博士論文

APPLICATION OF META-MODELING THEORY TO CONSTRUCTION OF CONSISTENT SEISMIC RESPONSE ANALYSIS MODELS CONSIDERING SOIL-STRUCTURE INTERACTION

(構造物-地盤の相互作用を考慮した整合する地震応答解析の構築のためのメタモデリング理論の適用)

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Abstract

Seismic safety of an important structure is re-examined for severe events or extremely rare events. There is a need for more advanced numerical analysis of estimating structural seismic response, particularly accounting for effects of soil-structure interaction (SSI). It is also necessary to make a link between the conventional analysis and such more advanced numerical analysis, in order to improve the former as well as to validate the latter. To this end, the meta-modeling theory, which relates structural mechanics to continuum mechanics is applied.

This thesis is aimed at investigating a possibility to improve current analysis models for structural seismic responses analysis considering SSI according to the meta-modeling theory. Proposed is a methodology of constructing an analysis model or determining model parameters such that the model be consistent with a massive solid element model of finite element method (FEM) of high fidelity.

The thesis first carries out literature surveys in the following three fields: 1) structural seismic response analysis; 2) evaluation of SSI effects; and 3) the meta-modeling theory. The surveys reveal that SSI has been rigorously considered based on continuum mechanics but simplified models (which are computable) are constructed.

The thesis next makes mathematical analysis of general soil-structure problems. A Lagrangian of soil and a structure is formulated, and an initial-boundary value problem is posed. An ideal rigid-body foundation is introduced to connect the soil and the structure in an ordinary model which considers SSI. The presence of this foundation simplifies the model, but, at the same time, it makes least transparent to understand what mathematical approximations are made in converting the initial-boundary value problem.

Based on the mathematical analysis, the thesis seeks to clarify SSI in a most rigorous manner. The targets are the rigid body foundation and the soil spring that changes depending on frequency. The requirements of the soil spring are studied. It is shown that the soil spring can work well as approximation, if the structure is symmetric and of simple configuration and if the foundation is most stiff and tightly connected to the soil.

The thesis proposes a methodology of constructing a consistent model which consists of a multiple mass-spring system (or a stick model) for a structure, a rigid-body foundation, and soil springs for soil, by using an FEM solution of a high fidelity model of the structure and the soil. According to the meta-modeling theory, a set of suitable approximate functions of displacement which account for seismic responses of the structure and the soil are chosen, and the consistent model is constructed by substituting the set into the Lagrangian. This methodology is applicable to a case when a structure is located on the ground or partially embedded in ground.

Next, construction of soil springs for soil according to the proposed methodology is studied, under an assumption that a suitable stick model is made for the structure. A simple structure of two-story building and a complicated structure which mimics a nuclear power plant buildings are considered. The performance of the constructed stick model and soil springs is examined.

Construction of a stick model for a structure according to the proposed methodology is studied. A modal analysis of a high fidelity model is used to this end. A methodology is proposed to convert a set of natural frequencies and dynamic modes to a stick model. It is shown that perfect conversion is not possible except for cases when the structure satisfies certain conditions, which is regarded as being "well designed" in the thesis.

In concluding the thesis, achievements of the research works are summarized. Proposal of a methodology of constructing a consistent model considering SSI is emphasized. Remarks are made for the future works.

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Chapter 1

Introduction

1.1 Background

After the recent strong earthquakes such as the Niigataken-Chuetsu-Oki earthquake of July 2007 and the Great Tohoku Earthquake of March 2011, there is a need for the re-evaluation of seismic design of important structures such as nuclear power plant (NPP) for severe events of extremely rare possibility. Establishing the methodologies for an accurate and reliable seismic analysis of such structures of critical importance is essential to ensure their safety. Due to the sheer size of these structures and the limitation of experimental setups, computational mechanics and numerical simulation is the only approach that can be used to determine the seismic response of these structures for different earthquake scenarios as it is not possible to conduct full scale experiments for such massive structures.

Conventionally, a mass spring modeling has been employed for the seismic analysis of structures with the structure modeled as a set of a few or a few tens of masses, connected by springs representing the stiffness of the structure. These models, basically developed for simple settings of structure in an era of lesser computational resources, require experience and need of making several simplifications and hence lack objectivity. There is an absence of a unified approach for checking the quality of different simplified models constructed for the same structure. Further, these conventional models have been based on conservative estimation of response to guarantee higher factors of safety and lack the ability to quantitatively show the concrete margin of safety for a structure which is important to know, especially after the recent earthquake experiences.

The change in the dynamic response of the structure because of the presence of underlying soil and vice versa is called as soil-structure interaction (SSI). There are two aspects of SSI. The first is the modification of the free-field ground motion due to presence of the structure and the second is the modification of the structural response due to the flexibility of the supporting soil. For the case of flexible structures founded on rigid ground, the SSI is not significant and has been ignored in the conventional structural analysis. However, for stiff and heavy structures, the soil flexibility can change the response of the structure significantly due to SSI.

For an accurate seismic response analysis of an important and massive structure such as an NPP, SSI is a fundamental issue and its careful consideration is necessary during analysis. Conventionally, due to lack of computational resources, SSI has been considered by considering the structure and soil separately and replacing the effect of soil as a soil-spring. Just like the mass spring modeling for structure, the soil-springs were developed for the simple soil system with simplifying assumptions for the foundation and the supported structure and lack the objectivity requiring experienced engineers and use of judgement.

On the other hand, when using the 3D solid elements based FEM analysis, there is no need to consider the SSI explicitly and the effect of SSI appears in the solution when a model of a soil-structure system is analyzed. The latest computers are able to perform seismic response analysis of a soil-structure model with DOF of the order of millions. Even with the availability of resources of high performance computing, the importance of simplified models cannot be neglected. These models are needed at the beginning of a complex analysis to perform preliminary simulations and to see the effect the different parameters. These simplified models are also useful during the probabilistic seismic risk analysis because of their simplicity and lesser requirement of computational resources.

Keeping the above facts in mind, there is a need of more advanced numerical analysis of estimating structural seismic response, particularly accounting for effects of SSI to get a concrete estimate of the safety of the structure for any future earthquake scenario which has not been considered previously. It is also necessary to make a link between the conventional analysis and such more advanced numerical analysis, in order to improve the former as well as to validate the latter. To this end, use of the meta-modeling theory is being proposed.

Meta-modeling theory is being proposed in order to strengthen a link between structural mechanics (the conventional models) and continuum mechanics. Meta-modelling theory is a modelling methodology to construct a model which is consistent with continuum mechanics. The key concept is that all modelings solve the same problem of continuum mechanics but use different mathematical approximations, without using any physical assumption. See Appendix A for a summary of meta-modeling theory.

Meta-modeling theory proves that some structural mechanics problems are mathematical approximation of continuum mechanics problem; for instance, beam problems which do use only Young's modulus are regarded as an approximation of continuum mechanics problem of elasticity which uses both Young's modulus and Poisson's ratio. The meta-modeling theory is simple in principle. It only uses a Lagrangian of continuum mechanics, from which a continuum mechanics problem or a structural mechanics problem is derived by applying no or some mathematical approximations, respectively.

In this thesis, meta-modeling theory is used to clarify the soil structure interaction in the view point of structural and continuum mechanics and to show that the conventional soil-spring model can be objectively derived from the continuum mechanics by using purely mathematical approximations and without using any physical assumption. Use of physical assumptions means solving a different problem and hence the consistency of simplified model with that of the continuum mechanics modeling is lost.

1.2 Objectives

In the past, simplified models have been developed and used for the SSI analysis. However with the resources of HPC becoming increasingly available, there is a possibility of improving the analysis models used in practice by making use of the numerical solution of the models of high fidelity and this can be achieved by using the meta-modeling theory to create a link between the conventional modeling and the modern modeling based on 3D solid FEM.

The field of SSI analysis, however, is diverse and involves several research areas such as the earthquake wave propagation, soil mechanics, foundation engineering, structural analysis, contact analysis etc. and each field can further have different cases and levels of complexity. For example different types of soil and structure of soil, spread or pile footing, embedded or surface foundation, complexity of structure, consideration of slip and detaching at the interface and automation of high fidelity analysis model development from CAD data.

The long term research plan is to improve the SSI evaluation for the real-life soil-structure interaction problems. However, keeping the diversity of related fields in mind, it's still a long time until we are able to improve the SSI evaluation just like with the mass-spring model which has a long history. This study is a first step to this improvement and hence the major focus is first of all on the clarification of SSI problem.

The objective of this study is to apply meta-modeling theory to construct a consistent seismic response analysis model to consider SSI. A structure which mimics the complexity of a nuclear power plant structure is taken as the target structure to show the usefulness of the constructed model. The scope of this study is summarized as follows:

- Clarifying SSI according to meta-modeling theory,
- Proposing a methodology to construct a consistent mass-spring model that can approximate the solution of solid element FEM model,
- Showing the usefulness of the proposed methodology with the help of a numerical experiment.

1.3 Thesis structure

The contents of this dissertation are summarized as follows. First, the findings of a detailed literature survey are presented in Chapter 2. Mathematical analysis of a general soil-structure problem is then made in Chapter 3. Then, based on the mathematical analysis, SSI is clarified in the most rigorous manner in Chapter 4. In Chapter 5, a methodology to construct a consistent soil-spring is proposed and a consistent soil-spring model is constructed. In Chapter 6, the usefulness of the constructed soil spring is shown with the help of numerical experiments. In Chapter 7, the construction of a consistent stick model for a structure using the modal analysis results is studied. Achievements of the research work are summarized and remarks for the future works are made in Chapter 8.

Chapter 2

Literature Survey

2.1 Overview

In this chapter, the findings of the literature survey in the fields of structural seismic response analysis, evaluation of SSI effects and the meta-modeling theory are presented. The surveys reveal that it is possible to do the seismic response analysis of a structure and to rigorously consider the effects of SSI during the seismic response analysis based on continuum mechanics. However, simplified models traditionally developed in the era of little computational resources for the seismic response analysis and the SSI analysis are still being used.

Various simplified models have been developed by different researchers for the simple structure and soil cases and various physical and mathematical assumptions are made for different problems requiring experience and personal judgement and there have been a lack of a unified approach to develop a simplified method of desired fidelity for the structural seismic analysis and the SSI analysis. However meta-modeling theory is being used in this regard which is a model construction technique in which the focus is to ensure the consistency between the different models constructed for the same problem. For the structural seismic response analysis, the consistent stick, beam, plate and shell models have been constructed using the meta-modeling theory and this thesis focuses on the application of this theory for the construction of consistent soil spring for the evaluation of SSI effects.

The contents of this chapter are organized as follows. First, the literature survey related to structural seismic response analysis is presented in Section 2.2. Then the literature survey about the evaluation of SSI effects is mentioned in Section 2.3. In the end, the literature survey related to the meta-modeling theory is presented in Section 2.4.

2.2 Structural seismic response analysis

Structural seismic response analysis involves the computation of deformation and stress of a structure when subjected to an earthquake ground motion. The physics behind this analysis is simple as the excitation of structure subjected to an input ground motion is completely described as a solution of linear or non-linear wave equation of solid continuum for the structure and mathematically this equation is given as 4D partial differential equation of space and time. Finite element method can be employed as a numerical analysis tool for the solution of this partial differential equation. A lot of research is being done in the field of structural engineering employing this approach and detailed FEM models are being developed with the DOF of the order of millions for important structures such as NPP structure [1, 2, 3], tunnel structure [4, 5], building structure [6, 7], bridges and dams [8] etc.

The 3D solid element based FEM modeling of structures is the best possible approach to perform the seismic response analysis [9]. This is because of its high accuracy, need of only few small and cheap material property tests and the uniform accuracy of results. However depending on the complexity of the structure and the size of the FEM model, the full FEM modeling generally requires long computational time and high performance computing resources. Further constructing a detailed analysis model is not an easy task, even though research work is being done on the automation of model construction from the CAD data. This approach also becomes difficult to apply for the case of seismic probabilistic risk analysis for which a significant number of earthquake scenarios need to be considered.

Conventionally, because of the lack of computational resources, simplified models have been developed for the seismic response analysis of both simple and complicated structures [10, 11, 12, 13, 14]. These lumped mass models involve the discretization of the structure into a set of lumped masses connected by springs by considering the structural configuration or the locations of interest where the response is needed. The magnitude of the mass for each node of the spring is calculated from the portion of the weight of the structure being contributed to that node and is termed as the “tributary area consideration”.

For the determination of the stiffness of the spring, mainly there are two ways, namely a static method and a geometric method [11, 15]. In the static method, an arbitrary static load is applied to the FEM model of the structure to determine its stiffness just like the static pushover analysis [16]. In the geometric method, the geometric shape of the cross section is considered to determine the sectional moment of inertia and the shear coefficients. Alternatively, a strain energy method [17] is also proposed to evaluate the equivalent stiffness for complex structures.

Constructing the lumped mass model as explained above does not ensure the consistency of the model with the actual physical problem or with the 3D FEM model of the problem and results in different dynamic characteristics of the model than those obtained from the 3D FEM analysis [9, 18]. However the simplicity should not destroy the consistency of the problem and different models constructed for the same structure should share the same fundamental dynamic characteristics. Some efforts have been made recently by Roh. *et al* [9, 19] to ensure the same dynamic characteristics of the lumped mass model and the solid element based FEM model by using the mode shapes obtained by solid element FEM model for the development of the lumped mass model. These studies use the equal number of lumped masses and the target modes and the location of mass points is decided by investigating the mode shapes of the structure.

However, these studies are limited to the simple cases of axially symmetric structures considering their lateral modes and elastic behavior. Tuning of the lumped mass models is easy by tuning the stiffness of the springs for the linear case, however in case of the non-linear case, since the modes and the frequencies change with the non-linearity of the material, using one tuned value for the spring constants is generally not possible.

Overall, for the conventional lumped mass modeling, fixing the locations of the mass points and the lumping of the mass requires experience and personal judgement and doesn't provide a unified approach for the simplified modeling. Even though the HPC resources are becoming increasingly available, the conventional lumped mass modeling is still being used widely in practice for important structures such as NPP [2, 9, 20, 21, 22]. Simplified models are useful especially at the initial stages of the analysis however it is necessary to ensure the consistency of the developed simplified models with the high fidelity models to ensure the accuracy.

2.3 Evaluation of soil-structure interaction

The consideration of the effect of soil-structure interaction during the seismic response analysis is an important issue that has been considered during the analysis since long as it can affect the actual behavior and design of the structure [23, 24]. Observations made during the large earthquakes in past have specifically emphasized the importance and need of considering SSI effect in seismic response analysis. SSI changes the response of the structure compared to that of the fixed base structure depending on the characteristics of the structure and the soil and its effect is generally significant for the case of heavy and stiff structures founded on relatively weaker soils [25].

The first attempt to study the phenomenon of dynamic SSI was carried out in Japan by K. Sezawa and K. Kanai [26, 27] in 1935. However, the theory of dynamic SSI has been properly formulated for the first time in 1936 by E. Reissner [28] in which the behavior of circular foundation lying on the elastic half-space and subjected to vertical time-harmonic loading is considered. Since then, a numerous amount of analytical, numerical and empirical researches have been made on the evaluation of SSI for more accurate seismic response analysis [29, 30, 31, 32].

There are two general approaches for considering the SSI effect [23]. One is the Direct Method in which the soil and structure are directly modeled and solved together by directly integrating numerically through the time domain. The other one is called the Substructure Method in which by defining a rigid interface between the structure and soil domains, both the domains are separately considered and solved in frequency domain. The concept of dynamic impedances is used for the soil part and it is represented as spring and dashpot. Details of the researches done in this method are discussed below.

2.3.1 Direct method for evaluation of SSI

In this approach, the soil and structure domains are both part of the same model and it is analysed in a single step by using a numerical discretization scheme such as FEM in the time domain. Just as the seismic response analysis of structure discussed in previous section, the physics of this approach is simple and involves the solution of linear or nonlinear solid wave equation for the soil-structure system.

One of the issues of this approach is the consideration of size of soil domain and the boundaries for the soil domain in order to simulate the effect of the surrounding soil as well as to avoid the trapping of waves inside the considered soil domain after being reflected from the boundaries. For this reason one option is to use a sufficiently large soil domain so that the wave is dissipated before reaching the boundary. Alternatively, several studies have been done to introduce various boundaries to reduce the scale of the problem and to simulate the radiation of energy in an unbound continuum. The studies include the introduction of viscous boundary [33], consistent boundary [34], unified boundary [35], transmitting boundary [36] and the viscous spring boundary [37]. These boundaries act either as non-reflecting boundaries or as adsorbing layers and avoid the reflections of the outward propagating waves from the soil-domain.

Other than FEM, boundary element method (BEM) is a numerical method that only discretizes the boundary of the domain of interest. BEM is advantageous in the sense that it requires only a surface discretization and doesn't require complicated non-reflecting boundaries, which are required for FEM [38, 39]. However application of BEM requires the evaluation of appropriate Green's function and its applicability is difficult in the case of heterogeneous soil domain. Further, the above mentioned advantage won't be there if BEM is used for non-linear problem due to the presence of the integral component in the total domain.

Despite the presence of artificial boundaries, for an accurate calculation, it is required for the model to have a significant volume of soil supporting the structure. As a result, the size and the DOF of the soil-structure system becomes significantly large. Further, to capture the response of the soil and structure at relatively large frequencies, a very fine discretization of the domains is required which in turn requires a significant computational effort, especially for the case of 3D non-linear analysis. For this reason, conventionally the direct approach hasn't been used much and simplified models were developed, which are discussed in next subsection.

However in recent years, the increased availability of the resources of HPC has made it possible to analyse a soil-structure model comprising of solid elements of the order of millions and the 3D non-linear analysis can be performed in the time domain without having to specially consider the SSI as the effect of SSI is automatically accounted for during the analysis. There have been several studies conducted making the detailed solid element based FEM models for the seismic response analysis of soil-structure systems, see for example [1, 40, 41, 42].

2.3.2 Substructure method for evaluation of SSI

In the substructure approach or the indirect method, the structure and soil are considered separately with the assumption of a rigid foundation in between. Seismic response analysis of structure considering SSI using this approach consists of 3 steps [43]. First is the determination of the foundation input motion which is different from the free-field motion due to presence of the structure and is termed as the kinematic interaction. Second step involves the calculation of frequency dependent impedance functions for the foundation. For this, a unit harmonic excitation of displacement/rotation at a particular frequency is given to the foundation, and the reactional force/moment of the soil is computed and the ratio of the two is an impedance function. The third step involves the calculation of dynamic response of the structure supported by the springs with impedance functions calculated in second step and subjected to the foundation input motion which is computed in first step. This is termed as the inertial interaction.

