On Polarized Surfaces of Sectional Genus Three

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(Received October 26, 1987)

(Introduced by Assoc. Prof. Fujita)

Abstract

The sectional genus g of a polarized surface (S, D) is defined by the formula $2g-2=(K+D)D$, where K is the canonical bundle. This paper gives a classification of (S, D) in case $g=3$.

Introduction

A *polarized surface* is a pair (S, D) of a nonsingular complete algebraic surface S defined over the complex number field and an ample divisor D on S. The *sectional genus* $g(S, D)$ of (S, D) is given by the formula $2g(S, D)-2=(K+D)D$, where K is the canonical divisor of S. We have a classification theory of (S, D) with $g(S, D)\leqq 2$ (see [1], [2], [3], [5]). In this paper we treat polarized surfaces with $g(S, D)=3$.

The author would like to express his hearty thanks to Professor T. Fujita for invaluable comments during the preparation of this paper.

Notation, convention and terminology

We shall work over the complex number field. Throughout this paper S stands for a nonsingular complete algebraic surface. We use the standard notation from algebraic geometry. The words "Cartier divisors", "line bundles" and "invertible sheaves" are used interchangeably, and "vector bundles" and "locally free sheaves", too.

$\equiv$: the numerical equivalence of Cartier divisors.
A_X: the pull-back of a Cartier divisor A on Y by a given morphism $X\to Y$. However, when there is no danger of confusion, we often write A instead of A_X by abuse of notation.
Σ_n: $=\boldsymbol{P}_{P^1}(\mathcal{O}_{P^1}\oplus\mathcal{O}_{P^1}(-n))$ for any nonnegative integer n.
K: the canonical divisor of S.

$\kappa(S)$: the Kodaira dimension of S.
$q(S)$: the irregularity of S.
$Q_p(S)$: the blowing-up of S at a point p.
E_p: the (-1)-curve on $Q_p(S)$ over p.

§1. Preliminaries

(1.1) DEFINITION [2]. Let (S, D) be a polarized surface and let p be a point on S. Set $S'=Q_p(S)$ and $D'=D_{S'}-E_p$. If D' is ample, the polarized surface (S', D') is called the *simple blowing-up* of (S, D) at p.

(1.2) DEFINITION [1]. A polarized surface (S, D) is a *scroll* over a curve C if S is a $\boldsymbol{P}^1$-bundle over C and $DF=1$ for any fiber F of $S\to C$.

(1.3) PROPOSITION. Let (S, D) be a polarized surface. Then $K+D$ is nef unless

(1) $(S, D)\simeq(\boldsymbol{P}^2, \mathcal{O}_{\boldsymbol{P}^2}(1))$,
(2) $(S, D)\simeq(\boldsymbol{P}^2, \mathcal{O}_{\boldsymbol{P}^2}(2))$, or
(3) (S, D) is a scrool over a nonsingular curve.

For a proof, we refer to [1] and [5].

(1.4) LEMMA. Let (S, D) be a polarized surface and suppose that the geometric genus $p_g(S)=0$. If (S, D) is not of the type (1.3; 3), then $g(S, D)\geqq 2q(S)$.

Proof. Since $g(S, D)\geqq 0$, we may assume $q(S)\geqq 1$. If $\kappa(S)\geqq 0$, then $q(S)=1$ because $\chi(\mathcal{O}_S)=1-q(S)+p_g(S)\geqq 0$. On the other hand $\kappa(S)\geqq 0$ yields $g(S, D)\geqq 2$, hence the assertion holds. If $\kappa(S)=-\infty$, then $K^2\leqq 8(1-q(S))$. Hence $(K+D)^2=K^2+2(KD+D^2)-D^2\leqq 8(1-q(S))+4(g(S, D)-1)-D^2=4(g(S, D)-2q(S)+1)-D^2$. Now $K+D$ is nef by (1.3), so $(K+D)^2\geqq 0$. From them $g(S, D)\geqq 2q(S)$. Q.E.D.

