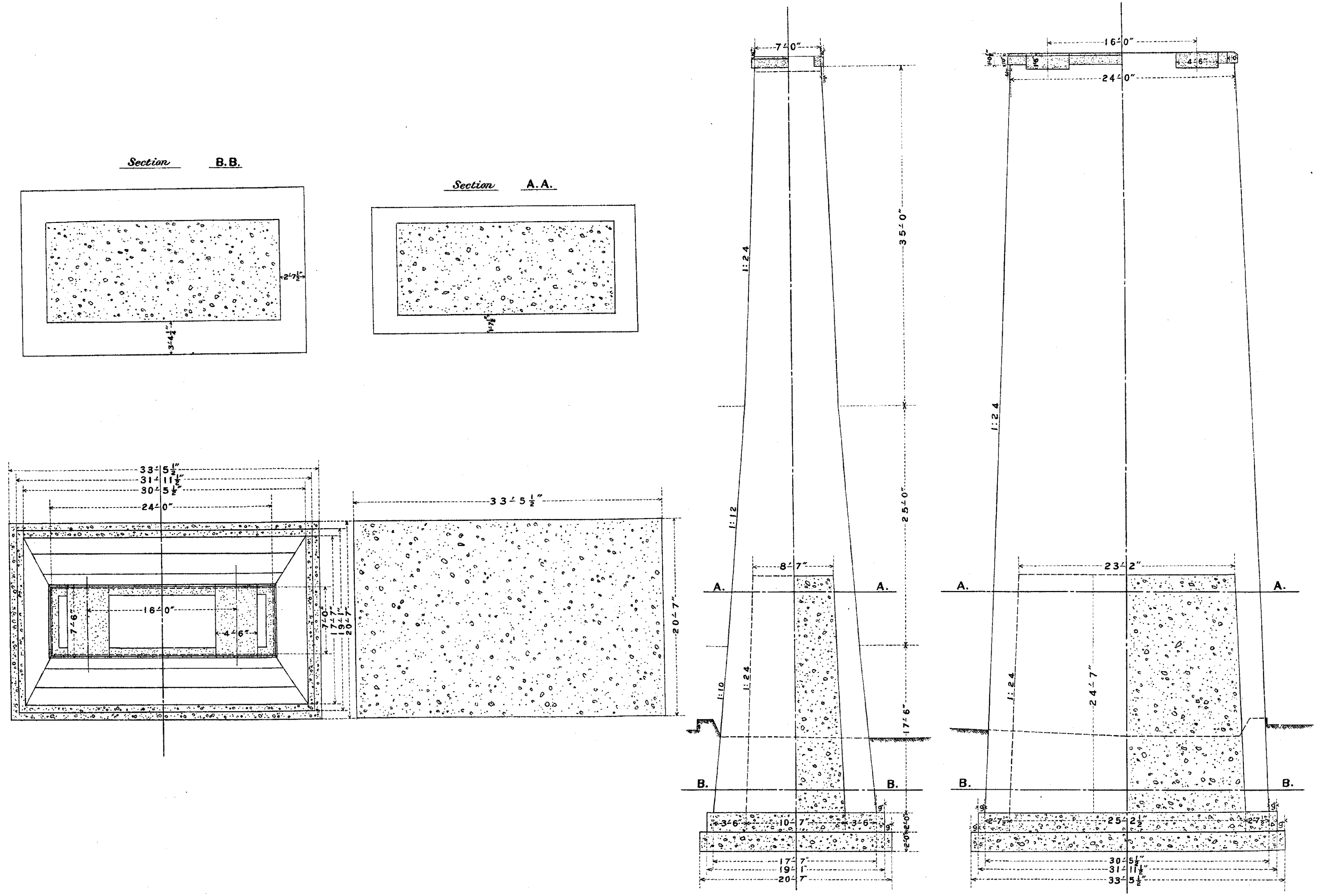