Conventionally, analytical solutions have been tried for the substructure approach for the simple shapes, configuration and material properties of the structure and soil. For the first step of substructure approach, until now there has been research done about the input ground motion that involves the surface, piled and embedded foundations [43, 44, 45]. Conventionally the 1D approach is used for the evaluation of the amplified ground motion [24] with the assumption of earthquake source at a sufficiently large depth and homogeneous soil or stratified soil and hence the earthquake wave is considered as a vertically propagating shear wave only. For the case of surface foundations, with the above assumption of shear waves only, the kinematic interaction effect has been ignored [46].

For the second step of substructure approach which involves finding the soil springs is mathematically a mixed boundary value problem and conventionally several empirical and analytical solutions have been found for this problem such as the empirical relations by Gazetas [47, 48] and the analytical solutions by Veletsos *et al* [49], Luco *et al* [50, 51] and Kausal *et al* [52, 53] and these solutions have been applied for the modelling of soil as sway-rocking springs for different structures including the NPP structures [2, 22, 54, 55] and for the development of commercial analysis softwares such as CLASSI [56], FLUSH [57] and SASSI [58].

However these studies have been limited to the cases of specific geometries involving circular and rectangular rigid foundations bonded to the linear-elastic, isotropic and homogenous half space subjected to the harmonic excitation at the centroid of the foundation which limits the applicability of these solutions. There are studies which show significant differences in response obtained by using these different analytical solutions [59]. The impedance functions can also be found by using the numerical modeling for the soil domain. It is important to mention here that for the direct method as mentioned before, the concept of determination of these functions becomes superfluous as there is no need to consider the interaction effect separately.

2.4 Meta-modeling theory

Even though the increased availability of resources of HPC has made it possible to perform the detailed 3D seismic response analysis of a soil-structure system, but simplified models are still needed and have significance especially at the initial stages of analysis and during the seismic probabilistic risk analysis or when the resources of HPC are not available. However it is important that the developed simplified analysis models should be consistent with the high fidelity models to ensure the accuracy and same dynamic characteristics. As explained before, the simplified analysis models which have been developed for the seismic response analysis of structure considering SSI often involve various simplifying assumptions which may not to be satisfied for the real soil-structure system. These conventional models require the experience and judgement of the engineer and there is a lack of a unified approach for simplified model development for SSI analysis.

The state as mentioned above is not limited to the SSI analysis only but it’s the same in overall structural mechanics. With this background, meta-modeling theory is developed by Hori *et al* [60], which is a model building technique with the focus on ensuring the consistency between the different models constructed for the same problem. This approach is based on continuum mechanics and models are constructed by solving variational problem of Lagrangian of continuum mechanics by using a displacement function which is approximated without any physical assumption. This gives a unified approach to objectively construct different fidelity models and the difference of each model can be evaluated as the difference of the approximated displacement functions used.

So far, the beam theory, plate theory and shell theory have been formulated using meta-modeling theory [60, 61, 62] and it is shown that certain structural mechanics modelings are approximation of the continuum mechanics modeling. Further, the meta-modeling theory has been used to construct consistent mass spring model for the seismic response analysis of structures, to rigorously convert the solid element solutions to the beam element solutions and for the analysis of buried pipelines [63, 64, 65].

2.5 Observations from literature survey

The literature survey reveals that even though it is becoming increasingly possible to do the detailed solid element based FEM analysis of real life civil engineering problem, yet the conventional simplified models are still being used even for the analysis of important structures such as nuclear power plants. The conventional models which have been developed for the simplified cases of structure and soil in the era of lesser computational resources have limitations in applicability and do not ensure the consistency with the continuum mechanics modeling. Meta-modeling theory is being used as a solution to this issue in structural mechanics to ensure the consistency of simplified models with the continuum mechanics.

Chapter 3

Mathematical Analysis of Soil-Structure Interaction Problem

3.1 Overview

The main aim of this chapter is to perform mathematical analysis of a general soil-structure interaction (SSI) problem. This analysis is needed to clarify the mathematical characteristics of the problem and to rigorously formulate the problem in the framework of the meta-modeling theory. The assumption of an ideal rigid foundation is explained, which is the basis of the sub-structuring approach that is used for SSI analysis.

The contents of this chapter are organized as follows. First, a Lagrangian of soil-structure system is formulated in Section 3.2, and an initial boundary value problem is posed in Section 3.3. The introduction of a rigid body foundation is explained in Section 3.4. Finally, the decomposition of the solution for the posed initial boundary value problem is discussed in Section 3.5.

3.2 Formulation of Lagrangian

In a general SSI problem, an analysis domain of a soil-structure system is regarded as a domain which consists of a structure and soil. More precisely, denoting the domain of a structure and soil by S and G, respectively, we denote the domain of the soil-structure by V,

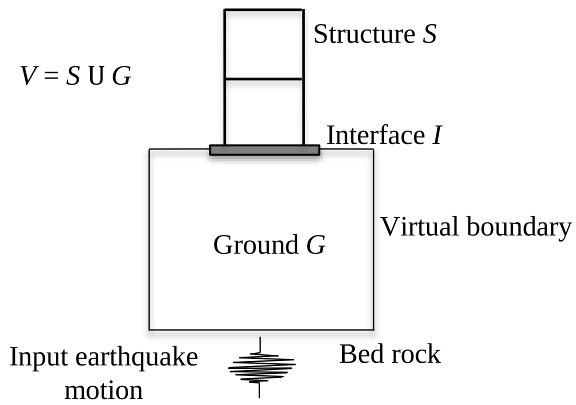


Figure 3.1: Analysis domain for the seismic response analysis considering SSI

the union of *S* and *G*; see Fig. 3.1. *G* is a finite region with an appropriate boundary condition with which there does not occur unnecessary reflection of input waves.

Assuming small deformation and linear elasticity, the Lagrangian for this soil-structure system is given as the kinetic energy minus the potential energy of the system. The Lagrangian is thus formulated in terms of a displacement function in *V*, denoted by $\boldsymbol{u}(\boldsymbol{x}, t)$**,** as

$$\mathcal{L}[\boldsymbol{v}, \boldsymbol{\epsilon}] = \int_V \frac{1}{2}\rho \boldsymbol{v} \cdot \boldsymbol{v} - \frac{1}{2}\boldsymbol{\epsilon} : \boldsymbol{c} : \boldsymbol{\epsilon}\, \mathrm{d}v \tag{3.1}$$

where ρ and $\boldsymbol{c}$ are density and elasticity tensor, respectively, $\cdot$ and : are inner product and the second order contraction, respectively, and $\boldsymbol{v}$ and $\boldsymbol{\epsilon}$ are velocity and strain, respectively. We compute $\boldsymbol{v} = \dot{\boldsymbol{u}}$ and $\boldsymbol{\epsilon} = sym\{\boldsymbol{\nabla u}\}$, using $\boldsymbol{u}$ which satisfies prescribed initial and boundary conditions; sym stands for the symmetric part of the second-order tensor, $\left(\dot{(\,.\,)}\right)$ and $\boldsymbol{\nabla}(\,.\,)$ being temporal derivative and gradient of $(\,.\,)$ respectively. Note that a bold character indicates a vector or tensor quantity, and $\boldsymbol{x}$ and *t* are spatial coordinate and time, respectively. The domain does not have to be uniform, and the value of ρ and $\boldsymbol{c}$ changes in soil and structure. For simplicity, the variables $\boldsymbol{x}$ and *t* are omitted in the equation.

The initial-boundary value problem for $\boldsymbol{u}(\boldsymbol{x}, t)$ in *V* together with the continuity condition at interface *I* is derived from the variational problem of the Lagrangian given in Eq. (3.1). The variational problem is formulated as

$$\delta \int_T \mathcal{L}[\dot{\boldsymbol{u}}, sym\{\boldsymbol{\nabla u}\}]\mathrm{d}t = 0 \tag{3.2}$$

where *T* is an appropriate time domain. For a general displacement function (which is not subjected to any restrictions), the variational problem of the Lagrangian results in a mathematical problem of continuum mechanics, i.e. the wave equation; a 4D partial differential equation, which is analyzed numerically by FEM and the continuity of the traction is automatically satisfied provided the continuity condition of displacement at *I* is guaranteed.

Another mathematical problem can be posed for a mass spring model from the variation problem by using a displacement function which is particularly specified. The continuity of displacement and traction at the interface needs to be satisfied for the displacement function. This is not an easy task at all, since the displacement function of the structure that is determined by the mass spring model is spatially uniform at the interface but this restriction of the displacement function does not guarantee that the accompanying traction at the interface is spatially uniform.

Because of the coupling of the structure and soil, the natural frequency of the structure changes in the soil-structure system; see Appendix B. Therefore, care should be taken about this change because the dynamic properties of structure cannot be determined by the parameters of the structure only. Designing a structure becomes more difficult in the case of considering SSI.

3.3 Introduction of rigid body foundation

According to the sub-structure approach, there is a room to pose conditions for I between S and G. Indeed, it is possible to introduce a rigid body plate of infinitely thin thickness for I so that the both displacement and traction are continuous there. This treatment simplifies the soil-structure system because the structure and soil domains can be treated separately.

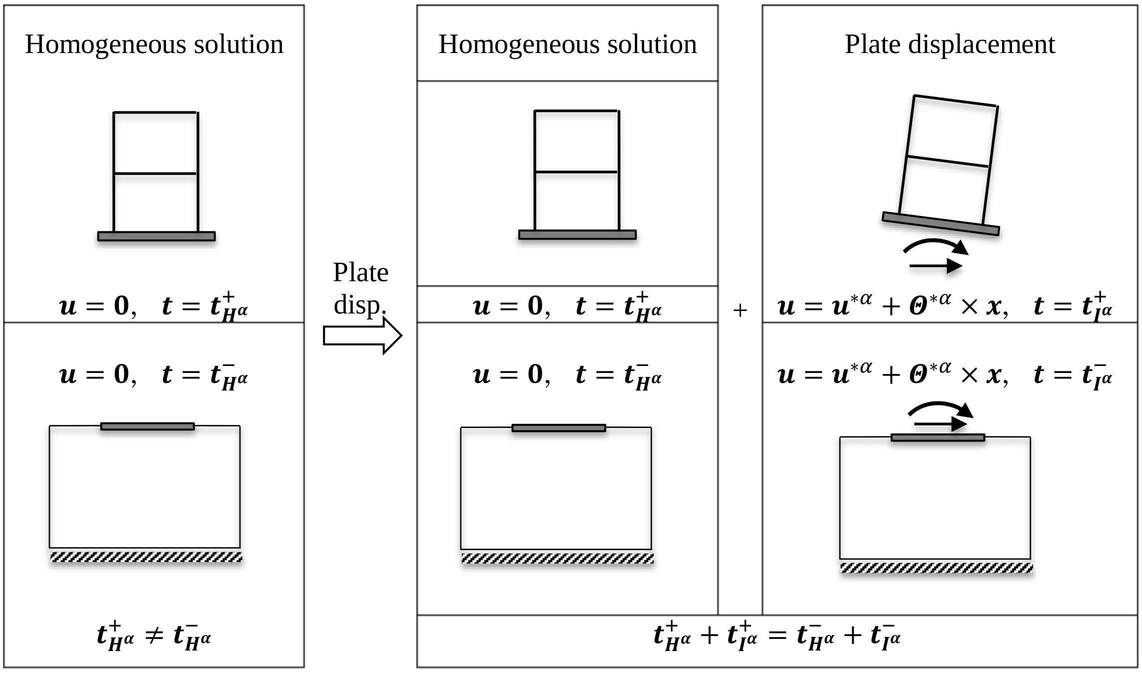
With the assumption of the rigid foundation, the displacement continuity at the interface is readily satisfied as it is shown in Fig. 3.2 and Fig. 3.3. However, the traction continuity condition has to be posed, which leads to SSI. Speaking specifically, we restrict the form of $\boldsymbol{u}$ which is associated with the movement of the interface. Denoting by $\boldsymbol{u}^*$ and $\boldsymbol{\Theta}^*$, i.e. the rigid body translation and rotation of the interface respectively, we restrict $\boldsymbol{u}$ at the interface as follows:

$$\boldsymbol{u}(\boldsymbol{x}, t) = \boldsymbol{u}^*(t) + \boldsymbol{\Theta}^*(t) \times \boldsymbol{x} \qquad \text{on } I, \tag{3.3}$$

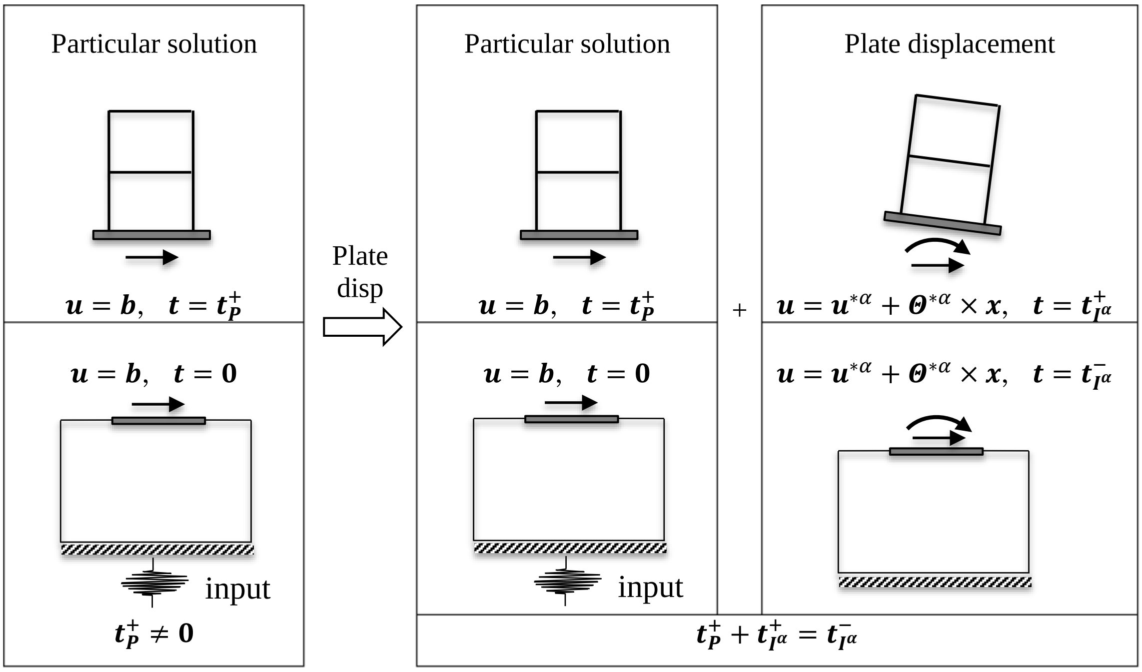
where $\times$ is the cross (outer) product. We assume that rotation is small and the coordinate origin is at the centre of I.

The $\boldsymbol{u}(\boldsymbol{x}, t)$ as given in Eqn. (3.3) can be regarded as boundary condition for displacement function of structure S because I is a part of the boundary of S. As a nature of the boundary value problem, we have to consider a homogeneous solution and a particular solution for both the structure and soil domains to ensure the continuity of displacement and traction at interface I. The rigid plate is horizontally moved and rotated by the $\boldsymbol{u}$ of Eq. (3.3) to return the reaction force from the soil to the structure. A physical model representing the relationship of this movement of rigid plate and the reaction force is the soil spring.

Figures 3.2a and 3.2b show the movement of the plate induced by the discontinuity of traction for the cases of homogeneous and particular solution respectively. $\boldsymbol{t}$ denotes the traction, a "+" superscript shows the traction between plate and structure whereas a "-" shows the traction between plate and soil. Subscripts "H" and "P" represent the case of homogeneous solution, and particular solution respectively. As will be mentioned in next Section, the homogeneous solution here is actually the modal displacement, whereas the particular solution is the amplified displacement induced by the input ground motion. $\boldsymbol{b}$ stands for the amplified ground motion in the soil domain at the interface I. Finally for each mode of the homogeneous solution and the movement of the plate, and for the particular solution, the continuity of the traction is guaranteed as shown in Fig. 3.3.



(a): For homogeneous solution



(b): For particular solution

Figure 3.2: Satisfaction of continuity of traction by movement of plate

3.4 Decomposition of solution

As mentioned before in this chapter, to consider the dynamic behaviour of a soil-structure system subjected to seismic wave as shown in Fig. 3.1, the variational problem of the

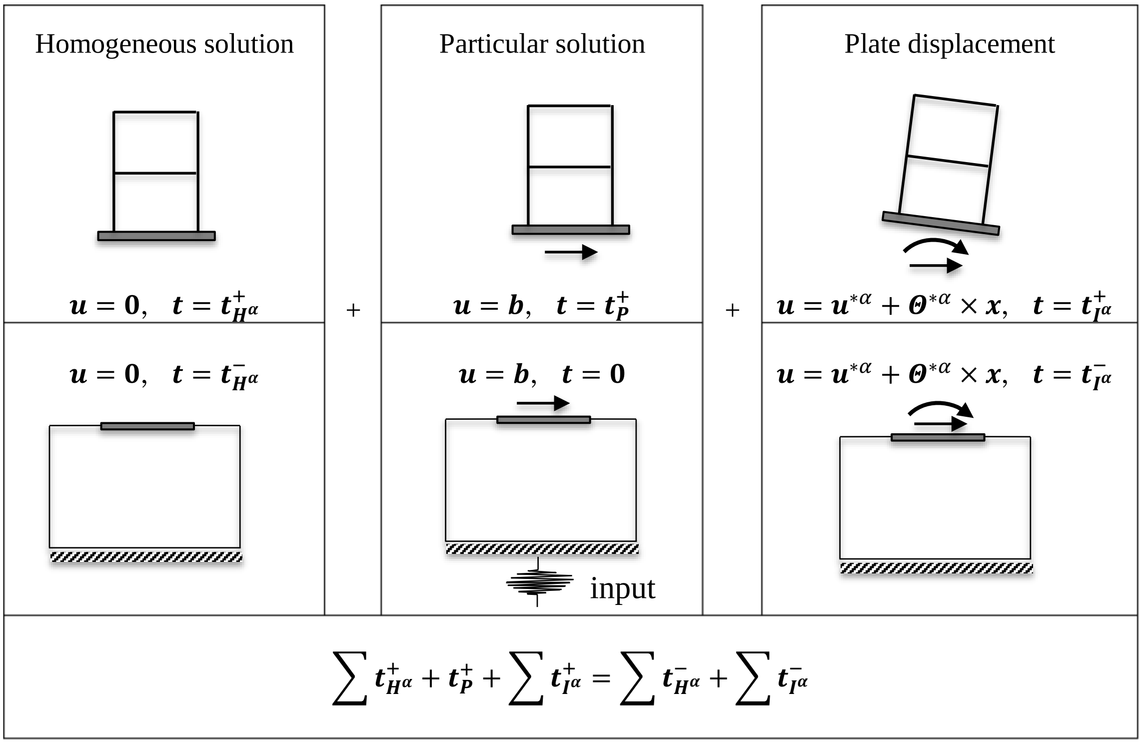


Figure 3.3: Continuity of traction at interface

Lagrangian of the soil-structure system results in an initial boundary value problem of solving the wave equation that is a partial differential equation of 4D. The general solution for this problem can be represented as a sum of homogeneous solution i.e. general solution of homogeneous equation and a particular solution of the inhomogeneous equation. With the introduction of rigid body foundation as explained in previous Section, we separate the structure and soil domains and decompose the solution of this soil-structure problem into homogeneous and particular solution of individual structure and soil-domains. This gives the ease of separately considering and solving the problem of soil-structure interaction for the structure and soil domains without having to consider the whole soil-structure system together. This decomposition is schematically shown in Fig. 3.4 and it is explained below.

