

# 学位論文

Baryon Asymmetry and Cosmological Moduli/Polonyi Problem

(物質・反物質非対称性と宇宙論的モジュライ問題)

平成29年12月博士（理学）申請

東京大学大学院理学系研究科

物理学専攻

早川 拓

*Ph.D. Thesis*

**Baryon Asymmetry and Cosmological Moduli/Polonyi Problem**

Taku Hayakawa

*Institute for Cosmic Ray Research  
Department of Physics, the University of Tokyo*

## Abstract

Supersymmetric models generally contain long-lived particles that could cause cosmological difficulties. In particular, moduli/Polonyi fields dominate total energy of the universe as coherent oscillation and spoil the success of the Big-Bang cosmology. It is known that the moduli/Polonyi abundance can be diluted sufficiently by thermal inflation. However, preexisting baryon asymmetry is also diluted in this scenario. In this thesis, we study whether it is possible to generate the observed baryon asymmetry with the dilution of the moduli/Polonyi abundance. When we consider baryogenesis before dilution of the moduli abundance, the Affleck-Dine mechanism is the most promising among known baryogenesis mechanisms. In gravity-mediated SUSY breaking models with the moduli mass of  $\mathcal{O}(1)$  TeV, the Affleck-Dine mechanism before dilution cannot explain the observed baryon asymmetry. In gauge-mediated SUSY breaking models, the Affleck-Dine fields except for  $LH_u$  flat direction inevitably form into Q-balls after the onset of their oscillation. The produced baryon number is absorbed into Q-balls, and it is difficult to extract the baryon number from Q-balls. We show that the Affleck-Dine fields cannot provide sufficient baryon number with dilution of moduli abundance because of Q-ball formation. In the case of the  $LH_u$  flat direction,  $\mu$ -term prevents the Q-ball formation. We propose alternative scenario using the  $LH_u$  direction, but we show that it cannot explain the observed baryon asymmetry either. When the moduli/Polonyi field is as heavy as  $\mathcal{O}(100)$  TeV, it can decay before the Big-Bang nucleosynthesis, but the lightest supersymmetric particles are generally overproduced from the decay. We show that the baryon asymmetry cannot be explained by the Affleck-Dine mechanism even if the moduli abundance is diluted by entropy production to prevent the LSP overproduction. In the case of the Polonyi field with a mass of  $\mathcal{O}(100)$  TeV, on the other hand, we show that the observed baryon-to-dark matter ratio is explained in sequestering models with a (pseudo-)Nambu-Goldstone boson.

# Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
1.1	Overview . . . . .	3
1.2	Outline of this thesis . . . . .	6
<b>2</b>	<b>Supersymmetry and cosmological problems</b>	<b>8</b>
2.1	Motivations for supersymmetry . . . . .	8
2.2	Minimal Supersymmetric Standard Model . . . . .	11
2.3	Mediation of SUSY breaking . . . . .	15
2.4	Cosmological problems of supersymmetry . . . . .	17
2.5	Solutions to cosmological moduli/Polonyi problems . . . . .	23
2.6	Moduli/Polonyi problem and baryon asymmetry . . . . .	31
<b>3</b>	<b>Baryogenesis before dilution</b>	<b>33</b>
3.1	Affleck-Dine mechanism . . . . .	33
3.2	Q-ball . . . . .	39
3.3	Case of gravity-mediation models . . . . .	44
3.4	Case of gauge-mediation models . . . . .	46
3.5	$LH_u$ direction in gauge-mediation models . . . . .	57
<b>4</b>	<b>Heavy moduli/Polonyi scenario</b>	<b>66</b>
4.1	AD mechanism in high scale SUSY models . . . . .	67
4.2	Baryon asymmetry and dark matter abundance . . . . .	70
4.3	Sequestering model and decay process of Polonyi field . . . . .	72
<b>5</b>	<b>Conclusion</b>	<b>79</b>

# Chapter 1

## Introduction

### 1.1 Overview

The origin of matter-antimatter asymmetry in the universe is one of long-standing questions in cosmology. The matters and antimatters are composed of elementary particles, and they seem to be created by the same amount in the early universe. However, all of astronomical objects are composed of not antimatters but matters, and there are no primordial antimatters in our universe. The question is how this asymmetry was created during the history of the universe. The cosmic microwave background (CMB) observation and the Big-Bang nucleosynthesis (BBN) offers an estimation of the asymmetry between baryonic and antibaryonic components. Both CMB observation and successful formation of light elements require the following baryon asymmetry [1–3]:<sup>1</sup>

$$\frac{n_B}{s} \simeq (8.7 \pm 0.1) \times 10^{-11}, \quad (1.1)$$

where  $n_B \equiv n_b - n_{\bar{b}}$  is the difference between the baryon density  $n_b$  and the antibaryon density  $n_{\bar{b}}$ , and  $s$  is the entropy density.

Generation of the baryon asymmetry is referred to as *baryogenesis* and has been studied in connection with particle physics for a long time. Sakharov firstly pointed out the following necessary conditions for baryogenesis, that are referred to as Sakharov's criteria [4]: (1) baryon number violating processes, (2)  $C$  and  $CP$  violating processes, (3) departure from thermal equilibrium. The earliest attempt to produce the baryon asymmetry was proposed in the context of the Grand Unified Theory (GUT) [5, 6]. The GUT [7] predicts the existence of baryon number violation, and  $CP$  violating decay of superheavy particles can produce baryon asymmetry. However, the GUT baryogenesis is

---

<sup>1</sup> The value is obtained from the analysis of the Planck collaboration using the data of TT, TE, EE+lowP+BAO (95%C.L.).

incompatible with the inflationary universe because it is difficult to achieve temperature of the GUT scale after the inflation.

Actually, the electroweak sector of the standard model itself has baryon number violation at the quantum level [8]. The violation is provided by a quantum anomaly, and these effects are very tiny since they occur by quantum tunneling effects in the present cold universe. At high temperature above the electroweak scale, however, it was found that thermal transitions over the barrier occur and that the baryon number violating processes, called sphaleron processes [9, 10], come to thermal equilibrium [11].

The sphaleron processes make it possible to produce baryon asymmetry at least in two ways. The first one is electroweak baryogenesis [11–13]. In this scenario, the electroweak transition must be first order since departure from thermal equilibrium is realized by walls of false vacuum bubbles. However, it is found that the electroweak transition of the standard model is a smooth crossover [14, 15]. Therefore, an extension of the standard model is needed for successful electroweak baryogenesis.

The second one is leptogenesis [16]. While the sphaleron processes violate the baryon ( $B + L$ ) symmetry, they conserve  $B - L$  symmetry. If lepton asymmetry is generated before the electroweak phase transition, it is converted into baryon asymmetry. In this case, heavy right-handed neutrinos other than the standard model particles are usually responsible for generating the lepton asymmetry.

These two scenarios can work in the inflationary universe since the baryon asymmetry is dynamically generated after the end of the primordial inflation. An important point is that the origin of the baryon asymmetry cannot be explained in the framework of the standard model. It is still an open question how the baryon asymmetry was generated in the early universe.

In particle physics, on the other hand, supersymmetry (SUSY)<sup>2</sup> is one of the most attractive candidates for extensions of the standard model. It can not only drastically relax the hierarchy problem but also achieve the unification of the gauge couplings, which implies the existence of the GUTs. In addition, supersymmetric models with conserved  $R$  parity predict the stability of the Lightest Supersymmetric Particle (LSP), which becomes a good candidate for the dark matter. Moreover, SUSY extensions of the SM contain a lot of flat directions with  $B - L$  charge [19], which can produce the  $B - L$  asymmetry [20]. In the early universe, one of the flat directions, which we call the Affleck-Dine field, may receive an angular kick from SUSY breaking and  $R$  symmetry breaking effects and rotates in its complex plane, which corresponds to the generation of the  $B - L$  asymmetry. The

---

<sup>2</sup> For reviews, see Refs. [17, 18]

$B - L$  asymmetry is converted into the baryon asymmetry through the sphaleron process. This mechanism, known as the "Affleck-Dine mechanism", can produce baryon number more efficiently than the electroweak baryogenesis and the leptogenesis [20, 21].

In spite of these advantages of the SUSY, some long-lived particles in supersymmetric models tend to have cosmological difficulties. In local SUSY, the existence of the gravitino is expected, which is a superpartner of the graviton. The gravitino interacts with other particles only through interactions suppressed by the Planck scale, hence has a very long lifetime. In the history of the universe, gravitinos are produced through scattering processes during the reheating [22]. When the gravitino mass is of  $\mathcal{O}(1)$  TeV, the decay of the produced gravitinos destroy the BBN [23–26]. When the gravitino mass is much lighter than the electroweak scale, it is stable and its abundance may give too much contribution to the cosmic density of the present universe [27]. The reheating temperature is then constrained from these cosmological difficulties [28–30].

The situation gets worse when we think of supersymmetric models as an effective theory of superstring. In superstring theories, a lot of flat directions, called *moduli fields*, generally appear at the low energy scale through compactifications of extra dimensions. It is known that many of them can be stabilized with heavy masses by flux compactifications [31–33], but some of them often remain relatively light, whose mass is about the gravitino mass [34]. In this thesis, I focus on these moduli fields whose mass is the order of the gravitino mass. Such moduli fields cause cosmological difficulties, called *cosmological moduli problems* [34–36], because they also interact with other particles only through the interaction suppressed by the Planck scale and hence have long lifetimes. Moreover, when the moduli mass is of  $\mathcal{O}(0.1)$  MeV– $\mathcal{O}(1)$  GeV, moduli generally decay into X-rays during the present epoch, and it may give too much contribution to the X-ray background spectrum, which constrains the moduli abundance more severely than the dark matter abundance [37, 38].

The moduli density mainly comes from its coherent oscillation. Since its initial amplitude is of the order of the Planck scale, its density soon dominates the energy density of the universe. As we will see, the moduli problems cannot be solved even if the reheating temperature is as low as  $\mathcal{O}(10)$  MeV, which is restricted from below to realize the BBN [39, 40]. Therefore, the moduli problems are more severe than the gravitino problems.

The same problem occurs when supersymmetric models contain a singlet field in a hidden sector. For example, singlet fields are often responsible for spontaneous breaking of the SUSY. The simplest SUSY breaking model with a singlet field is called the "Polonyi model" [41]. The singlet field whose  $F$ -term breaks the SUSY, called the "Polonyi field",

	gauge mediation	gravity/anomaly mediation	
moduli (gravitino) mass	$\lesssim \mathcal{O}(1) \text{ GeV}$	$\mathcal{O}(0.1\text{-}1) \text{ TeV}$	$\mathcal{O}(100) \text{ TeV}$
part	Section 3.4 Section 3.5 ( $LH_u$ direction)	Section 3.3	Chapter 4

Table 1.1: The contents of Chap. 3 and 4.

also causes cosmological problems as well as moduli fields [35].

There are some ways to solve the problems. One of the most probable candidates is the “thermal inflation” [42, 43]. It is a mini-inflation caused after the onset of the moduli oscillation, and dilutes the moduli density. The mechanism requires a scalar field called “the flaton”. The end of the thermal inflation is triggered by the thermal mass of the flaton, and the flaton decay can produce a large number of entropy enough to solve the moduli problems. Indeed, it has been studied that some specific models succeed in diluting the moduli density sufficiently [44–46].

However, this is not the end of the story. One is faced with another important problem with regard to the baryon asymmetry. When the thermal inflation dilutes the moduli density, it also dilutes preexisting baryon ( $B - L$ ) asymmetry. In order to explain the present baryon asymmetry, it is necessary to produce huge baryon ( $B - L$ ) number enough to survive dilution beforehand. The thermal leptogenesis cannot work because it cannot produce such huge lepton number beforehand. The electroweak baryogenesis also seems to be difficult to explain the observed baryon asymmetry since reheating temperature is often predicted to be below the electroweak scale in the thermal inflation. Therefore, we need a mechanism to generate the baryon number more efficiently.

The topic of this thesis is how to explain the observed baryon asymmetry in the context of the thermal inflation. As mentioned above, the Affleck-Dine mechanism is one of the most probable candidates for baryogenesis in supersymmetric models and can generate much larger baryon asymmetry than other baryogenesis scenarios such as the thermal leptogenesis. In this thesis, we consider the case where the Affleck-Dine mechanism is responsible for producing baryon number.

## 1.2 Outline of this thesis

The outline of this thesis is as follows. In Chap. 2, we review motivations for supersymmetry and its cosmological implications. In particular, we focus on cosmological moduli/Polonyi problems and introduce mechanisms to partially solve these problems. In



the former half of Chap. 3, we review the Affleck-Dine mechanism and the Q-ball, which is a non-topological soliton formed during the Affleck-Dine field oscillation. In the rest of the chapter, we study whether the Affleck-Dine mechanism can explain the observed baryon asymmetry with dilution of the moduli abundance. Section 3.4 and 3.5 are based on the work of Ref. [47]. In Chap. 4, we consider the case where the moduli/Polonyi field is as heavy as  $\mathcal{O}(100)$  TeV. This chapter is based on the work of Ref. [48]. The contents of Chap. 3 and 4 are summarized in the Table 1.1. Chapter 5 is devoted to conclusions.

# Chapter 2

## Supersymmetry and cosmological problems

In this chapter, we will review supersymmetry and its cosmological problems. We then explain conventional scenarios to solve the problems.

### 2.1 Motivations for supersymmetry

In particle physics, the idea of symmetry provides us with a description of interactions between elementary particles. The standard model successfully describes the strong and electroweak interactions based on the local symmetry of  $SU(3)_C \times SU(2)_L \times U(1)_Y$ , and has been experimentally confirmed with a high degree of precision to date, except for neutrino oscillation [49, 50]. The electromagnetic interaction appears as a result of the spontaneously breaking of the electroweak symmetry  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ , which requires the existence of the Higgs boson transforming as a  $SU(2)_L$  doublet with hypercharge. The symmetry breaking occurs when the Higgs acquires the Vacuum Expectation Value (VEV) of the order of 100 GeV, and then quarks, leptons and W and Z bosons acquire masses smaller than the electroweak scale. These masses are determined by the couplings between Higgs and standard model particles. It should be noted that the standard model particles except for the Higgs are massless unless the electroweak symmetry breaking occurs. Therefore, the Higgs VEV is an origin of masses of already known particles. How does the Higgs field acquire the VEV and why is it of the order of 100 GeV?

The potential for the electrically neutral part of the Higgs doublet is given by

$$V = m_H^2 |H|^2 + \lambda_H |H|^4, \quad (2.1)$$

where  $m_H^2$  is the mass of the Higgs field at the origin of the potential,  $\lambda_H$  is the Higgs

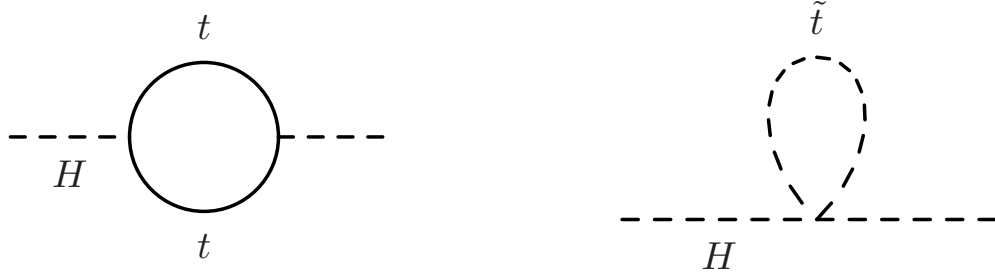


Figure 2.1: Some of the Feynman diagrams contributing to the Higgs mass corrections.

self-coupling of  $O(1)$  and  $H$  represents the Higgs field. The symmetry breaking requires  $m_H^2 < 0$  and  $\lambda_H > 0$ , and  $m_H^2$  should be of the order of  $-(100 \text{ GeV})^2$  in order to get the Higgs VEV of  $\mathcal{O}(100) \text{ GeV}$ . However, the parameter of  $m_H^2$  receives huge quantum corrections from loops containing fermions. For example, the top quarks  $t_L, t_R$  couples to the Higgs field with a term of  $-y_t H t_L t_R^\dagger$ , and then  $m_H^2$  receives the following corrections at the one-loop level:

$$\Delta m_H^2 = -\frac{|y_t|^2}{8\pi^2} \Lambda_{UV}^2 + \dots \quad (2.2)$$

Here  $\Lambda_{UV}$  is an ultraviolet momentum cutoff. Other fermions and gauge bosons coupling to the Higgs also contribute to the quantum corrections. If one assumes that the momentum cutoff is the Planck scale  $m_{pl}$ , the renormalized parameter  $m_H^2$  of the order of  $-(100 \text{ GeV})^2$  requires miraculous cancellation between the bare parameter and the quantum corrections of  $\mathcal{O}(m_{pl}^2)$ . In other words,  $\mathcal{O}(10^{-34})$  fine-tuning is necessary for the successful electroweak symmetry breaking. This is called the hierarchy problem in the standard model.

Supersymmetry (SUSY) is one of the most attractive candidates for solving the hierarchy problem. The SUSY is symmetry between bosons and fermions, and then predicts an equal number of boson and fermion degrees of freedom. Therefore, there exists supersymmetric partners of all the known standard model particles. For example, the supersymmetric standard model has superpartners of the top quarks,  $\tilde{t}_L$  and  $\tilde{t}_R$ , with terms like  $|y_t|^2 |H|^2 |\tilde{t}_L|^2$  and  $|y_t|^2 |H|^2 |\tilde{t}_R|^2$ . The quantum corrections proportional to  $\Lambda_{UV}^2$  in Eq. (2.2) are cancelled by the following contributions from loops containing these scalar particles at the one-loop level:

$$\Delta m_H^2 = 2 \times \frac{|y_t|^2}{16\pi^2} \Lambda_{UV}^2 + \dots \quad (2.3)$$

Some of the diagrams including top quarks and stops are shown in Fig. 2.1. These cancellations also occur between gauge bosons and their superpartners loops. The quadratic

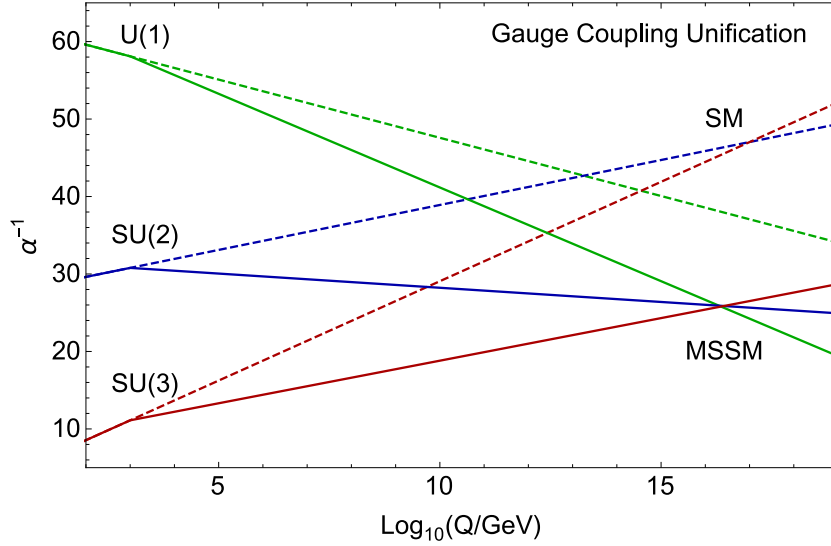


Figure 2.2: The energy dependence of the standard model gauge couplings,  $\alpha_a = g_a^2/4\pi$ , at the one-loop level, where  $a$  is a gauge index. We take the SUSY scale as 1 TeV.

divergences similarly vanish to all orders in perturbation theory unless the supersymmetry is broken [51–54]. The SUSY provides a solution to the hierarchy problem.

Actually, the SUSY must be broken in the vacuum state since superpartners of the standard model particles are not discovered. Fortunately, *soft* SUSY breaking ensures the cancellation of quadratic divergences even in the presence of the SUSY breaking. Softly breaking terms contain parameters with positive mass dimension, and then the SUSY is restored when these parameters vanish. We denote the scale of the mass parameters as  $m_{SUSY}$ . The contribution to the Higgs mass term from the soft SUSY breaking effects is as follows:

$$\Delta m_H^2 = m_{SUSY}^2 \left[ \frac{\lambda}{16\pi^2} \ln \frac{\Lambda_{UV}}{m_{SUSY}} + \dots \right]. \quad (2.4)$$

Here  $\lambda$  is a typical dimensionless coupling. One can find that this contribution vanishes in the limit of  $m_{SUSY} \rightarrow 0$  and that the quantum corrections to  $m_H^2$  is at most of the order of  $m_{SUSY}^2$  as long as the SUSY is softly broken. In order to solve or drastically relax the hierarchy problem,  $m_{SUSY}$  should be around the electroweak or TeV scale and not much larger than TeV scale.

Another important motivation for the SUSY is the unification of the gauge couplings at the high energy scale. We already know that the electromagnetic and the weak interactions are unified into the electroweak interaction. Similarly, it is also expected that three gauge couplings of  $SU(3)_C \times SU(2)_L \times U(1)_Y$  are unified at the higher energy scale, which implies

the existence of the GUT [7]. One can evolve the gauge couplings up to the high energy scale according to renormalization group equations. The unification of the gauge couplings cannot be achieved in the standard model as shown in Fig. 2.2. In supersymmetric models, however, the renormalization group equations are different from the standard model, and then one can actually see the unification of the gauge couplings just below the Planck scale.

Furthermore, we know that there exists four fundamental forces in nature: the gravitational force, the strong force, the weak force and the electromagnetic force. If three fundamental interactions other than the gravitation are unified by the SUSY GUT, our further goal is the unification of all the known interactions including the gravitation. The gravitational force can be described by the general relativity at the macroscopic scale, but how to combine the quantum theory and the general relativity is completely unknown. The superstring theory, that is one of supersymmetric theories, is known to be the most probable candidate for “the theory of everything” at present. If the superstring theory is a proper theory of the unification of all the known fundamental forces, there should exist the SUSY.

## 2.2 Minimal Supersymmetric Standard Model

The simplest supersymmetric extension for the standard model is called Minimal Supersymmetric Standard Model (MSSM). As mentioned in the previous section, all the known standard model particles must have superpartners with the same degrees of freedom. For example, each of two-component Weyl fermions has one complex scalar partner called a “sfermion”. The superpartner of the left-handed quark  $q_L$  is a left-handed squark  $\tilde{q}_L$ . Hereafter, we denote superpartners of fermions  $f$  as  $\tilde{f}$ . Gauge bosons and Higgs bosons also have fermions called “gauginos” and “higgsinos”. We show all the particle contents of the MSSM in Table 2.1.

A notable feature of the MSSM is that it contains two Higgs doublets from the following reasons. First, the gauge anomalies must vanish in order to ensure the gauge symmetry at the quantum level. In the standard model, one can find that these anomalies vanish by using the charge of the quarks and the leptons. In the MSSM, on the other hand, the higgsinos also contribute to the gauge anomalies, and then anomaly cancellation requires two higgsinos with opposite hypercharge,  $Y = 1/2$  and  $Y = -1/2$ . Second, two Higgs doublets are necessary for giving mass terms of both up-type and down-type quarks in the supersymmetric Lagrangian. In the following, we call the Higgs boson giving the masses of the up-type quarks “up-type Higgs” and one giving the down-type quark masses “down-

Chiral superfield		Scalar boson	Weyl fermion	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Squarks, Quarks	$Q$	$\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)^T$	$Q = (u_L, d_L)^T$	<b>3</b>	<b>2</b>	$\frac{1}{6}$
	$\bar{u}$	$\tilde{u}_R^* (\tilde{\bar{u}})$	$u_R^\dagger (\bar{u})$	<b>3</b>	<b>1</b>	$-\frac{2}{3}$
	$\bar{d}$	$\tilde{d}_R^* (\tilde{\bar{d}})$	$d_R^\dagger (\bar{d})$	<b>3</b>	<b>1</b>	$\frac{1}{3}$
Sleptons, Leptons	$L$	$\tilde{L} = (\tilde{e}_L, \tilde{\nu}_L)^T$	$L = (e_L, \nu_L)^T$	<b>1</b>	<b>2</b>	$-\frac{1}{2}$
	$\bar{e}$	$\tilde{e}_R^* (\tilde{\bar{e}})$	$e_R^\dagger (\bar{e})$	<b>1</b>	<b>1</b>	1
Higgs, Higgsinos	$H_u$	$H_u = (H_u^+, H_u^0)^T$	$\tilde{H}_u = (\tilde{H}_u^+, \tilde{H}_u^0)^T$	<b>1</b>	<b>2</b>	$\frac{1}{2}$
	$H_d$	$H_d = (H_d^0, H_d^-)^T$	$\tilde{H}_d = (\tilde{H}_d^0, \tilde{H}_d^-)^T$	<b>1</b>	<b>2</b>	$-\frac{1}{2}$

Vector superfield		Vector boson	Weyl fermion	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Gluon, Gluino		$g$	$\tilde{g}$	<b>8</b>	<b>1</b>	0
W boson, Wino		$W$	$\tilde{W}$	<b>1</b>	<b>3</b>	0
B boson, Bino		$B$	$\tilde{B}$	<b>1</b>	<b>1</b>	0

Table 2.1: The contents of MSSM particles.

type Higgs”.

Before introducing the MSSM Lagrangian, we will briefly see construction of supersymmetric Lagrangian. The supersymmetric Lagrangian is easily constructed based on superspace. Coordinates on superspace are  $x^\mu$ ,  $\theta^\alpha$  and  $\theta_\alpha^\dagger$ , where  $\alpha$  and  $\dot{\alpha}$  are spinor indices.  $\theta^\alpha$  and  $\theta_\alpha^\dagger$  are anti-commuting two-component spinors. On superspace, a complex scalar field  $\phi$  and its supersymmetric partner, Weyl fermion  $\psi$ , can be described as a chiral superfield:

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y), \quad (2.5)$$

where  $y^\mu \equiv x^\mu + i\theta^\dagger\bar{\sigma}^\mu\theta$ , and  $F$  is an auxiliary field. A real superfield  $V^a$  that contains gauge fields is written as

$$V^a = \theta^\dagger\bar{\sigma}^\mu\theta A_\mu^a(y) + \theta^\dagger\theta^\dagger\theta\lambda^a(y) + \theta\theta\theta^\dagger\lambda^{\dagger a}(y) + \frac{1}{2}\theta\theta\theta^\dagger\theta^\dagger [D^a(y) + i\partial^\mu A_\mu^a(y)], \quad (2.6)$$

where  $A_\mu^a$  is a gauge field,  $\lambda^a$  is a gaugino field and  $D^a$  is an auxiliary field. Here  $a$  is a gauge index. A chiral field strength superfield  $W_\alpha^a$  is given as

$$W_\alpha^a = \lambda_\alpha^a + \theta_\alpha D^a - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha F_{\mu\nu}^a + i\theta\theta(\sigma^\mu\nabla_\mu\lambda^{\dagger a})_\alpha, \quad (2.7)$$

where  $F_{\mu\nu}^a$  is a field strength of the gauge field  $A_\mu^a$ .  $\nabla_\mu$  is the gauge covariant derivative, but is the usual coordinate derivative when the gauge group is abelian. We can write the

supersymmetric Lagrangian by using the above superfields as follows:

$$\mathcal{L} = \int d\theta^2 d\theta^{\dagger 2} K(\Phi^{*i}, \tilde{\Phi}_j) + \left( \int d\theta^2 \left[ \frac{1}{4} \tau_{ab}(\Phi_i) \hat{W}_\alpha^a \hat{W}^{b\alpha} + W(\Phi_i) \right] + \text{c.c.} \right). \quad (2.8)$$

Here we used  $\tilde{\Phi}_i = (e^{2T^a \hat{V}^a})_i^j \Phi_j$ ,  $\hat{V}^a = g_a V^a$  and  $\hat{W}_\alpha^a = g_a W_\alpha^a$ .  $K$  is a gauge-invariant real function called “Kähler potential” with mass dimension 2.  $\tau_{ab}$  and  $W$  are gauge-invariant holomorphic functions called a “gauge kinetic function” and “superpotential”.

In the case of renormalizable Lagrangian, the Kähler potential and the gauge kinetic function are written as

$$K = \Phi^{*i} \tilde{\Phi}_i, \quad (2.9)$$

$$\tau_{ab} = \delta_{ab} \left( \frac{1}{g_a^2} - i \frac{\theta_a}{8\pi^2} \right), \quad (2.10)$$

where  $\theta_a$  is a CP violating parameter. The supersymmetric part of the MSSM Lagrangian can be described by the following superpotential:

$$W_{MSSM} = y_{u,ij} \bar{u}_i Q_j H_u - y_{d,ij} \bar{d}_i Q_j H_d - y_{e,ij} \bar{e}_i L_j H_d + \mu H_u H_d. \quad (2.11)$$

Here,  $y_u$ ,  $y_d$  and  $y_e$  are the Yukawa couplings, and  $i, j$  represent the family indices. We omit all the gauge indices. For example, the term of  $y \bar{u} Q H_u$  represents  $y \bar{u}^a Q_{\alpha a} (H_u)_\beta \epsilon^{\alpha\beta}$ , where  $a$  is a  $SU(3)_C$  gauge index, and  $\alpha$  and  $\beta$  are  $SU(2)_L$  gauge indices. Similarly, the term of  $\mu H_u H_d$ , that is called a “ $\mu$ -term”, represents  $\mu (H_u)_\alpha (H_d)_\beta \epsilon^{\alpha\beta}$ .

One can generally write other gauge-invariant terms, such as  $LL\bar{e}$  or  $\bar{u}d\bar{d}$ , in the renormalizable superpotential. However, these terms are  $L$  and  $B$  violating interactions and are constrained by experiments. For example, they cause relatively early proton decay [55, 56]. In order to forbid these harmful terms, we generally impose discrete  $R$ -symmetry defined as

$$P_R \equiv (-1)^{3(B-L)+2s}, \quad (2.12)$$

where  $s$  is spin of particles. This discrete symmetry is called “ $R$ -parity”. All the standard model particles and two Higgs bosons have  $R$ -parity charge of 1, and their superpartners have  $R$ -parity charge of  $-1$ . This symmetry provides another benefit to supersymmetric models in light of cosmology. The  $R$ -parity conservation predicts stability of the Lightest Supersymmetric Particle (LSP) since it cannot decay into the standard model particles, and then it is a probable candidate for the dark matter. The existence of the dark matter candidate is additional motivation for the SUSY.

The SUSY breaking effects give gaugino masses, sfermion masses and trilinear couplings for scalar fields. The SUSY breaking Lagrangian is given as

$$\begin{aligned}
\mathcal{L}_{soft} = & -\frac{1}{2} \left( M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B} + \text{c.c.} \right) \\
& -m_{\tilde{Q},ij}^2 \tilde{Q}_i^* \tilde{Q}_j - m_{\tilde{L},ij}^2 \tilde{L}_i^* \tilde{L}_j - m_{\tilde{u},ij}^2 \tilde{u}_i^* \tilde{u}_j - m_{\tilde{d},ij}^2 \tilde{d}_i^* \tilde{d}_j - m_{\tilde{e},ij}^2 \tilde{e}_i^* \tilde{e}_j \\
& - \left( a_{u,ij} \tilde{u}_i \tilde{Q}_j H_u - a_{d,ij} \tilde{d}_i \tilde{Q}_j H_d - a_{e,ij} \tilde{e}_i \tilde{L}_j H_d + \text{c.c.} \right) \\
& -m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 - (b H_u H_d + \text{c.c.}), \tag{2.13}
\end{aligned}$$

where  $M_3$ ,  $M_2$  and  $M_1$  are gluino, wino and bino masses, and  $m_{\tilde{Q},ij}^2$ ,  $m_{\tilde{L},ij}^2$ ,  $m_{\tilde{u},ij}^2$ ,  $m_{\tilde{d},ij}^2$  and  $m_{\tilde{e},ij}^2$  are sfermion squared mass matrices.  $a_{u,ij}$ ,  $a_{d,ij}$  and  $a_{e,ij}$  are complex matrices with mass dimension 1. These are SUSY breaking terms corresponding to the Yukawa matrices in Eq. (2.11).  $m_{H_u}^2$  and  $m_{H_d}^2$  are squared masses of up-type and down-type Higgs scalar fields.  $b$  is the SUSY breaking term called the ‘ $b$ -term’, and has mass dimension 2. This term is necessary for the successful electroweak symmetry breaking.