(1.5) ***P^1-bundles over a nonsingular curve*** Let S be a $\boldsymbol{P}^1$-bundle over a nonsingular curve C. Then there exists a vector bundle E of rank two on C such that $S\simeq\boldsymbol{P}_C(E)$. We may assume that E satisfies the conditions that $H^0(E)\neq 0$ but for all line bundles L on C with $\deg L<0$, $H^0(E\otimes L)=0$. In this case E is said to be *normalized*. Of course E is not necessarily determined uniquely, but the integer $e=-\deg(\det E)(=-c_1(E))$ is an invariant of S. If E is decomposable, then $E\simeq\mathcal{O}_C\oplus L$ for some line bundle L on C with $\deg L\leqq 0$. Therefore $e\geqq 0$. All the values of $e\geqq 0$ are possible. If E is indecomposable, then $-q(S)\leqq e\leqq 2q(S)-2$. Let H be the tautological line bundle and let F be a fiber of $S\to C$. Then every divisor D on S is numerically equivalent to $aH+bF$ for some integers a, b. We can determine the ample divisors on S. If the invariant $e \geqq 0$ (resp. $e<0$), then a divisor $D \equiv aH+bF$ is ample if and only if $a>0$ and $b>ae$ (resp. $a>0$ and $2b>ae$).

For details we refer to [4] and [9].

(1.6) LEMMA. Let (S, D) be a polarized surface and suppose $\kappa(S)=-\infty$ and $KD \geqq 0$. Then $K^2<0$.

Proof. When $KD=0$, $K \equiv 0$ or $K^2<0$ by the Hodge index theorem. However, the former does not occur, since S has an extremal rational curve. Hence we may assume from here $KD>0$. Suppose to the contrary that $K^2 \geqq 0$. Then, by the Riemann-Roch theorem for surfaces either $tK+D$ or $(1-t)K-D$ has sections for $t \gg 0$. But the second possibility is excluded, since $((1-t)K-D)D<0$ for $t \geqq 1$. So $tK+D$ has a section. Taking an arbitrary nef divisor N on S, we have $(tK+D)N \geqq 0$ for $t \gg 0$, thus $KN \geqq 0$ in particular. This implies that K is pseudo-effective, which contradicts $\kappa(S)=-\infty$. Q. E. D.

§ 2. Classification

In what follows, let (S, D) be a polarized surface with $g(S, D)=3$.

(2.1) Clearly (S, D) is neither $(\boldsymbol{P}^2, \mathcal{O}_{\boldsymbol{P}^2}(1))$ nor $(\boldsymbol{P}^2, \mathcal{O}_{\boldsymbol{P}^2}(2))$. So if $K+D$ is not nef, then (S, D) is of the type (1.3; 3). Since $g(S, D)=q(S)$ in this case, (S, D) is a scroll over a nonsingular curve of genus three.

From now on, we assume that $K+D$ is nef.

(2.2) Suppose that K is nef. Then $0 \leqq KD=4-D^2$, so $D^2=1, 2, 3$ or 4.

If $D^2=4$, then $KD=0$. Since $K^2 \geqq 0$, $K \equiv 0$ by the Hodge index theorem.

If $D^2=3$, then $KD=1$ and $(3K-D)D=0$. Obviously $3K-D \not\equiv 0$. Hence $0>(3K-D)^2=9K^2-3$ by the Hodge index theorem. From this $K^2=0$. So S is a minimal elliptic surface.

If $D^2=2$, then $KD=2$ and $(K-D)D=0$. The Hodge index theorem yields $K \equiv D$ or $0>(K-D)^2=K^2-2$. In the latter case S is either a minimal elliptic surface or a minimal surface of general type with $K^2=1$.

If $D^2=1$, then $KD=3$ and $(K-3D)D=0$. Similarly as above, unless $K \equiv 3D$, S is either a minimal elliptic surface or a minimal surface of general type with $1 \leqq K^2 \leqq 8$.

(2.3) Suppose that K is not nef. Then the following three cases occur by the theory of extremal rays:

(1) $S \simeq \boldsymbol{P}^2$.
(2) S is a $\boldsymbol{P}^1$-bundle over a nonsingular curve.
(3) $S \not\simeq \Sigma_1$ has a (-1)-curve.

(2.4) When (2.3 ; 1) holds, $(S, D)\simeq(\boldsymbol{P}^2, \mathcal{O}_{P^2}(4))$.

(2.5) We consider the case (2.3 ; 2). With the same notation as in (1.5), we have $K\equiv -2H+(2q(S)-2-e)F$. Set $D\equiv aH+bF$. Then $a\geqq 2$ since $K+D$ is nef. Hence (S, D) is not of the type (1.3 ; 3). Consequently $q(S)=1$ or 0 by (1.4). Note $D^2=a(2b-ae)$ and $4=(K+D)D=(a-1)(2b-ae)+2a(q(S)-1)$.