First of all for the soil part, the particular solution is the amplified ground motion in the soil domain which is the solution of wave equation in soil subjected to the input excitation. In Fig. 3.4, $\boldsymbol{b}(\boldsymbol{x},t)$ is the amplified ground motion in the soil domain and $\boldsymbol{b}^{*}(t)$ is the average amplified ground motion at the foundation level. The particular solution for the structure is the solution of wave equation in the structure domain subjected to the amplified ground motion at the foundation level.

Next the homogeneous solution for structure in the frequency domain is the solution of the wave equation, Eqn. (3.4a) in structure domain *S* with the displacement $\boldsymbol{u}=\boldsymbol{0}$ at the interface *I*.

$$
\begin{aligned}
\rho \ddot{\boldsymbol{u}} + \boldsymbol{\nabla}.(\boldsymbol{c}:\boldsymbol{\nabla}\boldsymbol{u}) = \boldsymbol{0} \quad & \text{in } S, \\
\boldsymbol{u} = \boldsymbol{0} \quad & \text{on } I.
\end{aligned}
\tag{3.4}
$$

As seen, a set of the above differential equation and the boundary condition poses a homogeneous problem, and a non-trivial solution of this problem is the mode for a certain natural frequency, i.e. a solution of

$$
\rho\omega^2 \boldsymbol{\phi}(\boldsymbol{x}) + \boldsymbol{\nabla}.(\boldsymbol{c}:\boldsymbol{\nabla}\boldsymbol{\phi}(\boldsymbol{x})) = \boldsymbol{0},
$$

where ω and $\boldsymbol{\phi}$ are the natural frequency and the corresponding mode. The structure vibrates with displacement at the base fixed, and the natural frequency and natural modes can be determined without considering SSI. So in this way we can separate the soil and structure domains without changing their dynamic characteristics, with the help of the assumption of rigid body foundation.

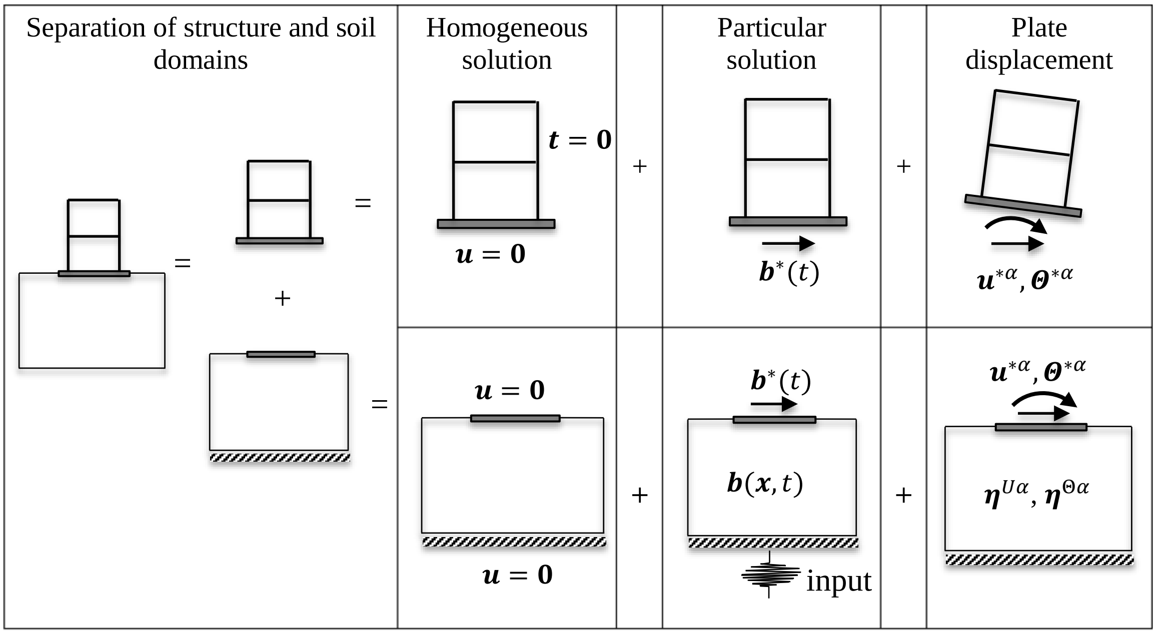


Figure 3.4: Decomposition of solution

For the movement of the plate to ensure continuity of traction, as given in Eq. (3.1), we get the displacement functions $\boldsymbol{\eta}^{U\alpha}$ and $\boldsymbol{\eta}^{\Theta\alpha}$ in soil domain corresponding to the translation and rotation of the rigid interface respectively, in frequency domain, see Fig. 3.4. By using these displacement functions, soil spring corresponding to the rigid body translation and rotation is calculated.

The homogeneous solution for soil, similar to the homogeneous solution for structure, is the solution of the homogeneous wave equation. However, it should be noted that in this case the natural mode of the soil is not the one with free boundary condition at soil surface, as shown in Fig. 3.4, rather it is the mode with boundary condition $\boldsymbol{u} = \boldsymbol{0}$ at I and is sometimes referred to as “surface wave” and it can be mathematically determined, independent of the structure, as the homogeneous solution of the wave equation for the soil domain with the boundary conditions as mentioned above.

Chapter 4

Clarification of SSI

4.1 Overview

In this chapter, the underlying approximation that is made in introducing the soil spring for SSI is clarified by rigorously formulating SSI. It is the approximation that determines the applicability and limitations of the soil spring. As mentioned in the previous chapter, there are some simplifications which are made when modeling the SSI in terms of the soil spring. Those simplifications should be understood to study the applicability and limitations of the soil spring. The key simplification, which is regarded as an approximation, is the introduction of rigid body foundation. It is natural to model soil as a spring if we consider the translation and the movement of the foundation that are induced by the reaction force and bending moment of the soil.

While the modeling is simple, it is not a trivial task to determine the properties of the soil spring. Indeed, the properties change depending of the frequency, if we consider SSI in frequency domain. The dependence on the soil spring properties on the frequency leads to quasi-non-linear characteristics of soil, which is favored by conventional analysis.

The contents of this chapter are organized as follows. First, the rigid-body foundation is discussed in Section 4.2. Then the frequency dependence of the soil spring is explained in Section 4.3. In the end, the requirements of soil spring for SSI evaluation are presented in Section 4.4.

4.2 Rigid-body foundation

As explained in Section 3.4, in order to estimate the dynamic characteristics of a structure which are estimated without considering SSI, we pose suitable null boundary conditions, one of which is zero displacement at the bases, which corresponds to the case that the foundation is a rigid body. This is because the introduction of an ideal rigid body leads to spatially fixed displacement boundary condition and zero displacement boundary condition does not disturb this boundary condition.

The introduction of the rigid-body foundation is reasonable if we consider that practically the foundations are stiffer for the parts of structures. We also point out an advantage of the rigid-body foundation is that the soil-structure problem can be separated into a structure problem and a soil problem, without changing the dynamic properties of the structure. In this way the soil and structure domains are separately solved by additionally considering SSI via the soil spring, instead of solving the soil-structure system as a whole, which will require larger computations.

Another advantage is that continuity of traction at every point at the interface does not have to be satisfied. The equilibrium of the reaction force and moment of the structure and the soil is sufficient (see Section 5.2). This reduces the problem size considerably.

In reality, however, there is no rigid body foundation; it is only theoretically possible to have an ideally rigid foundation. For the case of a realistic foundation which has sufficiently high (but not rigid) stiffness, the spatially non-uniform response of the foundation inevitably leads to the non-uniform floor response. It is expected that the quality of the prediction of the floor response becomes poor if a soil spring that is based on the introduction of the rigid-body foundation is used.

4.3 Frequency dependent soil spring

According to the meta-modeling theory, we interpret a soil spring is the consequence of introducing the rigid-body foundation. Hence, it is straightforward to compute the spring property by considering the rigid-body translation and rotation of the foundation.

In practice, it is often a case that such displacement functions for soil as explained in Section 3.5 are used for the evaluation of the soil spring properties. The displacement functions are considered in the frequency domain, rather than in the time domain, so that the computation needed for the evaluation is reduced. A unit harmonic excitation of displacement/rotation at a particular frequency is given to the foundation, and the reactional force/moment of the soil is computed so that the soil spring properties are determined from the ratio of the reaction force/moment to the displacement/rotation.

The properties of the soil spring change, depending on the selection of these displacement functions, and as long as the input ground motion is a simple harmonic motion, the soil spring is expected to give a good approximate solution. But since a real ground motion has wide range of frequencies, theoretically it is not possible to make a ground spring that can approximate the response because of any ground motion. However, considering the displacement functions for the movement of rigid plate for a range of frequencies, the application range of the soil spring can be improved.

4.4 Requirements of soil spring for SSI evaluation

Since soil spring modeling that is made for the evaluation of SSI is a simplification of the continuum mechanics problem, its applicability is limited; in other words, the soil spring modeling could be either a good or bad modeling, depending on the continuum mechanics problem. For a simple soil-spring to be a good modeling or to be applicable, there are some requirements which need to be satisfied for a structure, soil and an input ground motion.

However, we have to emphasize that this simplified modeling may not be applicable to any soil-structure problem and cannot give a good approximate solution for a particular set of structure, soil and input ground motion.

A structure which is suitable for the application of the soil spring modeling is the one which is simple and symmetric. These two conditions ensure that center of rotation of the plate remains at the center of the plate (as assumed in Section 3.4) and there is no significant shift in the center of rotation of the plate during the seismic excitation. However for a complicated and unsymmetrical structure, this condition is hardly satisfied and the soil spring may give a poor evaluation of SSI.

The next important requirement is the satisfaction of the condition of a rigid foundation. A foundation which is significantly stiffer than both the structure and the soil is required. Furthermore, the foundation should be rigidly connected to the structure and the soil to avoid any separation and slip which are not accounted for in constructing the soil spring.

In general, a site consisting of few geological rock layers has a more or less continuous distribution of natural frequencies of surface waves, and hence it is difficult to select a limited number of frequencies in which the soil spring is constructed. However, a uniform and stratified geological structure has a scattered distribution of surface waves, if the direction of the propagation is restricted to the vertical direction only. This condition ensures the applicability of the soil spring modeling. As for a site of soil layers which has a scattered distribution of vertical surface waves, the applicability of the soil spring modeling is ensured as well.

Chapter 5

Construction of

Consistent Mass-Spring Soil-Spring Model

5.1 Overview

This chapter seeks to develop a methodology of constructing a consistent mass-spring soil-spring model for the structural seismic response analysis that accounts for soil-structure interaction (SSI). Starting from the Lagrangian of continuum mechanics and selecting suitably approximated displacement functions, the variational problem of the Lagrangian is converted to an initial value problem of the mass-spring soil-spring model. A key issue is the explicit expressions of the mass and the stiffness constants, which are rigorously derived from the Lagrangian.

As the simplest case, the governing equation of a single mass-spring system for a structure with rigid body foundation and a soil spring is derived, based on an assumption of one-dimensional deformation (or uni-directional displacement). Since the objective of this dissertation is the seismic response analysis of an NPP building considering SSI, the characteristics of an NPP building and underlying soil are discussed in order to clarify the applicability and limitations of the mass-spring soil-spring model that is constructed according to the developed methodology.

The contents of this chapter are organized as follows. First, the approximation of displacement functions is discussed in Section 5.2. A governing equation for the simplest case of a mass-spring soil-spring model is derived in Section 5.3. A modified governing equation which includes the effects of surface waves is derived in Section 5.4. The characteristics of a typical NPP building and underlying soil are discussed in Sections 5.5 and 5.6, respectively.

5.2 Approximation for displacement function

The variational problem of the Lagrangian of Eq. (3.1) for a displacement function, denoted by $\boldsymbol{u}$, is regarded as a physical problem; various mathematical problems each of which has a distinct solution are derived from the physical problem by making suitable approximations for $\boldsymbol{u}$. Note that the exact solution of the variational problem, which corresponds to the mathematical problem of continuum mechanics, is numerically obtained as a converged solution of FEM that uses a sufficiently large number of solid elements.

Based on the meta-modelling theory, we construct a lower fidelity model for a mathematical problem which is derived with making suitable approximations. A mass-spring soil-spring model is one of this lower fidelity model. This model is constructed just by using a suitably approximated displacement function $\boldsymbol{u}$. Here, the mathematical approximation of $\boldsymbol{u}$ means the specification of the form of $\boldsymbol{u}$. Substitution of this approximated displacement function into the Lagrangian automatically leads to a consistent mass-spring soil-spring model; taking the variation for the resulting Lagrangian, the governing equation is derived, in which the mass and the spring constant are rigorously determined in terms of density and elasticity.

In the above methodology of constructing a consistent model, we should not make any physical assumptions. Setting physical assumptions means changing the Lagrangian, which results in another physical problem. We have to validate the assumption. Moreover, the altered physical problem has a solution different from the continuum mechanics problem, and the consistency with the continuum mechanics is lost.

In constructing a low fidelity model, we have to keep these points in mind; the mathematical approximations ought to be made for the displacement function, so that a mass-spring soil-spring model with rigorously determined mass and spring constants are constructed. The first point is the separation of displacement function in the structure and soil regions, denoted by *S* and *G*, respectively.; the continuity of the displacement at the interface, denoted by *I*, must be guaranteed. The second point is the input of the amplified ground motion to the base of the structure; the amplified ground motion is computed in the absence of the structure. The third point is that the natural frequency and the corresponding mode of the mass-spring model are required to match with those of the continuum mechanics problem, in order to achieve higher accuracy of the mass-spring soil-spring model.

Based on the above, the displacement function corresponding to the mass-spring soil-spring model in three domains, i.e. soil *G*, the interface *I* and the structure *S* is considered as follows;

$$\boldsymbol{u}(\boldsymbol{x},t) = \begin{cases} \boldsymbol{b}(\boldsymbol{x},\boldsymbol{t}) + \boldsymbol{u}^G(\boldsymbol{x},\boldsymbol{t}) & in\ G, \\ \boldsymbol{b}^*(t) + \boldsymbol{U}(t) + \boldsymbol{\Theta}(t) \times \boldsymbol{x} & on\ I, \\ \boldsymbol{b}^*(t) + \boldsymbol{U}(t) + \boldsymbol{\Theta}(t) \times \boldsymbol{x} + \boldsymbol{u}^S(\boldsymbol{x},t) & in\ S. \end{cases} \tag{5.1}$$

where $\boldsymbol{b}$ is the amplified ground motion in *G* without the presence of *S*; $\boldsymbol{u}^G$ is the additional displacement to $\boldsymbol{b}$ by the presence of *S*; $\boldsymbol{b}^*$ is the rigid body translation at the interface *I* induced by $\boldsymbol{b}$; $\boldsymbol{U}$ and $\boldsymbol{\Theta}$ are, respectively, the rigid body translation and rotation of *I* induced by SSI; and $\boldsymbol{u}^S$ is the additional displacement induced in the structure by $(\boldsymbol{b}^* + \boldsymbol{U} + \boldsymbol{\Theta} \times \boldsymbol{x})$ because of the inertia of the structure. Among these displacements, $\boldsymbol{b}$ and $\boldsymbol{b}^*$ are computed separately, and unknown functions are $\boldsymbol{u}^S(\boldsymbol{x},t)$, $\boldsymbol{u}^G(\boldsymbol{x},t)$, $\boldsymbol{U}(t)$ and $\boldsymbol{\Theta}(t)$.

When G is homogeneous or layered (parallel stratification), $\boldsymbol{b}$ varies only with depth and

$$\boldsymbol{b}^* = \frac{1}{I}\int_I \boldsymbol{b}\,\mathrm{d}S$$

matches $\boldsymbol{b}$ of the ground surface and the rigid body rotation becomes zero. $1/I$ in above equation is the area of the interface I. Hence, for a simple setting of stratified soil, $\boldsymbol{b}^*$ and $\boldsymbol{b}$ can be regarded as same. With this assumption, the interpretation of Eq. (5.1) is that the amplified ground motion $\boldsymbol{b}$ input into S through I induces displacement $\boldsymbol{u}^S$ in S, and its reaction force induces displacement $\boldsymbol{u}^G$ in G through I. The displacement of I corresponding to the rigid foundation is given as discussed in Chapter 3 in Eq. (3.3), as combination of rigid translation and rotation, in addition to $\boldsymbol{b}$.