Before proceeding to the next section, we will see the electroweak symmetry breaking in the case of the MSSM. At the tree level, the potential for two Higgs doublets is given as

$$\begin{aligned}
V = & (|\mu|^2 + m_{H_u}^2) |H_u|^2 + (|\mu|^2 + m_{H_d}^2) |H_d|^2 + (b H_u H_d + \text{c.c.}) \\
& + \frac{1}{8} (g^2 + g'^2) (|H_u|^2 - |H_d|^2)^2 + \frac{1}{2} g^2 |H_u^\dagger H_d|^2, \tag{2.14}
\end{aligned}$$

where  $g$  and  $g'$  are  $SU(2)_L$  and  $U(1)_Y$  gauge couplings, respectively. Note that  $H_u$  and  $H_d$  are Higgs doublets,  $H_u = (H_u^+, H_u^0)^T$  and  $H_d = (H_d^0, H_d^-)^T$ . The terms proportional to the gauge couplings come from the first term in Eq. (2.8), and are called  $D$ -term contributions since their origins are the auxiliary fields in Eq. (2.6). By using freedom of  $SU(2)_L$  gauge transformation, one can choose  $\langle H_u^+ \rangle = 0$  at the vacuum state. This choice determines the VEV of the down type Higgs as  $\langle H_d^- \rangle = 0$ . Then, the potential for the neutral parts of the Higgs doublets is written as

$$\begin{aligned}
V = & (|\mu|^2 + m_{H_u}^2) |H_u^0|^2 + (|\mu|^2 + m_{H_d}^2) |H_d^0|^2 - (b H_u^0 H_d^0 + \text{c.c.}) \\
& + \frac{1}{8} (g^2 + g'^2) (|H_u^0|^2 - |H_d^0|^2)^2. \tag{2.15}
\end{aligned}$$

Redefinition of the phases of  $H_u^0$  or  $H_d^0$  can rotate the phase of  $b$ , and we will take it to be real and positive for convenience.

The potential must be bounded from below at large field values of  $H_u^0$  and  $H_d^0$ . The  $D$ -term contributions give quartic couplings of the Higgs fields, and vanish when  $|H_u^0| = |H_d^0|$ .



Therefore, the quadratic term must be positive in this direction. This condition gives the following inequality:

$$2b < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2. \quad (2.16)$$

In order for the quarks and leptons to acquire the masses, both  $H_u^0$  and  $H_d^0$  must have the nonzero VEVs, which requires a negative squared mass of one linear combination of  $H_u^0$  and  $H_d^0$  around the origin of the potential. This condition leads to

$$b^2 > (|\mu|^2 + m_{H_u}^2) (|\mu|^2 + m_{H_d}^2). \quad (2.17)$$

Conditions to minimize the potential of Eq. (2.15) are given by

$$m_{H_u}^2 + |\mu|^2 - \frac{b}{\tan \beta} - \frac{g^2 + g'^2}{4} v^2 \cos(2\beta) = 0, \quad (2.18)$$

$$m_{H_d}^2 + |\mu|^2 - b \tan \beta + \frac{g^2 + g'^2}{4} v^2 \cos(2\beta) = 0. \quad (2.19)$$

Here,  $\beta$  and  $v$  are defined as

$$\tan \beta \equiv \frac{\langle H_u^0 \rangle}{\langle H_d^0 \rangle}, \quad (2.20)$$

$$v^2 \equiv \langle H_u^0 \rangle^2 + \langle H_d^0 \rangle^2 \simeq (174 \text{ GeV})^2. \quad (2.21)$$

Note that these equations satisfy the conditions given by Eqs. (2.16) and (2.17). Therefore, the electroweak symmetry breaking correctly occurs as long as the minimization conditions are satisfied.

## 2.3 Mediation of SUSY breaking

As mentioned above, the SUSY must be broken in the vacuum since the superpartners of the standard model particles are not discovered. In order to construct a phenomenologically consistent SUSY model, we need some mechanisms to provide the soft SUSY breaking terms given by Eq. (2.13). Order parameters of the SUSY breaking are the auxiliary fields of some superfields, in other words, non-vanishing  $F$ -terms in Eq. (2.5) or  $D$ -terms in Eq. (2.6) implying the SUSY breaking. As we will see, a viable model requires a hidden sector different from the MSSM sector, that is a SUSY breaking sector, and a way to communicate the SUSY breaking effects to the MSSM sector.

The SUSY breaking effects are expected to communicate to the observable sector indirectly because of the tree-level supertrace formula [57]:

$$\text{Str} \mathbf{M}^2 \equiv \sum_{\text{spin } j} (-1)^{2j} (2j+1) \text{Tr} \mathbf{M}_j^2 = -2g_a \text{Tr} [T^a] D^a \quad (2.22)$$

The supertrace is defined by a weighted sum over all particles with spin  $j$ . Since the traces on the generators of non-abelian gauge group vanish, the only  $U(1)$  gauge group contributes to the right-hand side. However, it also vanishes as long as we consider anomaly-free gauge theories. Therefore, one can find that the supertrace must vanish at the renormalizable level. If the observable sector is connected to the SUSY breaking sector at the tree level, some superpartners should be light enough to be discovered. This is the reason to require a mechanism for an indirect mediation of the SUSY breaking effects.

Here, we introduce two major scenarios to mediate the SUSY breaking effects. One possible way is the mediation by interactions suppressed by the Planck scale. We call these models “gravity-mediation models” [58–63]. Since the SUSY breaking effects are mediated by the Planck-suppressed operators, the soft SUSY breaking terms in Eq. (2.13) are roughly estimated as

$$m_{SUSY} \sim \frac{\langle F \rangle}{M_{pl}}, \quad (2.23)$$

where  $m_{SUSY}$  represents generic soft SUSY breaking terms, and we assume that the SUSY is broken by the  $F$ -term in the hidden sector. For TeV scale soft mass terms,  $\sqrt{\langle F \rangle} \simeq 10^{10} \text{ GeV}$  is required.

The gravity-mediation models generically provide new sources of flavor violations in addition to the standard model Yukawa couplings. If the standard model Yukawa couplings are generated at the scale below the Planck scale, there are no reasons for the sfermion mass matrices in Eq. (2.13) to be diagonal in the flavor basis. Off-diagonal components, in other words, mixings between sfermions with different flavor often cause flavor changing processes that are strongly constrained for TeV scalar masses. We call this problem the SUSY flavor problem.

The other way to mediate the SUSY breaking effects utilizes the standard model gauge interactions. We call these models “gauge-mediation models” [51, 52, 64–68]. These models require messenger fields that are charged under the standard model gauge symmetry  $SU(3)_C \times SU(2)_L \times U(1)_Y$  and mediate SUSY breaking effects to the observable sector through the gauge interaction. The superpotential in the messenger sector is generically given by

$$W_{\text{mess}} = k Z Q \bar{Q}, \quad (2.24)$$

where  $Q$  and  $\bar{Q}$  are messenger fields, and  $Z$  is the SUSY breaking field.  $k$  is a dimensionless coupling constant. We assume that the SUSY breaking field resides in the hidden sector,

and both the scalar component and its  $F$ -term acquire the VEVs. The VEV of the scalar component  $\langle Z \rangle$  provides masses for the messenger fields. The VEV of the  $F$ -term gives mass splitting of the messenger fields.

The SUSY violation is communicated to the observable sector through loop diagrams containing these messenger fields. Gaugino masses are provided by one-loop diagrams, and squared masses for scalar fields are provided by two-loop diagrams. These soft masses are then estimated as

$$m_{SUSY} \sim \frac{\alpha}{4\pi} \frac{\langle F_Z \rangle}{M_{\text{mess}}}, \quad (2.25)$$

where  $\alpha$  denotes a generic gauge coupling constant defined as  $\alpha \equiv g^2/4\pi$ , and  $M_{\text{mess}}$  denotes the messenger mass estimated as  $M_{\text{mess}} \simeq k\langle Z \rangle$ . Given that  $M_{\text{mess}} \gtrsim \sqrt{\langle F_Z \rangle}$ , the SUSY breaking scale is estimated as

$$\sqrt{\langle F_Z \rangle} \simeq 1.1 \times 10^5 \text{ GeV} \alpha^{-1/2} \left( \frac{M_{\text{mess}}}{10^6 \text{ GeV}} \right)^{1/2} \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{1/2}, \quad (2.26)$$

for TeV scale soft masses. Therefore, the gauge-mediation models predict the SUSY breaking scale to be much lower than the gravity-mediation models. Note that the gravity-mediated SUSY breaking effects exist even in the gauge-mediation models. In these models, we can neglect such contributions since the SUSY breaking terms of Eq. (2.23) are much smaller than the gauge-mediated SUSY breaking effects.

An attractive point of these models is to provide an explanation to the SUSY flavor problem. If the standard model Yukawa couplings are generated at higher scales than the messenger mass scale, the flavor violation is generated only through the Yukawa interactions since the gauge interactions are flavor-blind. Therefore, we can predict the sparticle masses without knowing physics at the higher energy scale than the messenger scale, and the gauge-mediation models can solve the SUSY flavor problem.

## 2.4 Cosmological problems of supersymmetry

Although the SUSY is attractive from the theoretical point of view, there are some problems in cosmology. As mentioned in Chapter 1, supersymmetric models contain some long-lived particles which cause cosmological difficulties. In this section, we will briefly explain such problems.

### 2.4.1 Gravitino problem

The existence of the gravitino, which is the superpartner of the graviton, is predicted in local SUSY. The gravitino appears as a fermionic gauge field with spin  $3/2$  in supergravity. From Eq. (2.12), one can find that the gravitino has  $R$ -parity of  $-1$ . The gravitino mass is given by [17, 18]

$$m_{3/2} = \frac{\langle F \rangle}{\sqrt{3}M_{pl}}, \quad (2.27)$$

where  $m_{3/2}$  denotes the gravitino mass. The gravity-mediation models generally predict that the gravitino mass is comparable to the soft mass scale. On the other hand, the gauge-mediation models predict the gravitino mass much lower than the soft mass scales. In these models, the gravitino would be the LSP and cannot decay into any particles.

During the reheating after the inflation, gravitinos are abundantly produced through scattering processes. The yield variable for the gravitino is proportional to the reheating temperature  $T_{RH}$ . When  $T_{RH}$  is of the order of  $10^9$  GeV, it is estimated as [22, 29]

$$Y_{3/2} \equiv \frac{n_{3/2}}{s} \simeq 1.4 \times 10^{-13} \left[ 1 + 0.6 \left( \frac{m_{1/2}}{m_{3/2}} \right)^2 \right] \left( \frac{T_{RH}}{10^9 \text{ GeV}} \right), \quad (2.28)$$

where  $n_{3/2}$  is the number density of the gravitino, and  $m_{1/2}$  is the gaugino mass at the unification scale.

If the gravitino mass is smaller than the electroweak scale, the gravitino is the LSP and stable when the  $R$ -parity is conserved. The abundance of gravitinos produced during the reheating process must be smaller than the observed dark matter abundance,  $\Omega_{DM}h^2 = 0.12$  [2]. Here  $h$  is the present Hubble parameter in units of  $100 \text{ km} \cdot \text{sec}^{-1} \cdot \text{Mpc}^{-1}$ , and the density parameter  $\Omega_{DM}$  is defined as a ratio of the present dark matter density  $\rho_{DM}$  to the critical density  $\rho_{cr}$ ,  $\rho_{DM}/\rho_{cr}$ . Therefore,  $\Omega_{3/2} < \Omega_{DM}$  constrains the reheating temperature from above. For example,  $T_{RH}$  must be lower than  $\mathcal{O}(10^5)$  GeV for  $m_{3/2}$  of  $\mathcal{O}(1)$  MeV. Furthermore, the NLSP (Next-to-LSP) decay into standard model particles must be taken into account since the produced daughter particles may spoil the BBN. The constraint depends on particle models since the lifetime and the branching ratio of the NLSP decay depend on the mass spectra and what the NLSP is.

If the gravitino is not the LSP, it is unstable. The gravitino generally has a very long lifetime since it interacts with other particles only through the gravitational interaction. When the gravitino is as heavy as TeV scales, its lifetime is estimated as [23–25]

$$\tau_{3/2} \simeq 10^5 \left( \frac{m_{3/2}}{1 \text{ TeV}} \right)^{-3} \text{ sec}. \quad (2.29)$$

If the gravitino is heavier than  $\mathcal{O}(10\text{--}100)$  TeV, it can decay before the BBN. In this case, the constraint from the BBN becomes much milder.<sup>1</sup> If the gravitino is lighter than  $\mathcal{O}(10)$  TeV and unstable, it decays during or after the BBN. In this case, there are two decay modes to spoil the success of the BBN: radiative decays and hadronic decays. In the radiative decays, energetic photons or charged particles produced in electromagnetic showers destroy synthesized light elements such as  ${}^4\text{He}$  and D. The energy of daughter particles generally dissipates into thermal bath, but the synthesized light elements are destroyed through the electromagnetic interaction before the thermalization. On the other hand, the hadronic decay of the gravitino causes  $p \leftrightarrow n$  conversion which leads to the increase of the neutron-to-proton ( $n/p$ ) ratio. This leads to the overproduction of the synthesized  ${}^4\text{He}$  abundance. Moreover, the hadronic decays at  $T \lesssim \mathcal{O}(0.1)$  MeV cause overproduction of D and  ${}^3\text{He}$  through the destruction of  ${}^4\text{He}$  by energetic nucleons. Taking into account these effects, the gravitino abundance produced during the reheating is constrained. In the case of  $m_{3/2}$  of  $\mathcal{O}(1)$  TeV, the reheating temperature is constrained to be lower than  $\mathcal{O}(10^{5-6})$  GeV [28–30].

### 2.4.2 Moduli problem

In superstring theories, there appears a lot of massless scalar fields, called moduli fields, whose VEV determines the scale of extra dimensions at the low energy scale. Several ways to stabilize these moduli fields are known [31–33, 70], but all of the moduli fields are not always stabilized at string scales. Some of them often remain flat, and their potential is generally lifted with the curvature of the order of the gravitino mass after the SUSY breaking.

Now, let us focus on one of the moduli fields, and consider its dynamics in the early universe. During and after the primordial inflation, the vacuum energy which causes the expansion of the universe largely breaks SUSY and lifts the moduli potential. The moduli field then acquires a mass of the order of the Hubble scale and sits down at the local minimum of the potential. The generic moduli potential is given as

$$V = \frac{1}{2}c_H H^2 (\eta - \eta_0)^2 + \frac{1}{2}m_\eta^2 \eta^2 + \cdots, \quad (2.30)$$

where we take the origin as the minimum of the moduli potential at present.  $\eta$  denotes the moduli field,  $m_\eta$  denotes its mass and  $H$  denotes the Hubble scale. In the following, we will assume that the moduli mass is about the gravitino mass ( $m_\eta \simeq m_{3/2}$ ).  $c_H$  is a

---

<sup>1</sup> Even if gravitinos decay before the onset of the BBN, the abundance of LSPs produced from the gravitino decay may exceed the dark matter abundance as explained in Sec. 2.5.2.

coefficient of the Hubble induced mass term and is expected to be  $\mathcal{O}(1)$ .  $\eta_0$  denotes the minimum of the potential determined by the Hubble induced mass term, and is expected to be of the order of the Planck scale. This is because the moduli field is a singlet field without any symmetry enhanced points and the gravitational scale is the only scale appearing in the supergravity action. Since this potential is different from the one at present, the moduli field is displaced from the origin when the Hubble scale is larger than the moduli mass scale. When  $H \simeq m_\eta$ , the moduli field starts to oscillate around the origin with the amplitude of the order of  $M_{pl}$  according to the following equation:

$$\frac{d^2\eta}{dt^2} + 3H\frac{d\eta}{dt} + V_\eta = 0, \quad (2.31)$$

where  $V_\eta$  denotes the derivative of the potential with the moduli field. The energy of the oscillating moduli field redshifts as  $a^{-3}$  like non-relativistic matters, where  $a$  denotes the scale factor. Therefore, its energy soon dominates the universe after the inflaton decays.

In the case where the reheating after the primordial inflation occurs after the onset of the moduli oscillation, the ratio of its energy density to the entropy density is estimated as

$$\frac{\rho_\eta}{s} \simeq \frac{1}{8} T_{\text{RH}}^{\text{inf}} \left( \frac{\eta_0}{M_{pl}} \right)^2 \simeq 1.3 \times 10^5 \text{ GeV} \left( \frac{T_{\text{RH}}^{\text{inf}}}{10^6 \text{ GeV}} \right) \left( \frac{\eta_0}{M_{pl}} \right)^2, \quad (2.32)$$

where  $\rho_\eta$  is the oscillating energy of the moduli field, and  $T_{\text{RH}}^{\text{inf}}$  denotes the reheating temperature after the primordial inflation. On the other hand, in the case where the reheating occurs before the onset of the moduli oscillation, the ratio is given by

$$\frac{\rho_\eta}{s} \simeq \frac{1}{8} T_\eta \left( \frac{\eta_0}{M_{pl}} \right)^2 \simeq 2.8 \times 10^9 \text{ GeV} \left( \frac{m_\eta}{1 \text{ TeV}} \right)^{1/2} \left( \frac{\eta_0}{M_{pl}} \right)^2. \quad (2.33)$$

$T_\eta$  is temperature at the onset of the moduli oscillation and is defined as

$$T_\eta \equiv \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{m_\eta M_{pl}} \simeq 2.2 \times 10^{10} \text{ GeV} \left( \frac{m_\eta}{1 \text{ TeV}} \right)^{1/2}, \quad (2.34)$$

where  $g_*$  is the effective number of degrees of freedom. Here we used  $g_* = 229$ . The moduli-to-entropy density ratio are conserved until present unless entropy production occurs. Since the ratio of the critical density  $\rho_{cr}$  to the present entropy density  $s_0$  is given by

$$\frac{\rho_{cr}}{s_0} \simeq 3.6 \times 10^{-9} h^2 \text{ GeV}, \quad (2.35)$$

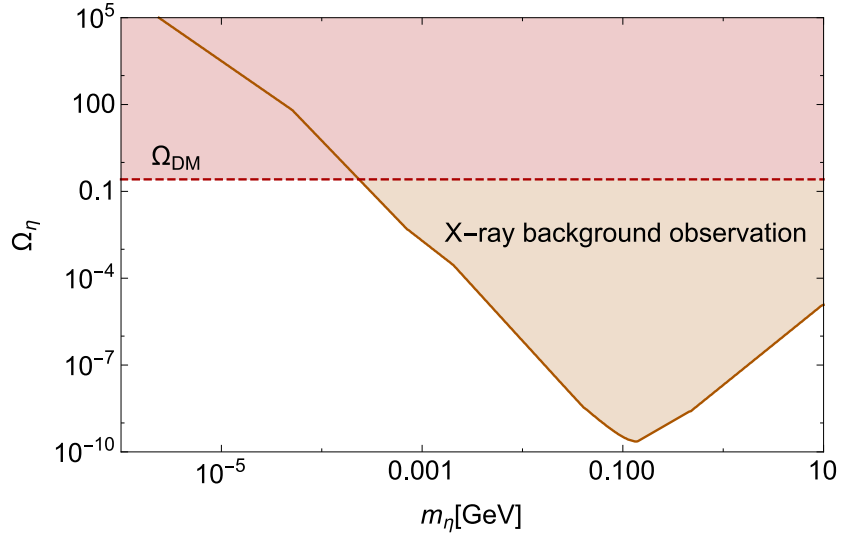


Figure 2.3: The constraints on the moduli abundance. The orange line shows the X-ray background constraint, and the red dashed line shows the dark matter abundance. We recast this figure using the results in Refs. [37, 38].

one can find that the produced moduli density is much larger than the critical density. Note that the moduli abundance cannot be smaller than the critical density even if the reheating temperature is as low as  $\mathcal{O}(10)$  MeV, which is restricted from below to realize the BBN.

Since the moduli field gravitationally interacts with other particles as well as the gravitino, it also has a very long lifetime. When the moduli mass is of the order of TeV scale, the moduli field decays during or after the BBN. As is the case for the unstable gravitino, the moduli abundance is severely constrained as [28, 30]

$$\frac{\rho_\eta}{s} \lesssim \mathcal{O}(10^{-14}) \text{ GeV} \quad (2.36)$$

when the branching ratio for hadronic decay channels is  $\mathcal{O}(1)$  and the moduli mass is of  $\mathcal{O}(1)$  TeV. On the other hand, when the moduli mass is smaller than  $\mathcal{O}(100)$  MeV, its lifetime is larger than the present age of the universe. Since its density contributes to the dark matter density, the produced moduli density must be smaller than the dark matter density as well as the stable gravitino.

Moreover, there is a more stringent upper bound on the moduli abundance from X-ray observations when  $\mathcal{O}(10^{-4}) \text{ GeV} \lesssim m_\eta \lesssim \mathcal{O}(1) \text{ GeV}$  [37, 38]. One of the most probable candidates for the moduli field is the dilaton whose VEV determines gauge coupling constants. The dilaton field  $\eta$  generically couples to kinetic terms of gauge fields as

follows:

$$\mathcal{L} = \frac{b}{4} \frac{\eta}{M_{pl}} F_{\mu\nu} F^{\mu\nu}, \quad (2.37)$$

where  $b$  is a coefficient of the order of 1. Due to this coupling, the dilaton field can decay into two photons or two gluons if the decay is kinetically allowed. In particular, the decay mode into photons is always open, and the emitted photons should not exceed the observed X-ray backgrounds. We then obtain the stringent constraint for the moduli mass of  $\mathcal{O}(10^{-4})$  GeV  $\lesssim m_\eta \lesssim \mathcal{O}(1)$  GeV (see Fig. 2.3).

### 2.4.3 Polonyi problem

A singlet field such as the moduli field may appear even if you do not promote supersymmetric models to superstring theories. The Polonyi field appearing in the Polonyi model is one of such fields. The Polonyi model is the simplest SUSY breaking model in which the  $F$ -term of the Polonyi field  $Z$  breaks the SUSY in the hidden sector [41].<sup>2</sup> In this model, the only ingredient in the SUSY breaking sector is an elementary field  $Z$ . The superpotential in the hidden sector is given by

$$W_{\text{hid}} = \mu_{SUSY}^2 M_{pl} \left( 1 + c \frac{Z}{M_{pl}} + \cdots \right), \quad (2.38)$$

where  $\mu_{SUSY}$  is a parameter with mass dimension 1, and  $c$  is a dimensionless parameter of  $\mathcal{O}(1)$ . Higher order terms are expressed by the ellipsis. Note that the parameter  $\mu_{SUSY}$  breaks the  $R$  symmetry since  $Z$  has  $R$  charge of 0.  $\mu_{SUSY}$  is related to the gravitino mass as  $|\mu_{SUSY}|^2 \simeq \langle |W_{\text{hid}}| \rangle / M_{pl} \simeq m_{3/2} M_{pl}$ . The mass of the Polonyi field is of the order of  $m_{3/2}$  for generic Kähler potentials. The  $F$ -term of  $Z$  is given by  $\langle |F_Z| \rangle \simeq m_{3/2} M_{pl}$ , which implies spontaneous SUSY breaking. This model is attractive in terms of its simplicity, but the Polonyi field causes cosmological difficulties called the “Polonyi problems” as well as the moduli field [35]. The Polonyi problem is also a severe problem because the Polonyi abundance cannot be suppressed enough even for low reheating temperature of  $T_{\text{RH}} \simeq \mathcal{O}(10)$  MeV.

---

<sup>2</sup> In the Polonyi model, the SUSY breaking scale can be easily obtained by dynamical transmutation [53], by assigning vanishing  $R$  charge to the Polonyi field and breaking the  $R$  symmetry by gaugino condensation [71, 72].



## 2.5 Solutions to cosmological moduli/Polonyi problems

There are several ways of solving the cosmological moduli/Polonyi problems. One possible solution is to dilute the moduli/Polonyi density by some mechanisms, for example, by thermal inflation [42, 43]. The thermal inflation is known to be one of the most probable candidates for dilution mechanisms. Another simple way is to make the moduli/Polonyi field heavy enough to decay before the BBN. In this case, scales of soft scalar masses often deviate from the electroweak scales, and then it becomes difficult to achieve the electroweak symmetry breaking. Moreover, the abundance of LSPs produced from the moduli decay may exceed the observed dark matter abundance as we will explain. In this section, we will explain how the moduli/Polonyi problems are solved in these scenarios.<sup>3</sup> For simplicity, we focus only on the moduli field, but the following scenarios are applicable to the case of the Polonyi field.

### 2.5.1 Thermal inflation

The thermal inflation is a short epoch of accelerated expansion of the universe at low energy scales. We show the evolution of the energy density in Fig. 2.4. This mechanism requires a scalar field corresponding to a flat direction at the renormalizable level, which is called the “flaton”. Firstly, let us introduce a simple model of the thermal inflation with a discrete symmetry.

#### A. Model with $Z_4$ symmetry

We assume an approximate  $Z_4$  symmetry in order to ensure the flatness. The superpotential is given by [43–47]

$$W = \frac{\lambda_X}{4M_{pl}} X^4 + g_\xi X \xi \bar{\xi} + C, \quad (2.39)$$

where  $\lambda_X$  and  $g_\xi$  are dimensionless coupling constants, and  $X$  is the superfield of the flaton field with a  $Z_4$  charge of 1. Here we assume that  $\xi$  and  $\bar{\xi}$  are massless fields charged under the standard model gauge symmetry.  $C$  is a constant term which cancels out the vacuum energy and is related to the gravitino mass by  $|C| \simeq m_{3/2} M_{pl}^2$ . Note that  $X$  is singlet under the standard model gauge symmetry. The massless charged fields,  $\xi$  and  $\bar{\xi}$ , interact with thermal bath, which generates the thermal mass term for  $X$ . Here, we

---

<sup>3</sup> For other solutions, see Refs. [73–78].

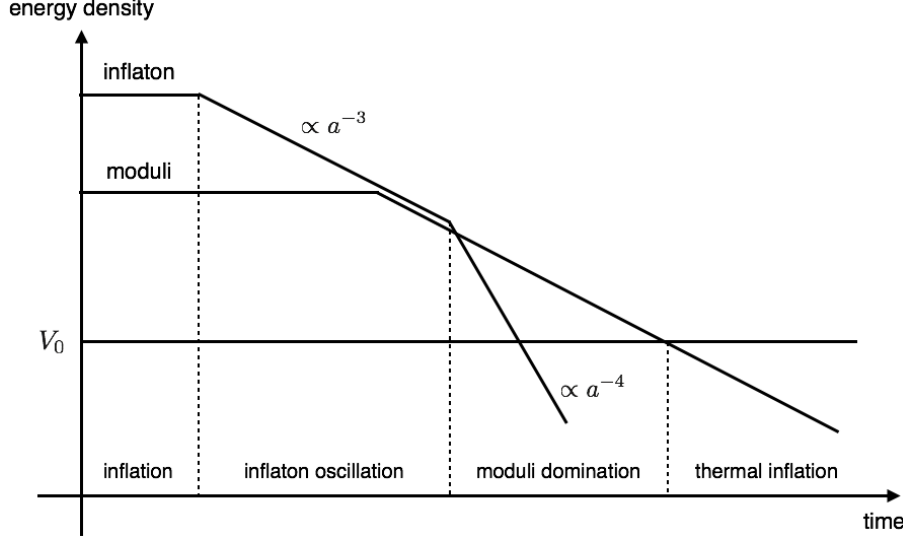


Figure 2.4: The evolution of the energy density in the thermal inflation scenario. When the vacuum energy  $V_0$  dominates total energy density of the universe, the thermal inflation starts.

ignore higher dimensional terms since we focus on the field value much smaller than the Planck scale.

Including SUSY breaking effects and the thermal mass term, the potential is given by

$$V(X) = V_0 + (c_T T^2 - m_X^2) |X|^2 + \left( \frac{a_X}{4M_{pl}} \lambda_X X^4 + \text{c.c.} \right) + \frac{|\lambda_X|^2}{M_{pl}^2} |X|^6 + \dots, \quad (2.40)$$

where  $V_0$  represents the vacuum energy which causes the thermal inflation, and  $c_T$  is a coefficient of the order of the  $g_\xi$  squared. When  $\xi$  and  $\bar{\xi}$  are  $\mathbf{5}$  and  $\bar{\mathbf{5}}$  in  $SU(5)$ ,  $c_T$  takes a value of  $5g_\xi^2/3$  [46].  $a_X$  is a dimensional parameter of the order of  $m_{3/2}$ . Hereafter, we assume that  $X$  has a tachyonic soft SUSY breaking mass term around the origin ( $m_X^2 > 0$ ),<sup>4</sup> which is essential for the thermal inflation to work.

The evolution of the flaton  $X$  is as follows: we assume that  $X$  obtains a positive Hubble induced mass term during the primordial inflation.  $X$  is then expected to sit around the origin just after the inflation. Even when  $H \lesssim m_X$ ,  $X$  can be trapped around the origin due to the thermal mass term, and then the vacuum energy  $V_0$  causes the accelerated expansion of the universe when the vacuum energy exceeds the total energy of the universe. The thermal inflation lasts until the temperature decreases to the critical value of  $T_{\text{end}} \simeq c_T^{-1/2} m_X$ . After the end of the thermal inflation,  $X$  starts to roll down to

<sup>4</sup> In gauge-mediation models, the soft SUSY breaking mass term arises from the coupling with massless gauge charged fields,  $\xi$  and  $\bar{\xi}$ .

the true minimum due to the negative mass term. When  $H$  decreases to the decay rate of the flaton, it decays into radiation with huge entropy production, and then the radiation dominated universe is realized.