(2.6) Suppose $q(S)=1$. Then $4a=(a-1)D^2$. So $(a, D^2)=(2, 8)$, $(3, 6)$ or $(5, 5)$.
In case $(a, D^2)=(2, 8)$, we use (1.5) to find $b-e=2$, $e=1, 0, -1$. Similarly, in case $(a, D^2)=(3, 6)$ (resp. $(5, 5)$), $(b, e)=(1, 0)$ (resp. $(-2, -1)$).

(2.7) REMARK. Let S be a $\boldsymbol{P}^1$-bundle over an elliptic curve. Then, similarly as in (2.6), we get the following classification table of polarized surfaces (S, D) with $g(S, D)=2, 4, 5$ or 6, which will be used in (2.16):

	$g(S, D)$	a	D^2	KD	b	e
(1)	2	2	4	-2	$1+e$	$0, -1$
(2)	2	3	3	-1	-1	-1
(3)	4	2	12	-6	$3+e$	$2, 1, 0, -1$
(4)	4	3	9	-3	0	-1
(5)	4	4	8	-2	$1+2e$	$0, -1$
(6)	4	7	7	-1	-3	-1
(7)	5	2	16	-8	$4+e$	$3, 2, 1, 0, -1$
(8)	5	3	12	-4	2	0
(9)	5	5	10	-2	1	0
(10)	5	9	9	-1	-4	-1
(11)	6	2	20	-10	$5+e$	$4, 3, 2, 1, 0, -1$
(12)	6	3	15	-5	$(5+3e)/2$	$1, -1$
(13)	6	6	12	-2	$1+3e$	$0, -1$
(14)	6	11	11	-1	-5	-1

(2.8) Suppose $q(S)=0$. Then $b-ae>0$ since $e\geqq 0$. Thus $4=(K+D)D\geqq (a-1)(b+1)-2a=(a-1)(b-1)-2$. So $(a, D^2)=(2, 16)$ or $(3, 15)$ if $e>0$. If $e=0$, then $(a-1)(b-1)=3$. Since $S\simeq \boldsymbol{P}^1\times\boldsymbol{P}^1$, we may assume $b\geqq a$. So $(a, D^2)=(2, 16)$.
In case $(a, D^2)=(2, 16)$, we have $b-e=4$, $e=3, 2, 1, 0$.
In case $(a, D^2)=(3, 15)$, $(b, e)=(4, 1)$. Therefore $S=Q_p(\boldsymbol{P}^2)$ and $D=\mathcal{O}_{P^2}(4)-E_p$. Note that in this case (S, D) is the simple blowing-up of $(\boldsymbol{P}^2, \mathcal{O}_{P^2}(4))$.

(2.9) REMARK. Let S be a $\boldsymbol{P}^1$-bundle over $\boldsymbol{P}^1$. Then, similarly as in (2.8), we obtain the following classification table of polarized surfaces (S, D) with $g(S, D)=2, 4, 5$ or 6, which will be used in (2.16):

	$g(S, D)$	a	D^2	KD	b	e
(1)	2	2	12	-10	$3+e$	2, 1, 0
(2)	4	2	20	-14	$5+e$	4, 3, 2, 1, 0
(3)	4	3	18	-12	3	0
(4)	5	2	24	-16	$6+e$	5, 4, 3, 2, 1, 0
(5)	5	3	21	-13	5	1
(6)	6	2	28	-18	$7+e$	6, 5, 4, 3, 2, 1, 0
(7)	6	3	24	-14	$(8+3e)/2$	2, 0
(8)	6	4	24	-14	5	1

In the last case (S, D) is the simple blowing-up of $(\boldsymbol{P}^2, \mathcal{O}_{\boldsymbol{P}^2}(5))$.

(2.10) We divide the case (2.3; 3) into two subcases according as $2K+D$ is nef or not.

(2.11) If $2K+D$ is not nef, then there exists an extremal rational curve E such that $(2K+D)E<0$. Since $S\not\simeq\Sigma_1$, E must be a (-1)-curve, so $DE=1$. We contract E to a nonsingular point on another surface S'. It is easy to see that $D+E$ is the pull-back of an ample divisor D' on S'. Note that $K'+D'$ is nef for the canonical divisor K' of S' and that $g(S, D)=g(S', D')$. Repeating this procedure if necessary, we eventually get a polarized surface (S_0, D_0) satisfying one of the following conditions.