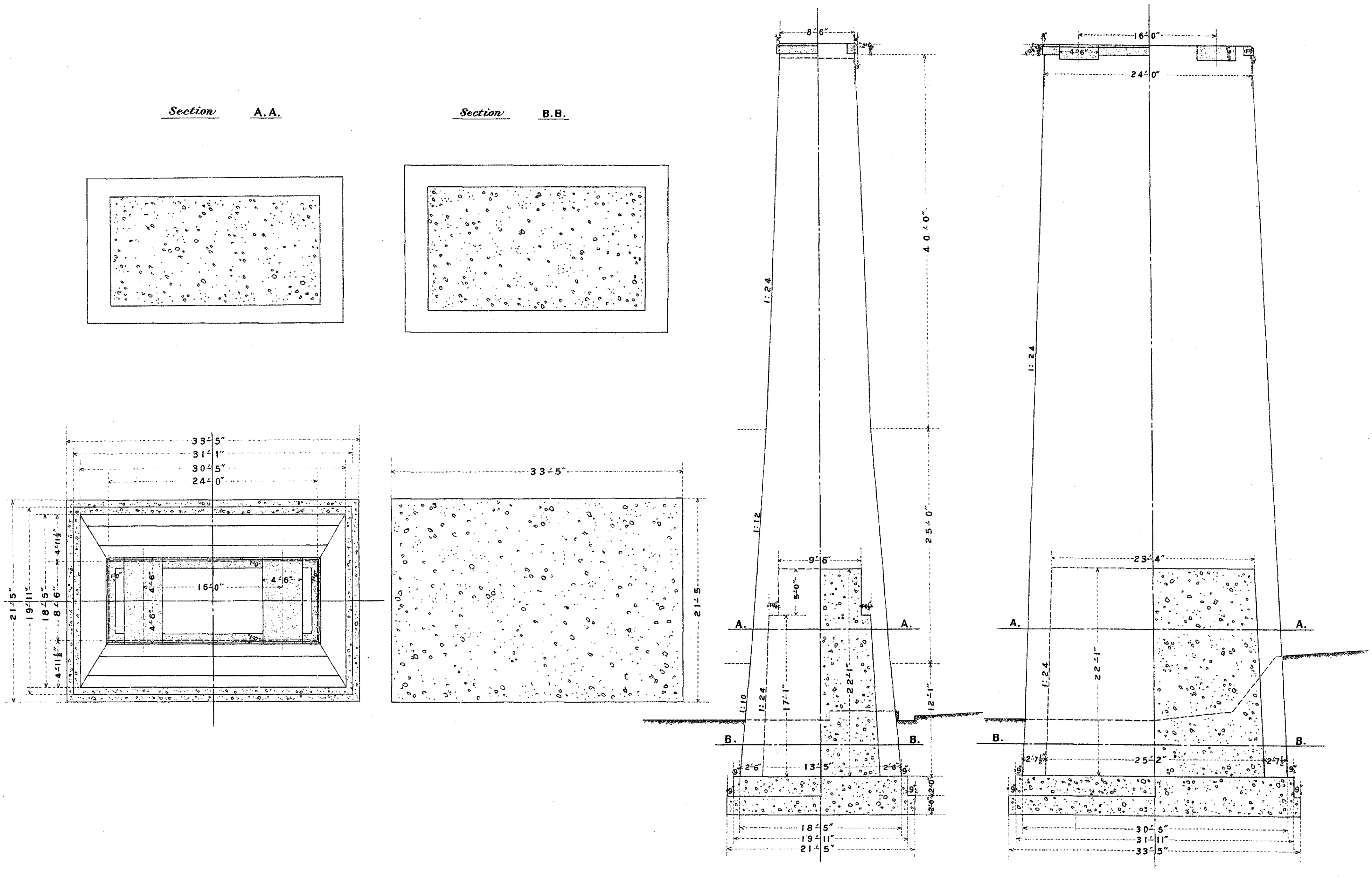