The true minimum of the potential is determined by the negative mass term, the  $A$ -term and non-renormalizable terms. The flaton VEV at present is given by

$$M^2 \equiv \langle |X| \rangle^2 = \frac{m_X M_{pl}}{6|\lambda_X|} \left[ \frac{|a_X|}{m_X} + \sqrt{\frac{|a_X|^2}{m_X^2} + 12} \right]. \quad (2.41)$$

For simplicity, we assume that  $\lambda_X$  is of  $\mathcal{O}(1)$ . One can find that the flaton VEV is much larger than the electroweak scale. Therefore, the charged matter  $\xi$  ( $\bar{\xi}$ ) with masses of the order of  $g_\xi \langle |X| \rangle$  is expected to be much heavier than the electroweak scale after the thermal inflation. By requiring that the vacuum energy vanishes at the true minimum, the vacuum energy  $V_0$  is determined as follows:

$$V_0 = \frac{2m_X^2 M^2}{3} \left[ 1 + \frac{|a_X|}{24m_X} \left( \frac{|a_X|}{m_X} + \sqrt{\frac{|a_X|^2}{m_X^2} + 12} \right) \right]. \quad (2.42)$$

When  $X$  has its large VEV, it is decomposed as

$$X = \left[ M + \frac{\chi}{\sqrt{2}} \right] \exp \left( i \frac{a_\chi}{\sqrt{2}M} \right), \quad (2.43)$$

where  $\chi$  and  $a_\chi$  are canonically normalized real scalar fields. The component  $\chi$  corresponds to the flaton that starts to oscillate after the thermal inflation. We obtain a mass of the radial component  $\chi$  around the true minimum:

$$m_\chi^2 = 4m_X^2 \left[ 1 + \frac{|a_X|}{12m_X} \left( \frac{|a_X|}{m_X} + \sqrt{\frac{|a_X|^2}{m_X^2} + 12} \right) \right]. \quad (2.44)$$

If  $C$  was absent in Eq. (2.39) and the  $R$  symmetry was not broken, the superpotential for  $X$  would have the  $R$  symmetry and the phase component  $a$  would be a massless  $R$  axion. However, the  $R$  symmetry breaking, namely the non-zero VEV of the superpotential, generates the  $R$  symmetry breaking  $A$ -terms and the phase component  $a_\chi$  also obtains a mass as

$$m_{a_\chi}^2 = \frac{m_X |a_X|}{3} \left( \frac{|a_X|}{m_X} + \sqrt{\frac{|a_X|^2}{m_X^2} + 12} \right). \quad (2.45)$$

In gravity-mediation models, the  $R$ -axion is as heavy as the flaton with a mass of  $\mathcal{O}(1)$  TeV. In gauge-mediation models, on the other hand, the  $R$ -axion is much lighter than the electroweak scale since the  $R$  symmetry breaking parameter is small.

After the end of the thermal inflation, the energy density of the oscillating flaton field dominates that of the universe. Thus, the reheating occurs when  $H$  decreases to the decay rate of the flaton  $\chi$ . The decay process highly depends on models. In the model given by Eq. (2.39), the flaton can decay into gluons through one loop diagrams of  $\xi$  and  $\bar{\xi}$ . The decay rate of this process is given by [46]

$$\Gamma(\chi \rightarrow 2g) = \frac{1}{4\pi} \left( \frac{\alpha_S}{4\pi} \right)^2 \frac{m_\chi^3}{M^2}, \quad (2.46)$$

where  $\alpha_S$  is the  $SU(3)_C$  gauge coupling constant defined as  $\alpha_S \equiv g_3^2/4\pi$ . If the reheating completes by this decay, the reheating temperature  $T_{\text{RH}}^\chi$  is estimated as

$$\begin{aligned} T_{\text{RH}}^\chi &\simeq \left( \frac{90}{\pi^2 g_*(T_{\text{RH}}^\chi)} \right)^{1/4} \sqrt{\Gamma(\chi \rightarrow 2g) M_{\text{pl}}} \\ &\simeq 6.1 \text{ GeV} \alpha_S \left( \frac{g_*(T_{\text{RH}}^\chi)}{100} \right)^{-1/4} \left( \frac{m_\chi}{1 \text{ TeV}} \right)^{3/2} \left( \frac{M}{10^{11} \text{ GeV}} \right)^{-1}. \end{aligned} \quad (2.47)$$

When  $m_\chi > 2m_{a_\chi}$ , the flaton can also decay into  $R$ -axions. At the tree level, the decay rate of this process is estimated as [46]

$$\Gamma(\chi \rightarrow 2a_\chi) = \frac{1}{64\pi} \frac{m_\chi^3}{M^2}. \quad (2.48)$$

The produced  $R$ -axions decay into standard model particles through one-loop diagrams containing  $\xi$  and  $\bar{\xi}$ . For example, the  $R$ -axions decay into photons and the decay rate is estimated as [46]

$$\Gamma(a_\chi \rightarrow 2\gamma) \simeq \frac{2}{9\pi} \left( \frac{\alpha_{em}}{4\pi} \right)^2 \frac{m_{a_\chi}^3}{M^2}, \quad (2.49)$$

where  $\alpha_{em}$  is the fine-structure constant defined as  $\alpha_{em} \equiv e^2/4\pi$ . If loop running particles are charged under  $SU(3)_C$  and  $m_{a_\chi} \gtrsim \mathcal{O}(1) \text{ GeV}$ , the  $R$ -axion can also decay into gluons. Note that the flaton decay into  $R$ -axions is always allowed in gauge-mediated SUSY breaking models. We then should consider the  $R$ -axion decay processes in such cases.

As seen above, in gauge-mediation models, the flaton decays into both the standard model particles and  $R$ -axions when its decay rate becomes comparable to the Hubble parameter. The produced standard model particles thermalize immediately and reheating occurs. However, the produced  $R$ -axions cannot thermalize soon since they have only one-loop suppressed interactions with particles in thermal bath. After the  $R$ -axions become non-relativistic particles, its density scales as  $a^{-3}$  and soon dominates the universe [45, 46].

Later,  $R$ -axions decay into the standard model particles and reheating occurs again. Using Eq. (2.49), the reheating temperature when the  $R$ -axions decay,  $T_{\text{RH}}^{a_\chi}$ , is estimated as [47]

$$\begin{aligned} T_{\text{RH}}^{a_\chi} &\simeq \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{\Gamma(a_\chi \rightarrow 2\gamma) M_{pl}} \\ &\simeq 1.6 \text{ keV} |\lambda_X|^{1/2} \left( \frac{\alpha_{em}}{1/137} \right) \left( \frac{m_X}{1 \text{ TeV}} \right)^{1/4} \left( \frac{|a_X|}{100 \text{ keV}} \right)^{3/4}, \end{aligned} \quad (2.50)$$

where we use  $g_* = 3.36$ . It is found that the  $R$ -axions decay at the temperature much lower than  $\mathcal{O}(10) \text{ MeV}$ . This implies that it destroys light elements synthesized at the BBN era and spoils the success of the BBN.

## B. Model with linear term

One way to avoid the late-time decay of  $R$ -axions in the  $Z_4$  symmetric model is a prohibition of the flaton decay into  $R$ -axions. It can be realized by adding a  $Z_4$  symmetry breaking term [45–47],

$$\delta W = \alpha X, \quad (2.51)$$

into the superpotential. Note that  $\alpha$  is a parameter with mass dimension 2. As we will explain below, this term leads to degeneracy of masses of the flaton and the  $R$ -axion even in gauge-mediated SUSY breaking models. The flaton decay into  $R$ -axions is forbidden kinematically.

Here, we should comment on another motivation to introduce the  $Z_4$  symmetry breaking term. When the superpotential has the  $Z_4$  symmetry, there exists four degenerate minima in the potential for  $X$ . The flaton randomly falls into one of them after the thermal inflation, and then domain walls are formed. The domain walls dominate the energy density of the universe, which leads to a cosmological disaster [79]. Hence, a bias for the degenerate minima is necessary so that domain walls collapse before they dominate the universe. In order to avoid the domain wall problem, the following condition should be satisfied:<sup>5</sup>

$$\delta V_{\text{bias}} \gg \frac{\sigma^2}{M_{pl}^2}, \quad (2.52)$$

---

<sup>5</sup> Assuming that the energy density of domain walls obeys the scaling relation,  $\rho_{\text{DW}} \sim \sigma H$ , domain walls dominate the universe when  $H_{\text{dom}} \sim \sigma/M_{pl}^2$ . On the other hand, domain walls decay due to the bias when  $H_{\text{dec}} \sim \delta V_{\text{bias}}/\sigma$ . The condition is derived by requiring  $H_{\text{dec}} \gg H_{\text{dom}}$ .

where  $\delta V_{\text{bias}}$  represents the bias of the energy density, and  $\sigma$  represents the tension of domain walls. In terms of the symmetry breaking parameter  $\alpha$ , the condition is expressed as

$$|\alpha| \gtrsim \frac{m_{3/2}^2 m_\chi M}{M_{pl}^2}. \quad (2.53)$$

The necessary bias is so small that it does not change the scenario of the thermal inflation. Hereafter, we consider the case that  $|\alpha|$  is large enough to satisfy Eq. (2.53).

The flaton potential is now given by

$$\begin{aligned} V(X) = & V_0 - (2\alpha a_X X + \text{h.c.}) - m_X^2 |X|^2 \\ & + \left( \frac{\alpha^* \lambda_X}{M_{pl}} X^3 + \text{h.c.} \right) + \left( \frac{1}{4} \frac{a_X}{M_{pl}} \lambda_X X^4 + \text{h.c.} \right) + \frac{|\lambda_X|^2}{M_{pl}^2} |X|^6. \end{aligned} \quad (2.54)$$

The negative mass term is induced from the gauge-mediated SUSY breaking effect and is applicable only for the flaton VEV smaller than the messenger mass scale. When the flaton VEV becomes larger than the messenger scale, the negative soft SUSY breaking term becomes suppressed at large amplitude [80]. In the following, we focus on the parameter region where the flaton VEV at the true minimum is larger than the messenger mass scale. We then ignore the negative mass term. When we assume that  $|a_X| \simeq m_{3/2} \simeq \mathcal{O}(100) \text{ keV}$  and  $|\lambda_X| \simeq \mathcal{O}(1)$ , the trilinear term dominates over both the linear term and the quartic term at the true minimum for the flaton VEV larger than  $\mathcal{O}(10^7) \text{ GeV}$ .<sup>6</sup> In this case, the flaton VEV at present is estimated as

$$\langle X \rangle \equiv M \simeq \left( \frac{|\alpha| M_{pl}}{|\lambda_X|} \right)^{1/3}, \quad (2.55)$$

and the vacuum energy at the origin is estimated as

$$V_0 \simeq |\alpha|^2. \quad (2.56)$$

$V_0$  is determined by requiring that the vacuum energy vanishes at the true minimum. The masses of the flaton and  $R$ -axion are now almost the same and are given by

$$m_\chi \simeq m_{a_\chi} \simeq 3 \left( \frac{|\alpha|^2 |\lambda_X|}{M_{pl}} \right)^{1/3}. \quad (2.57)$$

---

<sup>6</sup> As explained in Sec. 3.5, if there is a coupling between the flaton and Higgs such as Eq. (3.110), the flaton VEV larger than  $\mathcal{O}(10^{10}) \text{ GeV}$  can generate the  $\mu$ -term of the electroweak scale. We then focus on the parameter region where the flaton VEV is relatively large.

Note that the flaton decay into  $R$ -axions is forbidden since their masses are degenerate.

Taking into account the thermal effects, the flaton potential around the origin is rewritten by

$$V(X) \simeq (cT^2 - m_X^2) \left| X - \frac{2\alpha^* a_X^*}{c_T T^2 - m_X^2} \right|^2 + V_0 - \frac{4|\alpha|^2 |a_X|^2}{c_T T^2 - m_X^2}, \quad (2.58)$$

and the local minimum before and during the thermal inflation is given by

$$\langle |X| \rangle \simeq \frac{2|\alpha| m_{3/2}}{c_T T^2 - m_X^2}. \quad (2.59)$$

Then, one can find that the flaton does not sit at the origin due to the  $Z_4$  symmetry breaking term  $\alpha$ . This implies that the thermal inflation ends when the flaton VEV becomes so large that it decouples from the thermal plasma, i.e., when the temperature decreases to  $T_{\text{end}} \simeq c_T^{1/2} \langle |X| \rangle$ . Although the vacuum energy during the thermal inflation also deviates from  $V_0$ , we can ignore the deviation  $\delta V/V \simeq m_{3/2}^2/c_T T^2$ .

### C. Moduli abundance

Now, we can estimate a dilution factor as follows:

$$\begin{aligned} \Delta &\equiv \frac{s_f a_f^3}{s_i a_i^3} = \frac{4}{3} \frac{V_0}{2\pi^2/45 g_{*s}(T_{\text{end}}) T_{\text{end}}^3 T_{\text{RH}}^X} \\ &\simeq 3.0 \times 10^{20} \left( \frac{V_0}{10^{28} \text{ GeV}^4} \right) \left( \frac{g_{*s}(T_{\text{end}})}{100} \right)^{-1} \left( \frac{T_{\text{end}}}{100 \text{ GeV}} \right)^{-3} \left( \frac{T_{\text{RH}}^X}{1 \text{ GeV}} \right)^{-1}, \end{aligned} \quad (2.60)$$

where  $s_i$  and  $s_f$  are the entropy densities before and after the entropy production occurs, respectively. By using Eqs. (2.32) and (2.60), the yield variable for the moduli density after the dilution is estimated as

$$\begin{aligned} \frac{\rho_{\eta, \text{BB}}}{s} &\simeq \frac{1}{8\Delta} T_{\text{RH}}^{\text{inf}} \left( \frac{\eta_0}{M_{pl}} \right)^2 \\ &\simeq 4.1 \times 10^{-15} \text{ GeV} \left( \frac{\Delta}{3 \times 10^{20}} \right)^{-1} \left( \frac{T_{\text{RH}}^{\text{inf}}}{10^7 \text{ GeV}} \right) \left( \frac{\eta_0}{M_{pl}} \right)^2. \end{aligned} \quad (2.61)$$

In the following, we will call moduli produced before the thermal inflation “Big-Bang moduli”. Here we used the subscript of BB to represent the Big-Bang moduli.

In the scenario of the thermal inflation, there exists additional contribution to the Big-Bang moduli. We call the secondary produced moduli “thermal inflation moduli” [43].

As with Eq. (2.30), the moduli potential during the thermal inflation is given by

$$\begin{aligned} V &= \frac{1}{2}\tilde{c}_H H^2 (\eta - \tilde{\eta}_0)^2 + \frac{1}{2}m_\eta^2 \eta^2 + \dots \\ &\simeq \frac{1}{2}(m_\eta^2 + \tilde{c}_H H^2) \left( \eta - \frac{\tilde{c}_H H^2}{m_\eta^2 + \tilde{c}_H H^2} \tilde{\eta}_0 \right)^2 + \dots, \end{aligned} \quad (2.62)$$

where  $\tilde{c}_H$  is of  $\mathcal{O}(1)$  and  $\tilde{\eta}_0$  is of the order of the Planck scale. Equation (2.62) clarifies that the moduli field sits at the minimum depending on the Hubble induced mass term during the thermal inflation. After that, it starts to roll down to the true minimum with the amplitude of  $\eta \simeq \tilde{c}_H (H_{\text{th}}/m_\eta)^2 \tilde{\eta}_0$ .  $H_{\text{th}}$  is the Hubble parameter at the end of the thermal inflation. The ratio of the moduli to the entropy density at present is therefore given by

$$\begin{aligned} \frac{\rho_{\eta,\text{TH}}}{s} &= \frac{\rho_{\eta,\text{TH}}}{4V_0/3T_{\text{RH}}^X} = \frac{\tilde{c}_H^2 T_{\text{RH}}^X V_0}{24 m_\eta^2 M_{\text{pl}}^2} \left( \frac{\tilde{\eta}_0}{M_{\text{pl}}} \right)^2 \\ &\simeq 7.0 \times 10^{-17} \text{ GeV} \tilde{c}_H^2 \left( \frac{T_{\text{RH}}^X}{1 \text{ GeV}} \right) \left( \frac{V_0}{10^{28} \text{ GeV}^4} \right) \left( \frac{m_\eta}{1 \text{ TeV}} \right)^{-2} \left( \frac{\tilde{\eta}_0}{M_{\text{pl}}} \right)^2. \end{aligned} \quad (2.63)$$

Note that the thermal inflation moduli is produced more abundantly as the vacuum energy  $V_0$  becomes higher, which is contrary to the Big-Bang moduli.

The total moduli density is given by  $\rho_{\text{total}} = \rho_{\eta,\text{BB}} + \rho_{\eta,\text{TH}}$ , and then it is found that the thermal inflation can solve the moduli problem since  $\rho_{\text{total}}/s \lesssim \mathcal{O}(10^{-14}) \text{ GeV}$  for  $m_\eta \sim \mathcal{O}(1) \text{ TeV}$ . In the case of  $m_\eta \lesssim \mathcal{O}(10^{-4}) \text{ GeV}$ , the moduli density can also be sufficiently diluted out by taking model parameters properly.

### 2.5.2 Heavy moduli/Polonyi scenario

A simple way to avoid the moduli/Polonyi problems is to make the moduli/Polonyi field heavy enough to decay before the onset of the BBN, which requires the moduli mass of  $\mathcal{O}(10\text{-}100) \text{ TeV}$ . Hereafter we assume that the gravitino mass is as heavy as the moduli mass. The moduli decay produces huge entropy since its density dominates the cosmic density of the universe before its decay. With the decay rate of the moduli field,

$$\Gamma_\eta = \frac{d_\eta}{8\pi} \frac{m_\eta^3}{M_{\text{pl}}^2}, \quad (2.64)$$

the decay temperature is estimated as

$$T_\eta \equiv \left( \frac{90}{\pi^2 g_*(T_\eta)} \right)^{1/4} \sqrt{\Gamma_\eta M_{\text{pl}}} \simeq 4 \text{ MeV} d_\eta^{1/2} \left( \frac{m_\eta}{100 \text{ TeV}} \right)^{3/2}. \quad (2.65)$$



Here  $d_\eta$  is a numerical constant, and we used  $g_*(T_\eta) = 10.75$ . When the moduli mass is lighter than  $\mathcal{O}(10\text{-}100)$  TeV, the severe constraint,  $\rho_\eta/s \lesssim \mathcal{O}(10^{-14})$  GeV, is required since the moduli decays after the onset of the BBN,  $T \simeq \mathcal{O}(1\text{-}10)$  MeV [28–30]. When the moduli field is heavier than  $\mathcal{O}(100)$  TeV, it can decay before the BBN, and the constraint from the BBN becomes much milder.<sup>7</sup>

Even in this case, however, there is an incidental problem: LSPs are abundantly produced from the decay of the moduli field, and the LSP density tends to exceed the observed dark matter density. The abundance of the LSPs is given by [69, 85]

$$Y_{\text{LSP}} \equiv \frac{n_{\text{LSP}}}{s} \simeq \min \left[ \frac{\Gamma_\eta}{\langle \sigma v \rangle s(T_\eta)}, \frac{N_{\text{LSP}} n_\eta(T_\eta)}{s(T_\eta)} \right], \quad (2.66)$$

where  $n_{\text{LSP}}$  and  $n_\eta$  denote the number density of the LSP and the moduli field, respectively.  $\langle \sigma v \rangle$  represents a thermally averaged cross section of the pair annihilation between LSPs.  $N_{\text{LSP}}$  is the averaged number of superparticles produced by the decay of one moduli field. When the first term is relevant, the pair annihilation between LSPs effectively proceeds after the moduli decay, and the yield variable for relic LSPs,  $Y_{\text{LSP}}$ , is approximately proportional to  $T_\eta^{-1}$ . For example, when the neutral wino is the LSP with a mass of  $\mathcal{O}(0.1\text{-}1)$  TeV, it is found that the decay temperature  $T_\eta$  generally needs to be larger than  $\mathcal{O}(1\text{-}10)$  GeV [69] in order for the wino abundance not to exceed the observed dark matter density.<sup>8</sup> This requires the moduli mass heavier than about 5 PeV assuming that  $d_\eta \simeq \mathcal{O}(1)$ .

## 2.6 Moduli/Polonyi problem and baryon asymmetry

As explained above, cosmological moduli/Polonyi problems can be solved by several ways. However, it should be kept in mind that preexisting baryon asymmetry is also diluted out by entropy production. The dilution is provided by the flaton decay, the moduli/Polonyi decay and so on. Therefore, in order to obtain cosmologically consistent scenarios, we need a mechanism to generate huge baryon asymmetry beforehand or to generate the observed baryon asymmetry after the dilution.

As mentioned in Chapter 1, leptogenesis is one of well-known baryogenesis. In this scenario, lepton violating interactions generate lepton asymmetry at the higher energy

---

<sup>7</sup> When the gravitino mass is much lighter than the moduli mass and is about  $\mathcal{O}(1)$  TeV, the moduli decay into gravitinos is kinematically allowed with the branching fraction of  $\mathcal{O}(0.01\text{-}1)$  [81–84]. In this case, the constraint from the BBN becomes stringent.

<sup>8</sup> Reference [69] has taken into account the Sommerfeld effect and coannihilation among charged and neutral winos.

scale, i.e., before the entropy production. However, the produced lepton asymmetry is too small to survive after the dilution since the produced asymmetry is generally of the order of the observed baryon asymmetry before the dilution [86]. Then, the leptogenesis cannot work in the existence of moduli/Polonyi fields.

On the other hand, the electroweak baryogenesis might be able to generate the baryon asymmetry after the dilution. In this case, the reheating temperature after the dilution should be higher than the electroweak scale in order for the produced asymmetry not to be diluted. However, it seems to be difficult to achieve such high reheating temperature in both cases of the conventional models of thermal inflation and the heavy moduli/Polonyi scenarios. Moreover, the electroweak phase transition must be first order in order to generate the baryon asymmetry. Unless a particle model achieves the first order phase transition, the generated baryon asymmetry is washed out by the sphaleron process. In the MSSM, the parameter region where the electroweak phase transition becomes first order is strictly constrained [87, 88].

The Affleck-Dine (AD) mechanism is the most probable candidate for generating baryon asymmetry beforehand because it can produce  $B-L$  number much more efficiently than other baryogenesis scenarios, e.g., the leptogenesis and the electroweak baryogenesis. In the next chapter, we explore a parameter region to explain the observed baryon asymmetry by the AD mechanism in the existence of the moduli/Polonyi field.

# Chapter 3

## Baryogenesis before dilution

The thermal inflation dilutes out not only moduli density but also the baryon asymmetry which may be produced beforehand. When we consider that baryon number is produced before dilution, we need some viable mechanisms to produce sufficiently large amount of baryon asymmetry. The Affleck-Dine mechanism [20, 21] is a promising candidate in the framework of the SUSY. In the AD mechanism, flat directions rotating in the complex plane produce baryon number, and they decay into standard model particles in the early universe. In this chapter, we explain whether the AD mechanism can explain the observed baryon asymmetry with huge dilution.

### 3.1 Affleck-Dine mechanism

The SUSY predicts a lot of flat directions which have vanishing potentials at the renormalizable level [19]. In particular, the MSSM contains flat directions carrying baryon (and/or lepton) charge. They are referred to as “Affleck-Dine fields” in this scenario. One example of them is the  $\bar{u}\bar{d}\bar{d}$  flat direction. The  $\bar{u}\bar{d}\bar{d}$  flat direction is combination of right-handed squark fields with the same amplitude, for example,  $\tilde{u}_1^R = \frac{1}{\sqrt{3}}\Phi$ ,  $\tilde{d}_1^G = \frac{1}{\sqrt{3}}\Phi$  and  $\tilde{d}_2^B = \frac{1}{\sqrt{3}}\Phi$ , where  $\tilde{u}$  and  $\tilde{d}$  are the up-type and down-type right-handed squarks, respectively. The superscripts and the subscripts show color and family indices, respectively. In this direction, the  $D$ -term potentials indeed vanish and it also has no renormalizable terms in the  $F$ -terms if the  $R$ -parity is conserved. In the following, we explain the AD mechanism without restricting to the  $\bar{u}\bar{d}\bar{d}$  flat direction. The other flat directions with  $B - L$  charges are listed in Table 3.1.

Firstly, we will explain the AD potential responsible for generation of baryon number. The flat directions are lifted by non-renormalizable terms and SUSY breaking effects. The

flat directions	$B - L$
$LH_u$	-1
$\bar{u}\bar{d}\bar{d}$	-1
$LL\bar{e}$	-1
$Q\bar{d}L$	-1
$\bar{d}\bar{d}\bar{d}LL$	-3
$\bar{u}\bar{u}\bar{u}\bar{e}\bar{e}$	1
$Q\bar{u}Q\bar{u}\bar{e}$	1
$QQQQ\bar{u}$	1
$(QQQ)_4LLL\bar{e}$	-1
$\bar{u}\bar{u}\bar{d}Q\bar{d}Q\bar{d}$	-1

Table 3.1: The flat directions with  $B - L$  charges in the MSSM. The subscript of 4 shows that the combination of  $(QQQ)$  transforms as a **4** of  $SU(2)_L$  [19].

non-renormalizable superpotential is generically written as

$$W_{\text{NR}} = \frac{\lambda_\Phi}{nM_{pl}^{n-3}}\Phi^n, \quad (3.1)$$

where  $\lambda_\Phi$  is a coupling constant and  $n(\geq 4)$  is an integer.  $\Phi$  denotes the AD superfield. For example,  $n = 6$  in the case of the  $\bar{u}\bar{d}\bar{d}$  flat direction. The superpotential of Eq. (3.1) leads to the following AD field potential:

$$V_{A,\text{NR}} = \frac{m_{3/2}}{nM_{pl}^{n-3}}(a_n\lambda_\Phi\Phi^n + \text{c.c.}) + \frac{|\lambda_\Phi|^2}{M_{pl}^{2n-6}}|\Phi|^{2n-2}. \quad (3.2)$$

Here, we introduce a dimensionless parameter  $a_n$  of  $\mathcal{O}(1)$ , which depends on higher dimensional Kähler potentials. The terms proportional to  $a_n$  are  $A$ -terms induced from gravitational effects. The  $A$ -terms are global  $U(1)$  breaking terms and produce the  $B - L$  number during the evolution of the AD field as we will see.

The AD field also acquires the soft mass of the order of the gravitino mass, and its potential is expressed as

$$V_{\text{grav}} = m_{3/2}^2 \left( 1 + K \log \frac{|\Phi|^2}{M_*^2} \right) |\Phi|^2, \quad (3.3)$$

where  $M_*$  is a renormalization scale. This potential comes from the gravitational mediation effects including one-loop corrections [89]. The parameter  $K$  comes from one-loop

effects, and its absolute value is typically in the range of 0.01 to 0.1. If gaugino contribution to one-loop effects is larger than that of the top Yukawa interactions, the sign of  $K$  is negative, and vice versa.

When the SUSY breaking effects are mediated by the gauge interactions, the potential for the AD field is given by [90, 91]

$$V_{\text{gauge}} = M_F^4 \left( \log \frac{|\Phi|^2}{M_{\text{mess}}^2} \right)^2, \quad (3.4)$$

where  $M_{\text{mess}}$  is a mass scale of messenger fields which connect the observable sector with the SUSY breaking sector. This potential is applicable only for  $|\Phi| \gg M_{\text{mess}}$ . At lower scales than the messenger mass scale, the potential is replaced by a soft SUSY breaking mass term,  $m_{\text{SUSY}}^2 |\Phi|^2$ ,<sup>1</sup> where  $m_{\text{SUSY}} \sim \mathcal{O}(0.1\text{--}1) \text{ TeV}$ .  $M_F$  is related to the SUSY breaking scale as follows [90]:

$$M_F \simeq \frac{g^{1/2}}{4\pi} \sqrt{k\langle F \rangle}, \quad (3.5)$$

where  $g$  represents generic gauge couplings of the standard model, and  $\langle F \rangle$  denotes the SUSY breaking  $F$ -term. The parameter  $k$  is determined from coupling between the SUSY breaking sector and the messenger sector and satisfies  $k \lesssim \mathcal{O}(1)$ .

The SUSY breaking scale is constrained from below by the observed Higgs boson mass at around 125 GeV, which acquires radiative corrections from stop masses. Since the masses of scalar particles are proportional to the parameter of  $\Lambda \equiv k\langle F \rangle / M_{\text{mess}}$ , we obtain a lower bound of  $\Lambda$  as follows [92, 93]:<sup>2</sup>

$$\Lambda \equiv \frac{k\langle F \rangle}{M_{\text{mess}}} \gtrsim 6 \times 10^5 \text{ GeV}. \quad (3.6)$$

Moreover,  $M_{\text{mess}}^2 \gtrsim k\langle F \rangle$  is necessary to make masses of the messenger scalar particles positive. In combination with Eq. (3.6),  $\sqrt{k\langle F \rangle} \gtrsim \Lambda \gtrsim 6 \times 10^5 \text{ GeV}$  must be realized. Therefore,  $M_F$  is restricted from below,

$$M_F \gtrsim 5 \times 10^4 g^{1/2} \text{ GeV}. \quad (3.7)$$

Since the SUSY breaking scale  $\langle F \rangle$  is related to the gravitino mass as  $\langle F \rangle \simeq \sqrt{3} m_{3/2} M_{\text{pl}}$ ,  $M_F$  is restricted from the above as follows:

$$M_F \lesssim 1.6 \times 10^6 \text{ GeV} g^{1/2} \left( \frac{m_{3/2}}{100 \text{ keV}} \right)^{1/2}. \quad (3.8)$$

---

<sup>1</sup> The potential becomes logarithmic for  $|\Phi| \gg M_{\text{mess}}$  since a large amplitude of the AD field leads to suppression of the transmission of SUSY breaking effects [90].

<sup>2</sup> Hereafter, we consider the minimal gauge mediation model with one pair of the messenger particles [92]. The constraint becomes slightly milder in the case with more messenger particles, but the following results do not drastically change.

In the early universe, the vacuum energy largely breaks the SUSY and changes the potential for the AD field since it is generally coupled with the inflaton through Planck-suppressed interactions. Such SUSY breaking effects depend on the expansion rate of the universe, which affects the evolution of the AD field. In supergravity, the scalar potential is given by

$$V = e^{K/M_{pl}^2} \left[ (D_i W) K^{i\bar{j}} (D_{\bar{j}} W)^* - \frac{3}{M_{pl}^2} |W|^2 \right] + (D\text{-terms}), \quad (3.9)$$

where  $D_i W \equiv W_i + K_i W/M_{pl}^2$ . The subscripts represent the derivatives with a field  $i$ .  $K^{i\bar{j}}$  is an inverse matrix of  $K_{i\bar{j}}$ . The Kähler potential for the inflaton  $I$  and the AD field is generically written by

$$K = |\Phi|^2 + |I|^2 + \frac{c_I}{M_{pl}^2} |\Phi|^2 |I|^2 + \dots, \quad (3.10)$$

where  $c_I$  is a real coupling constant of  $\mathcal{O}(1)$ . Higher order terms are expressed by the ellipsis, and we assume that these terms are irrelevant to the dynamics of the AD field. Here, we consider that the  $F$ -term of the  $I$  causes the primordial inflation. Therefore, the Hubble parameter  $H$  is related to the  $F$ -term  $F_I$  as  $|F_I|^2 \simeq 3M_{pl}^2 H_{inf}^2$  during the inflation. From Eq. (3.9), the resulting potential is given by

$$V \supset 3(1 - c_I) H_{inf}^2 |\Phi|^2, \quad (3.11)$$

and we call it a Hubble-induced mass term. If  $H_{inf}$  is much larger than the soft mass of the AD field and  $c_I$  is larger than 1, one can easily find that the AD field acquires a large negative mass term during the inflation. Hereafter we consider the case of  $3(1 - c_I) < 0$  for the AD field to acquire a large field value.

After the primordial inflation ends, the inflaton starts to oscillate around the potential minimum. The energy density of the universe is dominated by its oscillation energy. Since the AD field is generically assumed to be coupled to the kinetic term of the inflaton, it acquires a Hubble-induced mass term even in the inflaton oscillating era. Because of the coupling between the inflaton and the AD field in the Kähler potential, the kinetic term contains the following term:

$$\mathcal{L} \supset \frac{c_I}{M_{pl}^2} \partial_\mu I \partial^\mu I^* |\Phi|^2. \quad (3.12)$$

Since time scale of the inflaton oscillation is much shorter than that of the AD field, we can take a time average of the inflaton oscillation. This term is then rewritten as

$$V \supset -\frac{3}{2} c_I H^2 |\Phi|^2, \quad (3.13)$$

by using the Virial theorem:  $\langle |\partial_0 I|^2 \rangle \simeq 3H^2 M_{pl}^2/2$ . In many models of inflation, the field whose  $F$ -term causes the accelerating expansion is different from the one whose oscillation energy dominates the universe after the inflation. Therefore, origins of the Hubble induced mass terms are generally different between Eqs. (3.11) and (3.13). Hereafter we assume that the AD field has the negative Hubble-induced mass terms both during the inflation and after the end of the inflation.