(1) The canonical divisor K_0 is nef.
(2) $S_0\simeq\boldsymbol{P}^2$.
(3) S_0 is a $\boldsymbol{P}^1$-bundle over a nonsingular curve.
(4) $S_0\not\simeq\Sigma_1$ has a (-1)-curve and $2K_0+D_0$ is nef.

Accordingly we conclude that (S, D) is a simple blowing-up of another polarized surface.

From now on, we assume that $2K+D$ is nef.

(2.12) Let E be a (-1)-curve on S. Then $m=DE\geqq 2$ and E is contractible to a nonsingular point on another surface S'. Since $D+mE$ is the pull-back of an ample divisor D' on S', we obtain a polarized surface (S', D'), in which $K'+D'$ and $2K'+D'$ are nef for the canonical divisor K' of S'. We claim that $\kappa(S)=-\infty$ unless K' is nef. Indeed, if K' is not nef, then one of the following conditions holds.

(1) $S'\simeq\boldsymbol{P}^2$.
(2) S' is a $\boldsymbol{P}^1$-bundle over a nonsingular curve.
(3) $S'\not\simeq\Sigma_1$ has a (-1)-curve E' and $2K'+D'$ is nef.

If (3) holds, then $n=D'E'\geqq 2$ and E' is contractible to a nonsingular point on another surface S''. Moreover, $D'+nE'$ is the pull-back of an ample divisor D'' on S''. Now, for the canonical divisor K'' of S'', $K''D''=(K'-E')(D'+nE')=K'D'-n=(K-E)(D+mE)-n=4-D^2-m-n<0$. So combining this with (1) and (2), we infer that $\kappa(S)=-\infty$ except in the case that K' is nef. If K' is nef, then $0\leqq K'D'=4-D^2-m$, hence $(D^2, m)=(1,2), (2,2)$ or $(1,3)$. In case $(D^2, m)=(1,2)$, $D'^2=5$ and $K'D'=1$. By the same method as in (2.2), we find that S' is a minimal elliptic surface. Similarly, in case $(D^2, m)=(2,2)$ (resp. $(1,3)$), $K'\equiv 0$ and $D'^2=6$ (resp. $K'\equiv 0$ and $D'^2=10$).

(2.13) Now we study the case $\kappa(S)=-\infty$. Suppose $q(S)>0$. Then $q(S)=1$ by (1.4), so $K^2\leqq -1$. On the other hand $0\leqq(2K+D)^2=4K^2+16-3D^2$. From them we can give the following table:

	D^2	KD	K^2
(1)	4	0	-1
(2)	3	1	-1
(3)	2	2	-1
(4)	2	2	-2
(5)	1	3	-1
(6)	1	3	-2
(7)	1	3	-3

(2.14) We deal with the case $q(S)=0$. Now $0\leqq(2K+D)D=8-D^2$ since $2K+D$ is nef.

If $D^2=8$, then $(2K+D)D=0$, which implies $2K+D\equiv 0$ by the Hodge index theorem. Hence $-K$ is ample, $K^2=2$ and $D=-2K$.

If $D^2=7$, then $KD=-3$ and $(7K+3D)D=0$. Thus $0\geqq(7K+3D)^2=49K^2-63$ by the Hodge index theorem. So $K^2\leqq 1$. On the other hand $0\leqq(2K+D)^2=4K^2-5$. From them we get a contradiction. Similarly, the case $D^2=6$ is ruled out.

If $D^2=5$, then $KD=-1$ and $(5K+D)D=0$. The Hodge index theorem gives $0\geqq(5K+D)^2=25K^2-5$, so $K^2\leqq 0$. On the other hand $0\leqq(2K+D)^2=4K^2+1$. Hence $K^2=0$.

If $D^2=4$, then $KD=0$, so $K^2<0$ by (1.6). On the other hand $0\leqq(2K+D)^2=4K^2+4$. From them $K^2=-1$.

If $D^2=3$, then $KD=1$ and $0\leqq(2K+D)^2=4K^2+7$. Combining this with (1.6) we obtain $K^2=-1$. Similarly, in case $D^2=2$ (resp. 1), $K^2=-1$ or -2 (resp. -1, -2 or -3).