As mentioned above, the use of a mode of the structure in discretizing $\boldsymbol{u}$ is a reasonable choice, in order to match the natural frequency and mode of the mass spring model with those of the continuum mechanics problem; they are the dynamic characteristics of the structure. Denoting by $\boldsymbol{\phi}^\alpha$ the α^{th} mode of the structure, we now approximate $\boldsymbol{u}^S$ as

$$\boldsymbol{u}^S(\boldsymbol{x},t) = \sum_\alpha a^\alpha(t)\boldsymbol{\phi}^\alpha(\boldsymbol{x}). \tag{5.2}$$

Here a^α is the amplitude of the α^{th} mode. In the mass spring model, $\boldsymbol{u}^G$, the displacement of soil, is induced when I is given forced displacement. Denoting by $\boldsymbol{\eta}^{U\alpha}$ and $\boldsymbol{\eta}^{\Theta\alpha}$ soil displacements that are induced by forced vibration of $\boldsymbol{U}$ and $\boldsymbol{\Theta}$, respectively, $\boldsymbol{u}^G$ is set as

$$\boldsymbol{u}^G(\boldsymbol{x},t) = \textstyle\sum_\alpha U^\alpha(t)\boldsymbol{\eta}^{U\alpha}(\boldsymbol{x}) + \Theta^\alpha(t)\boldsymbol{\eta}^{\Theta\alpha}(\boldsymbol{x}). \tag{5.3}$$

Here U^α and Θ^α are amplitudes of $\boldsymbol{\eta}^{U\alpha}$ and $\boldsymbol{\eta}^{\Theta\alpha}$, respectively.

With substitution of Eq. (5.1) using Eqs. (5.3) and (5.2), into Eq. (3.1), the expression for Lagrangian is as follows,

$$\mathcal{L}[a^\alpha, U^\alpha, \Theta^\alpha] = \mathcal{L}^S[a^\alpha] + \mathcal{L}^G[U^\alpha, \Theta^\alpha] + \mathcal{L}^I[a^\alpha, U^\alpha, \Theta^\alpha]. \tag{5.4}$$

Here $\mathcal{L}^S$ and $\mathcal{L}^G$ are the Lagrangian for the structure and soil alone respectively, whereas the third term $\mathcal{L}^I$ corresponds to SSI and the detailed expressions are given as follows,

$$\mathcal{L}^S = \sum_\alpha \frac{1}{2}M^{\alpha\alpha}(\dot{a}^\alpha)^2 - \frac{1}{2}K^{\alpha\alpha}(a^\alpha)^2 + M^{b\alpha}\dot{\boldsymbol{b}}^*\dot{a}^\alpha, \tag{5.5}$$

$$\mathcal{L}^G = \sum_{X,\alpha,Y,\beta} \frac{1}{2}M^{X\alpha Y\beta}\dot{X}^\alpha\dot{Y}^\beta - \frac{1}{2}K^{X\alpha Y\beta}X^\alpha Y^\beta + \sum_{X,\alpha} M^{bX\alpha}\dot{\boldsymbol{b}}^*\dot{X}^\alpha - K^{bX\alpha}\boldsymbol{b}^*X^\alpha. \tag{5.6}$$

Here X and Y are replaced by U or Θ. Mass $M^{\alpha\beta}$ and spring constant $K^{\alpha\beta}$ are computed by the following equation,

$$\begin{aligned} M^{\alpha\beta} &= \int_S \rho\, \boldsymbol{\phi}^\alpha \cdot \boldsymbol{\phi}^\beta \,\mathrm{d}v, \\ K^{\alpha\beta} &= \int_S \boldsymbol{\nabla}\boldsymbol{\phi}^\alpha : \boldsymbol{c} : \boldsymbol{\nabla}\boldsymbol{\phi}^\beta \,\mathrm{d}v \end{aligned} \tag{5.7}$$

Mass $M^{X\alpha Y\beta}$ and the stiffness $K^{X\alpha Y\beta}$ are calculated by above equations replacing from S to G for the integral domain and from $\boldsymbol{\phi}^\alpha$ to $\boldsymbol{\eta}^{U\alpha}$ or $\boldsymbol{\eta}^{\Theta\alpha}$ for integrand. Similarly $M^{b\alpha}$, $M^{bX\alpha}$ and $K^{bX\alpha}$ are also integrated in the S or G domain.

The third term of Lagrangian in Eq. (5.4) is,

$$\begin{aligned} \mathcal{L}^I = \sum_\alpha M^{U\alpha}\dot{a}^\alpha\left(\sum_\beta \dot{U}^\beta\right) + M^{\Theta\alpha}\dot{a}^\alpha\left(\sum_\beta \dot{\Theta}^\beta\right) + M\sum_\alpha \dot{U}^\alpha + M^I\sum_\alpha \dot{\Theta}^\alpha \\ + M^{SU\Theta}\sum_{\alpha,\beta}\dot{U}^\alpha\dot{\Theta}^\beta. \end{aligned} \tag{5.8}$$

Where M is the total mass of the structure and the remaining constants are as follows;

$$\begin{aligned} M^{U\alpha} &= \int_S \rho\, \boldsymbol{\phi}^{\boldsymbol{\alpha}} \,\mathrm{d}v \\ M^{\Theta\alpha} &= \int_S \rho\, \boldsymbol{\phi}^{\boldsymbol{\alpha}} \times \boldsymbol{x} \,\mathrm{d}v \\ M^I &= \int_S \rho\, \boldsymbol{x} \otimes \boldsymbol{x} \,\mathrm{d}v \\ M^{SU\Theta} &= \int_S \rho\, \boldsymbol{x} \,\mathrm{d}v \end{aligned}$$

It should be noted that $\boldsymbol{\phi}^{\boldsymbol{\alpha}}$ is the α-th mode, and hence we have $M^{\alpha\beta} = 0$ and $K^{\alpha\beta} = 0$ for the case of $\alpha \neq \beta$. For the case of $\alpha = \beta$, we have non-zero $M^{\alpha\alpha}$ and $K^{\alpha\alpha}$, satisfying $\sqrt{K^{\alpha\alpha}/M^{\alpha\alpha}} = \omega^\alpha$ with ω^α being the α-th natural frequency.

We consider the simplest case of one mode ($\alpha = 1$). The mass-spring model has one degree-of-freedom, a^1, and the rigid foundation has two degrees-of-freedom, (U^1, Θ^1). The expression for the resulting Lagrangian is given as follows:

$$\begin{aligned} \mathcal{L}[a, U, \Theta] = \frac{1}{2}M^S\dot{a}^2 - \frac{1}{2}K^S a^2 + M^b\dot{b}^*\dot{a} + \frac{1}{2}M^{UU}\dot{U}^2 + \frac{1}{2}M^{\Theta\Theta}\dot{\Theta}^2 + M^{U\Theta}\dot{U}\dot{\Theta} - \frac{1}{2}K^{UU}U^2 \\ - \frac{1}{2}K^{\Theta\Theta}\Theta^2 - K^{U\Theta}U\Theta + M^U\dot{a}\dot{U} + M^\Theta\dot{a}\dot{\Theta} + \frac{1}{2}M\dot{U}^2 + \frac{1}{2}M^I\dot{\Theta}^2 + M^{SU\Theta}\dot{U}\dot{\Theta}. \end{aligned} \tag{5.9}$$

Here, the superscript 1 is omitted.

The sum of the first three terms on the right side of above equation corresponds to $\mathcal{L}^S$ of Eq. (5.5) and is a Lagrangian of a one mass-spring model that receives input ground motion $\boldsymbol{b}^*$. The sum of the fourth to ninth terms in the above Lagrangian corresponds to $\mathcal{L}^G$ of Eq. (5.6) and the remaining terms correspond to $\mathcal{L}^I$ of Eq. (5.8), and these can be considered as the Lagrangian of the soil-spring model considering SSI. It should be noted that the explicit expressions for the mass and stiffness constants are derived and once given the displacement functions for the soil and structure domains, it is objectively and easily possible to determine the mass and spring constants without any need of experience or any physical assumption as the expressions only include the integration of the approximated displacement functions in the corresponding domains.

5.3 Governing equation for the 1D mass-spring soil-spring model

For simplicity, consider a ground structure system installed in a uniform soil. Considering the orthogonal coordinates with *x*-axis and *y*-axis in the horizontal direction and *z* -axis in the vertical direction. Since the soil is uniform, $\boldsymbol{b}$ of the ground *G* is a function only of *z* having a component in the *x*-direction only. We consider only the *x*-direction response of the structure by considering rigid body translation U of boundary *I* in the *x* direction only and rigid body rotation Θ only about the *y*-axis. The right side of the function of Eq. (5.1) is as follows.

$$\begin{aligned} \boldsymbol{u}^S &= a\phi(\boldsymbol{x})\boldsymbol{e}_x + (U(t) + \Theta(t)z)\boldsymbol{e}_x, \\ \boldsymbol{u}^G &= \left(U(t)\eta^U(\boldsymbol{x}) + \Theta(t)\eta^\Theta(\boldsymbol{x})\right)\boldsymbol{e}_x, \end{aligned} \tag{5.10}$$

where ϕ is the *x*-direction component of $\boldsymbol{\phi}^1$, the mode of the minimum natural frequency of the structure; η^U and η^Θ are the *x*-direction components of $\boldsymbol{\eta}^{U1}$ and $\boldsymbol{\eta}^{\Theta 1}$, the soil displacement induced by rigid body translation and rigid rotation of *I*; and $\boldsymbol{e}_x$ is the unit vector in the *x*-direction. Note that the natural frequency of the rigid body translation and rigid rotation is the same as that of the structure.

Substituting Eq. (5.10) into Eq. (5.1) and substituting the resulting $\boldsymbol{u}$ into Eq. (3.1), $\mathcal{L}[a, U, \Theta]$ of Eq. (5.9), a functional of three functions, is obtained. Using the Hamilton's principle, following governing equation is derived from the variational problem of $\delta \int \mathcal{L}\, \mathrm{d}t = 0$:

$$[M][\{\ddot{u}\} + [K]\{u\} = -\{f\}. \tag{5.11}$$

Here the vectors $\{u\}$ and $\{f\}$ are

$$\{u\} = \begin{Bmatrix} a \\ U \\ \Theta \end{Bmatrix}, \quad \{f\} = \begin{Bmatrix} M^U \ddot{b}^* \\ M\, \ddot{b}^* \\ M^{SU\Theta}\, \ddot{b}^* \end{Bmatrix}.$$

And the matrices $[M]$ and $[K]$ are

$$[M] = \begin{bmatrix} M^S & M^U & M^\Theta \\ & M + M^{UU} & M^{SU\Theta} \\ sym & & M^I + M^{\Theta\Theta} \end{bmatrix},$$

$$[K] = \begin{bmatrix} K^S & 0 & 0 \\ & K^{UU} & 0 \\ sym & & K^{\Theta\Theta} \end{bmatrix}.$$

The explicit expressions for the objective determination of mass and spring constants of the above matrices are given in the preceding section and since the expressions involve the spatial integration of the displacement functions, problem of any arbitrary shape of the structure, soil or foundation can be solved. This versatility and objectivity is an advantage of using the meta-modeling theory. Equation (5.11) takes on the same form as the governing equations of a conventional mass-spring model with a soil-spring; unknown functions are displacements a and U in the x direction and rotation Θ about y axis, as shown in Fig. 5.1. It is seen that K^{UU} and $K^{\Theta\Theta}$, which are derived from the purely mathematical procedures, correspond to the soil-spring.

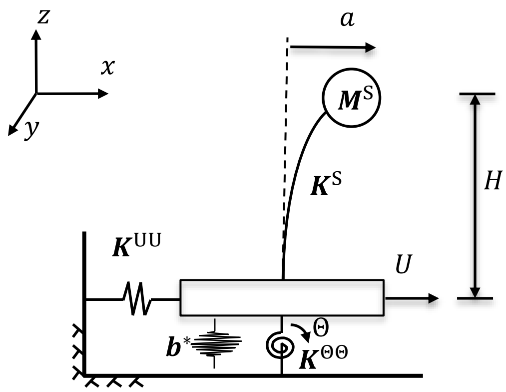


Figure 5.1: Conventional mass-spring soil spring model

5.4 Mass-spring soil-spring model accounting for natural modes of soil

It is possible to include the mode $\boldsymbol{\phi}^G$ of the soil to the displacement of ground $\boldsymbol{u}^G$ as given in Eq. (5.3) to satisfy the continuity of displacement. Then $\boldsymbol{u}^G$ is set as;

$$\boldsymbol{u}^G(\boldsymbol{x},t) = \sum_\alpha a^{G\alpha}(t)\boldsymbol{\phi}^{G\alpha}(\boldsymbol{x}) + U^\alpha(t)\boldsymbol{\eta}^{U\alpha}(\boldsymbol{x}) + \Theta^\alpha(t)\boldsymbol{\eta}^{\Theta\alpha}(\boldsymbol{x}), \quad (5.12)$$

where $a^{G\alpha}$ is the amplitude of the α^{th} mode. It should be noted that this natural mode of soil $\boldsymbol{\phi}^G$ is not the one with free boundary condition at soil surface, rather it is the mode with boundary condition $\boldsymbol{u} = \boldsymbol{0}$ at I and is sometimes referred to as surface wave and it can be mathematically determined, independent of the structure, as the homogeneous solution of the wave equation for the soil domain.

As in the previous section, we can derive the governing equation for the mass-spring system including surface wave by considering x-direction response only and the first mode of structure and soil. Then substituting the displacement functions of Eq. (5.2) and Eq. (5.12) into Eq. (3.1) and solving the variational problem of the resulting Lagrangian, the governing equation of the same form as Eq. (5.11) is obtained with the mass and stiffness matrices as;

$$[M] = \begin{bmatrix} M^S & M^U & M^\Theta & 0 \\ & M + M^{UU} & M^{SU\Theta} & M^{GU} \\ & & M^I + M^{\Theta\Theta} & M^{G\Theta} \\ sym & & & M^G \end{bmatrix},$$

$$[K] = \begin{bmatrix} K^S & 0 & 0 & 0 \\ & K^{UU} & 0 & K^{GU} \\ & & K^{\Theta\Theta} & K^{G\Theta} \\ sym & & & K^G \end{bmatrix},$$

and the displacement and force vectors are as follows

$$\{u\} = \begin{Bmatrix} a \\ U \\ \Theta \\ a^G \end{Bmatrix}, \quad \{f\} = \begin{Bmatrix} M^U \ddot{b}^* \\ M\, \ddot{b}^* \\ M^{SU\Theta} \ddot{b}^* \\ K^{Gb} + \dot{M}^{Gb} \end{Bmatrix},$$

where M^G, M^{GU}, $M^{G\Theta}$ and M^{Gb} are computed by replacing in the mass expression of Eq. (5.7) $\boldsymbol{\phi}^\alpha$ with $\boldsymbol{\phi}^G$ and $\boldsymbol{\phi}^\beta$ with $\boldsymbol{\phi}^G$ for M^G, $\boldsymbol{\eta}^U$ for M^{GU}, $\boldsymbol{\eta}^\Theta$ for $M^{G\Theta}$ and $\dot{\boldsymbol{b}}$ for M^{Gb} and integrating over the soil domain. Similarly, K^G, K^{GU}, $K^{G\Theta}$ and K^{Gb} are computed by replacing Eq. (5.7) $\boldsymbol{\phi}^\alpha$ with $\boldsymbol{\phi}^G$ and $\boldsymbol{\phi}^\beta$ with $\boldsymbol{\phi}^G$ for K^G, $\boldsymbol{\eta}^U$ for K^{GU}, $\boldsymbol{\eta}^\Theta$ for $K^{G\Theta}$ and $\boldsymbol{b}$ for K^{Gb} and integrating over the soil domain.

It should be noted that although Eq. (5.11) is an ordinary differential equation of an unknown 3D vector functions, this governing equation for the case of surface wave is an ordinary differential equation of an unknown 4D vector functions.

5.5 Stick (mass-spring) model for NPP building

An NPP building is a complex structure consisting of several structures such as the reactor enclosure building and the fuel storage building, etc. The most important structure of the NPP building is the nuclear reactor. Due to safety hazard associated with the NPP structure and the need of higher safety factors, the structure is usually massive with thick concrete sections of high stiffness.

The widely used simplified structural analysis model for an NPP building is a lumped mass stick model which is created by discretizing the actual structural and non-structural components by a series of massless beam elements considering the configuration of the structure i.e. beam to column connections or certain locations of interest. Each beam element has two nodes at its ends and the mass is lumped at those two nodes. Each beam element represents stiffness of the structure's section it approximates. These beam elements together form a stick representing a structure as a combination of several lumped masses and stiffness. Since a general nuclear power plant consists of more than one structure in the vicinity of each other, often more than one sticks are needed, which are joined at the base with the rigid foundation, to model the complete NPP structure as a lumped mass stick model.

There is a need for having a consistent stick model for NPP structure in order to have the same dynamic characteristics as that of a high fidelity model, so as to use it as a replacement of the high fidelity model at the initial stages of the design for the preliminary studies.

5.6 Soil-spring for NPP building

Nuclear power plants are generally founded on hard rock. Since the mass and the stiffness of the structure is significant, the SSI effect can be significant and for the simplified modeling, traditionally a sway-rocking model is used to consider the soil structure interaction effect with the foundation considered as the surface foundation and ignoring any effect of the embedment by the surrounding soil. Further the foundation is considered as ideally rigid. However, for a significant embedment depth, the effect of embedment may be significant and should be considered during modelling.

The methodology for the development of mass-spring soil-spring model proposed in this chapter is applicable for a case of surface foundation or a partial embedment with negligible effect of embedment. For, the consideration of effect of embedment of foundation, the improvement of this formulation is needed.

Chapter 6

Numerical Experiments for Soil-Spring Determination

6.1 Overview

In this chapter, numerical experiments are performed to show the usefulness of the mass-spring soil-spring models constructed in Chapter 5. A simple two-storey structure and an NPP structure with sufficiently large soil-domains are considered. The solid elements based FEM, the modal analysis and the mass-spring modeling is presented. The performance of the constructed simplified models is examined by comparing the frequency and the dynamic displacement response results of simplified model with those of the FEM model.

The contents of this chapter are organized as follows. First, the outline of the experiment procedure is described in Section 6.2. In Section 6.3, a simple two-storey building is considered and its frequency and dynamic response obtained from FEM and mass-spring soil-spring model is compared. Same comparison is made considering an NPP structure in Section 6.4.

6.2 Experiment procedure

The objective of the numerical experiment is to confirm that the response obtained from the developed mass-spring model is an appropriate approximation of the 3D FEM solution. For the two structures considered, first of all the modal analysis using solid element FEM analysis is carried out to determine the natural frequency and mode shapes of the structure and the soil and the approximated displacement functions of soil domain are determined by subjecting the rigid plate to unit harmonic translation and rotation. The amplified ground motion in the soil domain and the average amplified ground motion at the rigid plate level are determined by carrying out the dynamic analysis of soil domain subjected to input seismic excitation at the base, using solid element FEM analysis.