Thermal effects may also affect the dynamics of the AD field. Even before the inflaton completely decays, the radiation produced from the inflaton decay exists in the inflaton dominated era, which is called a dilute plasma. The temperature of the dilute plasma is given by [94]

$$T \simeq (T_{RH}^2 M_{pl} H)^{1/4}. \quad (3.14)$$

The effect from the dilute plasma must also be taken into account.

There are two types of thermal effects on the effective potential. Firstly, when fields  $\xi_k$  directly couples to the AD field and have effective masses lighter than temperature, thermal mass terms arise as [95]

$$V_{th,1} = \sum_{f_k|\Phi| \lesssim T} c_k f_k^2 T^2 |\Phi|^2, \quad (3.15)$$

where  $c_k$  is a constant parameter of  $\mathcal{O}(1)$ .  $f_k$  is a coupling constant between light fields  $\xi_k$  and the AD field  $\Phi$ . When the AD field has a field value, the fields  $\xi_k$  acquire effective masses as  $f_k|\Phi|$ . Here we assume that masses of  $\xi_k$  are much lighter than  $f_k|\Phi|$ . Therefore, only particles with effective masses  $f_k|\Phi| \lesssim T$  contribute to the thermal mass term of Eq. (3.15).

The other thermal effect exists at the two-loop level. Even when the AD field has a large field value, fields that do not couple to the AD field remain massless. These fields contribute to the free energy as radiation. When the AD field is the  $LH_u$  direction, for example, gluons and gluinos are massless fields. In this case, the potential is given as  $V \sim g_S^2(T)T^4$  at the two-loop level, where  $g_S(T)$  is a  $SU(3)_C$  gauge coupling constant and evolves with temperature. Its dependence changes with the nonzero AD field value because of fields with effective masses of  $f_k|\Phi| \gtrsim T$ . In the case of  $LH_u$  direction, the top quark actually satisfies the condition. The thermal effect is generally given by [96, 97]

$$V_{th,2} \simeq a_g \alpha_S(T)^2 T^4 \ln \left( \frac{|\Phi|^2}{T^2} \right), \quad (3.16)$$

where  $a_g$  is a constant parameter of  $\mathcal{O}(1)$ .

Let us explain the evolution of the Affleck-Dine field in the early universe. During the primordial inflation, the AD field sits down at the local minimum determined by the negative Hubble-induced mass term and the non-renormalizable terms. The field value is estimated as

$$|\Phi| \simeq \left( \frac{H_{inf} M_{pl}^{n-3}}{|\lambda_\Phi|} \right)^{1/(n-2)}. \quad (3.17)$$

After the end of the inflation, the energy of the universe is dominated by the oscillating inflaton. When the Hubble parameter decreases to  $H \simeq \sqrt{|V_\Phi/\Phi|}$ , the AD field starts to roll down to the origin of the potential. At this time, the phase of the AD field also starts to rotate in the complex plane due to the  $A$ -terms in Eq. (3.2). When the AD field  $\Phi$  carries the  $B-L$  charge, the rotation in the complex plane corresponds to the production of the  $B-L$  number. The  $B-L$  number density is given by

$$n_{B-L} = i\beta_{B-L} \left( \dot{\Phi}^* \Phi - \Phi^* \dot{\Phi} \right), \quad (3.18)$$

where  $\beta_{B-L}$  denotes the  $B-L$  charge of the AD field, and the dots denote the time derivative. As the oscillation amplitude of the AD field decreases due to the Hubble friction, higher order terms become irrelevant. Therefore, the  $B-L$  number violating operators, the  $A$ -terms, produce  $B-L$  number only at the onset of the oscillation. After that, the  $B-L$  number density scales as  $a^{-3}$ . The produced  $B-L$  number is finally converted into the standard model particles by the decay of the AD field.

The evolution equation for the  $B-L$  number density is expressed as

$$\dot{n}_{B-L} + 3Hn_{B-L} = 2\beta_{B-L} \text{Im} \left[ \frac{\partial V}{\partial \Phi} \Phi \right]. \quad (3.19)$$

The right-hand side is the source of the  $B-L$  asymmetry. By solving the equation, one can find that the asymmetry is produced most effectively at the onset of the oscillation. The produced  $B-L$  density is then estimated as

$$n_{B-L}(t_{\text{osc}}) \simeq 2\beta_{B-L} |a_n \lambda_\Phi| \sin[n\theta_i + \arg(a_n \lambda_\Phi)] \frac{m_{3/2}}{H_{\text{osc}}} \frac{|\Phi_{\text{osc}}|^n}{M_{pl}^{n-3}} \quad (3.20)$$

$$\equiv \epsilon n_{B-L}^{\text{max}}(t_{\text{osc}}), \quad (3.21)$$

where the subscripts of  $_{\text{osc}}$  show the values when the AD field starts to oscillate.  $\theta_i$  is the initial phase of the AD field.  $n_{B-L}^{\text{max}}(t_{\text{osc}})$  denotes the maximal baryon number produced at  $t = t_{\text{osc}}$  and is estimated as

$$n_{B-L}^{\text{max}}(t_{\text{osc}}) \equiv H_{\text{osc}} |\Phi_{\text{osc}}|^2. \quad (3.22)$$



$\epsilon$  is estimated as

$$\epsilon \simeq 2\beta_{B-L}|a_n\lambda_\Phi|\sin[n\theta_i + \arg(a_n\lambda_\Phi)]\frac{m_{3/2}}{H_{\text{osc}}^2}\frac{|\Phi_{\text{osc}}|^{n-2}}{M_{\text{pl}}^{n-3}}, \quad (3.23)$$

and shows the efficiency of the AD mechanism.  $\epsilon \simeq \mathcal{O}(1)$  means that the orbit of the AD field in the complex field plane is nearly circular. When  $\epsilon \lesssim \mathcal{O}(1)$ , its orbit becomes elliptic. The parameter  $\epsilon$  is called the ellipticity parameter.

The sphaleron process is in thermal equilibrium at temperature above the EW scale, and the produced  $B - L$  number is converted into the baryon number through anomalous  $B + L$  breaking processes. The baryon asymmetry is related to the  $B - L$  asymmetry as [98, 99]

$$n_B = \frac{8}{23}n_{B-L}, \quad (3.24)$$

where  $n_{B-L}$  expresses  $B - L$  number density. Since the produced baryon number is comparable to the number density of coherent oscillation with a large amplitude, the AD mechanism is much more efficient than the other conventional baryogenesis. The AD mechanism is a probable candidate for baryogenesis in cosmology with the long-lived particles such as the gravitino, the moduli field, and so on.

## 3.2 Q-ball

The baryon number production by the AD mechanism is closely related to Q-ball formation. Q-ball [100] is a non-topological soliton that could be formed during the oscillation of the AD field. The AD field fragments into Q-balls if its potential becomes flatter than a quadratic term for a larger field value. From the previous numerical calculation, it is known that almost all produced baryon charges are confined into Q-balls [101–103]. We then need to calculate baryon number released from these Q-balls to estimate the baryon asymmetry.

Firstly let us explain conditions that Q-balls are formed after the AD baryogenesis and profiles of Q-balls. The configuration of the Q-ball is determined by the condition of minimizing the energy with conserved baryon charge, where the energy and the baryon charge are given by

$$E = \int d^3x \left[ |\dot{\Phi}|^2 + |\nabla\Phi|^2 + V(|\Phi|) \right], \quad (3.25)$$

$$Q = 2 \int d^3x \text{Im} \left[ \Phi^* \dot{\Phi} \right], \quad (3.26)$$

respectively. Here, we assume that the AD field carries an unit of baryon charge for simplicity. The scalar field configuration is obtained by minimizing

$$E_\omega \equiv E + \omega \left( Q - 2 \int d^3x \text{Im} [\Phi^* \dot{\Phi}] \right), \quad (3.27)$$

where  $\omega$  is a Lagrangian multiplier.  $E_\omega$  is rewritten by

$$E_\omega = \int d^3x \left[ \left| \dot{\Phi} - i\omega\Phi \right|^2 - \omega^2 |\Phi|^2 + |\nabla\Phi|^2 + V(|\Phi|) \right] + \omega Q. \quad (3.28)$$

The time dependence of  $\Phi$  is determined as  $\Phi(\mathbf{x}, t) = \varphi e^{i\omega t}/\sqrt{2}$  from the first term in order to minimize  $E_\omega$ . We assume that the stable field configuration is spherically symmetric, which leads to the following equation of the field configuration:

$$\frac{\partial^2}{\partial r^2} \varphi + \frac{2}{r} \frac{\partial}{\partial r} \varphi + \omega^2 \varphi - \frac{\partial}{\partial \varphi} V(\varphi) = 0. \quad (3.29)$$

The boundary condition is  $\varphi'(0) = 0$  and  $\varphi(\infty) = 0$  in order to obtain a smooth and local configuration. The solution with the boundary condition exists for

$$\omega_0^2 \equiv \min \left[ \frac{2V(\varphi)}{\varphi^2} \right]_{\varphi=\varphi_0 \neq 0} < \frac{\partial^2 V(0)}{\partial \varphi^2}. \quad (3.30)$$

The inequality requires the existence of the field value where the potential is flatter than the quadratic potential. This condition is satisfied if the potential of the AD field is dominated by the terms of Eqs. (3.4) and (3.16). Even if the term of Eq. (3.3) dominates the potential, Eq. (3.30) is satisfied in the case of  $K < 0$ .

The profile of the Q-ball depends on the potential of the AD field. We show classification of the Q-ball in Table 3.2. When the AD field starts to oscillate by the potential determined by the gauge-mediated effect of Eq. (3.4), formed Q-balls are referred to as “gauge mediation type Q-balls”. The field configuration of the gauge-mediation type Q-ball is determined by solving Eq. (3.29) and is approximately given by [104]

$$\Phi(r) \simeq \frac{e^{i\omega t}}{\sqrt{2}} \times \begin{cases} \varphi_0 \frac{\sin \omega r}{\omega r} & \text{for } r < R \equiv \pi/\omega \\ 0 & \text{for } r > R \end{cases}, \quad (3.31)$$

where  $\omega$  and  $\varphi_0$  are given by

$$\omega \simeq \sqrt{2\pi} M_F Q^{-1/4}, \quad (3.32)$$

$$\varphi_0 \simeq M_F Q^{1/4}. \quad (3.33)$$

	gauge mediation			gravity mediation
$\phi_{\text{osc}}$	$\phi_{\text{osc}} < \phi_{\text{eq}}$	$\phi_{\text{osc}} > \phi_{\text{eq}}$		
$K$	$K < 0, K > 0$	$K < 0$	$K > 0$	$K < 0$
type	gauge mediation type	new type	delayed type	gravity mediation type

Table 3.2: Classification of the type of the Q-ball.

The energy of the Q-balls is calculated from Eq. (3.25) and is estimated as

$$E \simeq \frac{4\sqrt{2}\pi}{3} M_F Q^{3/4}. \quad (3.34)$$

One can find that the Q-ball energy per unit charge ( $\simeq dE/dQ$ ) is smaller for large  $Q$ . When it is smaller than the proton mass, i.e.  $dE/dQ \sim M_F Q^{-1/4} < 1 \text{ GeV}$ , Q-balls cannot decay into nucleons. The charge of the Q-ball can be determined by numerical simulations and is given by [103]

$$Q \sim \beta \left( \frac{\phi_{\text{osc}}}{M_F} \right)^4, \quad (3.35)$$

where  $\beta$  is a numerical coefficient and is determined as  $\beta \simeq 6 \times 10^{-4}$ .<sup>3</sup> Hereafter, we use  $\phi$  as the amplitude of the AD field and  $\phi_{\text{osc}}$  denotes the amplitude at the onset of the oscillation, in other words,  $\phi_{\text{osc}} \equiv |\Phi_{\text{osc}}|$ . Note that  $\phi_{\text{osc}}$  should be smaller than a threshold value  $\phi_{\text{eq}}$  given by

$$V_{\text{gauge}}(\phi_{\text{eq}}) = V_{\text{grav}}(\phi_{\text{eq}}), \quad (3.36)$$

in this case.  $\phi_{\text{eq}}$  is estimated as

$$\phi_{\text{eq}} \simeq 5.2 \times 10^{14} \text{ GeV} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{-1} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^2, \quad (3.37)$$

for  $M_{\text{mess}} \simeq 5 \times 10^5 \text{ GeV}$ . Note that  $\phi_{\text{eq}}$  has a logarithmic dependence on  $M_{\text{mess}}$ . The gauge-mediated effects dominate over the gravity-mediated effects below the threshold value  $\phi_{\text{eq}}$ .

In the case of  $\phi_{\text{osc}} \gtrsim \phi_{\text{eq}}$ , the gravity-mediated effects dominate over the gauge-mediated ones, and one can consider two types of scenarios:  $K > 0$  and  $K < 0$ . Firstly

<sup>3</sup> Precisely speaking, the numerical coefficient  $\beta$  depends on the orbit of the AD field in the complex plane.  $\beta \simeq 6 \times 10^{-4}$  for a circular orbit ( $\epsilon = 1$ ), while  $\beta \simeq 6 \times 10^{-5}$  for an oblate orbit ( $\epsilon \lesssim 0.1$ ). Hereafter, we use  $\beta \simeq 6 \times 10^{-4}$  for simplicity since our results do not change significantly.

we consider the case of  $K < 0$ . The potential of Eq. (3.3) with  $K < 0$  satisfies the condition for Q-ball formation. Q-balls formed in this potential are referred to as “new type Q-balls” [102]. Even in gravity-mediated SUSY breaking models, Q-ball formation occurs when the AD field starts to oscillate in the potential determined by the gravity-mediated effects of Eq. (3.3) with  $K < 0$ . Formed Q-balls are referred to as “gravity mediation type Q-balls”. The field configuration is approximately given by a Gaussian function [89]:

$$\Phi(r) \simeq \frac{1}{\sqrt{2}} \varphi_0 e^{-r^2/2R^2} e^{i\omega t}, \quad (3.38)$$

where  $R$ ,  $\omega$  and  $\varphi_0$  are given by

$$R \simeq \frac{1}{|K|^{1/2} m_{3/2}}, \quad (3.39)$$

$$\omega \simeq m_{3/2}, \quad (3.40)$$

$$\varphi_0 \simeq \left( \frac{|K|}{\pi} \right)^{3/4} m_{3/2} Q^{1/2}, \quad (3.41)$$

respectively. The energy of the Q-ball is given by

$$E \simeq m_{3/2} Q. \quad (3.42)$$

In gauge-mediated SUSY breaking models, this type of Q-balls is stable against the decay into nucleons since  $dE/dQ \simeq m_{3/2} < 1 \text{ GeV}$ . The charge of the new type Q-ball is given by [105, 106]

$$Q \sim \tilde{\beta} \left( \frac{\phi_{\text{osc}}}{m_{3/2}} \right)^2, \quad (3.43)$$

where  $\tilde{\beta} \simeq 2 \times 10^{-2}$ .

When  $K > 0$ , on the other hand, the condition of Eq. (3.30) is not satisfied, and then Q-balls are not formed. In this case, the oscillation of the AD field remains homogeneous. Its amplitude decreases as  $\phi \propto a^{-3/2}$  after it starts to oscillate. However, when the potential of the AD field becomes dominated by the gauge-mediated effects after the onset of the oscillation, in other words, its amplitude decreases to  $\phi_{\text{eq}}$ , Q-ball formation occurs. This type of Q-balls is referred to as “delayed type Q-balls” [103]. The profile and properties of the Q-ball are the same as those of the gauge-mediation type Q-ball (see Eqs. (3.31), (3.32), (3.33) and (3.34)), while the charge of the delayed type Q-ball is given by

$$Q \sim \beta \left( \frac{\phi_{\text{eq}}}{M_F} \right)^4. \quad (3.44)$$

The thermal logarithmic potential of Eq. (3.16) also satisfies the condition for Q-ball formation. If the AD field starts to oscillate by the thermal logarithmic potential, one can obtain charge of the Q-ball by replacing  $M_F$  with  $T_*$  in Eq. (3.35), where  $T_*$  is the temperature at Q-ball formation. The profile and properties become the same as those of gauge-mediation type Q-balls when temperature decreases sufficiently.

Finally, let us estimate baryon number released from the Q-balls. Even if the Q-ball is stable against its decay into nucleons, in other words,  $dE/dQ < 1 \text{ GeV}$ , baryon charge is released from the Q-ball surface via evaporation [107]. At finite temperature, the free energy of the AD field is minimized when the all confined baryon charges behave as free particles in the thermal plasma rather than inside the Q-ball. However, baryon charge inside the Q-ball cannot completely evaporate since the evaporation rate is slower than the cosmic expansion rate. The total evaporated charge of gauge-mediation type Q-balls is given by [103, 108]

$$\Delta Q \sim 10^{16} \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-2/3} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-1/3} Q^{1/12}. \quad (3.45)$$

In the case of new type Q-balls, it is given by [102, 103]

$$\Delta Q \sim 10^{20} \left( \frac{|K|}{0.01} \right)^{-2/3} \left( \frac{m_{3/2}}{100 \text{ keV}} \right)^{-1/3} \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-2/3}. \quad (3.46)$$

On the other hand, the Q-ball can decay into nucleons when  $dE/dQ > 1 \text{ GeV}$  and release baryon charges from its surface. The emission rate is determined by the Pauli blocking effect on its surface. It is given by [109–111]

$$\left| \frac{dQ}{dt} \right| \simeq \frac{(2\omega)^3 A}{96\pi^2}, \quad (3.47)$$

where  $A$  is the surface area of the Q-ball. In order to avoid destroying light elements formed during the BBN due to decay products of Q-balls, the decay temperature is constrained as follows:

$$T_{\text{dec}} \simeq \sqrt{\frac{1}{Q} \frac{dQ}{dt}} M_{pl} \gtrsim \mathcal{O}(10) \text{ MeV}. \quad (3.48)$$

In gauge-mediated SUSY breaking models, Q-ball formation is inevitable due to the logarithmic potential of Eq. (3.4). Therefore, we should take into account the evaporated charge and Q-ball decay temperature for the estimation of the baryon asymmetry.

### 3.3 Case of gravity-mediation models

In this section, we will estimate baryon number produced by the AD mechanism before dilution in gravity-mediation models with moduli mass of  $\mathcal{O}(1)$  TeV.<sup>4</sup> Since we need to dilute the moduli/Polonyi field density sufficiently, one can know whether the observed baryon asymmetry is explained by estimating the ratio of the produced baryon number to the moduli/Polonyi density. For simplicity, we will focus on one moduli field hereafter.

Firstly, we consider the case where the AD field starts to oscillate by the SUSY breaking terms ignoring the thermal potential. In gravity-mediation models, the AD potential at zero-temperature is given by Eq. (3.3). In this case, the Hubble parameter at the onset of the AD field oscillation is estimated as  $H_{\text{osc}} \simeq m_{3/2} \simeq \mathcal{O}(1)$  TeV. Since the field value of Eq. (3.17) is estimated by the balance between the negative Hubble induced mass term and the non-renormalizable term that is the last term in Eq. (3.2), it is applicable up to the Planck scale. Above the Planck scale, the exponential term with Kähler potential lifts the potential (see Eq. (3.9)). Therefore, the AD field value at the onset of the oscillation is given as

$$\phi_{\text{osc}} \equiv |\Phi_{\text{osc}}| \simeq \begin{cases} \left( \frac{H_{\text{osc}} M_{\text{pl}}^{n-3}}{|\lambda_{\Phi}|} \right)^{1/(n-2)} & \text{for } |\lambda_{\Phi}| \gtrsim \lambda_* \\ M_{\text{pl}} & \text{for } |\lambda_{\Phi}| \lesssim \lambda_* \end{cases}, \quad (3.49)$$

where  $\lambda_* \equiv H_{\text{osc}}/M_{\text{pl}}$ . The ellipticity parameter  $\epsilon$  is then estimated as

$$\epsilon \simeq \begin{cases} \frac{m_{3/2}}{H_{\text{osc}}} & \text{for } |\lambda_{\Phi}| \gtrsim \lambda_* \\ |\lambda_{\Phi}| \frac{m_{3/2} M_{\text{pl}}}{H_{\text{osc}}^2} & \text{for } |\lambda_{\Phi}| \lesssim \lambda_* \end{cases}. \quad (3.50)$$

Since the produced  $B - L$  asymmetry is about  $n_{B-L}(t_{\text{osc}}) \simeq \epsilon H_{\text{osc}} \phi_{\text{osc}}^2$ , one can find that the maximal  $B - L$  number density is achieved as  $n_{B-L}(t_{\text{osc}}) \simeq m_{3/2} M_{\text{pl}}^2$  when  $|\lambda_{\Phi}| \simeq \lambda_*$ , in other words,  $\phi_{\text{osc}} \simeq M_{\text{pl}}$  and  $\epsilon \simeq m_{3/2}/H_{\text{osc}} \simeq \mathcal{O}(1)$  in this case.

Ignoring the thermal potential, the ratio of the baryon number to moduli density is estimated as

$$\frac{n_B}{\rho_{\eta}} = \frac{8}{23} \frac{n_{B-L}}{3M_{\text{pl}}^2 H^2} \bigg|_{\text{AD osc}} \frac{3M_{\text{pl}}^2 H^2}{\rho_{\eta}} \bigg|_{\text{moduli osc}} \simeq \frac{16}{23} \frac{\epsilon}{H_{\text{osc}}} \left( \frac{\phi_{\text{osc}}}{M_{\text{pl}}} \right)^2 \left( \frac{\eta_0}{M_{\text{pl}}} \right)^{-2}, \quad (3.51)$$

where we used  $n_{B-L}(t_{\text{osc}}) \simeq \epsilon H_{\text{osc}} \phi_{\text{osc}}^2$ . Here, we assume that the moduli mass is about the gravitino mass and that the AD field and moduli field start to oscillate in the same

---

<sup>4</sup> The case where the moduli mass is about  $\mathcal{O}(100)$  TeV will be discussed in Chap. 4.

epoch. This ratio does not change after the moduli oscillation. The baryon asymmetry after the dilution is estimated as

$$\frac{n_B}{s} = \frac{n_B}{\rho_\eta} \frac{\rho_\eta}{s} \simeq 7.0 \times 10^{-18} \epsilon \left( \frac{\rho_\eta/s}{10^{-14} \text{ GeV}} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \left( \frac{H_{\text{osc}}}{1 \text{ TeV}} \right)^{-1} \left( \frac{\phi_{\text{osc}}}{M_{pl}} \right)^2. \quad (3.52)$$

One can find that the produced baryon asymmetry is much smaller than the observed one in gravity-mediation models.

Next, we consider the case where the AD field starts to oscillate by the thermal potential. When the thermal logarithmic potential dominates over the soft mass term at  $t = t_{\text{osc}}$ , the Hubble parameter is estimated as  $H_{\text{osc}} \simeq \alpha_S T_{\text{osc}}^2 / \phi_{\text{osc}}$ , where  $T_{\text{osc}} \simeq (T_{\text{RH}}^2 M_{pl} H_{\text{osc}})^{1/4}$ .  $H_{\text{osc}}$  and  $T_{\text{osc}}$  are then calculated as

$$H_{\text{osc}} \simeq \alpha_S^2 \frac{T_{\text{RH}}^2 M_{pl}}{\phi_{\text{osc}}^2}, \quad (3.53)$$

$$T_{\text{osc}} \simeq T_{\text{RH}} \sqrt{\frac{\alpha_S M_{pl}}{\phi_{\text{osc}}}}. \quad (3.54)$$

Since  $\alpha_S^2 T_{\text{osc}}^4 \log \frac{\phi_{\text{osc}}^2}{T_{\text{osc}}^2} \gtrsim m_{3/2}^2 \phi_{\text{osc}}^2$ , the AD field starts to oscillate before the beginning of the moduli oscillation, in other words,  $H_{\text{osc}} \gtrsim m_{3/2}$ .

Since the moduli field begins to oscillate after the onset of the AD field oscillation, there are two cases depending on when the reheating occurs. If the moduli field starts to oscillate before the reheating, the ratio of the baryon number to moduli density is estimated as Eq. (3.51). If the moduli field starts to oscillate after the reheating, the ratio becomes larger by the factor of  $T_{\text{RH}}/T_\eta$  since the radiation dominated universe is realized before the onset of the moduli oscillation.  $T_\eta$  is defined in Eq. (2.34). The ratio is estimated as

$$\frac{n_B}{\rho_\eta} \simeq \frac{16}{23} \frac{m_{3/2}}{H_{\text{osc}}^2} \left( \frac{\phi_{\text{osc}}}{M_{pl}} \right)^2 \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \times R, \quad (3.55)$$

where the factor of  $R$  is defined as

$$R \equiv \begin{cases} 1 & \text{for } T_{\text{RH}} \lesssim T_\eta \\ T_{\text{RH}}/T_\eta & \text{for } T_{\text{RH}} \gtrsim T_\eta \end{cases}. \quad (3.56)$$

We calculated the baryon number density before Q-ball formation for simplicity. Here we used  $\epsilon \simeq m_{3/2}/H_{\text{osc}}$  since the ellipticity parameter is maximized when  $|\lambda_\Phi| \gtrsim \lambda_*$ . When  $T_{\text{RH}} \lesssim T_\eta$ , the ratio is clearly smaller than the case of ignoring the thermal potential since  $H_{\text{osc}} \gtrsim m_{3/2}$  (see Eq. (3.51)). When  $T_{\text{RH}} \gtrsim T_\eta$ , one can find that the produced baryon

asymmetry decreases as the reheating temperature becomes higher (see Eqs. (3.53), (3.55) and (3.56)). Therefore, the produced baryon asymmetry is much smaller than the observed one even when the AD field starts to oscillate by the thermal potential. As a result, the AD mechanism cannot explain the observed baryon asymmetry with dilution in gravity-mediation models.

### 3.4 Case of gauge-mediation models

In gauge-mediated SUSY breaking models, Q-balls are inevitably formed during the oscillation of the AD field. Almost all baryon number is absorbed into Q-balls even if huge baryon asymmetry is produced by the AD mechanism. If Q-balls are stable, baryon charge is released from Q-balls via evaporation. In the case of unstable Q-balls, the decay of Q-balls can release baryon charge. However, it must occur before the BBN epoch in order to explain the baryon asymmetry. We study if the AD mechanism can work with dilution in both cases. In this section, we update the analysis in Ref. [112] by using the lower bound of Eq. (3.7).<sup>5</sup> The AD potential in the following cases is shown in Figs. 3.1 and 3.2.

#### A. Stable Q-ball formation when $V_{\text{gauge,grav}}(\phi_{\text{osc}}) \gtrsim V_{\text{th}}(\phi_{\text{osc}})$

Firstly, we consider the case where the AD potential is dominated by the zero-temperature potential. Since  $\Delta Q/Q$  becomes smaller as  $Q$  increases, it is more difficult to extract baryon charge from Q-balls with larger baryon number. In order to produce huge baryon number,  $\phi_{\text{osc}}$  should be large, which leads to increase of baryon charges confined in Q-balls. Then, the released baryon number generally becomes smaller as  $\phi_{\text{osc}}$  increases. In the case of the delayed type Q-ball, however, baryon charges of Q-balls do not increase even though  $\phi_{\text{osc}}$  becomes much larger than  $\phi_{\text{eq}}$ . Hence, the delayed type Q-ball seems to be able to most effectively provide baryon charge outside Q-balls. We then focus on the delayed type Q-ball.

The finally provided baryon asymmetry is calculated as

$$\frac{n_B}{s} = \frac{\tilde{n}_B}{s} \frac{\Delta Q}{Q}, \quad (3.57)$$

where  $\tilde{n}_B/s$  is the ratio estimated without considering Q-ball formation. When  $\phi_{\text{osc}} \gtrsim \phi_{\text{eq}}$ , the AD field starts to oscillate by  $V_{\text{grav}}$  that is given by Eq. (3.3) with  $K > 0$ . After that,

---

<sup>5</sup> This section is based on the work of Ref. [47]. In this thesis, we study all the cases including effects of the thermal potential while the analysis in Ref. [47] is incomplete.



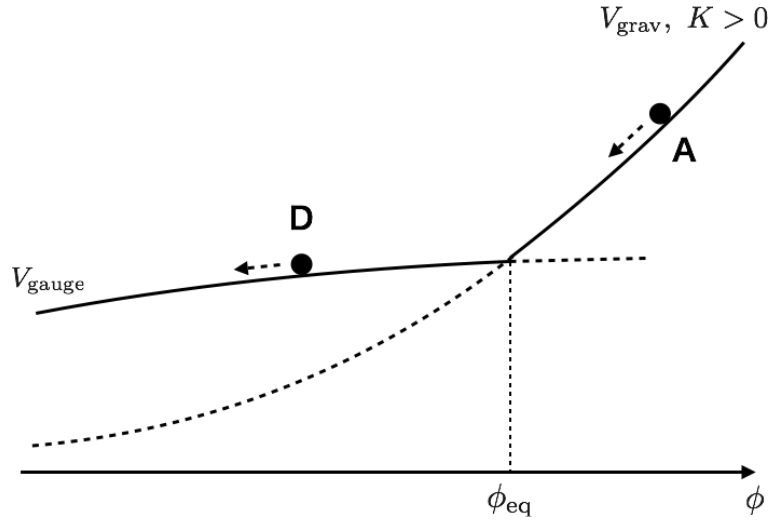


Figure 3.1: The potential of  $V_{\text{gauge}}$  and  $V_{\text{grav}}$ . The circles denote  $\phi_{\text{osc}}$  in the case of A and D.

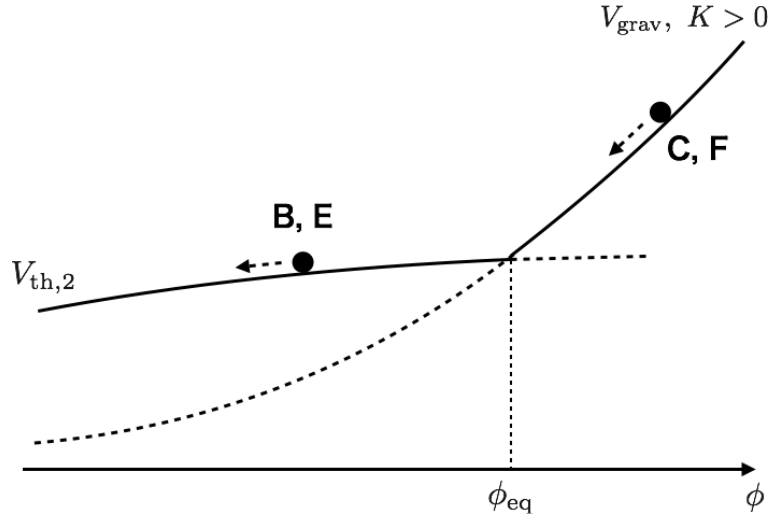


Figure 3.2: The potential of  $V_{\text{th},2}$  and  $V_{\text{grav}}$ . The circles denote  $\phi_{\text{osc}}$  in the case of B, C, E and F.

the oscillation amplitude decreases to  $\phi_{\text{eq}}$ , and then Q-balls are formed. The charge of the Q-ball is given by Eq. (3.44). At  $t = t_{\text{osc}}$ , the Hubble parameter becomes of the order of the gravitino mass,  $H_{\text{osc}} \simeq m_{3/2}$ . For the delayed type Q-ball,  $\Delta Q/Q$  is estimated as

$$\frac{\Delta Q}{Q} \sim \begin{cases} 2 \times 10^{-18} \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-2/3} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-4} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{11/3}, & \text{for } M_F \gtrsim M_{F*} \\ 1 & \text{for } M_F \lesssim M_{F*} \end{cases}, \quad (3.58)$$

where we have used Eqs. (3.37), (3.44) and (3.45). Since  $M_{F*}$  is estimated as

$$\frac{M_{F*}}{5 \times 10^4 \text{ GeV}} \simeq 4 \times 10^{-5} \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-1/6} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{11/12}, \quad (3.59)$$

we will consider the case for  $M_F \gtrsim M_{F*}$ . The stability condition,  $dE/dQ < 1 \text{ GeV}$ , leads to the upper bound on the gravitino mass:

$$m_{3/2} \lesssim 1.4 \text{ GeV}, \quad (3.60)$$

where we have used Eqs. (3.34), (3.37) and (3.44).