Therefore we get the following table for $D^2\leqq 5$:

	D^2	KD	K^2
(1)	5	-1	0
(2)	4	0	-1
(3)	3	1	-1
(4)	2	2	-1
(5)	2	2	-2
(6)	1	3	-1
(7)	1	3	-2
(8)	1	3	-3

(2.15) REMARK. Let things be as in (2.14). Then, by the same argument as above, we obtain the following table for $g(S, D)=2, 4, 5$ or 6 and $K^2 \geqq 0$:

	$g(S, D)$	D^2	KD	K^2
(1)	2	4	-2	1
(2)	4	12	-6	3
(3)	4	9	-3	1
(4)	4	8	-2	0
(5)	4	7	-1	0
(6)	5	16	-8	4
(7)	5	12	-4	1
(8)	5	10	-2	0
(9)	5	9	-1	0
(10)	6	20	-10	5
(11)	6	16	-6	2
(12)	6	14	-4	1
(13)	6	13	-3	0
(14)	6	12	-2	0
(15)	6	11	-1	0

(2.16) Taking the following steps, we can determine the structure of (S, D) in (2.13; 1)~(2.13; 7) and (2.14; 1)~(2.14; 8).

Step 1. We take $x=\min\{t \in \boldsymbol{Z}^+ | tK+D \text{ is not nef}\}$. Note $x \geqq 3$.

Step 2. Put $A=(x-2)K+D$. Then A is an ample divisor on S. Since $K+A$ is nef and $2K+A$ is not nef, (S, A) is of the type (2.11). Hence we get a polarized surface (S_0, A_0) which satisfies one of the conditions (2.11; 2), (2.11; 3) and (2.11; 4). Of course $g(S_0, A_0)=g(S, A)$ is not always equal to $g(S, D)$. Note $K_0(K_0+A_0)=K(K+A)$ since $(K_0+A_0)_S=K+A$.

Step 3. We study the structure of (S_0, A_0) using (2.6), (2.7), (2.8), (2.9), (2.13), (2.14) and (2.15). Here we cannot assume $g(S, A)=3$.

(2.17) As an example of the method (2.16), let us consider the case (2.14; 7). Let x, A, S_0, K_0 and A_0 be as above. Since $(4K+D)^2<0$, $x=3$ or 4.

If $x=4$, then $A^2=5$ and $KA=-1$, so $g(S_0, A_0)=g(S, A)=3$ and $K_0(K_0+A_0)=-3$. This implies $(S_0, A_0)\simeq(\boldsymbol{P}^2, \mathcal{O}_{\boldsymbol{P}}{}^2(4))$. Hence $S=Q_{p_{11}}\cdots Q_{p_1}(\boldsymbol{P}^2)$ and $D=\mathcal{O}_{\boldsymbol{P}}{}^2(10)-3E_{p_1}-\cdots-3E_{p_{11}}$.

If $x=3$, then $A^2=5$, $KA=1$ and $K_0(K_0+A_0)=-1$. Hence we have two possibilities:

(1) (S_0, A_0) is of the type (2.15; 5).
(2) $A_0{}^2=6$, $K_0A_0=0$ and $K_0{}^2=-1$.

We apply (2.16) to (S_0, A_0) again. We let $y=\min\{t\in\boldsymbol{Z}^+|tK_0+A_0$ is not nef$\}$ and put $B_0=(y-2)K_0+A_0$.

In case (1), $y=3$ or 4 since $(4K_0+A_0)^2<0$. If $y=4$, then $B_0{}^2=3$, $K_0B_0=-1$ and $K_{00}(K_{00}+B_{00})=-1$. Thus (S_{00}, B_{00}) is of the type (2.15; 1). From this $S=Q_{p_3}Q_{p_2}Q_{p_1}(S_{00})$ and $D=D_{00}-4E_{p_1}-2E_{p_2}-2E_{p_3}$, where S_{00} is a Del Pezzo surface with $K_{00}{}^2=1$, and $D_{00}=-5K_{00}$. If $y=3$, then $B_0{}^2=5$, $K_0B_0=-1$ and $K_{00}(K_{00}+B_{00})=-1$. However, this is excluded since $K_{00}{}^2>0$.

In case (2), $y=3$. So $B_0{}^2=5$, $K_0B_0=-1$ and $K_{00}(K_{00}+B_{00})=-2$. Accordingly $-K_{00}$ is ample, $K_{00}{}^2=2$ and $B_{00}=-2K_{00}$. From this $S=Q_{p_4}Q_{p_3}Q_{p_2}Q_{p_1}(S_{00})$ and $D=D_{00}-3E_{p_1}-3E_{p_2}-3E_{p_3}-2E_{p_4}$, where $D_{00}=-4K_{00}$.