The mode shapes and displacement functions along with the amplified ground motion are then used in the closed form expressions derived in Chapter 5 for the mass-spring soil-spring models and by integrating these functions over the corresponding domains, the mass and stiffness values for the mass-spring soil-spring models are determined. The Eigen and dynamic analyses of the soil-structure system are carried out using solid element FEM analysis and the results are compared with the results of the mass-spring soil-spring model.

6.3 Two storey building

The target system consists of a two-storey building and uniform soil as shown in Fig. 6.1 and Fig. 6.2. Each floor of the structure comprises of concrete columns supporting rigid

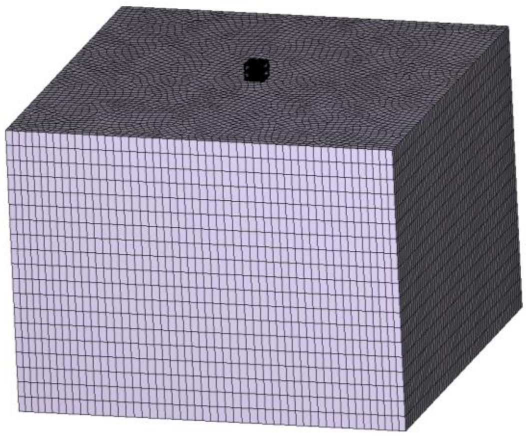


Figure 6.1: Two storey building with the soil domain considered

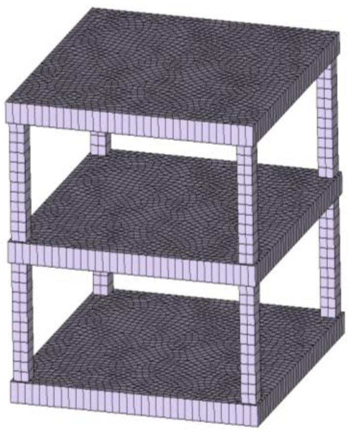


Figure 6.2: Two storey building considered

slab. The plan and cross-sectional dimensions for the structure are given in Fig. 6.2. Soil domain considered is in the shape of a cube with dimensions 200m×200m×150m. The structure rests directly on a stiff foundation laying on the surface without any embedment.

The mechanical properties of the materials used are listed in Table 6.1. Rayleigh damping is used with the damping ratio of 5% and the Rayleigh damping coefficients for mass and stiffness matrices are 0.9 and 0.002 respectively.

Table 6.1: Mechanical properties of materials used

	Column	Slab	Soil	Plate
ρ (kg/m^3)	2400	20000	2500	≈ 0
E (GPa)	30	2000	0.96	∞
ν	0.2	0.2	0.2	0.1

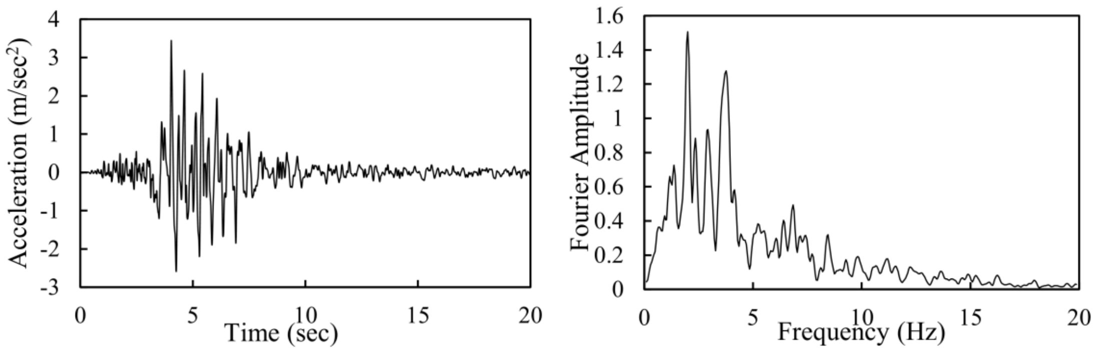


Figure 6.3: Input ground motion GM1 in time and frequency domains

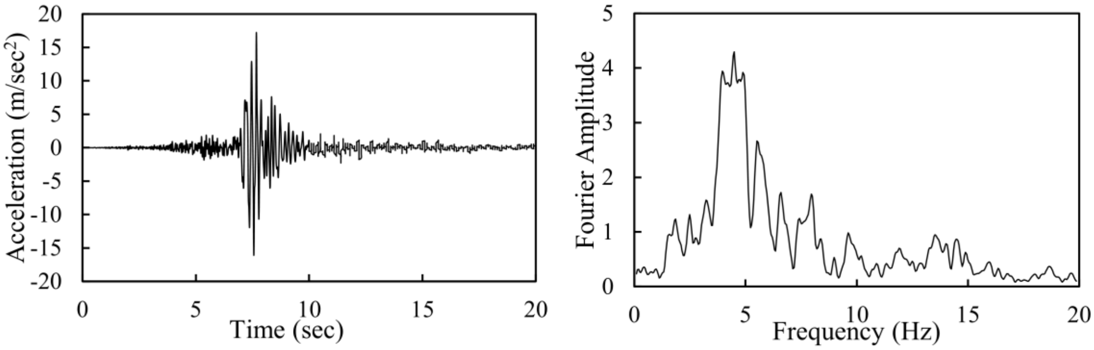


Figure 6.4: Input ground motion GM2 in time and frequency domains

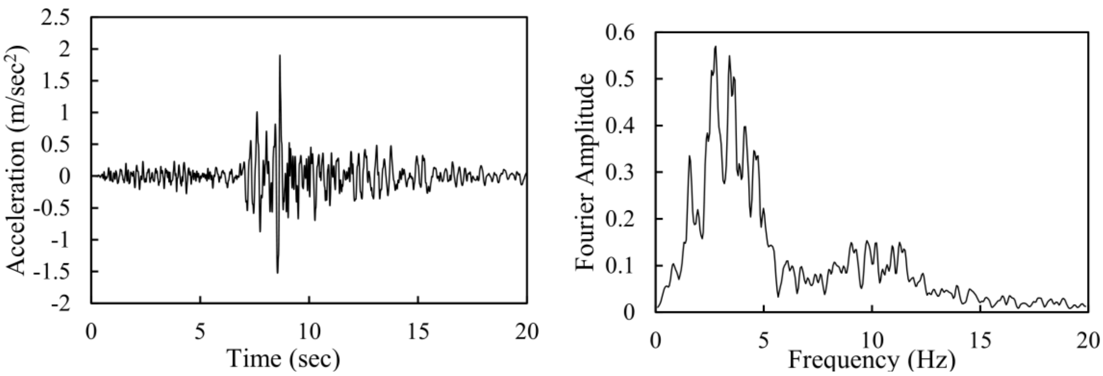


Figure 6.5: Input ground motion GM3 in time and frequency domains

The details of the mesh are given in Table 6.2. Three input ground motions, GM1 and GM2 and GM3 shown in Figs. 6.3, 6.4 and 6.5 respectively, having different distribution of dominant frequency, are input at the bottom of the soil domain. Using a sufficiently large soil domain, the one dimensional wave solution is imposed as boundary conditions on the side surfaces, in order to reduce reflection; the present dimension of the soil domain is sufficiently large so that little reflection is observed.

Table 6.2: Mesh details for FEM model of soil-structure system

	Structure	Soil	Total
No. of nodes	9738	67540	77278
No. of elements	4542	62016	66558
Element type	Linear hexahedron		

To calculate the values of the mass and the spring constants for the mass-spring model, dynamic modes of the structure and soil are determined using the 3D FEM analysis. The minimum natural frequency and the corresponding mode of the structure and the soil

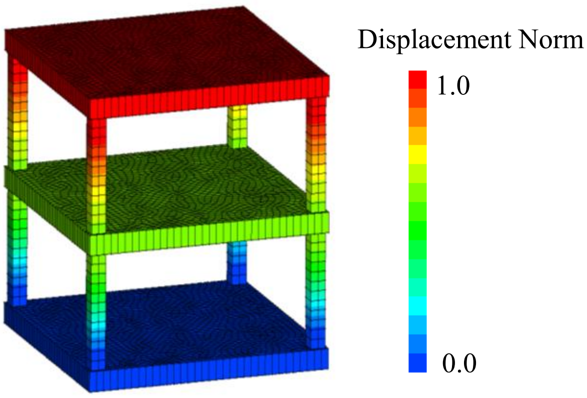


Figure 6.6: Minimum natural frequency of the structure =1.84 Hz

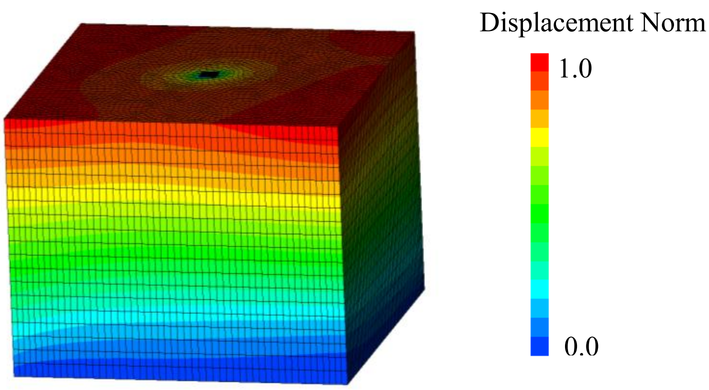


Figure 6.7: Minimum natural frequency of soil (surface wave) = 0.54 Hz

domain are shown in Figs. 6.6 and 6.7, respectively. The amplified ground motion $\boldsymbol{b}$ is computed by using the 3D FEM analysis, too. Time step used is 0.01sec.

The displacement functions of the ground η^U and η^Θ are determined from the dynamic analysis of the ground, which is vibrated by the rigid body translation and rigid rotation of I that oscillates at the minimum natural frequency of the structure. Using a different frequency is also possible. However for a common case of a structure founded on relatively stiffer rock, the excitation frequency of the structure is dominant and it is rational to use the natural frequency of the structure for the determination of displacement functions η^U and η^Θ. However for the opposite case, the natural frequency of the soil or some value in between the natural frequency of the structure and soil can be used for the determination of η^U and η^Θ functions. Depending on this frequency value used and the dominant frequency of the input ground motion, the performance of the soil spring can be different as mentioned in Chapter 4.

The mass and the spring constants are calculated using the closed form expressions that are derived in Chapter 5. The values are summarized in Table 6.3. Here, M^S is the modal mass for the first mode of structure, M is the total mass of the structure, K^S is the modal stiffness for the first mode of structure, M^G is the modal mass for the first mode of soil, K^G is the modal stiffness for the first mode of soil and similarly all the other constants are determined by the integration of the approximated displacement functions for the corresponding structure of soil domain. Recall that the normalized Eigen mode function (or mode shape) is applied, and the modal mass participation ratio for the first mode is 67%.

Table 6.3: Mass and stiffness constants

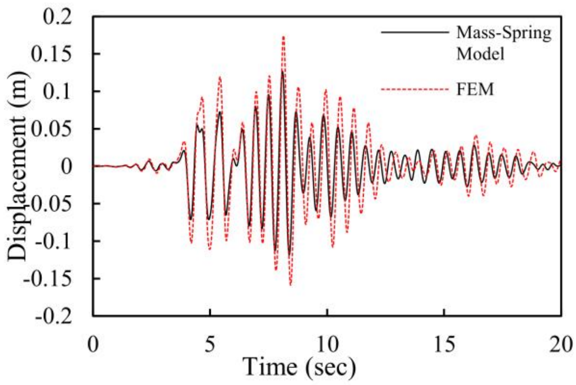
Constant	Value ($\times 10^6$)	Constant	Value ($\times 10^8$)
M^S (kg)	2.25	K^{UU} (N/ m^2)	132.0
M^U (kg)	1.91	$K^{\theta\theta}$ (N/ m^2)	1410.0
M^θ (kgm)	13.0	M^{GU} (kg)	1.25
M (kg)	2.42	$M^{G\theta}$ (kg)	0.166
M^{UU} (kg)	20.7	M^G (kg)	89.0
$M^{SU\theta}$ (kgm)	15.2	K^{GU} (N/ m^2)	113.0
M^I (kg m^2)	105.0	$K^{G\theta}$ (N/ m^2)	9.42
$M^{\theta\theta}$ (kg m^2)	16.4	K^G (N/ m^2)	1030.0
K^S (N/ m^2)	301.0		

Table 6.4 shows the primary natural frequency of the finite element model and the mass-spring model. It is seen that these approximately coincide, which is expected since the first natural mode of the structure is used. There is a reduction in the natural frequency of the structure from 1.84 to 1.73, because of the use of the stiff foundation and not an ideally rigid-body foundation, which is assumed in the construction of the mass-spring soil-spring model. The similarity of the frequency value obtained from FEM analysis and the mass spring models shows the consistency of the constructed mass spring model.

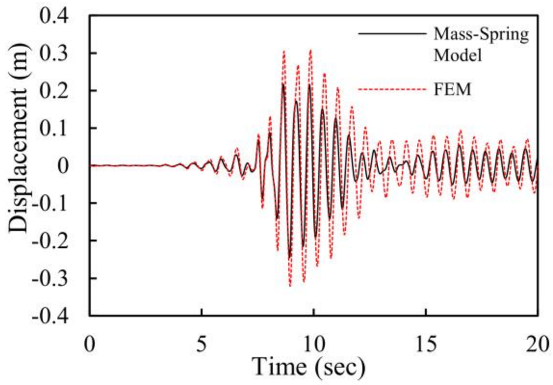
Table 6.4: Comparison of natural frequency

FEM model	Mass-spring model
1.73	1.76

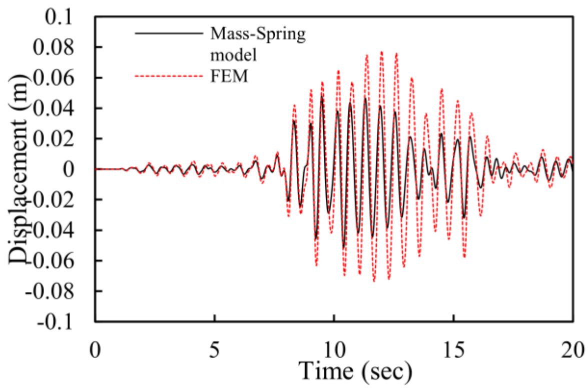
The constructed mass spring model with and without the effect of surface waves is used to determine the displacement response in the x-direction at the top of the structure, and the results are compared with those obtained from 3D FEM analysis. The comparison



a) Mass-Spring Model vs FEM for GM1



b) Mass-Spring Model vs FEM for GM2



c) Mass-Spring Model vs FEM for GM3

Figure 6.8: Comparison of x-direction displacement response at the top of structure without the inclusion of surface wave

for GM1, GM2 and GM3 without including the surface wave is shown in Figs. 6.8a, 6.8b and 6.8c respectively. It is seen that the solution of the mass-spring model is fairly consistent with that of the finite element model. The accuracy of the response can be further improved by considering more than one modes of the structure.

In Figs. 6.9a, 6.9b and 6.9c, the comparison for GM1, GM2 and GM3 with including the surface wave is shown, respectively. As seen, the inclusion of the surface wave results in an increased response because of the inclusion of the soil homogenous solution however there is not a significant difference in the response for this particular soil-structure setting. However, it is expected that with the increase in the scale of structure, this effect will be more significant, since larger forces have to be transmitted from the structure to the soil. It is important to mention here that for the case of inclusion of surface wave, as shown in Section 5.4, the two parameters M^{Gb} and K^{Gb} need to be evaluated at each time step of the input ground motion which results in an increased computational effort.

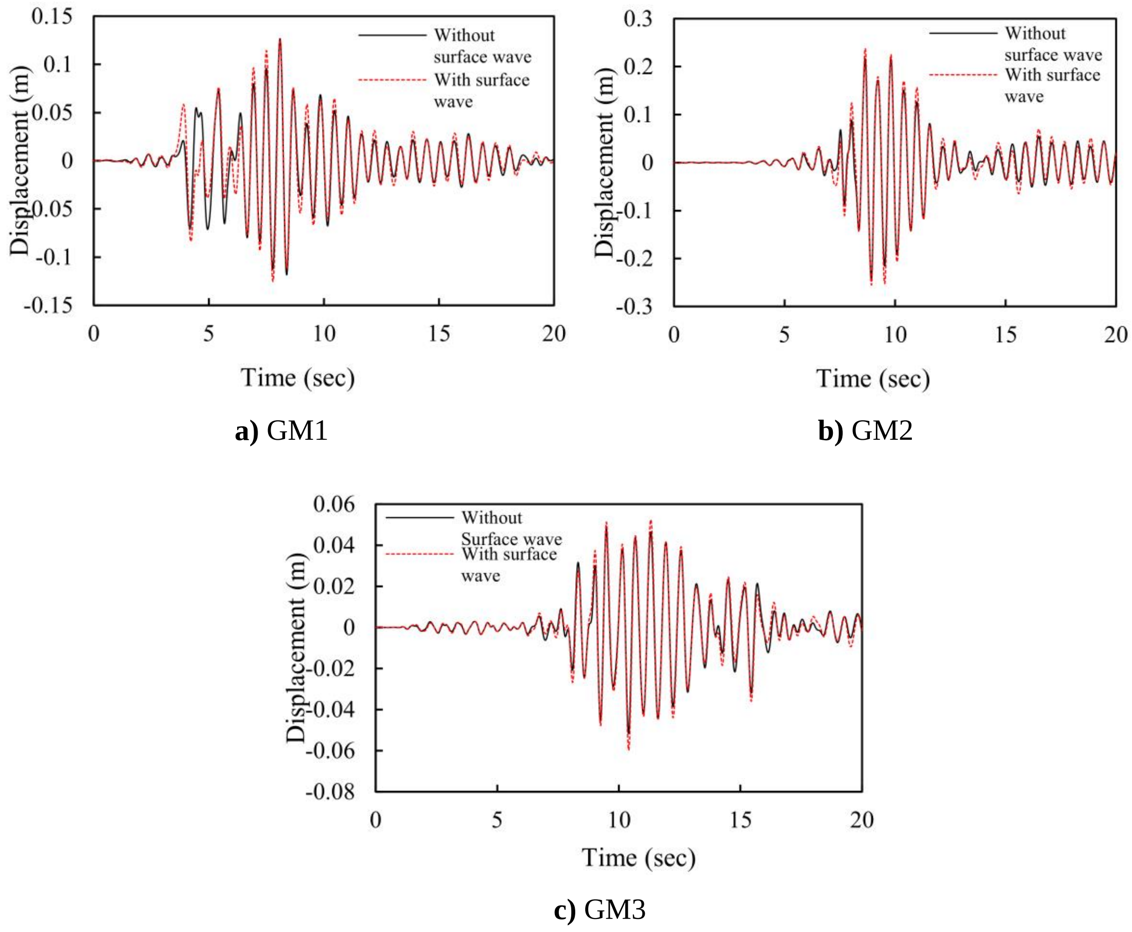


Figure 6.9: Comparison of x-direction displacement response at the top of structure for mass spring system with and without inclusion of surface wave

6.4 Nuclear power plant structure

The target soil-structure system consists of NPP structure and uniform soil as shown in Fig. 6.10. The structure is shown in detail along with the dimensions in Fig. 6.11. The part of the structure considered comprises of 7 floors resting on a thick foundation slab. The floors are numbered from bottom to top. Each floor comprises of slabs, walls and an inner concrete dome which accommodates the nuclear reactor. The mechanical properties of the materials used for structure model are shown in Table 6.5.