In this case, the ratio of the baryon number to moduli density is estimated as

$$\frac{\tilde{n}_B}{\rho_\eta} \simeq \frac{16}{23} \frac{m_{3/2}}{H_{\text{osc}}^2} \left( \frac{\phi_{\text{osc}}}{M_{pl}} \right)^2 \left( \frac{\eta_0}{M_{pl}} \right)^{-2}, \quad (3.61)$$

in the same way with Eq. (3.51). The produced baryon asymmetry is estimated as

$$\begin{aligned} \frac{n_B}{s} &\sim 3 \times 10^{-24} \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \frac{m_{3/2}}{H_{\text{osc}}} \left( \frac{H_{\text{osc}}}{200 \text{ keV}} \right)^{-1} \left( \frac{\phi_{\text{osc}}}{M_{pl}} \right)^2 \\ &\times \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-2/3} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-4} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{11/3}, \end{aligned} \quad (3.62)$$

where we take the gravitino mass as  $\mathcal{O}(100) \text{ keV}$  in order to avoid the X-ray background constraint (see Fig. 2.3). One can find that the estimated baryon asymmetry is too small to explain the present one,  $n_B/s \simeq 8 \times 10^{-11}$ , even if  $\phi_{\text{osc}} \sim M_{pl}$  and  $M_F$  is taken as the lower bound of Eq. (3.7).

## B. Stable Q-ball formation when $V_{\text{th},2}(\phi_{\text{osc}}) \gtrsim V_{\text{grav,gauge}}(\phi_{\text{osc}})$

We consider the case where the thermal logarithmic potential  $V_{\text{th},2}$  dominates over both  $V_{\text{gauge}}$  and  $V_{\text{grav}}$ . In this case,  $\alpha_S^2 T_{\text{osc}}^4 \gtrsim M_F^4$  and  $\alpha_S^2 T_{\text{osc}}^4 \gtrsim m_{3/2}^2 \phi_{\text{osc}}^2$  should be satisfied. The Hubble parameter  $H_{\text{osc}}$  and the temperature  $T_{\text{osc}}$  when the AD field starts to oscillate

are given by Eqs. (3.53) and (3.54). Therefore, the reheating temperature is constrained below as

$$T_{\text{RH}} \gtrsim \frac{M_F}{\alpha_S} \sqrt{\frac{\phi_{\text{osc}}}{M_{\text{pl}}}}, \quad (3.63)$$

and

$$T_{\text{RH}} \gtrsim \frac{\phi_{\text{osc}}}{\alpha_S M_{\text{pl}}} \sqrt{m_{3/2} M_{\text{pl}}}, \quad (3.64)$$

from  $V_{\text{th},2}(\phi_{\text{osc}}) \gtrsim V_{\text{gauge}}(\phi_{\text{osc}})$  and  $V_{\text{th},2}(\phi_{\text{osc}}) \gtrsim V_{\text{grav}}(\phi_{\text{osc}})$ , respectively.

Since the charge of the formed Q-ball is given by  $Q \simeq \beta(\phi_{\text{osc}}/T_{\text{osc}})^4$ ,  $\Delta Q/Q$  is estimated as

$$\begin{aligned} \frac{\Delta Q}{Q} &\sim 2 \times 10^{-12} \alpha_S^{11/6} \left( \frac{m_{\text{SUSY}}}{1 \text{ TeV}} \right)^{-2/3} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-1/3} \\ &\times \left( \frac{T_{\text{RH}}}{10^{10} \text{ GeV}} \right)^{11/3} \left( \frac{\phi_{\text{osc}}}{M_{\text{pl}}} \right)^{-11/2}, \end{aligned} \quad (3.65)$$

where we used Eq. (3.45). We can use this relation only when  $\Delta Q/Q < 1$ , in other words, only when

$$\frac{\phi_{\text{osc}}}{M_{\text{pl}}} \gtrsim 7 \times 10^{-3} \alpha_S^{1/3} \left( \frac{m_{\text{SUSY}}}{1 \text{ TeV}} \right)^{-4/33} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-2/33} \left( \frac{T_{\text{RH}}}{10^{10} \text{ GeV}} \right)^{2/3} \quad (3.66)$$

is satisfied, and the right-hand side does not exceed  $\mathcal{O}(1)$ :

$$T_{\text{RH}} \lesssim 1.7 \times 10^{13} \text{ GeV} \alpha_S^{-1/2} \left( \frac{m_{\text{SUSY}}}{1 \text{ TeV}} \right)^{2/11} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{1/11}. \quad (3.67)$$

Otherwise,  $\Delta Q/Q$  becomes 1.

Moreover,  $dE/dQ < 1 \text{ GeV}$  leads to

$$\frac{\phi_{\text{osc}}}{M_{\text{pl}}} \gtrsim 0.03 \alpha_S^{1/3} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{2/3} \left( \frac{T_{\text{RH}}}{10^{10} \text{ GeV}} \right)^{2/3}, \quad (3.68)$$

and the right-hand side is smaller than  $\mathcal{O}(1)$  when

$$T_{\text{RH}} \lesssim 1.7 \times 10^{12} \text{ GeV} \alpha_S^{-1/2} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-1}. \quad (3.69)$$

When  $\Delta Q/Q < 1$ , the produced baryon asymmetry is then estimated as

$$\begin{aligned} \frac{n_B}{s} &\simeq 6 \times 10^{-29} \alpha_S^{-13/6} \left( \frac{m_{3/2}}{200 \text{ keV}} \right) \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-2/3} \\ &\times \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-1/3} \left( \frac{T_{RH}}{10^{10} \text{ GeV}} \right)^{-1/3} \left( \frac{\phi_{osc}}{M_{pl}} \right)^{1/2} R, \end{aligned} \quad (3.70)$$

When  $T_{RH} \lesssim T_\eta$  ( $R = 1$ ), the lower bounds on the reheating temperature give an upper bound on the baryon asymmetry. Equation (3.63) leads to

$$\begin{aligned} \frac{n_B}{s} &\lesssim 3 \times 10^{-27} \alpha_S^{-11/6} \left( \frac{m_{3/2}}{200 \text{ keV}} \right) \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \\ &\times \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-2/3} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-2/3} \left( \frac{\phi_{osc}}{M_{pl}} \right)^{1/3}, \end{aligned} \quad (3.71)$$

and Eq. (3.64) leads to

$$\begin{aligned} \frac{n_B}{s} &\lesssim 4 \times 10^{-28} \alpha_S^{-11/6} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{5/6} \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \\ &\times \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-2/3} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-1/3} \left( \frac{\phi_{osc}}{M_{pl}} \right)^{1/6}. \end{aligned} \quad (3.72)$$

On the other hand, when  $T_{RH} \gtrsim T_\eta$  ( $R = T_{RH}/T_\eta$ ), the upper bounds on the reheating temperature give an upper bound on the produced baryon asymmetry. Equation (3.67) leads to

$$\begin{aligned} \frac{n_B}{s} &\lesssim 8 \times 10^{-24} \alpha_S^{-31/6} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{1/2} \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \\ &\times \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-6/11} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-3/11} \left( \frac{\phi_{osc}}{M_{pl}} \right)^{1/2}. \end{aligned} \quad (3.73)$$

When the equality of Eq. (3.67) is satisfied, Q-balls completely evaporate and the produced baryon asymmetry is maximized. Moreover, Eq. (3.69) leads to

$$\begin{aligned} \frac{n_B}{s} &\lesssim 2 \times 10^{-24} \alpha_S^{-31/6} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{1/2} \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \\ &\times \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-2/3} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-1} \left( \frac{\phi_{osc}}{M_{pl}} \right)^{1/2}. \end{aligned} \quad (3.74)$$

From these inequalities (Eqs. (3.71), (3.72), (3.73) and (3.74)), one can find that the produced baryon asymmetry is much smaller than the observed one when  $V_{th,2}$  dominates over both  $V_{grav}$  and  $V_{gauge}$ .

**C. Stable Q-ball formation when  $V_{\text{grav}}(\phi_{\text{osc}}) \gtrsim V_{\text{th},2}(\phi_{\text{osc}})$  and  $V_{\text{th},2}(\phi_{\text{eq}}) \gtrsim V_{\text{gauge}}(\phi_{\text{eq}})$**

Next, we consider the case where the AD field starts to oscillate by  $V_{\text{grav}}$  and fragments into Q-balls by the thermal logarithmic potential  $V_{\text{th},2}$ . In this case, the AD field starts to oscillate by  $V_{\text{grav}}$  with  $K > 0$ . The Hubble parameter at the onset of the oscillation is about  $H_{\text{osc}} \simeq m_{3/2}$ . After that, the oscillation amplitude decreases, and then the thermal logarithmic potential  $V_{\text{th},2}$  dominates. The threshold field value  $\phi_{\text{eq}}$  is determined by  $V_{\text{th},2}(\phi_{\text{eq}}) = V_{\text{grav}}(\phi_{\text{eq}})$ , and then  $\phi_{\text{eq}}$  is roughly given by  $\phi_{\text{eq}} \simeq T_{\text{eq}}^2/m_{3/2}$ , where  $T_{\text{eq}}$  is the temperature in this epoch. The oscillation amplitude scales as  $\phi \propto a^{-3/2}$  when it is larger than  $\phi_{\text{eq}}$ . Assuming that the universe is dominated by the inflaton oscillating energy, the oscillation amplitude scales as  $\phi \propto H$ , which leads to  $H_{\text{eq}} \simeq H_{\text{osc}} \frac{\phi_{\text{eq}}}{\phi_{\text{osc}}}$ . By using  $H_{\text{osc}} \simeq m_{3/2}$  and  $\phi_{\text{eq}} \simeq T_{\text{eq}}^2/m_{3/2}$ ,  $H_{\text{eq}}$  is expressed as

$$H_{\text{eq}} \simeq \frac{T_{\text{eq}}^2}{\phi_{\text{osc}}}. \quad (3.75)$$

Since the temperature of the dilute plasma is given by  $T_{\text{eq}} \simeq (T_{\text{RH}}^2 M_{\text{pl}} H_{\text{eq}})^{1/4}$ ,  $T_{\text{eq}}$  is expressed as

$$T_{\text{eq}} \simeq T_{\text{RH}} \sqrt{\frac{M_{\text{pl}}}{\phi_{\text{osc}}}} \quad (3.76)$$

We now consider the case where Q-balls are formed when  $V_{\text{th},2}(\phi_{\text{eq}})$  dominates over  $V_{\text{gauge}}(\phi_{\text{eq}})$ :

$$\frac{\phi_{\text{osc}}}{M_{\text{pl}}} \lesssim \alpha_S^2 \left( \frac{T_{\text{RH}}}{M_F} \right)^2. \quad (3.77)$$

Moreover,  $\phi_{\text{osc}}$  should be larger than  $\phi_{\text{eq}}$  in this scenario, which leads to

$$\frac{\phi_{\text{osc}}}{M_{\text{pl}}} \gtrsim \frac{T_{\text{RH}}}{\sqrt{m_{3/2} M_{\text{pl}}}}. \quad (3.78)$$

In order for  $\phi_{\text{osc}}$  to be smaller than the Planck scale, the reheating temperature is roughly lower than the moduli oscillation temperature, in other words,  $T_{\text{RH}} \lesssim \sqrt{m_{3/2} M_{\text{pl}}} \simeq T_\eta$ . Hence, we use  $R = 1$  in this case. Combining Eqs. (3.77) and (3.78), the reheating temperature is constrained below as

$$T_{\text{RH}} \gtrsim 1.1 \times 10^2 \text{ GeV} \alpha_S^{-2} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^2 \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{-1/2}. \quad (3.79)$$

Since the charge of the formed Q-ball is given by  $Q \simeq \beta(\phi_{\text{eq}}/T_{\text{eq}})^4$ ,  $\Delta Q/Q$  is estimated as

$$\begin{aligned} \frac{\Delta Q}{Q} &\sim 2 \times 10^{-17} \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-2/3} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-1/3} \\ &\times \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{11/3} \left( \frac{T_{\text{RH}}}{10^6 \text{ GeV}} \right)^{-11/3} \left( \frac{\phi_{\text{osc}}}{M_{pl}} \right)^{11/6}. \end{aligned} \quad (3.80)$$

We can use this relation only when  $\Delta Q/Q < 1$ , in other words, only when

$$\frac{\phi_{\text{osc}}}{M_{pl}} \lesssim 1 \times 10^9 \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{4/11} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{2/11} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{-2} \left( \frac{T_{\text{RH}}}{10^6 \text{ GeV}} \right)^2 \quad (3.81)$$

is satisfied. This condition is applied when the right-hand side does not exceed  $\mathcal{O}(1)$ :

$$T_{\text{RH}} \lesssim 30 \text{ GeV} \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-2/11} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-1/11} \left( \frac{m_{3/2}}{200 \text{ keV}} \right), \quad (3.82)$$

otherwise it just implies  $\phi_{\text{osc}} \lesssim M_{pl}$ .

Moreover,  $dE/dQ < 1 \text{ GeV}$  leads to

$$\frac{\phi_{\text{osc}}}{M_{pl}} \lesssim 1 \times 10^7 \left( \frac{T_{\text{RH}}}{10^6 \text{ GeV}} \right)^2 \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{-2} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-2}. \quad (3.83)$$

This condition is applied when the right-hand side is smaller than  $\mathcal{O}(1)$ :

$$T_{\text{RH}} \lesssim 3 \times 10^2 \text{ GeV} \left( \frac{m_{3/2}}{200 \text{ keV}} \right) \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right), \quad (3.84)$$

otherwise it just implies  $\phi_{\text{osc}} \lesssim M_{pl}$ .

The produced baryon asymmetry is estimated as

$$\begin{aligned} \frac{n_B}{s} &\simeq 4 \times 10^{-23} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{8/3} \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-2/3} \\ &\times \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-1/3} \left( \frac{T_{\text{RH}}}{10^6 \text{ GeV}} \right)^{-11/3} \left( \frac{\phi_{\text{osc}}}{M_{pl}} \right)^{23/6}, \end{aligned} \quad (3.85)$$

where we used  $R = 1$ . From the above inequalities, one can find that the produced baryon asymmetry is maximized when  $T_{\text{RH}} \simeq \alpha_S M_F$  (see Eq. (3.77)). Therefore, the baryon asymmetry is constrained as

$$\begin{aligned} \frac{n_B}{s} &\lesssim 2 \times 10^{-18} \alpha_S^{-11/3} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{8/3} \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \\ &\times \left( \frac{m_{SUSY}}{1 \text{ TeV}} \right)^{-2/3} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^{-4} \left( \frac{\phi_{\text{osc}}}{M_{pl}} \right)^{23/6}. \end{aligned} \quad (3.86)$$

Note that if  $\phi_{\text{osc}}$  is about the Planck scale the conditions of Eqs. (3.78), (3.81) and (3.83) are satisfied when  $T_{\text{RH}} \simeq \alpha M_F$ . One can find that the produced baryon asymmetry is too small to explain the observed baryon asymmetry when  $V_{\text{grav}}(\phi_{\text{osc}}) \gtrsim V_{\text{th},2}(\phi_{\text{osc}})$  and  $V_{\text{th},2}(\phi_{\text{eq}}) \gtrsim V_{\text{gauge}}(\phi_{\text{eq}})$ .

#### D. Unstable Q-ball formation when $V_{\text{gauge,grav}}(\phi_{\text{osc}}) \gtrsim V_{\text{th}}(\phi_{\text{osc}})$

In gauge-mediated SUSY breaking models, unstable Q-balls correspond to the “delayed type Q-ball” or the “gauge-mediation type Q-ball”. In the case of the delayed type Q-ball, the condition  $dE/dQ > 1 \text{ GeV}$  leads to the lower bound on the gravitino mass as  $m_{3/2} \gtrsim 1.4 \text{ GeV}$  from Eq. (3.60). As is the case for the gravity-mediation models (see Eq. (3.52)), the baryon asymmetry is estimated as

$$\frac{n_B}{s} \simeq 7.0 \times 10^{-15} \left( \frac{\rho_\eta/s}{10^{-14} \text{ GeV}} \right) \left( \frac{\eta_0}{M_{\text{pl}}} \right)^{-2} \left( \frac{m_{3/2}}{1 \text{ GeV}} \right)^{-1} \left( \frac{\phi_{\text{osc}}}{M_{\text{pl}}} \right)^2, \quad (3.87)$$

and it is too small to explain the present baryon asymmetry. We then focus on the gauge-mediation type Q-ball here.

In this case,  $V_{\text{gauge}}(\phi_{\text{osc}}) \gtrsim V_{\text{grav}}(\phi_{\text{osc}})$  should be satisfied, which leads to  $\phi_{\text{osc}} \lesssim \phi_{\text{eq}}$ .  $\phi_{\text{eq}}$  is roughly given by  $\phi_{\text{eq}} \simeq M_F^2/m_{3/2}$ , and then the oscillation amplitude is constrained as

$$\frac{\phi_{\text{osc}}}{M_{\text{pl}}} \lesssim \frac{M_F^2}{m_{3/2} M_{\text{pl}}}. \quad (3.88)$$

Moreover,  $V_{\text{gauge}}(\phi_{\text{osc}}) \gtrsim V_{\text{th},2}(\phi_{\text{osc}})$  leads to  $M_F^4 \gtrsim \alpha_S^2 T_{\text{osc}}^4$ , and this inequality is rewritten as

$$\frac{\phi_{\text{osc}}}{M_{\text{pl}}} \gtrsim \frac{\alpha_S^2 T_{\text{RH}}^2}{M_F^2}. \quad (3.89)$$

Combining Eqs. (3.88) and (3.89), we obtain a bound on reheating temperature as

$$T_{\text{RH}} \lesssim \frac{M_F^2}{\alpha_S \sqrt{m_{3/2} M_{\text{pl}}}}. \quad (3.90)$$

Since  $M_F$  is related to the SUSY breaking scale as Eq. (3.5), it is constrained as  $M_F \lesssim \sqrt{m_{3/2} M_{\text{pl}}}$ . Then, Eq. (3.90) implies  $T_{\text{RH}} \lesssim \sqrt{m_{3/2} M_{\text{pl}}} \simeq T_\eta$ , in other words,  $R = 1$ .

The charge of the formed Q-ball is given by  $Q \simeq \beta(\phi_{\text{osc}}/M_F)^4$ , and then  $dE/dQ > 1 \text{ GeV}$  leads to

$$\frac{\phi_{\text{osc}}}{M_{\text{pl}}} \lesssim 1.2 \times 10^{-5} \left( \frac{M_F}{10^6 \text{ GeV}} \right)^2. \quad (3.91)$$

Since the Q-ball should decay before the beginning of the BBN, the oscillation amplitude of the AD field is constrained as

$$\frac{\phi_{\text{osc}}}{M_{pl}} \lesssim 1.7 \times 10^{-6} \left( \frac{M_F}{10^6 \text{ GeV}} \right)^{6/5}, \quad (3.92)$$

where we use Eq. (3.48).

The resulting baryon asymmetry is then estimated as

$$\frac{n_B}{s} \simeq 0.36 \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \left( \frac{m_{3/2}}{200 \text{ keV}} \right) \left( \frac{M_F}{10^6 \text{ GeV}} \right)^{-4} \left( \frac{\phi_{\text{osc}}}{M_{pl}} \right)^4, \quad (3.93)$$

where we used  $H_{\text{osc}} = M_F^2/\phi_{\text{osc}}$ . Since  $\phi_{\text{osc}}$  is restricted from above by Eq. (3.92), the baryon asymmetry has an upper bound of

$$\frac{n_B}{s} \lesssim 3.0 \times 10^{-24} \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \left( \frac{m_{3/2}}{200 \text{ keV}} \right) \left( \frac{M_F}{10^6 \text{ GeV}} \right)^{4/5}. \quad (3.94)$$

Note that Eqs. (3.88) and (3.91) are satisfied when we take  $m_{3/2} = 200 \text{ keV}$  and  $M_F = 10^6 \text{ GeV}$ . One can find that the estimated baryon asymmetry is too small to explain the present baryon asymmetry.

### E. Unstable Q-ball formation when $V_{\text{th},2}(\phi_{\text{osc}}) \gtrsim V_{\text{grav,gauge}}(\phi_{\text{osc}})$

Next, we consider the case where the thermal logarithmic potential  $V_{\text{th},2}$  dominates over both  $V_{\text{gauge}}$  and  $V_{\text{grav}}$ . In this case, the reheating temperature is constrained from below as Eqs. (3.63) and (3.64). The charge of the Q-ball is given by  $Q \simeq \beta(\phi_{\text{osc}}/T_{\text{osc}})^4$ . The condition of  $dE/dQ > 1 \text{ GeV}$  leads to

$$\frac{\phi_{\text{osc}}}{M_{pl}} \lesssim 2.4 \times 10^{-3} \alpha_S^{1/3} \left( \frac{M_F}{10^6 \text{ GeV}} \right)^{2/3} \left( \frac{T_{\text{RH}}}{10^7 \text{ GeV}} \right)^{2/3}. \quad (3.95)$$

The BBN constraint (Eq. (3.48)) is expressed as

$$\frac{\phi_{\text{osc}}}{M_{pl}} \lesssim 6.6 \times 10^{-4} \alpha_S^{1/3} \left( \frac{M_F}{10^6 \text{ GeV}} \right)^{2/15} \left( \frac{T_{\text{RH}}}{10^7 \text{ GeV}} \right)^{2/3}, \quad (3.96)$$

and is applicable only when

$$T_{\text{RH}} \lesssim 5.9 \times 10^{11} \text{ GeV} \alpha_S^{-1/2} \left( \frac{M_F}{10^6 \text{ GeV}} \right)^{-1/5}. \quad (3.97)$$

Otherwise, Eq. (3.96) just implies  $\phi_{\text{osc}} \lesssim M_{pl}$ .



The produced baryon asymmetry is estimated as

$$\frac{n_B}{s} \simeq 3.6 \times 10^{-5} \alpha_S^{-4} \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \left( \frac{m_{3/2}}{200 \text{ keV}} \right) \left( \frac{T_{RH}}{10^7 \text{ GeV}} \right)^{-4} \left( \frac{\phi_{osc}}{M_{pl}} \right)^6 R. \quad (3.98)$$

When  $T_{RH} \lesssim T_\eta$  ( $R = 1$ ), Eq. (3.96) gives an upper bound on the baryon asymmetry as

$$\frac{n_B}{s} \lesssim 3.0 \times 10^{-24} \alpha_S^{-2} \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \left( \frac{m_{3/2}}{200 \text{ keV}} \right) \left( \frac{M_F}{10^6 \text{ GeV}} \right)^{4/5}. \quad (3.99)$$

Note that Eq. (3.95) is satisfied when we take  $m_{3/2} = 200 \text{ keV}$  and  $M_F = 10^6 \text{ GeV}$ . In the case of  $T_{RH} \gtrsim T_\eta$  ( $R = T_{RH}/T_\eta$ ), the produced baryon asymmetry is maximized when we take  $T_{RH}$  as the maximum value of Eq. (3.97), and it is given by

$$\frac{n_B}{s} \lesssim 1.8 \times 10^{-19} \alpha_S^{-2} \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \left( \frac{m_{3/2}}{200 \text{ keV}} \right) \left( \frac{M_F}{10^6 \text{ GeV}} \right)^{4/5}. \quad (3.100)$$

The resulting baryon asymmetry is then too small to explain the observed baryon asymmetry.

#### F. Unstable Q-ball formation when $V_{\text{grav}}(\phi_{\text{osc}}) \gtrsim V_{\text{th},2}(\phi_{\text{osc}})$ and $V_{\text{th},2}(\phi_{\text{eq}}) \gtrsim V_{\text{gauge}}(\phi_{\text{eq}})$

When the AD field starts to oscillate by  $V_{\text{grav}}$  and fragments into Q-balls by the thermal logarithmic potential  $V_{\text{th},2}$ , the condition  $V_{\text{th},2}(\phi_{\text{eq}}) \gtrsim V_{\text{gauge}}(\phi_{\text{eq}})$  requires

$$\frac{\phi_{\text{osc}}}{M_{pl}} \lesssim \frac{\alpha_S T_{RH}^2}{M_F^2}, \quad (3.101)$$

where  $\phi_{\text{eq}}$  is roughly estimated as  $\phi_{\text{eq}} \simeq T_{\text{eq}}^2/m_{3/2}$ .  $T_{\text{eq}}$  is given by Eq. (3.76). The condition  $dE/dQ > 1 \text{ GeV}$  leads to

$$\frac{\phi_{\text{osc}}}{M_{pl}} \gtrsim 3.1 \times 10^4 \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{-2} \left( \frac{M_F}{10^6 \text{ GeV}} \right)^{-2} \left( \frac{T_{RH}}{10^6 \text{ GeV}} \right)^2. \quad (3.102)$$

Combining Eqs. (3.101) and (3.102), the gravitino mass is constrained below as

$$m_{3/2} \gtrsim 1.4 \text{ GeV} \alpha_S^{1/2}. \quad (3.103)$$

Note that this condition is the same with the instability condition of the delayed type Q-ball (see Eq. (3.60)). The baryon asymmetry is estimated as Eq. (3.87), and is too small to explain the present baryon asymmetry.

## G. AD field oscillation by $V_{\text{th},1}$

When the AD field starts to oscillate by the thermal mass term  $V_{\text{th},1}$ , particles coupled with the AD field should be lighter than the temperature, which requires  $f\phi_{\text{osc}} \lesssim T_{\text{osc}}$ . Here  $f$  denotes generic couplings between the AD field and light particles in the thermal bath. Since  $T_{\text{osc}}$  is estimated as  $T_{\text{osc}} \simeq f^{1/3} T_{\text{RH}}^{2/3} M_{\text{pl}}^{1/3}$ , the condition is rewritten as

$$\frac{\phi_{\text{osc}}}{M_{\text{pl}}} \lesssim \left( \frac{T_{\text{RH}}}{f M_{\text{pl}}} \right)^{2/3}, \quad (3.104)$$

and the right-hand side is smaller than  $\mathcal{O}(1)$  when  $T_{\text{RH}} \lesssim f M_{\text{pl}}$ . Otherwise, Eq. (3.104) just implies  $\phi_{\text{osc}} \lesssim M_{\text{pl}}$ .

The produced baryon asymmetry is estimated as

$$\begin{aligned} \frac{n_B}{s} &\simeq 7.2 \times 10^{-21} \left( \frac{f}{10^{-5}} \right)^{-8/3} \left( \frac{m_{3/2}}{200 \text{ keV}} \right) \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{\text{pl}}} \right)^{-2} \\ &\times \left( \frac{T_{\text{RH}}}{10^6 \text{ GeV}} \right)^{-4/3} \left( \frac{\phi_{\text{osc}}}{M_{\text{pl}}} \right)^2 R. \end{aligned} \quad (3.105)$$

When  $T_{\text{RH}} \lesssim T_\eta$  ( $R = 1$ ), the condition of Eq. (3.104) gives an upper bound as

$$\frac{n_B}{s} \lesssim 1.0 \times 10^{-30} \left( \frac{f}{10^{-5}} \right)^{-4} \left( \frac{m_{3/2}}{200 \text{ keV}} \right) \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{\text{pl}}} \right)^{-2}. \quad (3.106)$$

On the other hand, when  $T_{\text{RH}} \gtrsim T_\eta$  ( $R = T_{\text{RH}}/T_\eta$ ), the baryon asymmetry is maximized for  $T_{\text{RH}} \simeq f M_{\text{pl}}$ . We then have the following bound:

$$\frac{n_B}{s} \lesssim 2.5 \times 10^{-24} \left( \frac{f}{10^{-5}} \right)^{-3} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{1/2} \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{\text{pl}}} \right)^{-2}. \quad (3.107)$$

Therefore, we cannot explain the observed baryon asymmetry even in this case.

## H. Summary

Q-ball formation is inevitable for the logarithmic potential induced from the gauge-mediated SUSY breaking effects and becomes a hinderance to the baryon number production. In order to produce huge baryon number, the amplitude of the AD field should be large, which renders the formed Q-ball stable. It is difficult to extract baryon charge from stable Q-balls and the evaporated baryon charge cannot explain the observed baryon asymmetry. On the other hand, unstable Q-balls can release all baryon charge. In this case, however, the amplitude of the AD field is restricted from above in order to prohibit Q-ball decay during and after the BBN epoch. Thus, sufficient baryon number cannot be produced. As a result, the AD mechanism is incompatible with dilution to solve the moduli problem in both cases.

### 3.5 $LH_u$ direction in gauge-mediation models

In the previous section, we focused on flat directions that have only soft SUSY breaking terms and non-renormalizable terms. However, there is an exception. The  $LH_u$  direction is special in terms of having the  $\mu$ -term which prevents it from forming into Q-balls. Moreover, one-loop correction gives a positive correction ( $K > 0$ ). This implies that the  $LH_u$  flat direction does not form into Q-balls.<sup>6</sup>

First, let us consider the case where the  $\mu$ -term exists before the thermal inflation and show that it is still difficult to explain the baryon asymmetry. Assuming that the AD field begins to oscillate because of the  $\mu$ -term before the dilution, the ratio of lepton number to moduli density is given by

$$\left| \frac{n_L}{\rho_\eta} \right| = \frac{2m_{3/2}}{\mu^2} \left( \frac{\phi_{\text{osc}}}{M_{pl}} \right)^2 \left( \frac{\eta_0}{M_{pl}} \right)^{-2}, \quad (3.108)$$

where  $\mu$  is the electroweak scale. Here we use Eq. (3.51) that is applicable when the total energy of the universe scales as  $a^{-3}$  between the eras when the AD field starts to oscillate and when the moduli field starts to oscillate. Even if the reheating completes before the onset of the moduli oscillation, the AD field immediately dominates the total energy of the universe when  $\phi_{\text{osc}} \simeq M_{pl}$  and the above estimation is valid in that case. The lepton asymmetry is partially converted into baryon asymmetry through the sphaleron process, and the produced baryon asymmetry is estimated as

$$\frac{n_B}{s} \simeq 6.0 \times 10^{-20} \left( \frac{\Omega_\eta h^2}{0.12} \right) \left( \frac{\eta_0}{M_{pl}} \right)^{-2} \left( \frac{m_{3/2}}{200 \text{ keV}} \right) \left( \frac{\mu}{1 \text{ TeV}} \right)^{-2} \left( \frac{\phi_{\text{osc}}}{M_{pl}} \right)^2. \quad (3.109)$$

The resulting baryon asymmetry is then too small to explain the present asymmetry. This is because the AD field oscillates earlier by the  $\mu$ -term whose scale is much larger than the gravitino mass scale ( $\mu \gg m_{3/2}$ ).