By the same way we can describe the precise structure of the others.

From the facts mentioned above we obtain the following

THEOREM. Let (S, D) and (S_0, D_0) be polarized surfaces such that $S=Q_{p_r}\cdots Q_{p_2}Q_{p_1}(S_0)$, $r\geqq 0$, and such that $D=D_0-m_1E_{p_1}-m_2E_{p_2}-\cdots-m_rE_{p_r}$, and let K_0 be the canonical divisor of S_0. Assume $g(S, D)=3$. Then one of the following eleven conditions is satisfied:

(0) (S, D) is a simple blowing-up of another polarized surface.
(I) $K\equiv D$ and $D^2=2$.
(II) $K\equiv 3D$ and $D^2=1$.
(III) S is a minimal surface of general type. D^2 and K^2 are as follows:

	D^2	K^2
(1)	2	1
(2)	1	$1\leqq K^2\leqq 8$

(IV) S_0 is a minimal elliptic surface. D^2, $D_0{}^2$, K_0D_0 and m_j are as follows:

	D^2	D_0^2	$K_0D_0:(m_1, m_2, \cdots, m_r)$
(1)	3		
(2)	2		
(3)	1		
(4)	1	5	1:2

(V) $K_0\equiv 0$. D^2, D_0^2 and m_j are as follows:

	D^2	$D_0^2:(m_1, m_2, \cdots, m_r)$
(1)	4	
(2)	2	6:2
(3)	1	10:3

(VI) There is a vector bundle E of rank two on an elliptic curve C with $e=-c_1(E)$ such that $S_0\simeq \boldsymbol{P}_C(E)$. $D_0\equiv aH+bF$, where H is the tautological line bundle and F is a fiber of $S_0\to C$. D^2, e, a, b and m_j are as follows:

	D^2	e	a	$b:(m_1, m_2, \cdots, m_r)$
(1)	8	1, 0, −1	2	$2+e$
(2)	6	0	3	1
(3)	5	−1	5	−2
(4)	4	0, −1	4	$1+2e$: 2
(5)	3	0, −1	6	$1+3e$: 3
(6)	3	−1	7	−3 : 2
(7)	2	0	9	1 : 4
(8)	2	−1	11	−5 : 3
(9)	2	0	5	1 : 2 2
(10)	1	0	13	1 : 5
(11)	1	−1	17	−8 : 4
(12)	1	0	7	1 : 3 2
(13)	1	−1	9	−4 : 2 2

(VII) $(S_0, D_0)\simeq(\boldsymbol{P}^2, \mathcal{O}_{\boldsymbol{P}^2}(a))$. D^2, a and m_j are as follows:

	D^2	$a:(m_1, m_2, \cdots, m_r)$
(1)	16	4
(2)	16	5:3
(3)	4	7:3 2 2 2 2 2 2 2 2 2
(4)	3	10:4 3 3 3 3 3 3 3 3
(5)	1	19:6 6 6 6 6 6 6 6 6
(6)	1	10:3 3 3 3 3 3 3 3 3 3
(7)	1	7:2 2 2 2 2 2 2 2 2 2 2

(VIII) $(S_0, D_0) \simeq (\Sigma_n, aH+bH')$, where H is the tautological line bundle and H' is the pull-back of $\mathcal{O}_{P^1}(1)$. D^2, n, a, b and m_j are as follows:

	D^2	n	a	$b:(m_1, m_2, \cdots, m_r)$
(1)	16	0, 2, 3	2	$4+n$
(2)	4	0, 2	4	$5+2n$: 2 2 2 2 2 2 2 2 2
(3)	3	0, 2	6	$7+3n$: 3 3 3 3 3 3 3 3 3

(IX) S_0 is a Del Pezzo surface with $K_0^2=1$ or 2, and $D_0=-aK_0$. D^2, K_0^2, a and m_j are as follows:

	D^2	K_0^2	$a:(m_1, m_2, \cdots, m_r)$
(1)	8	2	2
(2)	5	1	3 : 2
(3)	3	1	4 : 3 2
(4)	2	2	5 : 4 4 4
(5)	2	2	3 : 2 2 2 2
(6)	2	1	6 : 5 3
(7)	1	2	7 : 6 6 5
(8)	1	2	4 : 3 3 3 2
(9)	1	1	9 : 8 4
(10)	1	1	5 : 4 2 2

(X) (S, D) is a scroll over a nonsingular curve of genus three.

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