Fig. 10. Elevations and Sections of Pier No. 5 of the Shichimi-*gawa* Bridge.



Vibration of Pier No. 1 of the Takaya-*gawa* Bridge.

Magnification = 30. Up Train; Two Engines.

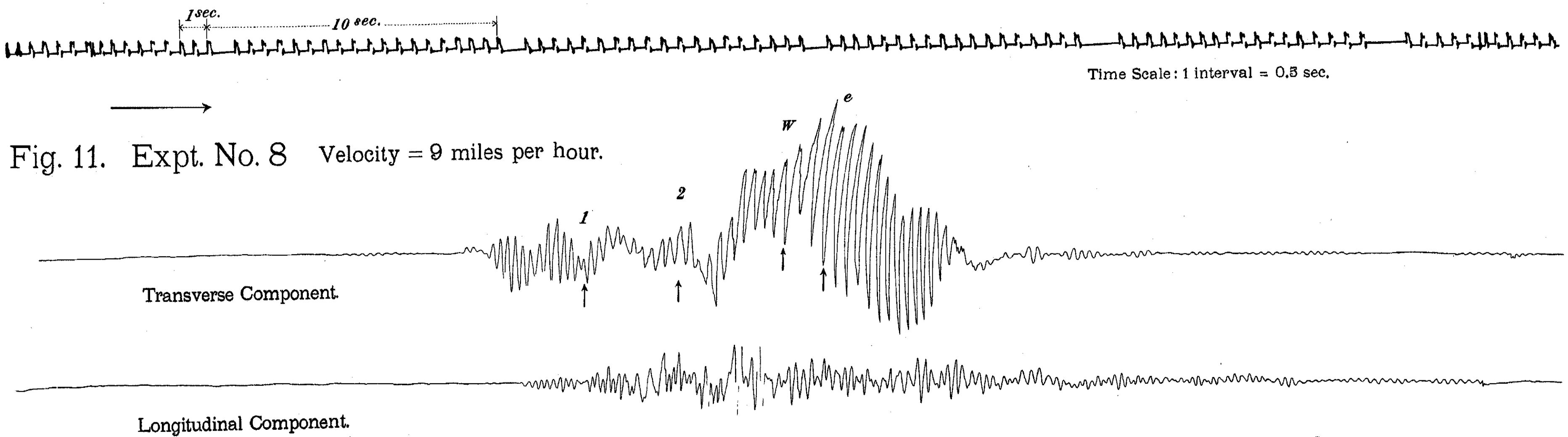


Fig. 11. Expt. No. 8 Velocity = 9 miles per hour.

(1)....1st Engine comes on to the Pier.
(2)....2nd " " " "
(w)....1st wagon passes over the Pier.
(e)....2nd " " " "

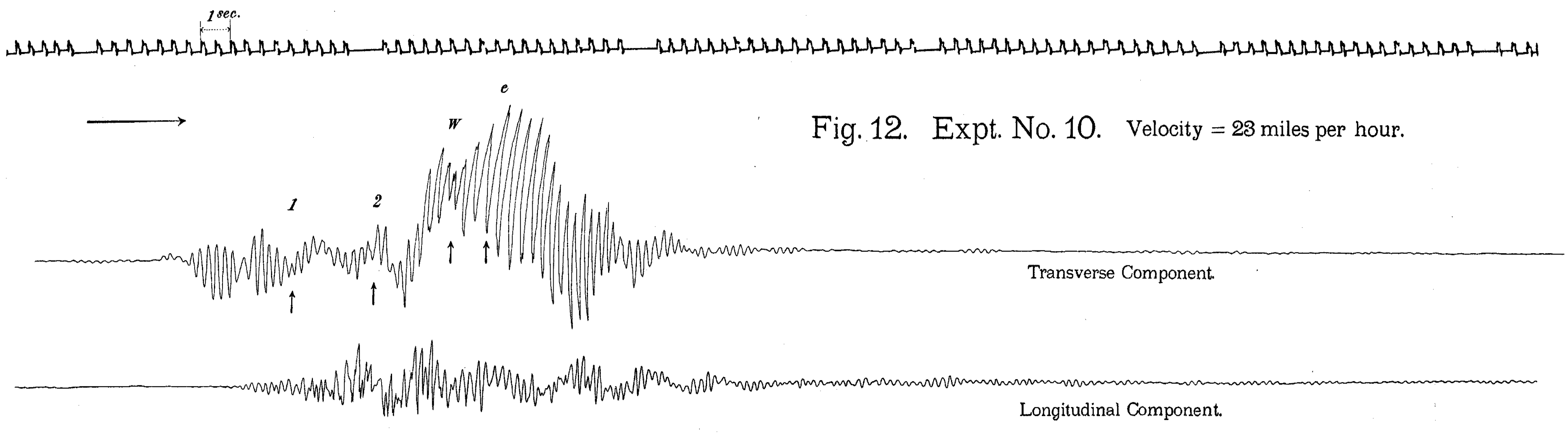


Fig. 12. Expt. No. 10. Velocity = 23 miles per hour.

Vibration of Pier No. 1 of the Takaya-*gawa* Bridge.

Magnification = 30. Down Train; 2 Engines.