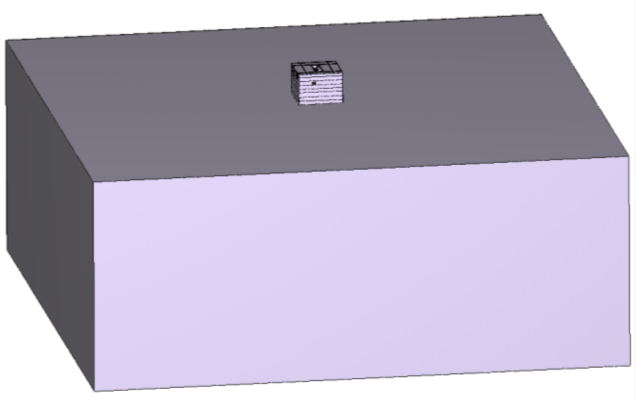


Figure 6.10: NPP structure with the soil domain considered

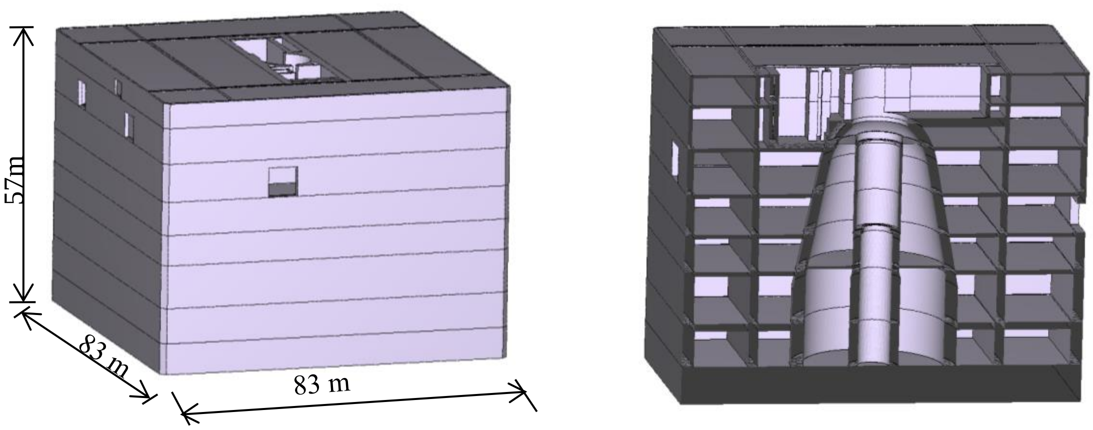


Figure 6.11: Elevation and cross-section view of the NPP structure

Soil domain considered is in the shape of a cube with dimensions of 1000m×1000m×400m and for this experiment, the structure rests directly on the soil and embedment is not considered. The mechanical properties of the soil are shown in Table 6.6 and the mesh details are given in Table 6.7.

Table 6.5: Mechanical properties of materials used for structure

Component	ρ (ton/mm^3) (x 10^{-9})	E (ton/mm^2) (x 10^3)	ν
1F walls	3.893	20.580	0.17
1F slab	3.893	20.580	0.17
1F inner concrete dome	0.251	2.139	0.3
2F walls	4.541	20.580	0.17
2F slab	4.541	20.580	0.17
2F inner concrete dome	0.940	2.943	0.3
3F walls	4.188	20.580	0.17
3F slab	4.188	20.580	0.17
3F inner concrete dome	0.274	2.878	0.3
4F walls	4.381	20.580	0.17
4F slab	4.381	20.580	0.17
4F inner concrete dome	1.908	1.967	0.3
5F walls	4.824	20.580	0.17
5F slab	4.824	20.580	0.17
5F inner concrete dome	0.415	2.444	0.3
6F walls	4.056	20.580	0.17
6F slab	4.056	20.580	0.17
6F inner concrete dome	0.458	1.124	0.3
7F walls	4.256	20.580	0.17
7F slab	4.256	20.580	0.17

Table 6.6: Mechanical properties of soil used

ρ (ton/mm^3)	E (ton/mm^2)	ν
2.5 x 10^{-9}	960	0.2

Rayleigh damping is used with the damping ratio of 5% and the Rayleigh damping coefficients for mass and stiffness matrices are 1.5 and 0.0017 respectively.

Table 6.7: Mesh details for FEM model of soil-structure system

	Structure	Soil	Total
No. of nodes	2070372	2159019	4229391
No. of elements	1086595	2090200	1295615
Element type	Quadratic tetrahedron	Linear hexahedron	

First 1000 time steps of the ground motion GM1 shown in Fig. 6.3 are input at the bottom of the soil domain. Same treatment for the boundary of soil domain is adopted as in previous example to avoid the reflection of waves. To calculate the values of the mass and the spring constants for the mass-spring model, dynamic mode of the structure is determined using the 3D FEM analysis as it is shown in Fig. 6.12.

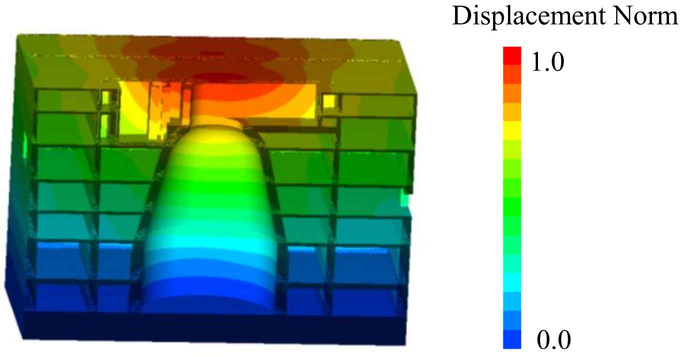


Figure 6.12: Minimum natural frequency of the structure = 3.80 Hz

The amplified ground motion $\boldsymbol{b}$ is computed by using the 3D FEM analysis, too. Time step used is 0.01sec. The displacement functions of the ground η^U and η^Θ are determined from the dynamic analysis of the ground, which is vibrated by the rigid body translation and rigid rotation of I that oscillates at the minimum natural frequency of the structure.

The mass and the spring constants are calculated using the closed form expressions that are derived in Chapter 5. The values are summarized in Table 6.8. Here also, M^S is the modal mass for the first mode of structure, M is the total mass of the structure, K^S is the modal stiffness for the first mode of structure, M^G is the modal mass for the first mode of soil, K^G is the modal stiffness for the first mode of soil and similarly all the other constants are determined by the integration of the approximated displacement functions for the corresponding structure of soil domain.

Table 6.8: Mass and stiffness constants

Constant	Value	Constant	Value
M^S (ton)	63142	K^{UU} (ton/mm^2)	7.79×10^8
M^U (ton)	53670.7	$K^{\theta\theta}$ (ton/mm^2)	6.04×10^{17}
M^θ (ton mm)	2.569×10^8	K^S (ton/mm^2)	3.66×10^7
M (ton)	353601	$M^{\theta\theta}$ (ton mm^2)	1.06×10^{15}
M^{UU} (ton)	1.35×10^6	M^I (ton mm^2)	8.47×10^{14}
$M^{SU\theta}$ (ton mm)	1.82×10^{10}		

Table 6.9 shows the primary natural frequency of the fixed base finite element model of structure and the mass-spring soil-spring model. The natural frequency of the soil-structure system could not be determined for the first 500 modes considered because of the high stiffness of the structure and the relatively weaker soil. There is however, a reduction in the natural frequency of the structure from 3.80 to the 2.61 value obtained by mass spring model which is because of the use of the stiff foundation and not an ideally rigid-body foundation.

Table 6.9: Comparison of natural frequency

Fixed base FEM model	Mass-spring model
3.80	2.61

The constructed mass spring model without the effect of surface waves is used to determine the displacement response in the x-direction at the top of the structure, and the

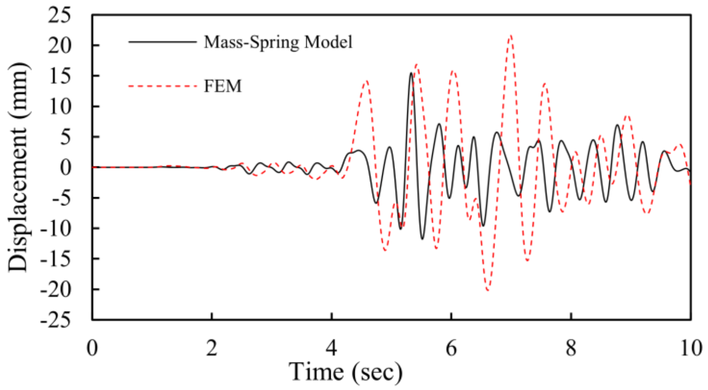


Figure 6.13: Comparison of x-direction displacement response at the top of structure obtained from mass spring model and FEM without the inclusion of surface wave

results are compared with those obtained from 3D FEM analysis. The comparison for first 1000 time steps of GM1 is shown in Fig. 6.13. It is seen that the solution of the mass-spring model is fairly consistent with that of the finite element model however a better response can be obtained by including more number of modes as in this model, only first mode of the structure has been used. The mass spring model with the inclusion of surface waves is not used for this example because the effort involved in integrating the functions for whole soil domain for each time step exceeds the benefit obtained from the simplicity of the model.

Chapter 7

Conversion of Modal Analysis to Mass-Spring Model

7.1 Overview

This chapter explains the possibility of using modal analysis performed using a high fidelity model to get a consistent mass-spring model for a complicated structure. A methodology to construct a consistent mass-spring model is already developed but it has certain limitations, such as the need to have equal number of Eigen modes and mass points of the mass-spring model. This requirement may easily be satisfied for a simple structure for which knowing the response at top of structure or each floor's response is sufficient, but for a complicated structure such as an NPP, for which the response at several locations where critical instruments are to be placed is needed other than the floor response, an improvement of the methodology is needed for the conversion of modal analysis to mass-spring model.

The target is to construct a consistent mass-spring model which, using the first few dominant modes of the structure, can approximate the solution of a high fidelity model. The characteristics of solution of a high fidelity model and modal analysis are explained and using the same methodology as discussed in previous chapter for the construction of a consistent model, a methodology for conversion of modal analysis to a mass-spring model is presented. However, a perfect conversion is not possible and the difficulties in the conversion are described.

The contents of this chapter are organized as follows. First, the solid element based FEM solution and its characteristics are discussed in Section 7.2. Physical and mathematical interpretation of modal analysis is presented in Section 7.3. General linear conversion of modal analysis to a mass-spring model and the difficulties faced in an exact conversion are discussed in Section 7.4. An example for the conversion of modal analysis according to the proposed methodology is presented in Section 7.5.

7.2 FEM solution

The variational problem of the Lagrangian of continuum mechanics of Eq. (3.1) results in a mathematical problem of continuum mechanics i.e. the wave equation. This 4D partial differential equation is solved numerically by FEM using sufficient solid elements to get the solid elements based FEM solution. This solution considers the effect of all the possible modes of a structure and needs the simple material properties as input.

Instead of using solid elements based FEM analysis, shell element models are often used. The major reason for this preference is the lesser computational time and disk space needed for a shell element analysis. However, the assumption of shell type structure i.e. having two dimensions greater than the third one, is not always possible especially for the massive structures such as NPP which have structural components with thick cross sections. Further the contact problems can also be better solved by the use of solid elements.

7.3 Modal analysis

Modal analysis is the determination of the dynamic properties of the structure which include the natural frequency and the dynamic modes. The deformation of a linear structural system can be expressed as a linear combination of these modes. The modes show how the structure will deform when excited. Mathematically, modal analysis of a structure in continuum mechanics is the solution of the homogeneous wave equation with the displacement at the base fixed, and a non-trivial solution of this equation is the mode for a certain natural frequency, i.e. a solution of

$$\rho(\omega^{\alpha})^2\boldsymbol{\phi}^{\alpha}(\boldsymbol{x}) + \boldsymbol{\nabla}.(\boldsymbol{c}:\boldsymbol{\nabla}\boldsymbol{\phi}^{\alpha}(\boldsymbol{x})) = \boldsymbol{0}, \tag{7.1}$$

where ω and $\boldsymbol{\phi}$ are the α-th natural frequency and the corresponding mode.

Modal analysis has been used to construct consistent lumped mass models for structures [11]. In case of simple structure, a particular mode shape generally shows the deformed shape of the whole structure and the movement of the structure is generally uniform along the plan dimension, especially for the case of relatively stiffer slabs as compared to the columns. However for a complicated structure, it is difficult to have this kind of global mode shape and different parts of the structure excite differently and the local behavior is dominant in the mode shapes. These local behaviors can easily be modeled in FEM analysis however to make a simplified model for such a complicated structure is a challenging task. For this reason, to determine the local behavior of the structure at the points of interest, which can be significantly different than the average floor response, more than one sticks in the stick model of a complicated structure are preferred and it is also preferable for the model to have the ability to capture the behavior of all the points of interest, and not just the average floor responses, which is the case for the mass-spring models for the simple structures.

7.4 Conversion of modal analysis

For a back ground of this conversion, consider a simple case of a structure modeled as a one mass-spring model. A Lagrangian for this mass-spring model is

$$\mathcal{L}_s[U,V] = \mathcal{K}_s[V] - \mathcal{P}_s[U], \tag{7.2}$$

where, *U* and *V* are the displacement and velocity of the mass in a particular direction with $V = \dot{U}$ and $\mathcal{K}_S$ and $\mathcal{P}_S$ are kinematic and strain energy defined as

$$\begin{aligned} \mathcal{K}_s[V] &= \frac{1}{2}MV^2, \\ \mathcal{P}_s[U] &= \frac{1}{2}KU^2, \end{aligned} \tag{7.3}$$

where *M* is the lumped mass and *K* is the spring constant for the lateral stiffness.

The same structure when modelled according to the continuum mechanics considering a three dimensional displacement function u, has a Lagrangian of continuum mechanics as given in Eq. (3.1) and here it is written as

$$\mathcal{L}_c[\boldsymbol{v},\boldsymbol{\epsilon}] = \mathcal{K}_c[\boldsymbol{v}] - \mathcal{P}_c[\boldsymbol{\epsilon}], \tag{7.4}$$

Where

$$\begin{aligned} \mathcal{K}_c[\boldsymbol{v}] &= \int_V \frac{1}{2}\rho\boldsymbol{v}\cdot\boldsymbol{v}\ \mathrm{dv}, \\ \mathcal{P}_c[\boldsymbol{\epsilon}] &= \int_V \frac{1}{2}\boldsymbol{\epsilon}:\boldsymbol{c}:\boldsymbol{\epsilon}\ \mathrm{dv}. \end{aligned} \tag{7.5}$$

The form of $\mathcal{L}_s$ and $\mathcal{L}_c$ given by Eqs. (7.2) and (7.4) is similar to each other, even though the functions and the functionals given by Eqs. (7.3) and (7.5) are totally different.

To derive a consistent simplified model, the meta-modeling theory takes advantage of this similarity of $\mathcal{L}_s$ and $\mathcal{L}_c$ and derives $\mathcal{L}_s$ from $\mathcal{L}_c$. The procedure of deriving $\mathcal{L}_s$ from $\mathcal{L}_c$ is simple, as we have to approximate the vector valued displacement function $\boldsymbol{u}$. if we approximate it as the product of an unknown temporal function, $U(t)$, and a known three-dimensional vector-valued function, $\boldsymbol{\Psi}(\boldsymbol{x})$, as

$$\boldsymbol{u}(\boldsymbol{x},t) = U(t)\boldsymbol{\Psi}(\boldsymbol{x}), \tag{7.6}$$

then substituting Eq. (7.6) into $\mathcal{L}_c$, $\mathcal{L}_s$ is obtained. Here the mass and spring constants, *M* and *K*, are explicitly and objectively computed in terms of ρ and $\boldsymbol{c}$ together with the assumed $\boldsymbol{\Psi}(\boldsymbol{x})$ as follows,

$$\begin{aligned} M &= \int_V \rho\boldsymbol{\Psi}\cdot\boldsymbol{\Psi}\ \mathrm{dv}, \\ K &= \int_V \boldsymbol{\nabla\Psi}:\boldsymbol{c}:\boldsymbol{\nabla\Psi}\ \mathrm{dv}. \end{aligned} \tag{7.7}$$

As explained above, it is clear that the derivation of $\mathcal{L}_s$ from $\mathcal{L}_c$ is rigorously made in a mathematical manner, without the need to make any physical assumptions and the Lagrangian problem of a continuum model with ρ and $\boldsymbol{c}$ is reduced to another Lagrangian problem of mass spring model of M and K by using an approximated displacement function. Therefore the solution of $\mathcal{L}_s$ is regarded as an approximation of the solution of $\mathcal{L}_c$.