In this section, we consider an alternative scenario where the  $\mu$ -term is negligible at the onset of the oscillation of the AD field and is generated at the end of the thermal inflation. The coupling between the flaton and Higgs supermultiplets prohibits the  $\mu$ -term before the thermal inflation and plays the role of generating the  $\mu$ -term by the flaton VEV after the thermal inflation. In this case, we expect that the scenario changes as follows. The  $LH_u$  flat direction produces lepton asymmetry by the SUSY breaking terms when  $H_{\text{osc}} \simeq m_{3/2}$ . Then, Q-balls are formed when the oscillation amplitude of the  $LH_u$  direction decreases to  $\phi_{\text{eq}}$ . After the thermal inflation, the generated  $\mu$ -term

---

<sup>6</sup> This section is based on the work of Ref. [47].

supermultiplets	$Q$	$\bar{u}$	$\bar{d}$	$L$	$\bar{e}$	$H_u$	$H_d$	$X$
$Z_4$ charge	2	2	0	0	2	0	2	1

Table 3.3: Assignment of  $Z_4$  charge

violates the condition for the existence of the Q-ball. Q-balls decay and the absorbed lepton number is released. In the following, we show if the released lepton number can explain the observed baryon asymmetry.

In order to generate the  $\mu$ -term, we assume that the flaton  $X$  couples with the Higgs supermultiplets as

$$W_\mu = \frac{\lambda_\mu}{M_{pl}} X^2 H_u H_d, \quad (3.110)$$

where  $\lambda_\mu$  is a dimensionless coupling constant. This term is also motivated by solving the  $\mu$  problem in the Higgs sector: why is the  $\mu$ -term electroweak scale and so small compared to the GUT or Planck scale? The coupling is allowed if we assign  $Z_4$  charge to the MSSM particles as shown in Table 3.3. Then, the following  $F$ -term potential arises:

$$V_F = \frac{|\lambda_\mu|^2 |X|^4}{M_{pl}^2} |\Phi|^2 \equiv \mu^2(X) |\Phi|^2, \quad (3.111)$$

where  $\Phi$  denotes the up-type Higgs scalar field, namely the AD field. For convenience, we introduce the  $\mu$  parameter defined as Eq. (3.111). Considering a model with the  $Z_4$  symmetry breaking term (see Sec. 2.5.1), the flaton potential is then expressed as

$$V(X) \simeq V_0 + \left( \frac{\alpha^* \lambda_X}{M_{pl}} X^3 + \text{h.c.} \right) + \frac{|\lambda_\mu|^2 |\Phi|^2}{M_{pl}^2} |X|^4 + \frac{|\lambda_X|^2}{M_{pl}^2} |X|^6, \quad (3.112)$$

where we neglect the linear terms and the quartic terms in Eq. (2.54). Before the thermal inflation, the flaton VEV is so small that  $\mu$ -term (Eq. (3.111)) is negligible for the dynamics of the AD field compared with SUSY breaking terms, which implies that the AD field forms into Q-balls when the oscillation amplitude is about  $\phi_{\text{eq}}$ . The flaton then acquires the VEV after the thermal inflation, and the  $\mu$ -term is provided for the AD field potential. Q-balls decay if the  $\mu$ -term breaks the condition for the existence of the Q-ball. The lepton charge is released to the thermal plasma.

The  $\mu$  parameter must increase from the outside to the inside of the Q-ball in order to violate the condition for the existence of the Q-ball. Namely, the following relation must be realized:

$$\frac{\mu^2(X(|\Phi_{\text{in}}|))}{\mu^2(X(|\Phi_{\text{out}}|))} > 1. \quad (3.113)$$

$\Phi_{\text{in}}$  and  $\Phi_{\text{out}}$  show the AD field values inside and outside the Q-ball, respectively. From Eq. (3.111), one can find that the  $\mu$  parameter explicitly depends on  $|X|$ .  $|X|$  is determined by the potential of Eq. (3.112), and depends on  $|\Phi|$  via the interaction of Eq. (3.111). Thus, one can find that the  $\mu$  parameter depends on the AD field value.

Since  $|\Phi| \neq 0$  inside the Q-ball, the coupling term between the flaton and the AD field lifts the flaton potential and  $|X|$  inside the Q-ball is smaller than that outside the Q-ball. Hence, Eq. (3.113) is not satisfied at the tree level. Taking into account the one-loop correction, however, we find that the  $\mu$ -term can be steeper than a quadratic mass term for a larger VEV of the AD field  $|\Phi|$ . It is expressed as

$$V = \left[ \frac{|\lambda_\mu|^2 M^4}{M_{pl}^2} \left( 1 + K \log \frac{|\Phi|^2}{M_*^2} \right) \right] |\Phi|^2, \quad (3.114)$$

where we assume that  $K > 0$  and  $|K| \simeq \mathcal{O}(0.1-0.01)$ . The parameter  $M$  is the flaton VEV. For different amplitudes of the AD field ( $|\Phi_{\text{in}}| > |\Phi_{\text{out}}|$ ), the ratio of the  $\mu$ -term is estimated as

$$\frac{\mu^2(|\Phi_{\text{in}}|)}{\mu^2(|\Phi_{\text{out}}|)} \simeq \frac{M_{\text{in}}^4}{M_{\text{out}}^4} \left( 1 + K \log \frac{|\Phi_{\text{in}}|^2}{|\Phi_{\text{out}}|^2} \right), \quad (3.115)$$

where  $M_{\text{in}}$  and  $M_{\text{out}}$  are the flaton field values at  $|\Phi_{\text{in}}|$  and  $|\Phi_{\text{out}}|$ , and satisfy  $M_{\text{in}} < M_{\text{out}}$ . Precisely speaking, when the second term in the parenthesis is  $\mathcal{O}(1)$ , the perturbation breaks down and one should solve renormalization group equations. For simplicity, we assume that the one-loop correction factor is 2 at most<sup>7</sup> and require

$$\frac{1}{2} \lesssim \frac{M_{\text{in}}^4}{M_{\text{out}}^4} < 1 \quad (3.116)$$

to realize the condition of Eq. (3.113). Here,  $M_{\text{in}}$  and  $M_{\text{out}}$  correspond to the flaton VEVs inside and outside the Q-ball. Note that  $M_{\text{in}}/M_{\text{out}} < 1$  is always satisfied because of the  $\mu$ -term in Eq. (3.112).

Because the  $\mu$ -term damps the flaton VEV inside the Q-ball, it should be smaller than the higher dimensional terms in order for  $M_{\text{in}}$  not to be highly damped compared with  $M_{\text{out}}$ . Thus, the AD field value inside the Q-ball,  $\phi_0$ , should be small to suppress the  $\mu$ -term.  $\phi_0$  is proportional to the oscillation amplitude in the case of the gauge-mediation type Q-ball and the new type Q-ball. On the other hand, in order to produce huge baryon number,  $\phi_{\text{osc}}$  should be large. In the case of the delayed type Q-ball,  $\phi_0$  is determined

---

<sup>7</sup> Even if we assume that the correction factor is larger than 2, our result does not change significantly.

only by  $\phi_{\text{eq}}$ . Then,  $\phi_0$  does not get larger even if we take  $\phi_{\text{osc}}$  larger than  $\phi_{\text{eq}}$ . Hence, we focus on the delayed type Q-ball hereafter.

Let us rewrite the condition for Q-ball formation in terms of the model parameters. In the case of the delayed type Q-ball,  $\phi_0$  is determined by the field value, where the gauge-mediation effect is comparable to the gravity-mediation effect, and is estimated as

$$\phi_0 \simeq \frac{1}{\sqrt{2}} \beta^{1/4} \phi_{\text{eq}} \simeq 5.8 \times 10^{13} \text{ GeV} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{-1} \left( \frac{M_F}{5 \times 10^4 \text{ GeV}} \right)^2. \quad (3.117)$$

Here we used Eq. (3.37). Substituting this estimated value into the flaton potential of Eq. (3.112), the flaton VEV inside the Q-ball is obtained by solving the following equation:

$$-3 \frac{|\alpha \lambda_X|}{M_{pl}} M_{\text{in}}^2 + 2 \frac{|\lambda_\mu|^2 \phi_0^2}{M_{pl}^2} M_{\text{in}}^3 + 3 \frac{|\lambda_X|^2}{M_{pl}^2} M_{\text{in}}^5 = 0. \quad (3.118)$$

In order to estimate the parameter  $\alpha$ , we introduce a dimensionless parameter  $\zeta$  as follows:

$$|\alpha| \equiv \zeta \frac{|\lambda_X| M_{\text{in}}^3}{M_{pl}}. \quad (3.119)$$

Note that  $\zeta = 1$  corresponds to the case without the  $\mu$ -term (see Eq. (2.55)) and that  $\zeta > 1$  is satisfied. Using Eqs. (2.55) and (3.119), one can find that the ratio of the flaton VEV inside to outside the Q-ball is given by

$$\frac{M_{\text{in}}}{M_{\text{out}}} = \frac{1}{\zeta^{1/3}}, \quad (3.120)$$

and the condition of Eq. (3.116) leads to  $1 < \zeta < 1.7$ . By solving the equation of Eq. (3.118), one can obtain

$$M_{\text{in}}^2 = \frac{2|\lambda_\mu|^2 \phi_0^2}{3(\zeta - 1)|\lambda_X|^2} \quad (3.121)$$

in terms of  $\zeta$ . Then, from Eq. (3.119), the parameter  $\alpha$  can be estimated as

$$|\alpha| = \zeta \left[ \frac{2}{3(\zeta - 1)} \right]^{3/2} \frac{|\lambda_\mu|^3 \phi_0^3}{|\lambda_X|^2 M_{pl}}. \quad (3.122)$$

By using the expression of the  $\mu$ -term as Eq. (3.111) and  $1 < \zeta < 1.7$ , one can obtain the following constraint on the parameter  $\alpha$ :

$$\begin{aligned} |\alpha| &= \zeta^{1/3} \left[ \frac{2}{3(\zeta - 1)} \right]^{1/2} \mu (|\Phi_{\text{out}}|) \phi_0 \\ &\gtrsim 2.0 \times 10^{16} \text{ GeV}^2 \left( \frac{\mu (|\Phi_{\text{out}}|)}{300 \text{ GeV}} \right) \left( \frac{\phi_0}{5.8 \times 10^{13} \text{ GeV}} \right). \end{aligned} \quad (3.123)$$

Hence, when the symmetry breaking parameter  $\alpha$  satisfies the above constraint, the condition of Eq. (3.113) can be realized including one-loop corrections. Hereafter, we assume the  $Z_4$  symmetry breaking term of the order of  $|\alpha| \simeq 2 \times 10^{16} \text{ GeV}^2$ .

From Eq. (2.56), the energy of the thermal inflation is estimated as

$$V_0 \simeq 4 \times 10^{32} \text{ GeV}^4 \left( \frac{|\alpha|}{2 \times 10^{16} \text{ GeV}^2} \right)^2. \quad (3.124)$$

The flaton and  $R$ -axion masses are estimated as

$$m_\chi \simeq m_{a_\chi} \simeq 600 \text{ GeV} \left( \frac{|\alpha|}{2 \times 10^{16} \text{ GeV}^2} \right)^{2/3} \left( \frac{|\lambda_X|}{5 \times 10^{-8}} \right)^{1/3}, \quad (3.125)$$

where the coupling constant  $|\lambda_X|$  is assumed to be small enough for the flaton mass to be smaller than sparticle mass, which keeps it from decaying into sparticle pairs. From Eq. (2.55), the flaton VEV at the true minimum is given by

$$M_{\text{out}} \simeq 10^{14} \text{ GeV} \left( \frac{|\alpha|}{2 \times 10^{16} \text{ GeV}^2} \right)^{1/3} \left( \frac{|\lambda_X|}{5 \times 10^{-8}} \right)^{-1/3}. \quad (3.126)$$

Then, the  $\mu$ -term outside the Q-ball is given by

$$\mu(|\Phi_{\text{out}}|) \simeq 330 \text{ GeV} \left( \frac{|\lambda_\mu|}{8 \times 10^{-8}} \right) \left( \frac{M_{\text{out}}}{10^{14} \text{ GeV}} \right)^2, \quad (3.127)$$

where the coupling constant  $|\lambda_\mu|$  is also assumed to be as small as  $10^{-7}$  to obtain the  $\mu$ -term of the electroweak scale.

After the thermal inflation, the flaton VEV violates the condition for the existence of the Q-ball and Q-balls decay. Before the thermal inflation, the produced lepton asymmetry is estimated as

$$\left| \frac{\tilde{n}_L}{s_i} \right| = \left| \frac{\tilde{n}_L}{\rho_{\eta, \text{BB}}} \right| \frac{\rho_{\eta, \text{BB}}}{s_i} = \frac{T_{\text{RH}}^{\text{inf}}}{4m_{3/2}} \left( \frac{\phi_{\text{osc}}}{M_{\text{pl}}} \right)^2, \quad (3.128)$$

where we used Eqs. (2.32) and (3.108) assuming that  $H_{\text{osc}} \simeq m_{3/2}$ . The released lepton number is converted into baryon number through the sphaleron process. From Eqs. (2.60), (3.24) and (3.128), the baryon asymmetry provided after the thermal inflation is estimated as

$$\begin{aligned} \frac{n_B}{s} &\simeq 1.4 \times 10^{-10} \left( \frac{T_{\text{RH}}^{\text{inf}}}{5 \times 10^7 \text{ GeV}} \right) \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{-1} \left( \frac{V_0}{4 \times 10^{32} \text{ GeV}^4} \right)^{-1} \\ &\times \left( \frac{T_{\text{RH}}^\chi}{10 \text{ MeV}} \right) \left( \frac{T_{\text{end}}}{20 \text{ TeV}} \right)^3 \left( \frac{\phi_{\text{osc}}}{M_{\text{pl}}} \right)^2. \end{aligned} \quad (3.129)$$

One can find that the observed baryon asymmetry,  $n_B/s \simeq 8 \times 10^{-11}$ , could be explained if the AD field begins to oscillate near the Planck scale. Q-balls collapse irrespective of the oscillation amplitude of the AD field, which is contrary to the case of unstable Q-balls.

Next, we estimate the temperature at the end of the thermal inflation,  $T_{\text{end}}$ , and the reheating temperature of the flaton decay,  $T_{\text{RH}}^X$ . The thermal inflation ends when  $T_{\text{end}} \sim c_T^{1/2} \langle |X| \rangle$  because thermal particles which couple with the flaton become massive. From Eq. (2.59), the temperature at the end of the thermal inflation is estimated as

$$T_{\text{end}} \simeq 20 \text{ TeV} \left( \frac{|\alpha|}{2 \times 10^{16} \text{ GeV}^2} \right)^{1/3} \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{1/3}, \quad (3.130)$$

where we assume  $c_T \simeq \mathcal{O}(1)$ . Note that the released lepton number is successfully converted to baryon number through the sphaleron process since the thermal inflation ends before the electroweak symmetry breaking.

As for the reheating temperature,  $T_{\text{RH}}^X$  is estimated from the decay rate of the flaton. Since  $m_\chi > 2m_h$ , the flaton mainly decays into two Higgs bosons at the tree level and they decay into the standard model particles. The decay rate is estimated as

$$\Gamma(\chi \rightarrow 2h) \simeq \frac{1}{16\pi} \left( \frac{|\lambda_\mu| M_{\text{out}}^2}{M_{pl} m_\chi} \right)^4 \frac{m_\chi^3}{M_{\text{out}}^2} = \frac{1}{16\pi} \left( \frac{\mu}{m_\chi} \right)^4 \frac{m_\chi^3}{M_{\text{out}}^2}, \quad (3.131)$$

where  $\mu \equiv |\lambda_\mu| M_{\text{out}}^2 / M_{pl}$ . Note that the flaton can decay into two higgsino if it is kinematically allowed, which may lead to overproduction of the LSP. We therefore assume that such a decay is kinematically forbidden, i.e.,  $m_\chi \lesssim 2\mu$ . The reheating temperature,  $T_{\text{RH}}^X$  is then expressed as

$$\begin{aligned} T_{\text{RH}}^X &\simeq \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{\Gamma(\chi \rightarrow 2h) M_{pl}} \\ &\simeq 7.8 \text{ MeV} \left( \frac{\mu(|\Phi_{\text{out}}|)}{300 \text{ GeV}} \right)^2 \left( \frac{m_\chi}{600 \text{ GeV}} \right)^{-1/2} \left( \frac{M_{\text{out}}}{10^{14} \text{ GeV}} \right)^{-1}, \end{aligned} \quad (3.132)$$

where we use  $g_* = 10.8$ . Hence, the reheating temperature can be higher than  $\mathcal{O}(1\text{-}10)$  MeV, which can avoid spoiling the success of the BBN.

We turn to estimate the density parameter of the moduli. From Eqs. (2.60) and (2.61), the density parameter of the Big-Bang moduli after the entropy dilution is given by

$$\begin{aligned} \Omega_{\eta, \text{BB}} h^2 &\simeq 1.1 \times 10^{-5} \left( \frac{T_{\text{RH}}^{\text{inf}}}{5 \times 10^7 \text{ GeV}} \right) \left( \frac{V_0}{4 \times 10^{32} \text{ GeV}^4} \right)^{-1} \\ &\quad \times \left( \frac{T_{\text{RH}}^X}{10 \text{ MeV}} \right) \left( \frac{T_{\text{end}}}{20 \text{ TeV}} \right)^3 \left( \frac{\eta_0}{M_{pl}} \right)^2. \end{aligned} \quad (3.133)$$



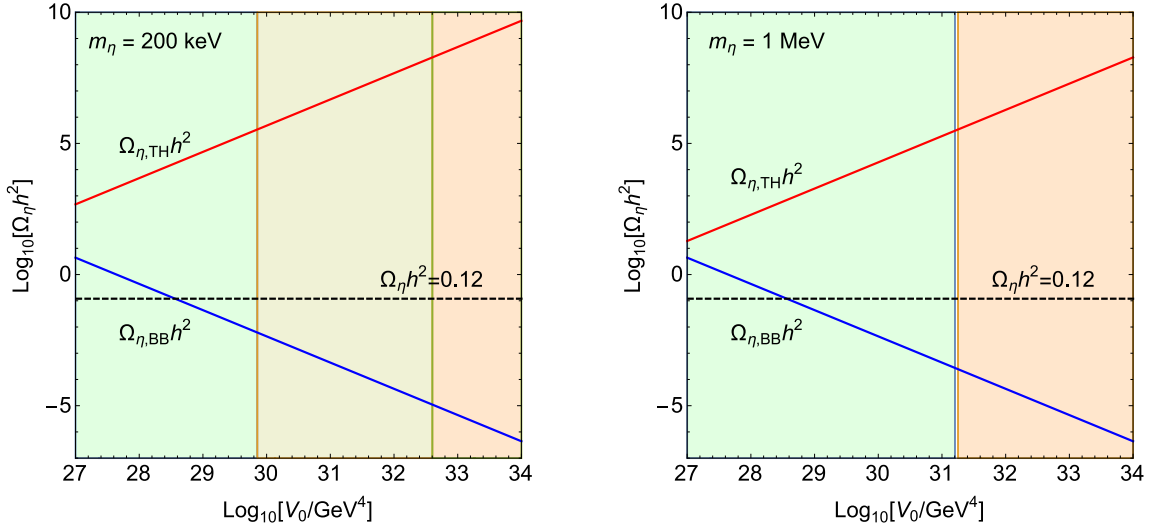


Figure 3.3: The density parameters of the Big-Bang moduli (blue line) and the thermal inflation moduli (red line) are plotted by taking  $T_{\text{RH}}^{\text{inf}} = 5 \times 10^7 \text{ GeV}$ ,  $T_{\text{end}} = 20 \text{ TeV}$ ,  $T_{\text{RH}}^{\text{x}} = 10 \text{ MeV}$  and  $\eta_0 = M_{\text{pl}}$ . The left (right) panel corresponds to the case where  $m_\eta = 200 \text{ keV}$  ( $1 \text{ MeV}$ ). In the green shaded region, the condition for Q-ball formation is not violated (see Eq. (3.123)), and Q-balls do not disappear. In the orange shaded region, the moduli field could not start to oscillate when the thermal inflation occurs ( $(V_0/3M_{\text{pl}}^2)^{1/2} \gtrsim m_\eta$ ). The dashed line shows the observed dark matter density parameter. One can find that even when  $m_\eta = 1 \text{ MeV}$ , the fine-tuning of  $\eta_0/M_{\text{pl}} \simeq \mathcal{O}(10^{-3})$  is needed to suppress the thermal inflation moduli since it scales as  $\eta_0^2$ .

It is found that the Big-Bang moduli is sufficiently diluted. On the other hand, the density parameter of the thermal inflation moduli is given by

$$\begin{aligned} \Omega_{\eta, \text{TH}} h^2 &\simeq 1.9 \times 10^8 c_H^2 \left( \frac{V_0}{4 \times 10^{32} \text{ GeV}^4} \right) \\ &\times \left( \frac{m_{3/2}}{200 \text{ keV}} \right)^{-2} \left( \frac{T_{\text{RH}}^{\text{x}}}{10 \text{ MeV}} \right) \left( \frac{\eta_0}{M_{\text{pl}}} \right)^2, \end{aligned} \quad (3.134)$$

where we used Eq. (2.63). We show the density parameters of these relic moduli in Fig. 3.3. One can find that the density of the thermal inflation moduli is much larger than the observed dark matter density. Therefore, in order for this scenario to work, the separation between the local minimum determined by the Hubble induced term and the true minimum should be of the order of  $\eta_0 \sim 10^{-4} M_{\text{pl}}$ .

Although this may result from 0.01% fine-tuning of the moduli potential, we have no motivation for the thermal inflation in that case since the moduli problem can be solved when  $T_{\text{RH}} \simeq 10 \text{ MeV}$  and  $\eta_0/M_{\text{pl}} \simeq \mathcal{O}(10^{-3})$  (see Eq. (2.32)). Furthermore, in

the orange shaded region where  $(V_0/3M_{pl}^2)^{1/2} \gtrsim m_\eta$  the moduli field could not start to oscillate when the potential energy of the flaton dominates the energy of the universe. This implies that the thermal inflation cannot work. Note that when  $m_\eta = 200$  keV, the orange shaded region is overlapped with the green shaded region, where the condition for Q-ball formation is not violated. Hence, it is found that this scenario does not work.

If the moduli field has no couplings with photons, the constraint from the X-ray background spectra is irrelevant. In this case, the moduli field with mass of  $\mathcal{O}(1)$  MeV, which cannot decay into electrons, could be the dark matter and the fine-tuning can be relaxed to 0.1%.

The situation does not get better even when the  $\mu$ -term is generated in other ways, e.g., in the case that  $\mu(X) \propto |X|^{2n}$ . The thermal inflation moduli is overproduced because of the high scale vacuum energy  $V_0$ . In order to satisfy the condition of Eq. (3.116),  $V_0$  should be larger than the  $\mu$ -term inside the Q-ball. This is because the flaton VEV inside the Q-ball drastically changes when  $V_0 \lesssim \mu^2(M_{\text{out}})\phi_0^2$  (see Eqs. (3.111) and (3.112)).  $\phi_0$  is constrained from below since  $M_F$  has the lower bound (see Eqs. (3.7) and (3.117)). The  $\mu$  parameter should be of the electroweak scale. Therefore, there always exists a lower bound to  $V_0$  similar to Eq. (3.123).

In summary, we have considered the alternative scenario where the  $LH_u$  direction plays the role of producing the baryon asymmetry with destruction of Q-ball in this section. In order to destroy Q-balls, the coupling term between the Higgs and the flaton should be smaller than the vacuum energy  $V_0$  inside the Q-ball. The lower bound of  $V_0$  implies that the thermal inflation begins before the moduli field starts to oscillate, or thermal inflation moduli are always overproduced even when  $m_\eta \simeq \mathcal{O}(1)$  MeV. Hence, we conclude that the observed baryon asymmetry cannot be explained with dilution of the moduli density in these scenarios.

Finally, we comment on other possible baryogenesis scenario after dilution. In gravity-mediation models with  $m_{3/2} \simeq \mathcal{O}(1)$  TeV, the modified AD mechanism, which has been proposed by Ref. [113], could explain the observed baryon asymmetry. In this model, the soft mass of the  $LH_u$  flat direction is negative ( $m_L^2 + m_{H_u}^2 < 0$ ). The  $LH_u$  flat direction begins to roll down to the large VEV at the end of thermal inflation and receives the angular kick when the flaton gets the large VEV. The  $LH_u$  flat direction provides lepton number after thermal inflation. The dynamics of the  $LH_u$  direction is so complicated that numerical simulations are necessary. Some works have revealed that this modified AD mechanism can work in gravity-mediation models. In Refs. [114, 115], the  $A$ -terms of the flaton field are taken around the electroweak scale, but these terms are suppressed in

gauge mediation models when the flaton field exceed the VEV larger than the messenger scale. Therefore, it is unclear if it can also work well in gauge-mediation models.

# Chapter 4

## Heavy moduli/Polonyi scenario

In the previous chapter, we found that the AD mechanism cannot produce baryon number enough to survive after dilution even though it is one of the most promising candidates for baryogenesis with such dilution. In this chapter, we consider a scenario that the moduli/Polonyi field is heavy enough to decay before the onset of the BBN. As mentioned in Sec. 2.5.2, if the moduli/Polonyi mass is about  $\mathcal{O}(100)$  TeV, the density of LSPs with mass of  $\mathcal{O}(0.1-1)$  TeV often exceeds the dark matter density.

There are several ways to avoid the overproduction. One way is to make all SUSY particles to be as heavy as moduli field and to forbid the moduli decay into LSP particles. In this case, however, successful electroweak symmetry breaking requires a somewhat severe fine-tuning because all the soft mass parameters are about  $\mathcal{O}(100)$  TeV. The other way is to make the moduli decay temperature higher than  $\mathcal{O}(1)$  GeV. This generically requires that the moduli field is heavier than the 5 PeV assuming that  $d_\eta \simeq \mathcal{O}(1)$ . The  $R$ -parity violation would also be a solution to the overproduction problem. If the overproduction problem is solved, the baryon asymmetry could be explained by the AD mechanism though the produced  $B - L$  number density is diluted by the entropy production from the moduli decay.

In this chapter, we focus on the scenario where the moduli mass is about  $\mathcal{O}(100)$  TeV and the LSP mass is about  $\mathcal{O}(0.1-1)$  TeV.<sup>1</sup> Since the non-thermal production of LSPs leads to the cosmological problem, we need a dilution mechanism with some baryogenesis mechanism. Actually, it is known that the AD mechanism faces a difficulty if the soft mass is smaller than the gravitino mass. In the anomaly-mediation models, for example, SUSY breaking effects are transmitted by the super-Weyl anomaly [116, 117]. The soft masses of the observable sector are generated at the loop-suppressed order of the gravitino

---

<sup>1</sup> This chapter is based on the work of Ref. [48].

mass in these models. We firstly explain that the AD mechanism has a problem in these cases and introduce one way to avoid the problem in the next section. We assume that the soft mass of the AD field,  $m_\Phi$ , is smaller than the gravitino mass ( $m_\Phi \ll m_{3/2}$ ) in this chapter.

## 4.1 AD mechanism in high scale SUSY models

When the AD potential is lifted by the non-renormalizable superpotential, the AD field potential is given by

$$V(\Phi) = (m_\Phi^2 - c_H H^2) |\Phi|^2 + \frac{m_{3/2}}{n M_{pl}^{n-3}} (a_n \lambda_\Phi \Phi^n + \text{c.c.}) + \frac{|\lambda_\Phi|^2}{M_{pl}^{2n-6}} |\Phi|^{2n-2}, \quad (4.1)$$

where  $c_H$  is a coefficient of the Hubble induced mass term. The origins of these terms are mentioned in Chap. 3 except for the soft mass term. The soft mass is assumed to be generated by loop-suppressed mediation effects ( $m_\Phi \ll m_{3/2}$ ) as seen below. When  $m_\Phi \ll m_{3/2}$ , the AD potential always has global minima as

$$|\Phi_{\min}| \simeq \left[ \frac{|a_n|}{(n-1)|\lambda_\Phi|} m_{3/2} M_{pl}^{n-3} \right]^{1/(n-2)}, \quad (4.2)$$

because of the relatively large  $A$ -terms. At the global minima, masses of the phase direction,  $m_\theta$ , is estimated as

$$m_\theta \simeq \sqrt{\frac{n}{n-1}} |a_n| m_{3/2}. \quad (4.3)$$

As with the conventional AD mechanism explained in the previous chapter, the AD field is trapped in the local minima determined by the negative Hubble induced mass and the non-renormalizable operator before the AD field oscillation. When  $H \simeq m_{3/2}$ , the amplitude of the AD field is about  $\Phi_{\min}$ , and the phase direction starts to roll down to the global minima. This leads to the charge/color breaking universe [118–121].

In order to avoid the problem, we assume that SUSY breaking effects including the  $A$ -terms are provided by the Kähler potential. To obtain the large field value, we assume that the AD field does not appear in the superpotential.<sup>2</sup> In supergravity Lagrangian, we

---

<sup>2</sup> For example,  $U(1)_R$  symmetry can prohibit appearance of the AD field in the superpotential.

consider the following terms in the Kähler potential:<sup>3</sup>

$$\begin{aligned}\mathcal{L}_\Phi &= \int d^2\theta d^2\bar{\theta} \left[ -3M_{pl}^2 \exp\left(-\frac{K}{3M_{pl}^2}\right) \right] \\ &\supset \int d^2\theta d^2\bar{\theta} \left[ f_1 |\Phi|^2 + \left( f_2 \frac{\Phi^n}{nM_{pl}^{n-2}} + \text{c.c.} \right) + f_3 \frac{|\Phi|^4}{M_{pl}^2} + \dots \right],\end{aligned}\quad (4.4)$$

where  $f_i$  ( $i = 1, 2, 3$ ) is an arbitrary real function of SUSY breaking fields, which satisfies  $f_i = f_i^\dagger$  for  $i = 1, 3$ . From these terms, the potential for the AD field  $\Phi$  is given by

$$V(\Phi) = m_\Phi^2 |\Phi|^2 - \frac{m_{3/2}^2}{nM_{pl}^{n-2}} (a_n \Phi^n + \text{c.c.}) + c_4 m_{3/2}^2 \frac{|\Phi|^4}{M_{pl}^2} + \dots, \quad (4.5)$$

where we assume that  $c_4$  is a positive dimensionless parameter. The quartic term proportional to  $m_{3/2}^2$  is originated from the quartic term in Eq. (4.4). Note that the quartic term in Eq. (4.4) is generically present since any symmetry cannot forbid such terms.

Because of the positive quartic term of the AD field, the amplitude of the global minima is estimated to be about the Planck scale in this case. In supergravity, a scalar potential is lifted around the Planck scale by the exponential factor of the Kähler potential (see Eq. (3.9)). Therefore, the global minima are expected to disappear when  $c_4$  is positive.

Let us focus on the dynamics of the AD field in this potential. Due to the negative Hubble induced mass term, the AD field takes its field value of the order of the Planck scale until  $H \simeq m_{3/2}$ . When  $H \lesssim m_{3/2}$ , the position of the local minimum of the AD field, that is determined by a balance between the negative Hubble induced mass term and the positive quartic term, becomes smaller than the Planck scale. Since the position is quickly driven towards the origin of the potential, the AD field cannot track the local minimum and starts to roll down to the origin when  $H \simeq m_{3/2}$  [21, 122].

We will explain the evolution of the AD field when  $H \simeq m_{3/2}$  in more detail. When  $m_\Phi \lesssim H \lesssim m_{3/2}$ , the AD potential for the radial component is given by

$$V(\Phi) \simeq -c_H H^2 |\Phi|^2 + c_4 m_{3/2}^2 \frac{|\Phi|^4}{M_{pl}^2} + \dots. \quad (4.6)$$

Here, we omit the soft SUSY breaking mass term assuming that  $H \gtrsim m_\Phi$ . The amplitude of the local minimum determined by the potential is given by

$$|\Phi_{\text{local}}| \simeq \sqrt{\frac{c_H}{2c_4}} \frac{M_{pl} H}{m_{3/2}}. \quad (4.7)$$

---

<sup>3</sup> The second term in the second line is equivalent to a superpotential term suppressed by the gravitino mass.