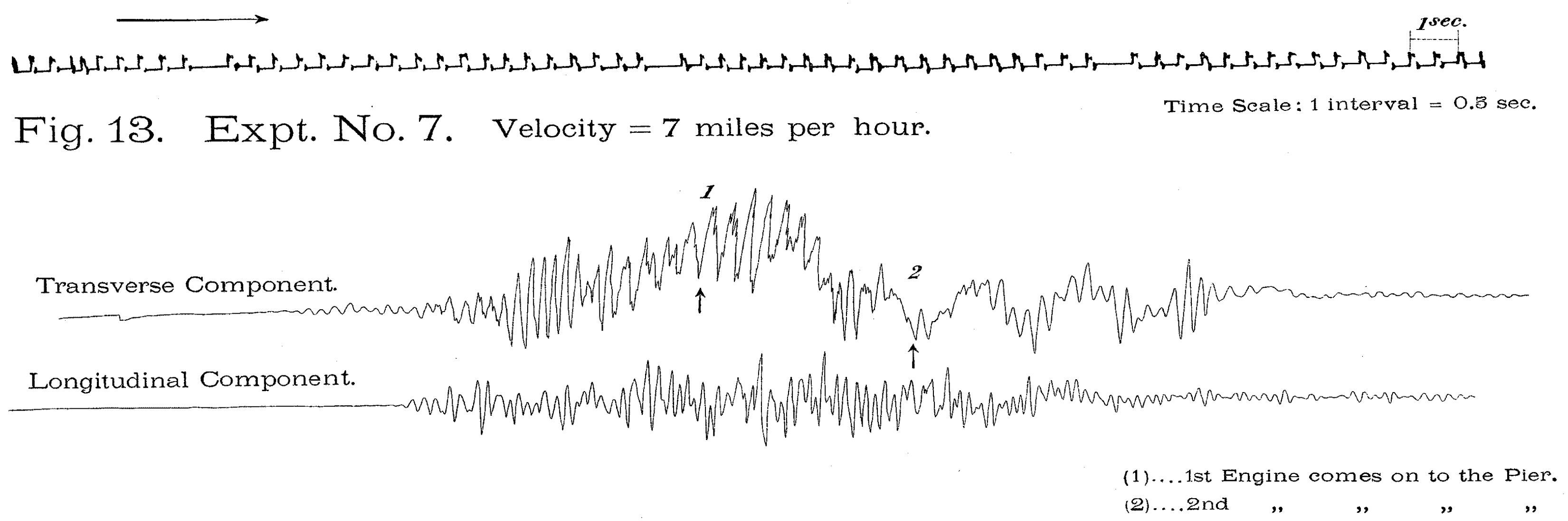


Fig. 13. Expt. No. 7. Velocity = 7 miles per hour.

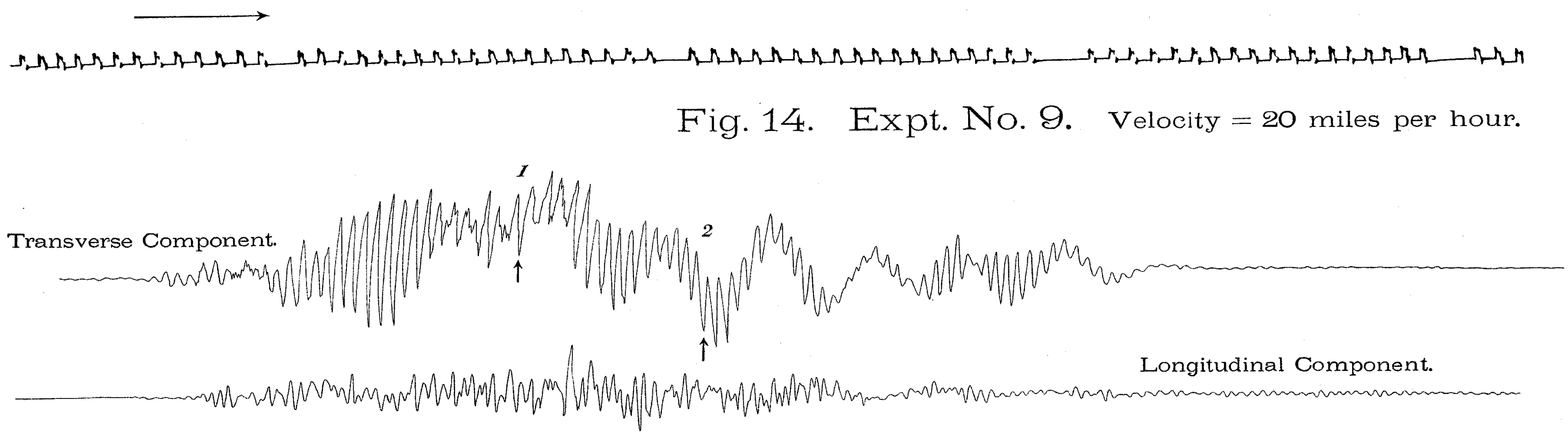


Fig. 14. Expt. No. 9. Velocity = 20 miles per hour.

Vibration of Pier No. 1 of the Takaya-*gawa* Bridge. Magnification = 30.