Since the target is to make a simplified model having same dynamic properties as those of continuum mechanics modeling, the use of mode shapes obtained from solid elements based FEM analysis to get the displacement function $\boldsymbol{\Psi}$ is rational. A methodology to approximately convert the modal analysis results to the approximated displacement functions for a mass-spring model has been proposed by Jayasinghe *et al* [18]. However, that conversion has the limitation of the need to have equal number of mode shapes and mass points of the mass-spring model. This requirement may easily be satisfied for a simple structure for which knowing the response at top of structure or each floor's response is sufficient, however as discussed in the previous section, for a complicated structure such as an NPP, for which the response at several locations where critical instruments are to be placed, is needed other than the average floor response, an improvement is needed for the conversion of modal analysis to mass-spring model for such structures. Hence there is a need to construct a consistent mass-spring model which, using the first few dominant modes of the structure, can approximate the solution of a high fidelity model.

For the above mentioned reason, we start with the Lagrangian of continuum mechanics, writing it as

$$\mathcal{L}[\boldsymbol{v}, \boldsymbol{\epsilon}] = \frac{1}{2}\langle \boldsymbol{v}, \boldsymbol{v}\rangle^{\rho} - \frac{1}{2}\langle \boldsymbol{\epsilon}, \boldsymbol{\epsilon}\rangle^{c}, \tag{7.8}$$

Where

$$\begin{aligned} \langle \boldsymbol{v}, \boldsymbol{v}\rangle^{\rho} &= \int_V \rho \boldsymbol{v} \cdot \boldsymbol{v} \, \mathrm{dv}, \\ \langle \boldsymbol{\epsilon}, \boldsymbol{\epsilon}\rangle^{c} &= \int_V \boldsymbol{\epsilon} : \boldsymbol{c} : \boldsymbol{\epsilon} \, \mathrm{dv}. \end{aligned} \tag{7.9}$$

Let $(\omega^{\alpha}, \boldsymbol{\phi}^{\alpha})$ be a pair of α-th natural frequency and mode satisfying Eq. (7.1), then the modal mass and stiffness for each mode are as follows

$$\begin{aligned} m^{\alpha} &= \langle \boldsymbol{\phi}^{\alpha}, \boldsymbol{\phi}^{\alpha}\rangle^{\rho}, \\ k^{\alpha} &= \langle \boldsymbol{\nabla}\boldsymbol{\phi}^{\alpha}, \boldsymbol{\nabla}\boldsymbol{\phi}^{\alpha}\rangle^{c}. \end{aligned} \tag{7.10}$$

Further the modal matrix $\{\boldsymbol{\phi}^\alpha\}$ satisfies $\langle \boldsymbol{\phi}^\alpha, \boldsymbol{\phi}^\beta \rangle^\rho = 0$ and $\langle \boldsymbol{\nabla\phi}^\alpha, \boldsymbol{\nabla\phi}^\beta \rangle^c = 0$ for $\alpha \neq \beta$.

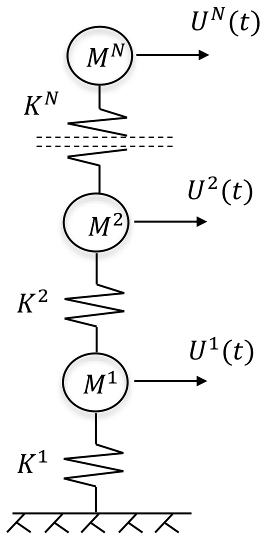


Figure 7.1: Mass-spring model

The Lagrangian for a mass spring model with N number of masses as shown in Fig. 7.1 is as follows;

$$\mathcal{L}^{(N)} = \sum_{I=1}^{N} \frac{1}{2} M^I \, (\dot{U}^I)^2 - \frac{1}{2} K^1 (U^1)^2 - \sum_{I=2}^{N} \frac{1}{2} K^I \, (U^I - U^{I-1})^2, \tag{7.11}$$

To derive Lagrangian of Eq. (7.11) from that of Eq. (7.8), as mentioned in previous section, we approximate the displacement function $\boldsymbol{u}$ as the product of an unknown temporal function, $U^I(t)$, and a known three-dimensional vector-valued function, $\boldsymbol{\Psi}^I(\boldsymbol{x})$, as

$$\boldsymbol{u}(\boldsymbol{x}, t) = \sum_{I=1}^{N} U^I(t) \boldsymbol{\Psi}^{\boldsymbol{I}}(\boldsymbol{x}). \tag{7.12}$$

Here, unknown $\{\boldsymbol{\Psi}^I\}$ are determined in terms of $\{\boldsymbol{\phi}^\alpha\}$ as follows

$$\boldsymbol{\Psi}^{\boldsymbol{I}}(\boldsymbol{x}) = \sum_{\alpha} C^{I,\alpha} \boldsymbol{\phi}^\alpha(\boldsymbol{x}), \tag{7.13}$$

Where $C^{I,\alpha}$ is a constant. Substituting Eq. (7.12) into the Lagrangian of Eq. (7.8), we get the continuum Lagrangian of the following form

$$\mathcal{L} = \sum_{I,J=1}^{N} \frac{1}{2} \left(\sum_{\alpha} m^\alpha \, C^{I,\alpha} C^{J,\alpha} \right) \dot{U}^I \dot{U}^J - \frac{1}{2} \left(\sum_{\alpha} k^\alpha \, C^{I,\alpha} C^{J,\alpha} \right) U^I U^J, \tag{7.14}$$

For the case of a two mass spring system with $I = 2$, if the Lagrangian of Eq. (7.14) is to be of the same form as the Lagrangian of Eq. (7.11), $\{C^{I,\alpha}\}$ ought to satisfy

$$\begin{gathered}\sum_{\alpha} m^{\alpha}\, C^{I,\alpha} C^{J,\alpha} = 0, \;\; \text{for } (I,J) = (1,2) \\ \sum_{\alpha} k^{\alpha} C^{2,\alpha}(C^{2,\alpha} + C^{1,\alpha}) = 0\end{gathered} \tag{7.15}$$

And for a three mass spring system with $I = 3$, $\{C^{I,\alpha}\}$ ought to satisfy

$$\begin{gathered}\sum_{\alpha} m^{\alpha}\, C^{I,\alpha} C^{J,\alpha} = 0, \;\; \text{for } (I,J) = (1,2),(2,3),(3,1) \\ \sum_{\alpha} k^{\alpha} C^{I,\alpha}(C^{I,\alpha} + C^{I-1,\alpha}) = 0\,, \quad \text{for } I = 2,3 \\ \sum_{\alpha} k^{\alpha} C^{1,\alpha} C^{3,\alpha} = 0.\end{gathered} \tag{7.16}$$

Unlike the previous formulation done by Jayasinghe *et al* considering the modes as 3D vectors, in this study, the one direction component of modes at a time is considered. Suppose that $\{\boldsymbol{\Psi}^{I}\}$ satisfy

$$P^{\alpha} \sum_{I} \phi_1^{\alpha}(\boldsymbol{x}^{I}) \boldsymbol{\Psi}^{I}(\boldsymbol{x}) = \boldsymbol{\phi}^{\alpha}(\boldsymbol{x}), \tag{7.17}$$

Where $\{\boldsymbol{x}^{I}\}$ are a set of suitable points in the structure domain and P^{α} is a suitable number. In terms of $\{C^{I,\alpha}\}$, the left side of Eq. (7.17) is

$$P^{\alpha} \sum_{\beta} \left(\sum_{I} \phi_1^{\alpha}(\boldsymbol{x}^{I})\, C^{I,\beta} \right) \boldsymbol{\phi}^{\beta}(\boldsymbol{x}),$$

And due to the independence of the modes, it is necessary that the following should hold,

$$\sum_{I=1}^{N} \phi_1^{\alpha}(\boldsymbol{x}^{I}) C^{I,\beta} = 0\,, \quad \text{for } \alpha \neq \beta \tag{7.18}$$

Equation (7.10) for m^{α} and k^{α} and Eq. (7.18) implies that

$$\begin{gathered}(P^{\alpha})^2 [\phi_1^{\alpha}(\boldsymbol{x}^{I})]^{T} [\langle \boldsymbol{\Psi}^{I}, \boldsymbol{\Psi}^{I} \rangle^{\rho}] [\phi_1^{\alpha}(\boldsymbol{x}^{J})] = m^{\alpha}, \\ (P^{\alpha})^2 [\phi_1^{\alpha}(\boldsymbol{x}^{I})]^{T} \left[\langle \boldsymbol{\nabla\Psi}^{I}, \boldsymbol{\nabla\Psi}^{I} \rangle^{c}\right] [\phi_1^{\alpha}(\boldsymbol{x}^{J})] = k^{\alpha},\end{gathered} \tag{7.19}$$

Where $[\phi_1^{\alpha}(\boldsymbol{x}^{I})]$ is an N component vector, and $[\langle \boldsymbol{\Psi}^{I}, \boldsymbol{\Psi}^{I} \rangle^{\rho}]$ and $[\langle \boldsymbol{\nabla\Psi}^{I}, \boldsymbol{\nabla\Psi}^{I} \rangle^{c}]$ are $N \times N$ matrices.

Writing Eq. (7.19) as

$$(P^{\alpha})^2[\phi_1^{\alpha}(\boldsymbol{x}^I)]^T[M][\phi_1^{\alpha}(\boldsymbol{x}^J)] = m^{\alpha},$$
$$(P^{\alpha})^2[\phi_1^{\alpha}(\boldsymbol{x}^I)]^T[K][\phi_1^{\alpha}(\boldsymbol{x}^J)] = k^{\alpha}, \tag{7.20}$$

shows that $(\omega^{\alpha}, [\phi_1^{\alpha}(\boldsymbol{x}^I)])$ are a pair of natural frequency and mode. And $M^{IJ} = 0$, and $K^{II-1} = -K^{II}$ give conditions for $\{C^{I,\alpha}\}$.

Further it is also required that the total mass of the structure i.e. $M = \int_V \rho(\boldsymbol{x})\, \mathrm{dv}$, is conserved which gives

$$\sum_{\alpha,I} m^{\alpha}(C^{1,\alpha})^2 = M. \tag{7.21}$$

Where $M^{IJ} = 0$ for $I \neq J$ is used. Hence for a perfect conversion of modal analysis results to a mass-spring model, the conditions given in Eqs. (7.15), (7.16) and (7.18) and the conditions $M^{IJ} = 0$, and $K^{II-1} = -K^{II}$should be satisfied.

As an example, for the conversion of two modes of the structure to a mass spring model with two masses, the conditions given above result in the following equations.

$$\begin{gathered} \sum_{I=1}^{2} \phi_1^1(\boldsymbol{x}^I)C^{I,2} = 0, \\ \sum_{I=1}^{2} \phi_1^2(\boldsymbol{x}^I)C^{I,1} = 0\,, \\ M^{12} = 0, \\ K^{21} = -K^{22}, \\ M^{11} + M^{22} = M. \end{gathered} \tag{7.22}$$

The explicit forms of these equations are given in Appendix C. Here $\{C^{I,\alpha}\}$ are four unknowns. It can be seen that the first two equations are linear equations whereas the last three equations are the non-linear equations of the unknowns $C^{I,\alpha}$.

Similarly for the conversion of two modes to a mass spring model with three masses, the resulting equations are

$$\begin{gathered} \sum_{I=1}^{3} \phi_1^1(\boldsymbol{x}^I)C^{I,2} = 0, \\ \sum_{I=1}^{3} \phi_1^2(\boldsymbol{x}^I)C^{I,1} = 0\,, \\ M^{12} = 0, \\ M^{23} = 0, \\ M^{31} = 0, \\ K^{33} = -K^{32}, \end{gathered} \tag{7.23}$$

$$K^{22} = -K^{21} + K^{33},$$
$$K^{13} = 0,$$
$$M^{11} + M^{22} + M^{33} = M.$$

The explicit forms of these equations are also given in Appendix C. Here $\{C^{I,\alpha}\}$ are six unknowns and the first two equations are linear while the seven equations are non-linear.

From the two cases of two and three mass spring models considered above, we have two or four unknowns and three or seven non-linear equations by suitably choosing two or three points for $\{\boldsymbol{x}^I\}$, respectively. This existence of non-linear equations and the over-determined system of equations for the determination of $\{C^{I,\alpha}\}$ makes it necessary to approximately solve these equations with some acceptable error. However, the satisfaction of first two linear equations is essential to reproduce the two modes.

7.5 Example for the construction of consistent mass spring model

As explained in the previous section, an ideal conversion of dynamic modes to a mass spring model is not possible and we need to approximately convert with relaxation of some conditions and some acceptable error. As an example of such conversion, a mass spring model as shown in Fig. 7.2 is constructed for the NPP structure as shown in Fig. 6.11. The detailed FEM model for this NPP is given in [66]. This mass spring model comprises of 6 masses and 7 springs. Spring K^4 is introduced for demonstration of method's versatility and for this example $K^4 = 0$.

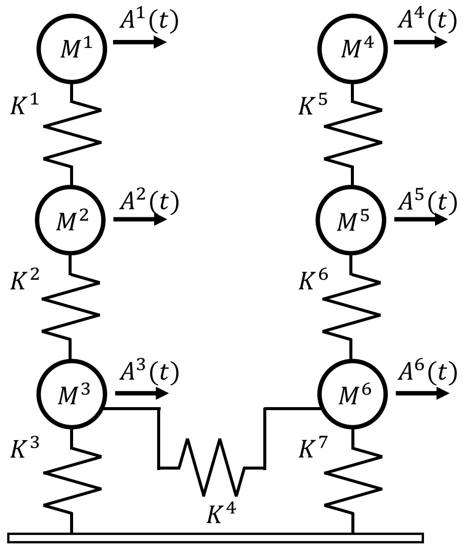


Figure 7.2: Mass-spring model to be constructed

Starting with the Lagrangian of Eq. (7.8), we need to determine a suitable function set $\boldsymbol{\psi}^\beta(\boldsymbol{x})$, such that it satisfies $\boldsymbol{u}(\boldsymbol{x},t) \approx \sum A^\beta(t)\boldsymbol{\psi}^\beta(\boldsymbol{x})$. The Lagrangian for the mass spring model of Fig. 7.2 is as follows

$$\mathcal{L} = \frac{1}{2}[\dot{A}]^T[M][A] - \frac{1}{2}[A]^T[K][A] \tag{7.24}$$

where the mass and stiffness matrices are given as follows

$$[M] = \begin{bmatrix} M^1 & & & & & \\ & M^2 & & & & \\ & & M^3 & & & \\ & & & M^4 & & \\ & & & & M^5 & \\ & & & & & M^6 \end{bmatrix}$$

$$[K] = \begin{bmatrix} K^1 & -K^1 & & & & \\ & K^1+K^2 & -K^2 & & & \\ & & K^2+K^3+K^4 & & & -K^4 \\ & & & K^5 & -K^5 & \\ & & & & K^5+K^6 & -K^6 \\ & & & & & K^6+K^7+K^4 \end{bmatrix}$$

Using the dynamic modes (ω^α, $\boldsymbol{\phi}^{\boldsymbol{\alpha}}$), $\boldsymbol{\psi}^\beta(\boldsymbol{x})$ is determined in terms of $\boldsymbol{\phi}^{\boldsymbol{\alpha}}$ as $\boldsymbol{\psi}^\beta(\boldsymbol{x}) = \sum_\alpha c^{\beta\alpha}\boldsymbol{\phi}^\alpha(\boldsymbol{x})$, with the unknowns $c^{\beta\alpha}$. Here β is the number of mass points and α is the number of modes. Substituting $\boldsymbol{u}(\boldsymbol{x},t) \approx \sum A^\beta(t)\boldsymbol{\psi}^\beta(\boldsymbol{x})$, into Lagrangian of Eq. (7.8) yields,

$$\mathcal{L} = \sum \frac{1}{2} m^{\beta\beta'} \dot{A}^\beta \dot{A}^{\beta'} - \frac{1}{2} k^{\beta\beta'} A^\beta A^{\beta'}, \tag{7.25}$$

with $m^{\beta\beta'} = \int_V \rho\boldsymbol{\psi}^\beta \cdot \boldsymbol{\psi}^{\beta'}\,\mathrm{d}v$ and $k^{\beta\beta'} = \int_V \nabla\boldsymbol{\psi}^\beta : \boldsymbol{c} : \nabla\boldsymbol{\psi}^{\beta'}\,\mathrm{d}v$, where $A^\beta(t)$ is the displacement of β-th mass at $\boldsymbol{x}^\beta$.