One can find that  $|\Phi_{\text{local}}|$  decreases as  $a^{-3/2}$  from the Planck scale after the Hubble scale becomes about the gravitino mass scale,  $H \simeq m_{3/2}$ .

We can write down the equation of motion for  $\phi \equiv |\Phi|/\sqrt{2}$  as

$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} - c_H H^2 \phi + c_4 m_{3/2}^2 \frac{\phi^3}{M_{pl}^2} = 0. \quad (4.8)$$

Using the number of e-folding  $N \equiv \ln(a/a_i)$  as a time variable, this is rewritten as

$$\frac{d^2\phi}{dN^2} + \frac{3}{2} \frac{d\phi}{dN} - c_H \phi + \frac{c_4 m_{3/2}^2}{M_{pl}^2 H^2} \phi^3 = 0, \quad (4.9)$$

where we take  $a_i$  as the scale factor when  $H_i \simeq m_{3/2}$ . Rescaling the AD field value  $\phi$  as

$$\psi \equiv \frac{\phi}{\phi_{\text{local},i}} e^{\frac{3N}{2}}, \quad \phi_{\text{local},i} = \sqrt{\frac{c_H}{c_4}} \frac{M_{pl} H_i}{m_{3/2}}, \quad (4.10)$$

we can eliminate the dependence on the time variable in the coefficients. In terms of  $\psi$ , the equation of motion is rewritten as

$$\frac{d^2\psi}{dN^2} - \frac{3}{2} \frac{d\psi}{dN} - c_H \psi + c_H \psi^3 = 0. \quad (4.11)$$

Note that the coefficient of the friction term is negative. This implies that the AD field cannot track the local minimum and starts to oscillate around the origin<sup>4</sup> when  $H \simeq m_{3/2}$ .  $B - L$  asymmetry is effectively produced at the onset of the oscillation. After that, the asymmetry is conserved since the amplitude of the AD field decreases, and the  $U(1)_{B-L}$  symmetry breaking terms become ineffective.

As with Eq. (3.21), the produced  $B - L$  asymmetry is estimated as

$$n_{B-L}(t_{\text{osc}}) \simeq 2\beta |a_n| \sin[n\theta_i + \arg(a_n)] \frac{m_{3/2}^2}{H_{\text{osc}}} \frac{|\Phi_{\text{osc}}|^n}{M_{pl}^{n-2}}, \quad (4.12)$$

and the ellipticity parameter  $\epsilon$  is given by

$$\epsilon \simeq 2\beta |a_n| \sin[n\theta_i + \arg(a_n)] \frac{m_{3/2}^2}{H_{\text{osc}}^2} \frac{|\Phi_{\text{osc}}|^{n-2}}{M_{pl}^{n-2}}. \quad (4.13)$$

The AD field starts to roll down to the origin from the field value of  $\mathcal{O}(M_{pl})$  when  $H_{\text{osc}} \simeq m_{3/2}$ . Therefore, the  $B - L$  asymmetry is estimated as  $n_{B-L}(t_{\text{osc}}) \simeq \epsilon m_{3/2} |\Phi_{\text{osc}}|^2$ , where  $\epsilon \simeq \mathcal{O}(1)$  and  $|\Phi_{\text{osc}}| \simeq M_{pl}$ , if the potential for the AD field is given as Eq. (4.5).

---

<sup>4</sup> Reference [122] numerically confirms this behavior in the context of the evolution of the PQ field.

## 4.2 Baryon asymmetry and dark matter abundance

In this section, we show that the baryon to dark matter ratio is simply given by the LSP mass and a branching fraction of the moduli/Polonyi decay into superparticles in this scenario. Before calculation, let us summarize our scenario. When  $H \simeq m_\eta \simeq m_{3/2} \simeq \mathcal{O}(100)$  TeV, both the moduli and the AD fields roll down to their origins with the amplitudes of the order of the Planck scale. At that time, the AD field generates the  $B - L$  asymmetry which is later converted to the baryon asymmetry through the sphaleron process. Then, entropy production occurs by some mechanism and dilutes both the moduli density and the baryon asymmetry. After the dilution, the moduli field decays into superparticles which consequently decay into LSPs before the epoch of the BBN. Thus, the dark matter density is determined by the abundance of the non-thermally produced LSPs, assuming that the thermal relic density of LSPs is negligible.

Firstly, we estimate the produced baryon asymmetry. Assuming that the inflaton decays after the onset of the oscillation of the AD field, the baryon to entropy density ratio is estimated as

$$\frac{n_B}{s} = \frac{8}{23} \frac{1}{\Delta} \left. \frac{3T_{\text{RH}}^{\text{inf}} n_{B-L}}{4\rho_{\text{inf}}} \right|_{\text{osc}} \simeq \frac{2}{23} \frac{\epsilon}{\Delta} \frac{T_{\text{RH}}^{\text{inf}}}{m_{3/2}} \left( \frac{|\Phi_{\text{osc}}|}{M_{\text{pl}}} \right)^2, \quad (4.14)$$

where  $\rho_{\text{inf}}$  denotes the energy density of the oscillating inflaton, and  $\Delta$  is the dilution factor defined in Eq. (2.60).  $T_{\text{RH}}$  is the reheating temperature after the inflaton decays. Note that the baryon number density is comparable to the density of the moduli field because both the AD field and moduli field simultaneously begin their oscillation with the same amplitude of the order of the Planck scale.

Let us make a comment on Q-ball formation. In our scenario, the AD field value at the onset of the oscillation is as large as the Planck scale. Thus, the formed Q-balls may be too large to decay before the BBN if Q-ball formation occurs, which renders the AD mechanism ineffective. Hence, the beta function for the soft mass of the AD field may need to be positive in order to prohibit the Q-ball formation. This requires the AD field to involve scalar fields which have large Yukawa couplings.

Next, we estimate the dark matter abundance. From Eq. (2.32), the LSP-to-entropy ratio is estimated as

$$\frac{\rho_{\text{LSP}}}{s} = m_{\text{LSP}} \frac{2\text{Br}_{\text{SUSY}}}{\Delta} \frac{n_\eta}{s_i} \simeq \frac{\text{Br}_{\text{SUSY}}}{\Delta} \frac{T_{\text{RH}}^{\text{inf}} m_{\text{LSP}}}{4m_\eta} \left( \frac{\eta_0}{M_{\text{pl}}} \right)^2, \quad (4.15)$$

where  $\text{Br}_{\text{SUSY}}$  denotes a branching fraction for the moduli/Polonyi decay into two super-



particles,<sup>5</sup> and  $m_{\text{LSP}}$  denotes the LSP mass. The number of the produced superparticles is almost equal to that of the LSPs due to the  $R$ -parity conservation. Here we assume that the moduli field decays after the dilution and that the pair annihilation between LSPs is not efficient. This assumption depends on the annihilation cross section (see Eq. (2.66)). When the neutral wino with a mass of  $\mathcal{O}(1)$  TeV is the LSP and the moduli decay temperature  $T_\eta$  is about  $\mathcal{O}(10)$  MeV, the first term of Eq. (2.66) exceeds the observed dark matter abundance as explained in Sec. 2.5.2. Therefore, if the LSP abundance estimated in Eq. (4.15) does not exceed the dark matter abundance, the pair annihilation is not efficient. Moreover, we assume that the decay products other than the LSPs do not contribute to the dark matter abundance. Hence, the dark matter abundance is explained by the non-thermally produced LSPs in this scenario.

Let us compare the dark matter abundance with the baryon asymmetry created by the AD mechanism. The ratio of moduli to  $B - L$  number density remains the same after they begin their oscillations since the densities of both components decreases as  $a^{-3}$ . From Eqs. (4.14) and (4.15), we obtain the following relation:

$$\begin{aligned} \frac{\Omega_B}{\Omega_{\text{LSP}}} &= \frac{8}{23} \frac{\epsilon}{\text{Br}_{\text{SUSY}}} \frac{m_p m_\eta}{m_{\text{LSP}} m_{3/2}} \left( \frac{|\Phi_{\text{osc}}|}{\eta_0} \right)^2 \\ &\simeq 0.33 \epsilon \left( \frac{\text{Br}_{\text{SUSY}}}{10^{-3}} \right)^{-1} \left( \frac{m_{\text{LSP}}}{1 \text{ TeV}} \right)^{-1} \left( \frac{|\Phi_{\text{osc}}|}{\eta_0} \right)^2, \end{aligned} \quad (4.16)$$

where  $m_p$  represents the proton mass ( $m_p \simeq 0.938$  GeV). Here, we assume  $m_\eta \simeq m_{3/2}$ . Note that  $\eta_0$  and  $|\Phi_{\text{osc}}|$  are of the order of the Planck scale and that  $\epsilon$  is of  $\mathcal{O}(1)$ . One can find that the baryon to dark matter ratio is determined by the LSP mass and the branching fraction of the moduli decay into superparticles. Assuming that the LSP mass is of  $\mathcal{O}(1)$  TeV,  $\text{Br}_{\text{SUSY}}$  is required to be of  $\mathcal{O}(10^{-3})$  in order to realize the observed value,  $\Omega_B/\Omega_{\text{DM}} \simeq 0.18$  [2].

This scenario needs the entropy production, e.g., the thermal inflation. Let us estimate the required amount of the entropy production. The observed dark matter to entropy density ratio is given by [2]

$$\frac{\rho_{\text{DM}}^{(\text{obs})}}{s_0} \simeq 4.4 \times 10^{-10} \text{ GeV}, \quad (4.17)$$

where  $\rho_{\text{DM}}^{(\text{obs})}$  denotes the observed dark matter energy density. Comparing Eq. (4.15) with

---

<sup>5</sup> Gravitinos are not produced from the Polonyi decay assuming that the decay is kinematically forbidden ( $m_Z < 2m_{3/2}$ ). The abundance of gravitinos produced during the reheating becomes negligible after the dilution.

Eq. (4.17), the dilution factor of the required entropy production is estimated as

$$\Delta \simeq 1.9 \times 10^{12} \left( \frac{T_{\text{RH}}^{\text{inf}}}{10^9 \text{ GeV}} \right) \left( \frac{m_{\text{LSP}}}{1 \text{ TeV}} \right) \left( \frac{\text{Br}_{\text{SUSY}}}{10^{-3}} \right) \left( \frac{m_\eta}{300 \text{ TeV}} \right)^{-1} \left( \frac{\eta_0}{M_{\text{pl}}} \right)^2, \quad (4.18)$$

where we assume that the inflaton decays after the moduli and the AD field start to oscillate. This is the case for  $T_{\text{RH}}^{\text{inf}} \lesssim 10^{12} \text{ GeV} (m_\eta/300 \text{ TeV})^{1/2}$ .  $m_\eta$  should be of  $\mathcal{O}(100) \text{ TeV}$  in order to relax the constraint from the BBN. Note that the dilution factor is estimated assuming that the entropy production occurs before the moduli decays.

When the moduli field decays through dimension 5 operators suppressed by the Planck scale, the branching fraction of the decay into SUSY particles is generally comparable to that into the standard model particles ( $\text{Br}_{\text{SUSY}} \simeq \mathcal{O}(1)$ ). Moreover, the AD mechanism discussed in the previous section works the most efficiently since  $\epsilon \simeq \mathcal{O}(1)$  and  $|\Phi_{\text{osc}}| \simeq M_{\text{pl}}$ . Therefore, it is found that the AD mechanism cannot explain the observed baryon asymmetry even if the moduli mass is as heavy as  $\mathcal{O}(100) \text{ TeV}$  (see Eq. (4.16)).

### 4.3 Sequestering model and decay process of Polonyi field

In the previous section, we concluded that the AD mechanism cannot explain the baryon asymmetry assuming that the branching fraction of the moduli decay into SUSY particles is  $\mathcal{O}(1)$ . In this section, we focus on the Polonyi field and study a way to suppress the branching fraction into SUSY particles ( $\text{Br}_{\text{SUSY}} \simeq \mathcal{O}(10^{-3})$ ). Here, we consider the so-called sequestering model [116, 123],<sup>6</sup> in which the SUSY breaking (Polonyi) sector is sequestered from the visible sector.

The Kähler potential and the superpotential are given by

$$K = -3M_{\text{pl}}^2 \log \left[ 1 - \frac{f_{\text{vis}} + f_{\text{hid}}}{3M_{\text{pl}}^2} \right], \quad (4.19)$$

and

$$W = W_{\text{vis}} + W_{\text{hid}}, \quad (4.20)$$

respectively. The subscripts of <sub>vis</sub> and <sub>hid</sub> denote the visible and the hidden sectors (SUSY breaking sector), respectively. We also assume that the standard model sector does not

---

<sup>6</sup> The sequestering model has been introduced in the context of extra dimension [116]. It is also realized in a four dimensional strongly coupled CFT [124–127].

directly couple to the hidden sector:

$$\mathcal{L}_{\text{gauge}} = \int d^2\theta \left[ \frac{1}{4} \tau_{\text{vis}} \hat{W}^a \hat{W}_a + \text{h.c.} \right], \quad (4.21)$$

where  $\hat{W}^a$  denotes field strength supermultiplets of the visible standard model gauge sector, and  $\tau_{\text{vis}}$  is a holomorphic function which depends only on visible sector fields.

In this setup, gaugino masses vanish at tree levels because the Polonyi field does not appear in the gauge sector. The quantum corrections to the gaugino masses arise only at loop-suppressed levels, which mainly come from the anomaly mediation [116, 117]. Then, the lightest gaugino is the neutral wino with a mass of  $\mathcal{O}(1)$  TeV when  $m_{3/2} \simeq \mathcal{O}(100)$  TeV. This is compatible with our scenario with the neutral wino LSP.

Soft scalar masses also vanish at tree levels when the Kähler potential is given by Eq. (4.19). They acquire loop-suppressed contribution from the anomaly mediation [116, 117], Planck-suppressed interactions [128] and so on. If the MSSM scalars acquire their masses only from the anomaly mediation, slepton masses would become negative. This is problematic in terms of the phenomenology. Thus, there should be other sources to give them positive masses. One of such candidates is one-loop corrections from the Planck suppressed interactions [116, 128]. When a cut-off scale is taken around the gravitational scale, one-loop correction can exceed the anomaly-mediated masses which appear at the two-loop level.<sup>7</sup> In this case, sfermion masses are of  $\mathcal{O}(10)$  TeV when  $m_{3/2} \simeq \mathcal{O}(100)$  TeV.<sup>8</sup>

When the soft masses and the supersymmetric masses ( $\mu$ -term) of the Higgs fields are of  $\mathcal{O}(10)$  TeV and of  $\mathcal{O}(1)$  TeV, respectively, the  $B$ -term ( $\sim \mu m_{3/2}$ ) is comparable to the scalar masses, which leads to the successful electroweak symmetry breaking. The higgsino with mass of  $\mu \sim \mathcal{O}(1)$  TeV could be the LSP instead of the neutral wino. When the soft masses are of  $\mathcal{O}(1)$  TeV, the  $B$ -term is generally too large to realize the electroweak symmetry breaking. In the Next-to-MSSM,<sup>9</sup> however, the supersymmetric Higgs mass term is generated as the breaking term of the scale invariance, and the (effective)  $B$ -term appears at loop-suppressed levels.

Since the SUSY breaking sector is now sequestered from the AD field, the functions  $f_i$  ( $i = 1, 2, 3$ ) in Eq. (4.4) do not contain the Polonyi field.<sup>10</sup> Even in this case, the potential

---

<sup>7</sup> When the one-loop correction determines scalar masses, the mass spectra of MSSM scalar particles become UV sensitive, which is contrary to the anomaly-mediated masses. Thus, we lose a solution to the SUSY FCNC problem unless the universality condition is imposed at the UV scale. There also exists other UV insensitive models which solve the negative slepton mass problem [129–133].

<sup>8</sup> The lightest Higgs boson mass acquires radiative corrections from stop one-loop diagrams [134–138]. Stop mass of  $\mathcal{O}(10)$  TeV is compatible with the relatively heavy observed Higgs boson mass of 125 GeV.

<sup>9</sup> For a review, see Ref. [139].

<sup>10</sup> In the early universe, the inflaton sector breaks the SUSY, which generates the Hubble induced mass

for the AD field involves the holomorphic  $A$ -terms and the quartic term of  $\mathcal{O}(m_{3/2}^2)$  due to the explicit breaking of the conformal symmetry. By requiring that the vacuum energy vanishes, the coefficients in Eq. (4.5),  $a_n$  and  $c_4$ , are estimated as  $a_n \simeq -f_2(n-1)$  and  $c_4 \simeq f_3$  when  $f_1 = 1$ . Note that the estimated values contain uncertainties of  $\mathcal{O}(1)$ .

Let us consider the decay process of the Polonyi field,  $Z$  (for details, see [140, 141]). Firstly, the Polonyi field generally decays into 2 gravitinos at tree levels when  $m_Z > 2m_{3/2}$ , where  $m_Z$  denotes the mass of the Polonyi field. This decay process is incompatible with our scenario since the branching fraction of the decay of the Polonyi into SUSY particles is required to be of  $\mathcal{O}(10^{-3})$ . Hence, we assume that the decay into 2 gravitinos is kinematically forbidden ( $m_Z < 2m_{3/2}$ ).

The decay into matter scalars comes from the kinetic terms for the sequestered potential:

$$\mathcal{L}_K = g_{ij*} \partial_\mu \phi^i \partial^\mu \phi^{*j}, \quad (4.22)$$

where  $\phi^i$  denotes the matter scalar fields, and  $g_{ij*} = \frac{\partial^2 K}{\partial \phi^i \partial \phi^{*j}}$ . The kinetic terms are converted into the following form up to a total derivative:

$$\mathcal{L}_K \sim \frac{Z}{M_{pl}} \phi^i \partial^2 \phi^{*j} + \text{h.c.} \quad (4.23)$$

Using the equation of motion, interaction terms from the kinetic terms are proportional to the scalar mass squared. Thus, the branching fraction of the decay mode  $Z \rightarrow \phi^i \phi^{*j}$  is suppressed by a factor of  $\mathcal{O}(m_\phi^4/m_Z^4) \sim \mathcal{O}(10^{-4}-10^{-5})$  when the scalar mass is smaller than the Polonyi mass. The Polonyi field also decays into matter scalar fields through one-loop diagrams by Planck-suppressed interactions, but the rates of these decays are the same order with that of the tree-level decay. Similarly, the branching fraction into matter fermions is proportional to fermion mass squared and is negligible. The decay rate into higgsinos with masses of  $\mu \sim \mathcal{O}(1)$  TeV is the same order with that into matter scalar fields since it is suppressed by a factor of  $\mathcal{O}(\mu^2/m_Z^2) \sim \mathcal{O}(10^{-4})$ .

Decay into three-body final states is suppressed for the sequestered potential. In general, the decay of  $Z \rightarrow \phi^i \chi^j \chi^k$ , where  $\chi^i$  denotes the matter fermions, occurs through the following interaction:

$$\mathcal{L}_{\text{three}} = -\frac{1}{2} e^{\frac{K}{2M_{pl}^2}} \left( \frac{K_Z}{M_{pl}^2} W_{ijk} - 3\Gamma_{Zi}^l W_{jkl} \right) Z \phi^i \chi^j \chi^k + \text{h.c.}, \quad (4.24)$$

---

term. In order to generate the negative Hubble induced mass term for the AD field, the inflaton sector should not be sequestered from the visible sector.

where the subscripts represent the derivative by the scalar fields, and  $\Gamma_{jk}^i = g^{il*} g_{jl*k}$ . One can find that this term vanishes if the Kähler potential is given by the form of Eq. (4.19). For the same reason, the decay of  $Z \rightarrow \phi^i \phi^j \phi^k$  does not occur, either.

Since the Polonyi field is not directly coupled with the gauge sector, it does not decay into gauge bosons and gauginos at tree levels. However, it can decay into them through the anomaly-mediated effects. When the mass of the Polonyi field is dominated by a supersymmetric mass term, the interaction term between  $Z$  and the gaugino  $\lambda$  are given by [142]

$$\mathcal{L}_{\text{anomaly}} = \frac{\alpha b_0 m_Z}{24\pi M_{pl}} \frac{K_Z}{M_{pl}} Z^* \lambda \lambda + \text{h.c.}, \quad (4.25)$$

where  $\alpha = g^2/4\pi$  represents a gauge coupling constant, and  $b_0 = 3T_G - T_R$  is the coefficient of the beta function. Since the SUSY breaking mass term is comparable to the supersymmetric mass term, the interaction terms are deviated from Eq. (4.25) by  $\mathcal{O}(1)$ . From Eq. (4.25), the decay rate is estimated as [142]

$$\Gamma(Z \rightarrow 2\lambda) \simeq \frac{N_g \alpha^2 b_0^2}{4608\pi^3} \frac{|K_Z|^2}{M_{pl}^2} \frac{m_Z^3}{M_{pl}^2}, \quad (4.26)$$

where  $N_g$  is the number of gauginos. The decay rate of  $Z$  into 2 gauge bosons is also the same as Eq. (4.26). The most important process is the decay into gluons and gluinos. We can estimate its rate by using  $N_g = 8$  and  $b_0 = 3$ .

In summary, the Polonyi field mainly decays into gluinos and gluons through the anomaly-mediated effects for the sequestered Kähler potential. If it is the leading process, however, the Polonyi field becomes long-lived, and the constraint from the BBN is again severe even with  $m_Z \simeq \mathcal{O}(100)$  TeV. We need some other efficient decay processes. Note that those decay processes should not yield large dark matter abundance. As a suitable decay process, we consider the decay of the Polonyi field into a (pseudo-)Nambu Goldstone Boson (NGB). To be specific, we introduce the QCD axion [143–146]. The axion is a pseudo-NGB associated with the spontaneous breaking of the Peccei-Quinn (PQ) symmetry and appears as a phase direction of the PQ field. Here we assume that the PQ field also belongs to the visible sector, which is natural as the PQ field must directly couple to standard model charged particles.

Let us consider the following supersymmetric axion model [133, 147]:

$$W_{PQ} = \kappa Y (P\bar{P} - v_{PQ}^2) + \lambda P X \bar{X}, \quad (4.27)$$

where  $Y$  is a gauge singlet superfield with no PQ charge.  $P$  and  $\bar{P}$  are PQ fields with PQ charge of +1 and -1.  $X$  and  $\bar{X}$  are superfields that have the standard model gauge

charge and global PQ charge of  $-1/2$ .  $\kappa$  and  $\lambda$  are dimensionless coupling constants.  $v_P$  denotes the scale of the PQ symmetry breaking. The  $F$ -term scalar potential is given by

$$V_{PQ,F} = |\kappa|^2 |P\bar{P} - v_{PQ}^2|^2 + |\kappa Y|^2 (|P|^2 + |\bar{P}|^2), \quad (4.28)$$

where we omitted the contribution from the second term in Eq. (4.27) assuming that the scalar fields of  $X$  and  $\bar{X}$  are stabilized at the origin. From the first term, one can find that  $\langle P\bar{P} \rangle = v_{PQ}^2$  at the global minimum. On the other hand, the scalar field of  $Y$  acquires the VEV of  $\langle |\kappa Y| \rangle \simeq m_{3/2}$  considering linear terms from the supergravity effects,  $V \simeq a_Y \kappa Y v_{PQ}^2 + \text{h.c.}$ , where  $a_Y$  is a dimensional parameter of the order the gravitino mass. Therefore,  $P$  and  $\bar{P}$  acquires equal masses of the order of  $m_{3/2}$ . In this case, the VEVs of  $P$  and  $\bar{P}$  are almost the same, and they are expanded as

$$P = v_{PQ} \exp\left(\frac{s + ia}{2v_{PQ}^2}\right), \quad (4.29)$$

$$\bar{P} = v_{PQ} \exp\left(-\frac{s + ia}{2v_{PQ}^2}\right), \quad (4.30)$$

where  $s$  and  $a$  denote the saxion field and the axion field, respectively.

As is the same with Eq. (4.23), the PQ fields interact with the Polonyi field through the kinetic terms as follows:

$$\mathcal{L}_K = \left( c \frac{Z}{M_{pl}} P \partial^2 P^* + \text{h.c.} \right) + \left( \bar{c} \frac{Z}{M_{pl}} \bar{P} \partial^2 \bar{P}^* + \text{h.c.} \right), \quad (4.31)$$

where  $c$  and  $\bar{c}$  are dimensionless constants assumed to be complex. Substituting Eqs. (4.29) and (4.30) for Eq. (4.31), one can find that there exists kinetic mixing terms between the Polonyi field and (s)axion. In other words, Eq. (4.31) contains the following terms:

$$\mathcal{L}_{\text{mixing}} = -\epsilon_R \partial_\mu z_R \partial^\mu s - \epsilon_R \partial_\mu z_I \partial^\mu a - \epsilon_I \partial_\mu z_R \partial^\mu a + \epsilon_I \partial_\mu z_I \partial^\mu s, \quad (4.32)$$

where  $\epsilon_R$  and  $\epsilon_I$  are given by

$$\epsilon_R = \frac{(c_R - \bar{c}_R)v_{PQ}}{\sqrt{2}M_{pl}}, \quad \epsilon_I = \frac{(c_I - \bar{c}_I)v_{PQ}}{\sqrt{2}M_{pl}}. \quad (4.33)$$

Here,  $z_R$  and  $z_I$  denote a real and an imaginary component of the Polonyi field ( $Z = \frac{1}{\sqrt{2}}(z_R + iz_I)$ ), respectively.  $c_R$  ( $\bar{c}_R$ ) and  $c_I$  ( $\bar{c}_I$ ) also represent a real and imaginary part of  $c$  ( $\bar{c}$ ), respectively. Note that  $\epsilon_R \ll 1$  and  $\epsilon_I \ll 1$  when the PQ breaking scale is much

smaller than the Planck scale. On the other hand, these fields are considered to have the following mass terms:

$$V_{\text{mass}} = \frac{1}{2}m_s^2 s^2 + \frac{1}{2}m_{z_R}^2 z_R^2 + \frac{1}{2}m_{z_I}^2 z_I^2, \quad (4.34)$$

where  $m_s$ ,  $m_{z_R}$  and  $m_{z_I}$  represent the saxion mass, and the Polonyi masses of the order of the gravitino mass. This term is obtained from the second term in Eq. (4.28). In order to estimate the rate of the Polonyi decay into axions, we need to diagonalize the kinetic mixing terms and transform the bases into mass eigenstates.<sup>11</sup> From Eq. (4.31) in these bases, we obtain the following interactions:

$$\mathcal{L} = -\xi_R \frac{\hat{z}_R}{\sqrt{2}M_{pl}} \partial_\mu \hat{a} \partial^\mu \hat{a} + \xi_I \frac{\hat{z}_I}{\sqrt{2}M_{pl}} \partial_\mu \hat{a} \partial^\mu \hat{a}, \quad (4.35)$$

where  $\xi_R$  and  $\xi_I$  are given by

$$\xi_R \simeq \frac{c_R + \bar{c}_R}{2}, \quad \xi_I \simeq \frac{c_I + \bar{c}_I}{2}, \quad (4.36)$$

at the leading order of  $\epsilon_R$  and  $\epsilon_I$ . Here, we used  $\hat{a}$  to show the mass eigenstate of the massless direction.  $\hat{z}_R$  and  $\hat{z}_I$  also denote the mass eigenstates of the Polonyi field.

The rate of the Polonyi decay into axions<sup>12</sup> is estimated as

$$\Gamma(z_i \rightarrow 2\hat{a}) = \frac{\xi_i^2}{64\pi} \frac{m_{z_i}^3}{M_{pl}^2}, \quad (4.37)$$

where the subscript of  $i$  represents  $R$  or  $I$ . Assuming that  $\xi_i$  is of  $\mathcal{O}(1)$ , the Polonyi decay temperature  $T_Z$  is estimated as

$$T_Z \simeq 7.1 \text{ MeV} \left( \frac{m_Z}{300 \text{ TeV}} \right)^{3/2}. \quad (4.38)$$

Note that the Polonyi density does not dominate the universe at its decay since we assume that the dilution occurs before the decay. Even when the Polonyi field is a subdominant component of the universe, it must decay before the BBN in order not to destroy synthesized light elements. Hence, the Polonyi should be as heavy as  $\mathcal{O}(100)$  TeV.

---

<sup>11</sup> Diagonalizing the kinetic mixing terms leads to the non-diagonal mass matrix of the canonical scalar fields. After that, the mass matrix is diagonalized by the rotation matrix.

<sup>12</sup> The decay products of the Polonyi field could contain the saxion and the axino that is a superpartner of the axion. Since the decay into them could lead to the overproduction of LSPs, we assume that such a decay process is kinematically forbidden, in other words,  $2m_s > m_Z$  and  $2m_{\tilde{a}} > m_Z$ , where  $m_{\tilde{a}}$  represents the axino mass.

The rate of the loop-suppressed decay (Eq. (4.26)) is much smaller than that of the tree-level decay (Eq. (4.37)), and we obtain the branching fraction of the Polonyi decay into superparticles as

$$\text{Br}_{\text{SUSY}} = \frac{\Gamma(Z \rightarrow 2 \text{ superparticles})}{\Gamma(Z \rightarrow 2 \text{ axions})} \sim 1 \times 10^{-3}. \quad (4.39)$$

Since the axion mass is typically much smaller than the LSP mass, the abundance of axions produced from the Polonyi decay is negligible compared with the LSP abundance.<sup>13</sup> Therefore, the dark matter abundance is determined by the abundance of the decay products of the suppressed decay into superparticles. The axion also gives a negligible contribution to the dark radiation. From Eqs. (4.16) and (4.39), it is found that the observed baryon-to-dark matter ratio of  $\Omega_B/\Omega_{\text{DM}} \simeq 0.18$  is explained in the sequestering model with the (pseudo-)NGB.

In summary, we have considered the cosmologically consistent scenario in the presence of a heavy moduli/Polonyi field. When the moduli/Polonyi field is as heavy as  $\mathcal{O}(100)$  TeV, it can decay before the onset of the BBN, but the abundance of LSPs produced by the decay is often overproduced. In the case of the moduli field, even the AD mechanism cannot explain the observed baryon asymmetry when some mechanisms dilute the moduli abundance in order to avoid the LSP overproduction. In the case of the Polonyi field, the baryon asymmetry can be explained by the AD mechanism if the visible sector and the SUSY breaking sector is sequestered and the visible sector contains a (pseudo-) NGB, which can be identified with the QCD axion. In this model, the Polonyi field decays into NGBs at the tree level, which do not contribute to the dark matter abundance. On the other hand, it decays into superparticles mainly through anomaly-induced interactions and hence is suppressed compared with the decay into NGBs.

---

<sup>13</sup> We also assume that the density of the coherent oscillation of the axion field does not exceed the observed dark matter density, which implies  $v_{PQ}/N_{\text{DW}} \sim 10^{9-13}$  GeV.



# Chapter 5

## Conclusion

In this thesis, we considered the cosmological consistent scenarios explaining the observed baryon asymmetry in the presence of the moduli/Polonyi field. As mentioned in Chap. 2, although the thermal inflation can dilute the moduli abundance sufficiently, well-known baryogenesis scenarios such as the leptogenesis cannot produce baryon number enough to survive after dilution.

In particular, the AD mechanism is the most promising baryogenesis with such huge dilution. In Sec. 3.3 and 3.4, we reviewed that the AD mechanism cannot explain the baryon asymmetry in both gravity-mediation and gauge-mediation models for the moduli mass of  $m_\eta \lesssim \mathcal{O}(1)$  TeV. It has been pointed out that Q-ball formation makes it difficult to explain the baryon asymmetry in Ref. [112]. We refined the estimated baryon number using the new lower bound of the SUSY breaking scale (Eq. (3.7)), and show that this scenario is more unlikely to work. In Sec. 3.5, we considered alternative scenario without Q-ball formation using the  $LH_u$  direction, which has not been considered in Ref. [112]. However, it was found that the thermal inflation moduli are overproduced in this scenario.