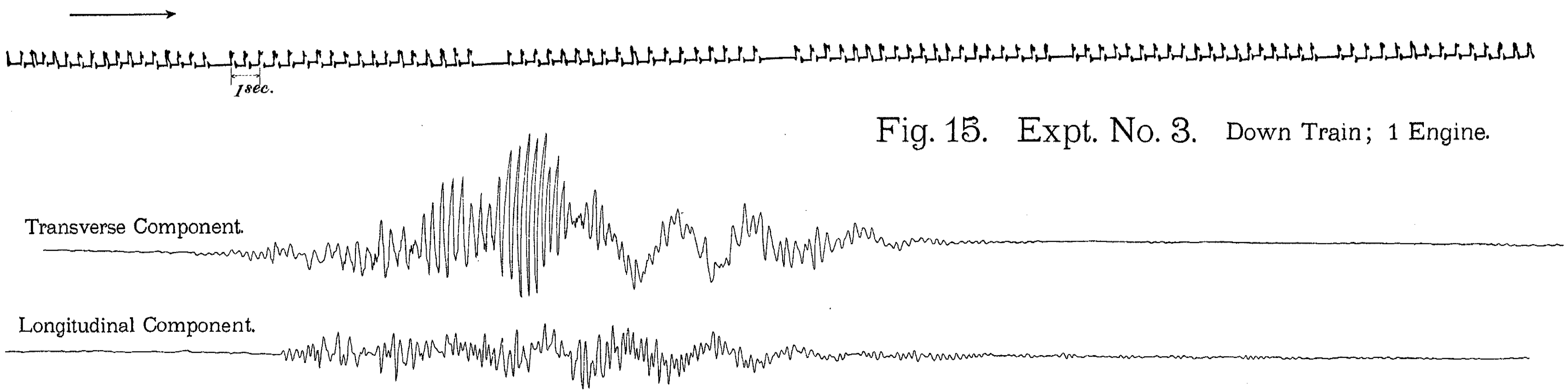


Fig. 15. Expt. No. 3. Down Train; 1 Engine.

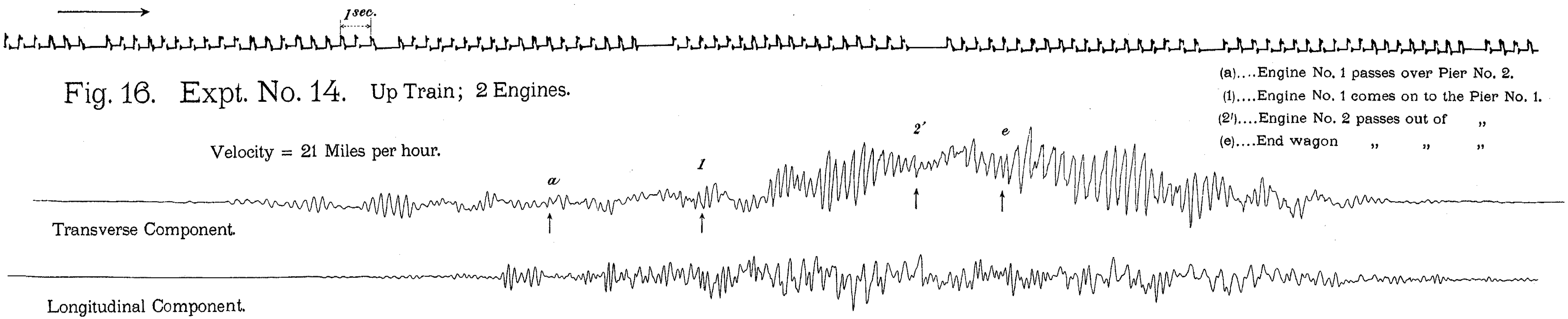
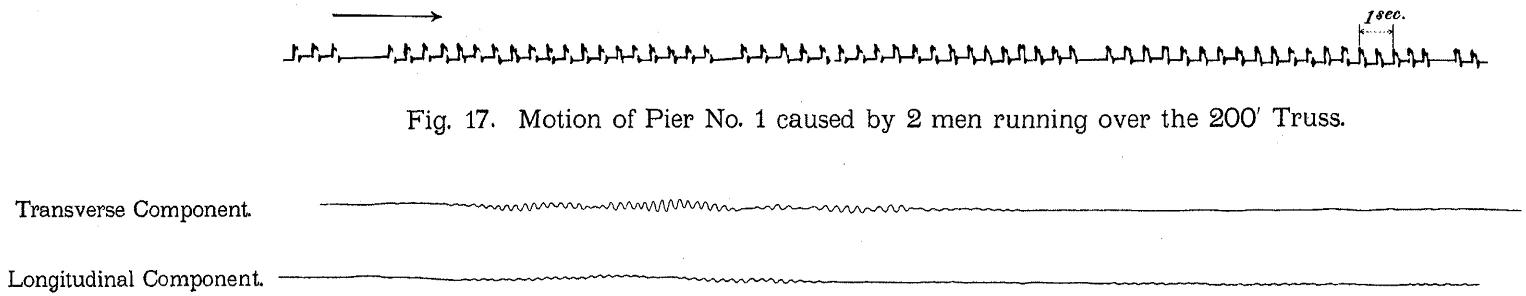


Fig. 16. Expt. No. 14. Up Train; 2 Engines.

Velocity = 21 Miles per hour.

(a)....Engine No. 1 passes over Pier No. 2.
(1)....Engine No. 1 comes on to the Pier No. 1.
(2′)....Engine No. 2 passes out of ,,
(e)....End wagon ,, ,, ,,



Time Scale: 1 interval = 0.5 sec.

Fig. 17. Motion of Pier No. 1 caused by 2 men running over the 200′ Truss.