The computation of $m^{\beta\beta'}$ and $k^{\beta\beta'}$ in terms of unknown $c^{\beta\alpha}$ and known (m^α, k^α) is made as follows

$$m^{\beta\beta'} = \textstyle\sum_\alpha c^{\beta\alpha} c^{\beta'\alpha} m^\alpha \quad \text{and} \quad k^{\beta\beta'} = \sum_\alpha c^{\beta\alpha} c^{\beta'\alpha} k^\alpha. \tag{7.26}$$

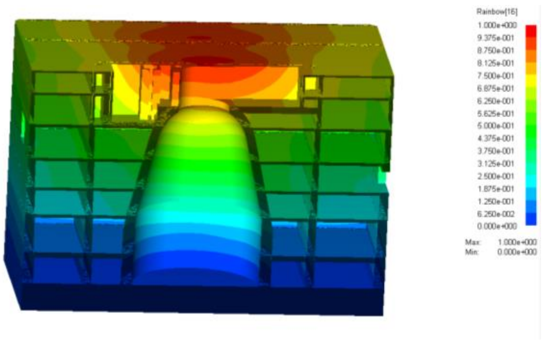
The conditions for the $c^{\beta\alpha}$ are obtained by ensuring the orthogonality of approximated displacement functions and comparing the mass and stiffness matrices of mass spring system with those obtained from the expressions of Eq. (7.26). These conditions are given as follows

$$\begin{gathered} \textstyle\sum_\beta c^{\beta\alpha'} \phi_1^\alpha(\boldsymbol{x}^\beta) = 0, \quad (\text{for } \alpha \neq \alpha') \\ \textstyle\sum_\alpha c^{\beta\alpha} c^{\beta'\alpha} m^\alpha = 0, \quad (\text{for } \beta \neq \beta') \\ k^{12} + k^{11} = 0, \\ k^{45} + k^{44} = 0, \\ k^{22} + k^{12} + k^{23} = 0, \\ k^{55} + k^{45} + k^{56} = 0. \end{gathered} \tag{7.27}$$

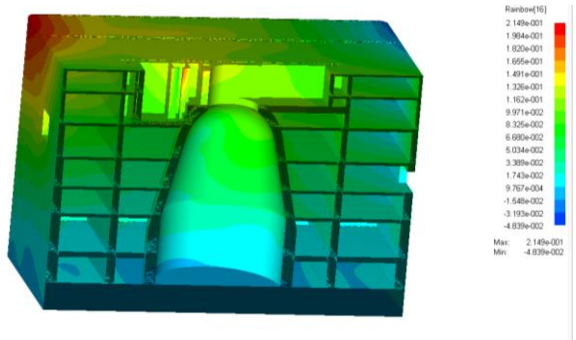
Then the mass and stiffness values for the mass-spring system in terms of $m^{\beta\beta'}$ and $k^{\beta\beta'}$ of Eq. (7.26) are as follows,

$$
\begin{gathered}
M^{\beta} = m^{\beta\beta} \qquad (\beta = 1,\ 2,\ \cdots,\ 6) \\
K^{1} = k^{11}, \\
K^{2} = -k^{23}, \\
K^{3} = k^{33} + k^{23} + k^{36}, \\
K^{4} = -k^{36}, \\
K^{5} = k^{44}, \\
K^{6} = -k^{56}, \\
K^{7} = k^{66} + k^{56} + k^{36}.
\end{gathered} \tag{7.28}
$$

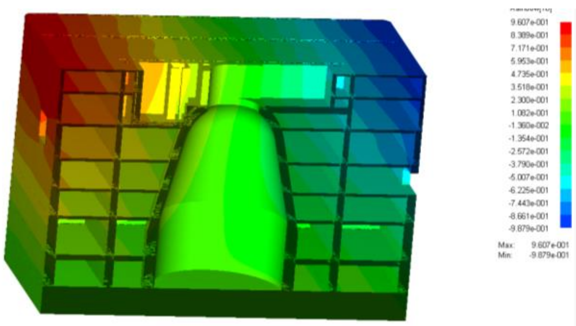
For this example, we take three dynamic modes of the NPP structure as shown in Fig. (7.3), with the mass and stiffness values given in Table 7.1.



f = 3.61 Hz



f = 3.64 Hz



f = 5.46 Hz

Figure 7.3: The three dynamic modes for the structure used

Table 7.1: Parameters for the three dynamic modes used

Mode	ω^{α}	m^{α} (ton)	k^{α} (ton/mm^2)
1	22.68	78553.76	40420400
2	22.88	81025.78	42415300
3	34.33	111063.13	130875000

The 25 conditions given in Eq. (7.22) are solved approximately to determine the 18 $c^{\beta\alpha}$ values, by minimizing the error. The mass and stiffness matrices for the mass-spring model are given as follows.

$$[M] = \begin{bmatrix} 78.6 & -28.8 & -15.4 & -70.0 & 33.9 & -20.1 \\ & 18.83 & -0.1 & 27.8 & -20.0 & 5.1 \\ & & 7.8 & 13.4 & -1.8 & 22.2 \\ & & & 64.7 & -33.0 & 12.4 \\ & sym & & & 21.9 & -4.5 \\ & & & & & 19.9 \end{bmatrix} \text{x } 10^3$$

$$[K] = \begin{bmatrix} 4.06 & -1.47 & -0.785 & -3.6 & 1.72 & -1.12 \\ & 0.98 & 0.0001 & 1.46 & -1.06 & 0.17 \\ & & 0.42 & 0.72 & -0.12 & 0 \\ & & & 3.41 & -1.7 & 0.34 \\ & sym & & & 1.18 & 0 \\ & & & & & 2.0 \end{bmatrix} \text{x } 10^7$$

The obtained mass and stiffness matrices result in the same natural frequency as that of the 3D FEM analysis. However, the presence of non- zero and negative terms in the mass matrix are due to the approximate solution and the purely mathematical treatment of the problem. One option can be to further force the off diagonal terms of mass matrix to zero by minimization and checking the effect on the frequency, however it is expected to significantly change the frequency since some of the off-diagonal terms are of the same order as those of the diagonal terms.

Chapter 8

Concluding Remarks

8.1 Overview

In this chapter, the achievements of this research work are summarized and recommendations are given for the possible improvement of the developed models and for achieving the long-term research plan of improving the conventional seismic response analysis approach.

8.2 Achievements

This thesis presents an application of meta-modeling theory to the construction of consistent seismic response analysis model which can consider the effect of SSI during analysis. Such structural response analysis modeling can be used as a more rational and accurate estimation method for possible earthquake damage to structures.

Following are the three main achievements of this study which were the objectives: 1) Clarifying SSI analysis according to the meta-modeling theory in structural mechanics and continuum mechanics and pointing out the benefits and limitations of the simplified mass spring modeling approach for the SSI analysis. 2) Proposition of a methodology for the construction of a consistent mass-spring model that can approximate the solution of solid element FEM model, with and without the consideration of SSI effect. 3) Showing the useful of the proposed methodology with the help of numerical experiments.

It is shown that there are simplifications which are made when modelling SSI in terms of soil-spring and it is these approximations which determine the applicability and limitations of the soil-spring and should be understood before applying the soil-spring modeling to any soil-structure system. The key simplifications are the assumptions of a rigid body foundation and a symmetric structure resting on uniform or stratified soil domain.

Next a methodology is developed for constructing a consistent mass-spring soil-spring model. Starting from the Lagrangian of continuum mechanics and selecting suitably approximated displacement functions, the variational problem of the Lagrangian is converted to an initial value problem of the mass-spring soil-spring model. As the simplest case, the governing equation of a single mass-spring system for a structure with rigid body foundation and a soil spring is derived, based on an assumption of uni-directional displacement. Explicit expressions of the mass and the stiffness constants are rigorously derived from the Lagrangian and can be applied to any shape of the foundation/structure.

Two simple numerical experiments performed to show the usefulness of the developed methodology and expressions show that the solution of the mass-spring model is fairly consistent with that of the finite element model.

Next a possibility of using modal analysis performed using a high fidelity model to get a consistent mass-spring model for a complicated structure and to improve the already existing consistent mass spring model for a simple structure is shown. Mass and spring constants for a mass spring model for a complicated structure are determined ensuring its consistency with the 3D FEM model results in terms of the dynamic characteristics.

8.3 Future works

The developed mass spring soil spring model should be extended to reproduce more than one modes of the structure as well as the soil which is the case in this study. The applicability and limitations of the stick model should be studied and its performance for the evaluation of the dynamic response of the structure should be examined. The developed models should be extended to consider the non-linear cases to get more benefits of the simplicity and the lesser computational effort needed for a simplified model. It is straightforward to apply meta-modeling to non-linear elasto-plasticity problem in which strain and stress increments are linearly related.

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Appendix A

Summary of Meta-modeling Theory

In the meta-modeling theory, the variational problem using a Lagrangian is called as a physical problem, and obtaining an approximated solution of this physical problem is called a modeling. Many kinds of modelings can be made for the same physical problem. Hence, a theory of making such modelings is called meta-modeling in the sense that modeling is modeled.

There are many ways to develop a distinct mathematical problem, depending on the accuracy that is expected in solving the physical problem. The meta-modeling theory delivers a set of consistent modelings which produce an approximate solution of the original modeling. As an example in structural mechanics problems, the meta-modeling theory uses continuum mechanics modeling as the basic modeling. Some of structural mechanics modelings are specified as consistent with continuum mechanics modeling. Then, those consistent structure mechanics modelings produce an approximate solution of the continuum mechanics modeling.

For simplicity, we assume a homogeneous elastic body (V) with an isotropic elasticity tensor and density, denoted by $\mathbf{c}$ and ρ. If velocity and strain are denoted by $\mathbf{v}$ and $\boldsymbol{\epsilon}$ respectively, the Lagrangian of V is

$$\mathcal{L}[\mathbf{v}, \boldsymbol{\epsilon}] = \int_V \frac{1}{2}\rho\mathbf{v}\cdot\mathbf{v} - \frac{1}{2}\boldsymbol{\epsilon}:\mathbf{c}:\boldsymbol{\epsilon}\,\mathrm{d}v, \tag{A.1}$$

where $\cdot$ and $:$ are the inner product and second-order contraction, respectively. We compute $\boldsymbol{v} = \dot{\boldsymbol{u}}$ and $\boldsymbol{\epsilon} = sym\{\boldsymbol{\nabla u}\}$, using a displacement function $\mathbf{u}$ which satisfies prescribed boundary and initial conditions; sym stands for the symmetric part of the second-order tensor, $(\dot{(\,.\,)})$ and $\boldsymbol{\nabla}(\,.\,)$ being temporal derivative and gradient of $(\,.\,)$.

Structure mechanics employs a one dimensional stress-strain relation that is not validated in any experiment. That is, Young's modulus, E, is used rather than the fourth-order tensor $\mathbf{c}$ as a material property of V. As an example, in the Cartesian coordinate of (x_1, x_2, x_3), the normal stress and strain components in the x_1-direction are related as

$$\sigma_{11} = E\epsilon_{11},$$

rather than $\boldsymbol{\sigma} = \mathbf{c}:\boldsymbol{\epsilon}$ or

$$\sigma_{11} = c_{1111}\epsilon_{11} + \cdots = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\epsilon_{11} + \cdots,$$

where $\boldsymbol{\sigma}$ is stress.

According to the meta-modeling theory, we do not have to assume the one-dimensional stress-strain relation, but we employ the following alternative Lagrangian:,

$$\mathcal{L}^*[\mathbf{v},\boldsymbol{\epsilon},\boldsymbol{\sigma}] = \int_V \frac{1}{2}\rho\mathbf{v}\cdot\mathbf{v} - \left(\boldsymbol{\sigma}:\boldsymbol{\epsilon} - \frac{1}{2}\boldsymbol{\sigma}:\mathbf{c}^{-1}:\boldsymbol{\sigma}\right)\,\mathrm{d}v, \tag{A.2}$$

where $\mathbf{c}^{-1}$ is the inverse tensor of $\mathbf{c}$. Since the terms in the parenthesis in Eq. (A.2) equal $\frac{1}{2}\boldsymbol{\epsilon}:\mathbf{c}:\boldsymbol{\epsilon}$, for $\boldsymbol{\sigma}$ satisfying $\boldsymbol{\epsilon} = \mathbf{c}^{-1}:\boldsymbol{\sigma}$, this Lagrangian is equivalent to the ordinary one of Eq. (A.1). It is easy to show that, if non-zero components of $\boldsymbol{\epsilon}$ and $\boldsymbol{\sigma}$ are ϵ_{11} and σ_{11} only, the second term in the integrand of $\mathcal{L}^*$ becomes $\sigma_{11}\epsilon_{11} - \frac{1}{2}\sigma_{11}^2/E$, and the variation with respect to σ_{11} is

$$\delta\left(\sigma_{11}\epsilon_{11} - \frac{1}{2}\frac{\sigma_{11}^2}{E}\right) = \frac{\delta\sigma_{11}}{E}(E\epsilon_{11} - \sigma_{11}).$$

As is seen, the one-dimensional stress strain relation is derived from the mathematical operation of taking variation, without making any assumption such as the one-dimensional stress-strain relation.

The meta-modeling theory leads to consistent modeling which solves the variational problem of $\mathcal{L}^*$. If no approximation is made for $\mathbf{u}$ (that produces $\mathbf{v}$ and $\boldsymbol{\epsilon}$) and $\boldsymbol{\sigma}$, it results in continuum mechanics modeling, and the governing equation for $\mathbf{u}$ is the wave equation, i.e.,

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t) - \nabla\cdot\left(\mathbf{c}(\mathbf{x}):\nabla\mathbf{u}(\mathbf{x},t)\right) = 0. \tag{A.3}$$

If certain approximations are made for $\mathbf{u}$ and $\boldsymbol{\sigma}$, it results in a consistent modeling that solves a different mathematical problem. (See Fig. A.1). However, this problem is to solve the same physical problem (that is described in terms of the variational problem) using the mathematical approximations.

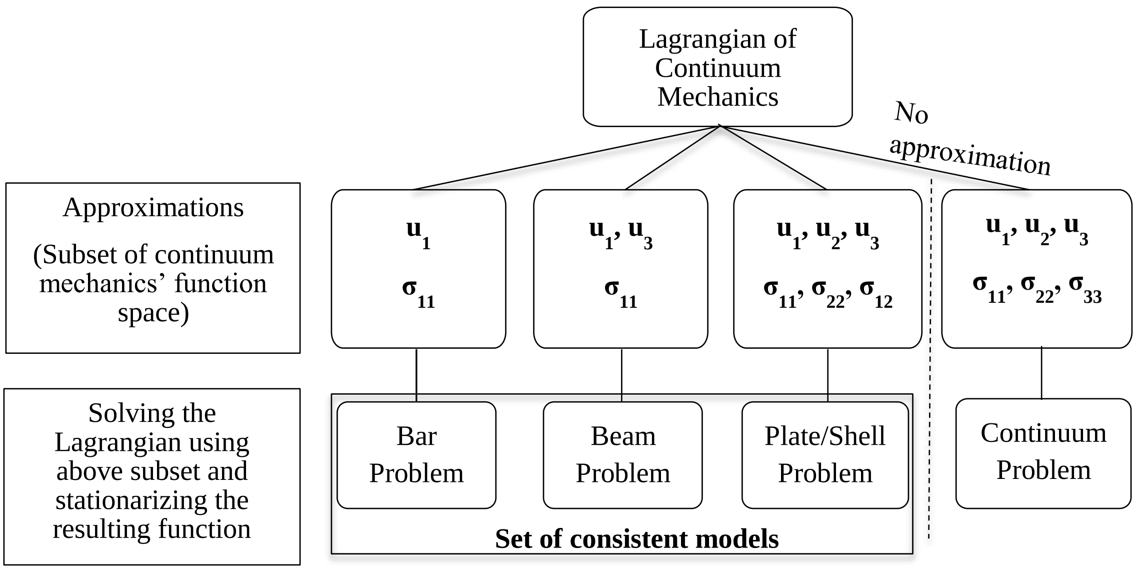

Figure A.1: Development of a set of consistent models using meta-modeling theory.

Appendix B

Effect of SSI on Natural Frequency of Structure

Consider a mass spring model with a single mass, $M^{(1)}$, connected to a spring of spring constant, $K^{(1)}$. A natural frequency of this model, denoted by ω, is given by the following equation;

$$\omega^2 M^{(1)} - K^{(1)} = 0.$$

Now, if this mass spring model is connected with another mass-spring model with mass and stiffness of $M^{(2)}$ and $K^{(2)}$, the natural frequency of this two-mass model is calculated by solving the following equation:

$$\det\left\{\omega^2 \begin{bmatrix} M^{(1)} & 0 \\ 0 & M^{(2)} \end{bmatrix} - \begin{bmatrix} K^{(1)} & -K^{(1)} \\ -K^{(1)} & K^{(1)} + K^{(2)} \end{bmatrix}\right\} = 0.$$

The natural frequency of the first mass spring model, i.e. $\omega = \sqrt{K^{(1)}/M^{(1)}}$ changes when it is connected with a second mass-spring model. This change is considered as interaction between the two masses.

We now regard the two masses of the spring model as the structure and the soil. The natural frequency of the structure is changed depending on the soil. This change could be interpreted as another interaction of the structure with the soil. This change, however, can be fully ignored by introducing the rigid body plate foundation, which corresponds to sufficiently stiff foundation of the structure.

Appendix C

Explicit Expressions for the Conversion of Modal Analysis

The explicit form of the conditions given in Eq. (6.22) is as follows

$\sum_{I=1}^{2} \phi_1^1(\boldsymbol{x}^I)C^{I,2} = 0$:

$$\phi_1^1(\boldsymbol{x}^1)C^{1,2} + \phi_1^1(\boldsymbol{x}^2)C^{2,2} = 0$$

$\sum_{I=1}^{2} \phi_1^2(\boldsymbol{x}^I)C^{I,1} = 0$:

$$\phi_1^2(\boldsymbol{x}^1)C^{1,1} + \phi_1^2(\boldsymbol{x}^2)C^{2,1} = 0$$

$M^{12} = 0$:

$$m^1C^{1,1}C^{2,1} + m^2C^{1,2}C^{2,2} = 0$$

$K^{21} = -K^{22}$:

$$k^1C^{2,1}(C^{2,1} + C^{1,1}) + k^2C^{2,2}(C^{2,2} + C^{1,2}) = 0$$

$M^{11} + M^{22} = M$:

$$m^1((C^{1,1})^2 + (C^{2,1})^2) + m^2((C^{1,2})^2 + (C^{2,2})^2) = M$$

The explicit form of the conditions given in Eq. (6.23) is as follows

$\sum_{I=1}^{3} \phi_1^1(\boldsymbol{x}^I)C^{I,2} = 0$:

$$\phi_1^1(\boldsymbol{x}^1)C^{1,2} + \phi_1^1(\boldsymbol{x}^2)C^{2,2} + \phi_1^1(\boldsymbol{x}^3)C^{3,2} = 0$$

$\sum_{I=1}^{3} \phi_1^2(\boldsymbol{x}^I)C^{I,1} = 0$:

$$\phi_1^2(\boldsymbol{x}^1)C^{1,1} + \phi_1^2(\boldsymbol{x}^2)C^{2,1} + \phi_1^2(\boldsymbol{x}^3)C^{3,1} = 0$$

$M^{12} = 0$:

$$m^1C^{1,1}C^{2,1} + m^2C^{1,2}C^{2,2} = 0$$

$M^{23} = 0$:

$$m^1C^{2,1}C^{3,1} + m^2C^{2,2}C^{3,2} = 0$$

$M^{31} = 0$:

$$m^1C^{3,1}C^{1,1} + m^2C^{3,2}C^{1,2} = 0$$

$K^{33} = -K^{32}$:

$$k^1C^{3,1}(C^{3,1} + C^{2,1}) + k^2C^{3,2}(C^{3,2} + C^{2,2}) = 0$$

$K^{22} = -K^{21} + K^{33}$:

$$k^1((C^{3,1})^2 - (C^{2,1})^2 - C^{2,1}C^{1,1}) + k^2((C^{3,2})^2 - (C^{2,2})^2 - C^{2,2}C^{1,2}) = 0$$

$K^{13} = 0$:

$$k^1C^{1,1}C^{3,1} + k^2C^{1,2}C^{3,2} = 0$$

$M^{11} + M^{22} + M^{33} = M$:

$$m^1((C^{1,1})^2 + (C^{2,1})^2 + (C^{3,1})^2) + m^2((C^{1,2})^2 + (C^{2,2})^2 + (C^{3,2})^2) = M$$