In Chap. 4, we considered the case of the heavy moduli/Polonyi field with  $m_\eta \simeq \mathcal{O}(100)$  TeV. We introduced one simple way for the AD mechanism to work the most efficiently in high scale SUSY models. However, we showed that the AD mechanism cannot explain the baryon asymmetry when the moduli abundance is diluted in order to avoid the LSP overproduction. In the case of the Polonyi field, we showed that the correct baryon-to-dark matter ratio can be achieved in sequestering models with a (pseudo-) NGB.

## Acknowledgments

First of all, the author would like to express his sincere gratitude to his supervisor Masahiro Kawasaki for various instructive suggestions, collaborations and kind support during his master and doctoral course. He would like to thank Keisuke Harigaya and Masaki Yamada, who are collaborators in Chap. 3 and 4, for useful discussions. He also thanks Kenji Kadota for kind hospitality during his stay at KAIST. He thanks all the members of the theory group at the Institute for Cosmic Ray Research (ICRR) for their hospitality. Finally, he would like to express his appreciation to his family for their support.

This work was supported by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan and the JSPS Research Fellowships for Young Scientists No. 17J07288. This work was also supported by the Leading Graduate Course for Frontiers of Mathematical Science and Physics (FMSP), MEXT, Japan.

# Bibliography

- [1] C. Patrignani *et al.* [Particle Data Group], “Review of Particle Physics,” *Chin. Phys. C* **40**, no. 10, 100001 (2016).
- [2] P. A. R. Ade *et al.* [Planck Collaboration], “Planck 2015 results. XIII. Cosmological parameters,” *Astron. Astrophys.* **594**, A13 (2016) [arXiv:1502.01589 [astro-ph.CO]].
- [3] R. H. Cyburt, B. D. Fields, K. A. Olive and T. H. Yeh, “Big Bang Nucleosynthesis: 2015,” *Rev. Mod. Phys.* **88**, 015004 (2016) [arXiv:1505.01076 [astro-ph.CO]].
- [4] A. D. Sakharov, “Violation of CP Invariance, c Asymmetry, and Baryon Asymmetry of the Universe,” *Pisma Zh. Eksp. Teor. Fiz.* **5**, 32 (1967) [*JETP Lett.* **5**, 24 (1967)] [*Sov. Phys. Usp.* **34**, 392 (1991)] [*Usp. Fiz. Nauk* **161**, 61 (1991)].
- [5] M. Yoshimura, “Unified Gauge Theories and the Baryon Number of the Universe,” *Phys. Rev. Lett.* **41**, 281 (1978) Erratum: [*Phys. Rev. Lett.* **42**, 746 (1979)].
- [6] A. Y. Ignatiev, N. V. Krasnikov, V. A. Kuzmin and A. N. Tavkhelidze, “Universal CP Noninvariant Superweak Interaction and Baryon Asymmetry of the Universe,” *Phys. Lett.* **76B**, 436 (1978).
- [7] H. Georgi and S. L. Glashow, “Unity of All Elementary Particle Forces,” *Phys. Rev. Lett.* **32**, 438 (1974).
- [8] G. ’t Hooft, “Symmetry Breaking Through Bell-Jackiw Anomalies,” *Phys. Rev. Lett.* **37**, 8 (1976).
- [9] N. S. Manton, “Topology in the Weinberg-Salam Theory,” *Phys. Rev. D* **28**, 2019 (1983).
- [10] F. R. Klinkhamer and N. S. Manton, “A Saddle Point Solution in the Weinberg-Salam Theory,” *Phys. Rev. D* **30**, 2212 (1984).

- [11] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, “On the Anomalous Electroweak Baryon Number Nonconservation in the Early Universe,” *Phys. Lett.* **155B**, 36 (1985).
- [12] A. G. Cohen, D. B. Kaplan and A. E. Nelson, “Progress in electroweak baryogenesis,” *Ann. Rev. Nucl. Part. Sci.* **43**, 27 (1993) [hep-ph/9302210].
- [13] V. A. Rubakov and M. E. Shaposhnikov, “Electroweak baryon number nonconservation in the early universe and in high-energy collisions,” *Usp. Fiz. Nauk* **166**, 493 (1996) [*Phys. Usp.* **39**, 461 (1996)] [hep-ph/9603208].
- [14] A. I. Bochkarev and M. E. Shaposhnikov, “Electroweak Production of Baryon Asymmetry and Upper Bounds on the Higgs and Top Masses,” *Mod. Phys. Lett. A* **2**, 417 (1987).
- [15] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, “The Electroweak phase transition: A Nonperturbative analysis,” *Nucl. Phys. B* **466**, 189 (1996) [hep-lat/9510020].
- [16] M. Fukugita and T. Yanagida, “Baryogenesis Without Grand Unification,” *Phys. Lett. B* **174**, 45 (1986).
- [17] H. P. Nilles, “Supersymmetry, Supergravity and Particle Physics,” *Phys. Rept.* **110**, 1 (1984).
- [18] S. P. Martin, “A Supersymmetry primer,” *Adv. Ser. Direct. High Energy Phys.* **21**, 1 (2010) [*Adv. Ser. Direct. High Energy Phys.* **18**, 1 (1998)] [hep-ph/9709356].
- [19] T. Gherghetta, C. F. Kolda and S. P. Martin, “Flat directions in the scalar potential of the supersymmetric standard model,” *Nucl. Phys. B* **468**, 37 (1996) [hep-ph/9510370].
- [20] I. Affleck and M. Dine, “A New Mechanism for Baryogenesis,” *Nucl. Phys. B* **249**, 361 (1985).
- [21] M. Dine, L. Randall and S. D. Thomas, “Baryogenesis from flat directions of the supersymmetric standard model,” *Nucl. Phys. B* **458**, 291 (1996) [hep-ph/9507453].
- [22] M. Bolz, A. Brandenburg and W. Buchmuller, “Thermal production of gravitinos,” *Nucl. Phys. B* **606**, 518 (2001) Erratum: [*Nucl. Phys. B* **790**, 336 (2008)] [hep-ph/0012052].

- [23] S. Weinberg, “Cosmological Constraints on the Scale of Supersymmetry Breaking,” *Phys. Rev. Lett.* **48**, 1303 (1982).
- [24] M. Y. Khlopov and A. D. Linde, “Is It Easy to Save the Gravitino?,” *Phys. Lett.* **138B**, 265 (1984).
- [25] J. R. Ellis, J. E. Kim and D. V. Nanopoulos, “Cosmological Gravitino Regeneration and Decay,” *Phys. Lett.* **145B**, 181 (1984).
- [26] M. Kawasaki and T. Moroi, “Gravitino production in the inflationary universe and the effects on big bang nucleosynthesis,” *Prog. Theor. Phys.* **93**, 879 (1995) [[hep-ph/9403364](#)].
- [27] T. Moroi, H. Murayama and M. Yamaguchi, “Cosmological constraints on the light stable gravitino,” *Phys. Lett. B* **303**, 289 (1993).
- [28] M. Kawasaki, K. Kohri and T. Moroi, “Big-Bang nucleosynthesis and hadronic decay of long-lived massive particles,” *Phys. Rev. D* **71**, 083502 (2005) [[astro-ph/0408426](#)].
- [29] M. Kawasaki, K. Kohri, T. Moroi and A. Yotsuyanagi, “Big-Bang Nucleosynthesis and Gravitino,” *Phys. Rev. D* **78**, 065011 (2008) [[arXiv:0804.3745 \[hep-ph\]](#)].
- [30] M. Kawasaki, K. Kohri, T. Moroi and Y. Takaesu, “Revisiting Big-Bang Nucleosynthesis Constraints on Long-Lived Decaying Particles,” [arXiv:1709.01211 \[hep-ph\]](#).
- [31] M. Grana, “Flux compactifications in string theory: A Comprehensive review,” *Phys. Rept.* **423**, 91 (2006) [[hep-th/0509003](#)].
- [32] M. R. Douglas and S. Kachru, “Flux compactification,” *Rev. Mod. Phys.* **79**, 733 (2007) [[hep-th/0610102](#)].
- [33] R. Blumenhagen, B. Kors, D. Lust and S. Stieberger, “Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes,” *Phys. Rept.* **445**, 1 (2007) [[hep-th/0610327](#)].
- [34] B. de Carlos, J. A. Casas, F. Quevedo and E. Roulet, “Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings,” *Phys. Lett. B* **318**, 447 (1993) [[hep-ph/9308325](#)].
- [35] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross, “Cosmological Problems for the Polonyi Potential,” *Phys. Lett.* **131B**, 59 (1983).

- [36] T. Banks, D. B. Kaplan and A. E. Nelson, “Cosmological implications of dynamical supersymmetry breaking,” *Phys. Rev. D* **49**, 779 (1994) [hep-ph/9308292].
- [37] M. Kawasaki and T. Yanagida, “Constraint on cosmic density of the string moduli field in gauge mediated supersymmetry breaking theories,” *Phys. Lett. B* **399**, 45 (1997) [hep-ph/9701346].
- [38] J. Hashiba, M. Kawasaki and T. Yanagida, “Cosmological constraint on the string dilaton in gauge mediated supersymmetry breaking theories,” *Phys. Rev. Lett.* **79**, 4525 (1997) [hep-ph/9708226].
- [39] M. Kawasaki, K. Kohri and N. Sugiyama, “Cosmological constraints on late time entropy production,” *Phys. Rev. Lett.* **82**, 4168 (1999) [astro-ph/9811437].
- [40] M. Kawasaki, K. Kohri and N. Sugiyama, “MeV scale reheating temperature and thermalization of neutrino background,” *Phys. Rev. D* **62**, 023506 (2000) [astro-ph/0002127].
- [41] J. Polonyi, “Generalization of the Massive Scalar Multiplet Coupling to the Supergravity,” Hungary Central Inst Res - KFKI-77-93 (77,REC.JUL 78) 5p
- [42] K. Yamamoto, “Phase Transition Associated With Intermediate Gauge Symmetry Breaking in Superstring Models,” *Phys. Lett.* **168B**, 341 (1986).
- [43] D. H. Lyth and E. D. Stewart, “Thermal inflation and the moduli problem,” *Phys. Rev. D* **53**, 1784 (1996) [hep-ph/9510204].
- [44] T. Barreiro, E. J. Copeland, D. H. Lyth and T. Prokopec, “Some aspects of thermal inflation: The Finite temperature potential and topological defects,” *Phys. Rev. D* **54**, 1379 (1996) [hep-ph/9602263].
- [45] T. Asaka, J. Hashiba, M. Kawasaki and T. Yanagida, “Cosmological moduli problem in gauge mediated supersymmetry breaking theories,” *Phys. Rev. D* **58**, 083509 (1998) [hep-ph/9711501].
- [46] T. Asaka and M. Kawasaki, “Cosmological moduli problem and thermal inflation models,” *Phys. Rev. D* **60**, 123509 (1999) [hep-ph/9905467].
- [47] T. Hayakawa, M. Kawasaki and M. Yamada, “Can thermal inflation be consistent with baryogenesis in gauge-mediated SUSY breaking models?,” *Phys. Rev. D* **93**, no. 6, 063529 (2016) [arXiv:1508.03409 [hep-ph]].

- [48] K. Harigaya, T. Hayakawa, M. Kawasaki and M. Yamada, “Cosmology with a Heavy Polonyi Field,” JCAP **1606**, no. 06, 015 (2016) [arXiv:1601.02140 [hep-ph]].
- [49] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], “Evidence for oscillation of atmospheric neutrinos,” Phys. Rev. Lett. **81**, 1562 (1998) [hep-ex/9807003].
- [50] Q. R. Ahmad *et al.* [SNO Collaboration], “Measurement of the rate of  $\nu_e + d \rightarrow p + p + e^-$  interactions produced by  $^8B$  solar neutrinos at the Sudbury Neutrino Observatory,” Phys. Rev. Lett. **87**, 071301 (2001) [nucl-ex/0106015].
- [51] M. Dine, W. Fischler and M. Srednicki, “Supersymmetric Technicolor,” Nucl. Phys. B **189**, 575 (1981).
- [52] S. Dimopoulos and S. Raby, “Supercolor,” Nucl. Phys. B **192**, 353 (1981).
- [53] E. Witten, “Dynamical Breaking of Supersymmetry,” Nucl. Phys. B **188**, 513 (1981).
- [54] S. Dimopoulos and H. Georgi, “Softly Broken Supersymmetry and SU(5),” Nucl. Phys. B **193**, 150 (1981).
- [55] H. K. Dreiner, “An Introduction to explicit R-parity violation,” Adv. Ser. Direct. High Energy Phys. **21**, 565 (2010) [hep-ph/9707435].
- [56] R. Barbier *et al.*, “R-parity violating supersymmetry,” Phys. Rept. **420**, 1 (2005) [hep-ph/0406039].
- [57] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, “Yang-Mills Theories with Local Supersymmetry: Lagrangian, Transformation Laws and SuperHiggs Effect,” Nucl. Phys. B **212**, 413 (1983).
- [58] A. H. Chamseddine, R. L. Arnowitt and P. Nath, “Locally Supersymmetric Grand Unification,” Phys. Rev. Lett. **49**, 970 (1982).
- [59] R. Barbieri, S. Ferrara and C. A. Savoy, “Gauge Models with Spontaneously Broken Local Supersymmetry,” Phys. Lett. **119B**, 343 (1982).
- [60] L. J. Hall, J. D. Lykken and S. Weinberg, “Supergravity as the Messenger of Supersymmetry Breaking,” Phys. Rev. D **27**, 2359 (1983).
- [61] N. Ohta, “Grand Unified Theories Based On Local Supersymmetry,” Prog. Theor. Phys. **70**, 542 (1983).

- [62] J. R. Ellis, D. V. Nanopoulos and K. Tamvakis, “Grand Unification in Simple Supergravity,” *Phys. Lett.* **121B**, 123 (1983).
- [63] L. Alvarez-Gaume, J. Polchinski and M. B. Wise, “Minimal Low-Energy Supergravity,” *Nucl. Phys. B* **221**, 495 (1983).
- [64] M. Dine and W. Fischler, “A Phenomenological Model of Particle Physics Based on Supersymmetry,” *Phys. Lett.* **110B**, 227 (1982).
- [65] L. Alvarez-Gaume, M. Claudson and M. B. Wise, “Low-Energy Supersymmetry,” *Nucl. Phys. B* **207**, 96 (1982).
- [66] M. Dine and A. E. Nelson, “Dynamical supersymmetry breaking at low-energies,” *Phys. Rev. D* **48**, 1277 (1993) [hep-ph/9303230].
- [67] M. Dine, A. E. Nelson and Y. Shirman, “Low-energy dynamical supersymmetry breaking simplified,” *Phys. Rev. D* **51**, 1362 (1995) [hep-ph/9408384].
- [68] M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, “New tools for low-energy dynamical supersymmetry breaking,” *Phys. Rev. D* **53**, 2658 (1996) [hep-ph/9507378].
- [69] T. Moroi, M. Nagai and M. Takimoto, “Non-Thermal Production of Wino Dark Matter via the Decay of Long-Lived Particles,” *JHEP* **1307**, 066 (2013) [arXiv:1303.0948 [hep-ph]].
- [70] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, “De Sitter vacua in string theory,” *Phys. Rev. D* **68**, 046005 (2003) [hep-th/0301240].
- [71] H. P. Nilles, “Dynamically Broken Supergravity and the Hierarchy Problem,” *Phys. Lett.* **115B**, 193 (1982).
- [72] S. Ferrara, L. Girardello and H. P. Nilles, “Breakdown of Local Supersymmetry Through Gauge Fermion Condensates,” *Phys. Lett.* **125B**, 457 (1983).
- [73] A. D. Linde, “Relaxing the cosmological moduli problem,” *Phys. Rev. D* **53**, R4129 (1996) [hep-th/9601083].
- [74] F. Takahashi and T. T. Yanagida, “Strong dynamics at the Planck scale as a solution to the cosmological moduli problem,” *JHEP* **1101**, 139 (2011) [arXiv:1012.3227 [hep-ph]].



- [75] F. Takahashi and T. T. Yanagida, “Why have supersymmetric particles not been observed?,” *Phys. Lett. B* **698**, 408 (2011) [arXiv:1101.0867 [hep-ph]].
- [76] K. Nakayama, F. Takahashi and T. T. Yanagida, “Cosmological Moduli Problem in Low Cutoff Theory,” *Phys. Rev. D* **86**, 043507 (2012) [arXiv:1112.0418 [hep-ph]].
- [77] K. Nakayama, F. Takahashi and T. T. Yanagida, “Gravity mediation without a Polonyi problem,” *Phys. Lett. B* **714**, 256 (2012) [arXiv:1203.2085 [hep-ph]].
- [78] K. Harigaya, M. Ibe, K. Schmitz and T. T. Yanagida, “A Simple Solution to the Polonyi Problem in Gravity Mediation,” *Phys. Lett. B* **721**, 86 (2013) [arXiv:1301.3685 [hep-ph]].
- [79] Y. B. Zeldovich, I. Y. Kobzarev and L. B. Okun, “Cosmological Consequences of the Spontaneous Breakdown of Discrete Symmetry,” *Zh. Eksp. Teor. Fiz.* **67**, 3 (1974) [*Sov. Phys. JETP* **40**, 1 (1974)].
- [80] H. Murayama, H. Suzuki and T. Yanagida, “Radiative breaking of Peccei-Quinn symmetry at the intermediate mass scale,” *Phys. Lett. B* **291**, 418 (1992).
- [81] M. Endo, K. Hamaguchi and F. Takahashi, “Moduli-induced gravitino problem,” *Phys. Rev. Lett.* **96**, 211301 (2006) [hep-ph/0602061].
- [82] S. Nakamura and M. Yamaguchi, “Gravitino production from heavy moduli decay and cosmological moduli problem revived,” *Phys. Lett. B* **638**, 389 (2006) [hep-ph/0602081].
- [83] M. Dine, R. Kitano, A. Morisse and Y. Shirman, “Moduli decays and gravitinos,” *Phys. Rev. D* **73**, 123518 (2006) [hep-ph/0604140].
- [84] M. Endo, K. Hamaguchi and F. Takahashi, “Moduli/Inflaton Mixing with Supersymmetry Breaking Field,” *Phys. Rev. D* **74**, 023531 (2006) [hep-ph/0605091].
- [85] T. Moroi and L. Randall, “Wino cold dark matter from anomaly mediated SUSY breaking,” *Nucl. Phys. B* **570**, 455 (2000) [hep-ph/9906527].
- [86] W. Buchmuller, P. Di Bari and M. Plumacher, “Leptogenesis for pedestrians,” *Annals Phys.* **315**, 305 (2005) [hep-ph/0401240].
- [87] S. Liebler, S. Profumo and T. Stefaniak, “Light Stop Mass Limits from Higgs Rate Measurements in the MSSM: Is MSSM Electroweak Baryogenesis Still Alive After All?,” *JHEP* **1604**, 143 (2016) [arXiv:1512.09172 [hep-ph]].

- [88] J. M. Cline, “Is electroweak baryogenesis dead?,” *Phil. Trans. Roy. Soc. Lond. A* (2017) [arXiv:1704.08911 [hep-ph]].
- [89] K. Enqvist and J. McDonald, “Q balls and baryogenesis in the MSSM,” *Phys. Lett. B* **425**, 309 (1998) [hep-ph/9711514].
- [90] A. de Gouvea, T. Moroi and H. Murayama, “Cosmology of supersymmetric models with low-energy gauge mediation,” *Phys. Rev. D* **56**, 1281 (1997) [hep-ph/9701244].
- [91] A. Kusenko and M. E. Shaposhnikov, “Supersymmetric Q balls as dark matter,” *Phys. Lett. B* **418**, 46 (1998) [hep-ph/9709492].
- [92] K. Hamaguchi, M. Ibe, T. T. Yanagida and N. Yokozaki, “Testing the Minimal Direct Gauge Mediation at the LHC,” *Phys. Rev. D* **90**, no. 1, 015027 (2014) [arXiv:1403.1398 [hep-ph]].
- [93] S. Kasuya, M. Kawasaki and T. T. Yanagida, “IceCube potential for detecting Q-ball dark matter in gauge mediation,” *PTEP* **2015**, no. 5, 053B02 (2015) [arXiv:1502.00715 [hep-ph]].
- [94] See, for example, E. W. Kolb and M. S. Turner, “The Early Universe,” *Front. Phys.* **69**, 1 (1990).
- [95] R. Allahverdi, B. A. Campbell and J. R. Ellis, “Reheating and supersymmetric flat direction baryogenesis,” *Nucl. Phys. B* **579**, 355 (2000) [hep-ph/0001122].
- [96] A. Anisimov and M. Dine, “Some issues in flat direction baryogenesis,” *Nucl. Phys. B* **619**, 729 (2001) [hep-ph/0008058].
- [97] M. Fujii, K. Hamaguchi and T. Yanagida, “Reheating temperature independence of cosmological baryon asymmetry in Affleck-Dine leptogenesis,” *Phys. Rev. D* **63**, 123513 (2001) [hep-ph/0102187].
- [98] S. Y. Khlebnikov and M. E. Shaposhnikov, “The Statistical Theory of Anomalous Fermion Number Nonconservation,” *Nucl. Phys. B* **308**, 885 (1988).
- [99] J. A. Harvey and M. S. Turner, “Cosmological baryon and lepton number in the presence of electroweak fermion number violation,” *Phys. Rev. D* **42**, 3344 (1990).
- [100] S. R. Coleman, “Q Balls,” *Nucl. Phys. B* **262**, 263 (1985) Erratum: [*Nucl. Phys. B* **269**, 744 (1986)].

- [101] S. Kasuya and M. Kawasaki, “Q ball formation through Affleck-Dine mechanism,” *Phys. Rev. D* **61**, 041301 (2000) [hep-ph/9909509].
- [102] S. Kasuya and M. Kawasaki, “A New type of stable Q balls in the gauge mediated SUSY breaking,” *Phys. Rev. Lett.* **85**, 2677 (2000) [hep-ph/0006128].
- [103] S. Kasuya and M. Kawasaki, “Q ball formation: Obstacle to Affleck-Dine baryogenesis in the gauge mediated SUSY breaking?,” *Phys. Rev. D* **64**, 123515 (2001) [hep-ph/0106119].
- [104] G. R. Dvali, A. Kusenko and M. E. Shaposhnikov, “New physics in a nutshell, or Q ball as a power plant,” *Phys. Lett. B* **417**, 99 (1998) [hep-ph/9707423].
- [105] S. Kasuya and M. Kawasaki, “Q Ball formation in the gravity mediated SUSY breaking scenario,” *Phys. Rev. D* **62**, 023512 (2000) [hep-ph/0002285].
- [106] T. Hiramatsu, M. Kawasaki and F. Takahashi, “Numerical study of Q-ball formation in gravity mediation,” *JCAP* **1006**, 008 (2010) [arXiv:1003.1779 [hep-ph]].
- [107] M. Laine and M. E. Shaposhnikov, “Thermodynamics of nontopological solitons,” *Nucl. Phys. B* **532**, 376 (1998) [hep-ph/9804237].
- [108] S. Kasuya and M. Kawasaki, “Baryogenesis from the gauge-mediation type Q-ball and the new type of Q-ball as the dark matter,” *Phys. Rev. D* **89**, no. 10, 103534 (2014) [arXiv:1402.4546 [hep-ph]].
- [109] S. S. Clark, “Particle production from Q-balls,” *Nucl. Phys. B* **756**, 38 (2006) [hep-ph/0510078].
- [110] M. Kawasaki and M. Yamada, “Q ball Decay Rates into Gravitinos and Quarks,” *Phys. Rev. D* **87**, no. 2, 023517 (2013) [arXiv:1209.5781 [hep-ph]].
- [111] K. Harigaya, A. Kamada, M. Kawasaki, K. Mukaida and M. Yamada, “Affleck-Dine Baryogenesis and Dark Matter Production after High-scale Inflation,” *Phys. Rev. D* **90**, no. 4, 043510 (2014) [arXiv:1404.3138 [hep-ph]].
- [112] S. Kasuya, M. Kawasaki and F. Takahashi, “On the moduli problem and baryogenesis in gauge mediated SUSY breaking models,” *Phys. Rev. D* **65**, 063509 (2002) [hep-ph/0108171].

- [113] E. D. Stewart, M. Kawasaki and T. Yanagida, “Affleck-Dine baryogenesis after thermal inflation,” *Phys. Rev. D* **54**, 6032 (1996) [hep-ph/9603324].
- [114] D. h. Jeong, K. Kadota, W. I. Park and E. D. Stewart, “Modular cosmology, thermal inflation, baryogenesis and predictions for particle accelerators,” *JHEP* **0411**, 046 (2004) [hep-ph/0406136].
- [115] M. Kawasaki and K. Nakayama, “Late-time Affleck-Dine baryogenesis after thermal inflation,” *Phys. Rev. D* **74**, 123508 (2006) [hep-ph/0608335].
- [116] L. Randall and R. Sundrum, *Nucl. Phys. B* **557**, 79 (1999) doi:10.1016/S0550-3213(99)00359-4 [hep-th/9810155].
- [117] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, *JHEP* **9812**, 027 (1998) doi:10.1088/1126-6708/1998/12/027 [hep-ph/9810442].
- [118] M. Kawasaki, T. Watari and T. Yanagida, “Vacuum instability in anomaly mediation models with massive neutrinos,” *Phys. Rev. D* **63**, 083510 (2001) [hep-ph/0010124].
- [119] M. Fujii, K. Hamaguchi and T. Yanagida, “Affleck-Dine baryogenesis / leptogenesis with a gauged  $U(1)(B-L)$ ,” *Phys. Rev. D* **64**, 123526 (2001) [hep-ph/0104186].
- [120] M. Fujii and K. Hamaguchi, “Nonthermal dark matter via Affleck-Dine baryogenesis and its detection possibility,” *Phys. Rev. D* **66**, 083501 (2002) [hep-ph/0205044].
- [121] M. Kawasaki and K. Nakayama, “Affleck-Dine baryogenesis in anomaly-mediated SUSY breaking,” *JCAP* **0702**, 002 (2007) [hep-ph/0611320].
- [122] K. Harigaya, M. Ibe, M. Kawasaki and T. T. Yanagida, “Dynamics of Peccei-Quinn Breaking Field after Inflation and Axion Isocurvature Perturbations,” *JCAP* **1511**, no. 11, 003 (2015) [arXiv:1507.00119 [hep-ph]].
- [123] K. Inoue, M. Kawasaki, M. Yamaguchi and T. Yanagida, “Vanishing squark and slepton masses in a class of supergravity models,” *Phys. Rev. D* **45**, 328 (1992).
- [124] M. A. Luty and R. Sundrum, “Supersymmetry breaking and composite extra dimensions,” *Phys. Rev. D* **65**, 066004 (2002) [hep-th/0105137].
- [125] M. Luty and R. Sundrum, “Anomaly mediated supersymmetry breaking in four-dimensions, naturally,” *Phys. Rev. D* **67**, 045007 (2003) [hep-th/0111231].

- [126] M. Ibe, K.-I. Izawa, Y. Nakayama, Y. Shinbara and T. Yanagida, “Conformally sequestered SUSY breaking in vector-like gauge theories,” *Phys. Rev. D* **73**, 015004 (2006) [hep-ph/0506023].
- [127] M. Ibe, K.-I. Izawa, Y. Nakayama, Y. Shinbara and T. Yanagida, “More on conformally sequestered SUSY breaking,” *Phys. Rev. D* **73**, 035012 (2006) [hep-ph/0509229].
- [128] I. Antoniadis and M. Quiros, “Supersymmetry breaking in M theory and gaugino condensation,” *Nucl. Phys. B* **505**, 109 (1997) [hep-th/9705037].
- [129] A. E. Nelson and N. J. Weiner, “Extended anomaly mediation and new physics at 10-TeV,” hep-ph/0210288.
- [130] K. Hsieh and M. A. Luty, “Mixed gauge and anomaly mediation from new physics at 10-TeV,” *JHEP* **0706**, 062 (2007) [hep-ph/0604256].
- [131] A. Gupta, D. E. Kaplan and T. Zorawski, “Gaugomally Mediation Revisited,” *JHEP* **1311**, 149 (2013) [arXiv:1212.6969 [hep-ph]].
- [132] A. Pomarol and R. Rattazzi, “Sparticle masses from the superconformal anomaly,” *JHEP* **9905**, 013 (1999) [hep-ph/9903448].
- [133] N. Abe, T. Moroi and M. Yamaguchi, “Anomaly mediated supersymmetry breaking with axion,” *JHEP* **0201**, 010 (2002) [hep-ph/0111155].
- [134] Y. Okada, M. Yamaguchi and T. Yanagida, “Upper bound of the lightest Higgs boson mass in the minimal supersymmetric standard model,” *Prog. Theor. Phys.* **85**, 1 (1991).
- [135] Y. Okada, M. Yamaguchi and T. Yanagida, “Renormalization group analysis on the Higgs mass in the softly broken supersymmetric standard model,” *Phys. Lett. B* **262**, 54 (1991).
- [136] J. R. Ellis, G. Ridolfi and F. Zwirner, “Radiative corrections to the masses of supersymmetric Higgs bosons,” *Phys. Lett. B* **257**, 83 (1991).
- [137] H. E. Haber and R. Hempfling, “Can the mass of the lightest Higgs boson of the minimal supersymmetric model be larger than  $m(Z)$ ?,” *Phys. Rev. Lett.* **66**, 1815 (1991).

- [138] J. R. Ellis, G. Ridolfi and F. Zwirner, “On radiative corrections to supersymmetric Higgs boson masses and their implications for LEP searches,” *Phys. Lett. B* **262**, 477 (1991).
- [139] U. Ellwanger, C. Hugonie and A. M. Teixeira, “The Next-to-Minimal Supersymmetric Standard Model,” *Phys. Rept.* **496**, 1 (2010) [arXiv:0910.1785 [hep-ph]].
- [140] M. Endo and F. Takahashi, “Non-thermal Production of Dark Matter from Late-Decaying Scalar Field at Intermediate Scale,” *Phys. Rev. D* **74**, 063502 (2006) [hep-ph/0606075].
- [141] S. Nakamura and M. Yamaguchi, “A Note on Polonyi Problem,” *Phys. Lett. B* **655**, 167 (2007) [arXiv:0707.4538 [hep-ph]].
- [142] M. Endo, F. Takahashi and T. T. Yanagida, “Anomaly-induced inflaton decay and gravitino-overproduction problem,” *Phys. Lett. B* **658**, 236 (2008) [hep-ph/0701042].
- [143] R. D. Peccei and H. R. Quinn, “CP Conservation in the Presence of Instantons,” *Phys. Rev. Lett.* **38**, 1440 (1977).
- [144] R. D. Peccei and H. R. Quinn, “Constraints Imposed by CP Conservation in the Presence of Instantons,” *Phys. Rev. D* **16**, 1791 (1977).
- [145] S. Weinberg, “A New Light Boson?,” *Phys. Rev. Lett.* **40**, 223 (1978).
- [146] F. Wilczek, “Problem of Strong p and t Invariance in the Presence of Instantons,” *Phys. Rev. Lett.* **40**, 279 (1978).
- [147] M. Kawasaki and K. Nakayama, “Axions: Theory and Cosmological Role,” *Ann. Rev. Nucl. Part. Sci.* **63**, 69 (2013) [arXiv:1301.1123 [hep-ph]].