Vibration of Pier No. 1 of the Takaya-*gawa* Bridge: Effect caused by stopping the Train on the 200′ Truss.

Down Train: 1 Engine. Magnification = 30.

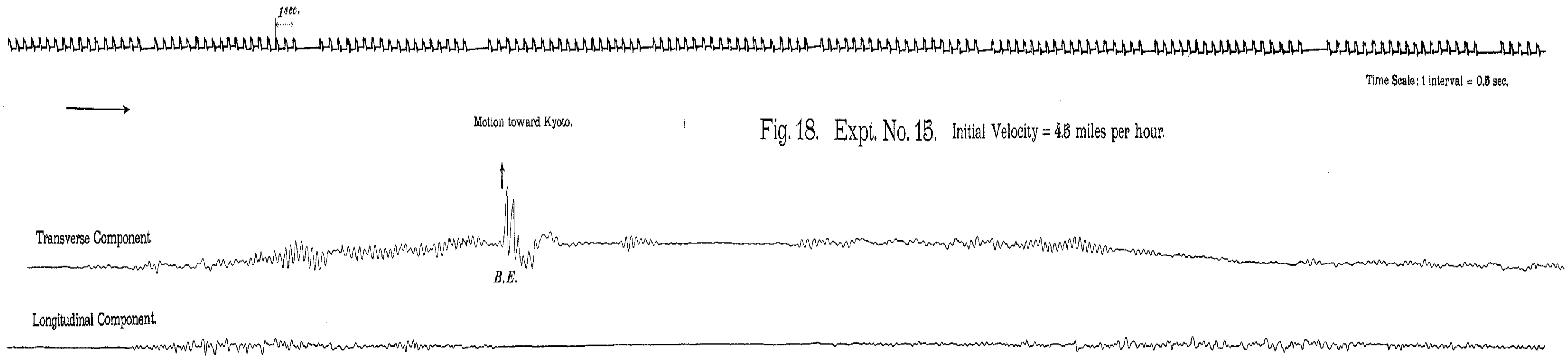


Fig. 18. Expt. No. 15. Initial Velocity = 4.5 miles per hour.

B.E.....Effect caused by the stopping of Engine.

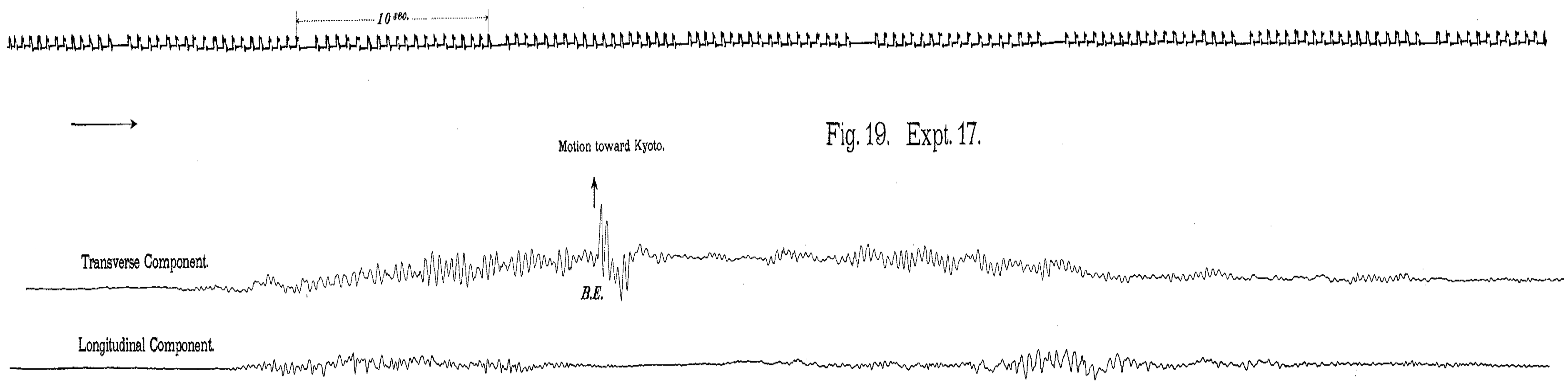


Fig. 19. Expt. 17.

Vibration of Pier No. 1 of the Takaya-*gawa* Bridge: Effect caused by stopping the Train on the 200′ Truss.

Up Train; 1 Engine. Magnification = 30.

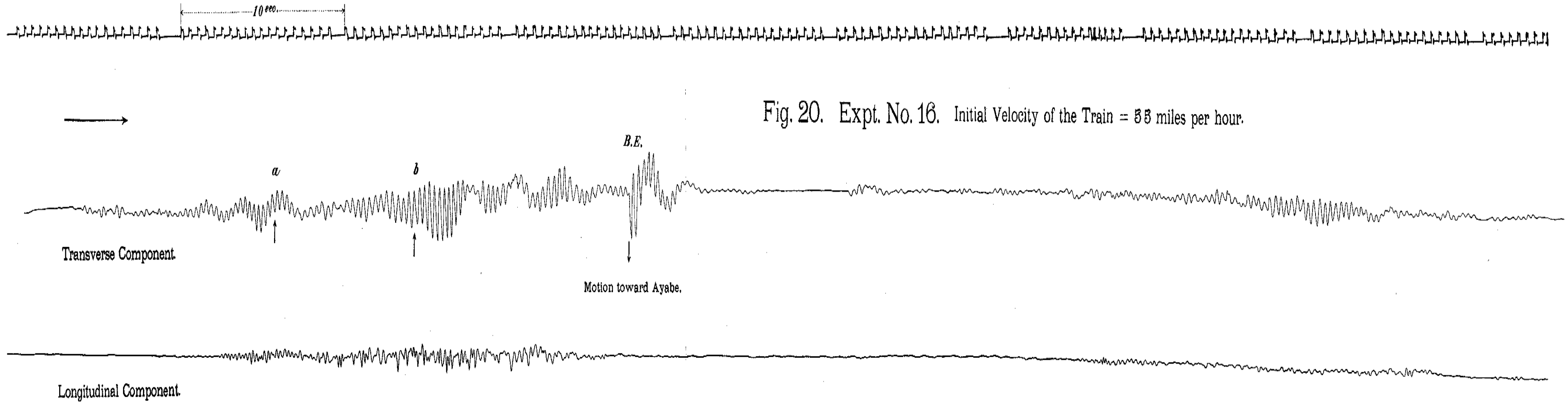
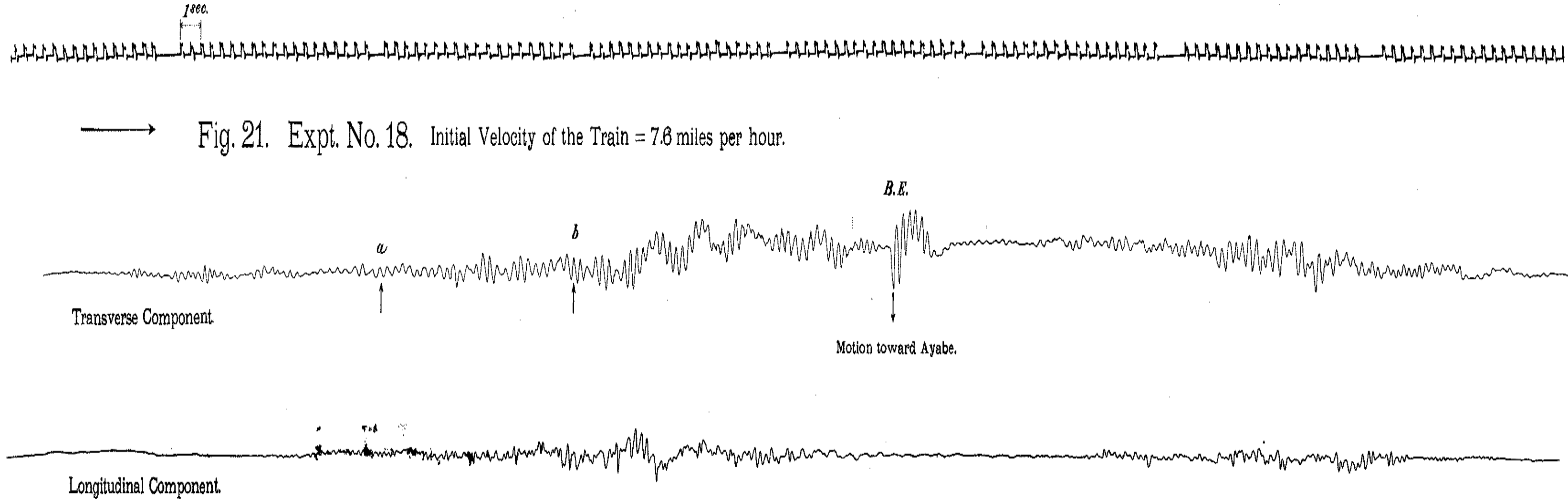


Fig. 20. Expt. No. 16. Initial Velocity of the Train = 5 5 miles per hour.

(B. E.)....Effect caused by the stopping of Engine.
(a)........Engine comes on to Pier No. 2.
(b).... " " " Pier No. 1.



Time Scale: 1 interval = 0.5 sec.

Fig. 21. Expt. No. 18. Initial Velocity of the Train = 7.6 miles per hour.

Pl. XVI.

Vibration of Pier No. 5 of the Shichimi-*gawa* Bridge.

Magnification = 30.

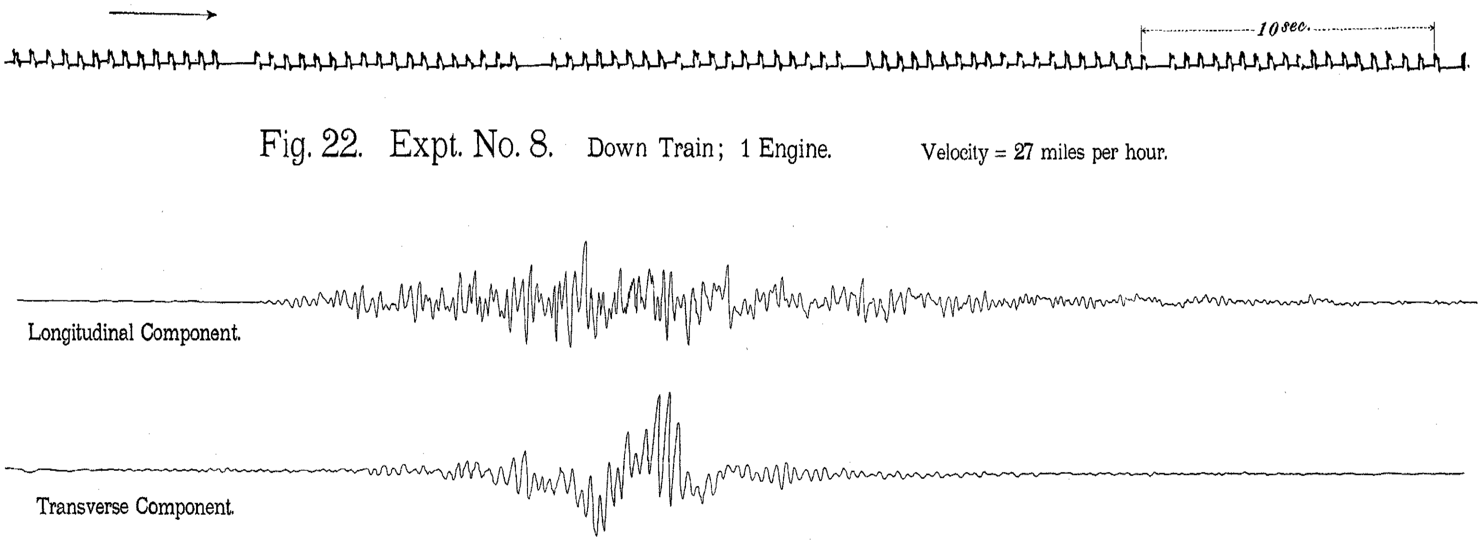


Fig. 22. Expt. No. 8. Down Train; 1 Engine. Velocity = 27 miles per hour.

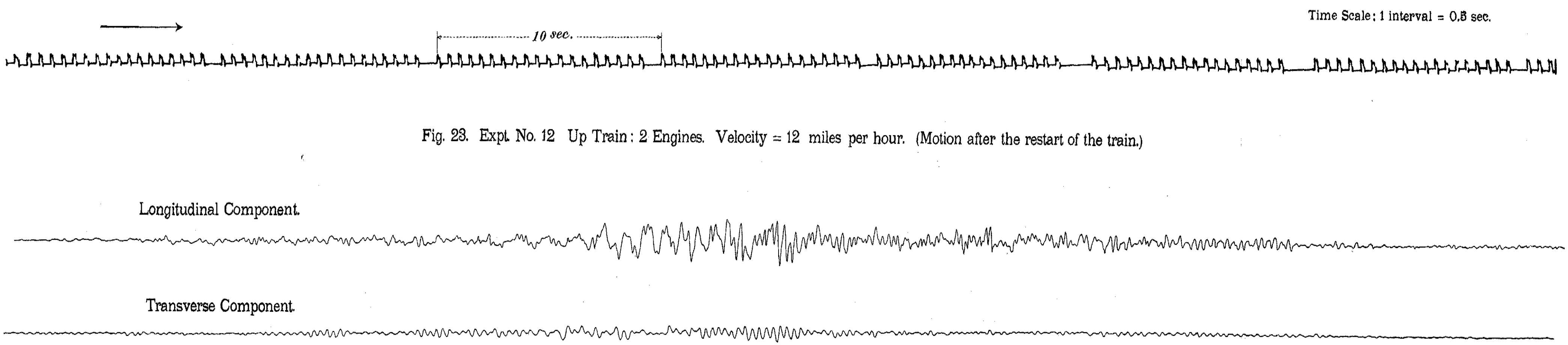


Fig. 23. Expt. No. 12 Up Train: 2 Engines. Velocity = 12 miles per hour. (Motion after the restart of the train.)