The Seventh Asian-Pacific Symposium on Structural Reliability and Its Applications (APSSRA2020)
October 4–7 2020, Tokyo, Japan
T. Takada, I. Yoshida & T. Itoi (editors)

Reliability Analysis of Structures Equipped with Friction-Based Isolators under Stochastic Ground Motion

H. Jensen[1], F. Mayorga[1] and C. Figueroa[1]

[1]*Federico Santa Maria Technical University. Email:* hector.jensen@usm.cl

Abstract: This work presents a general framework for performing reliability analyses of involved structural models equipped with friction-based devices under stochastic excitation. An experimentally validated model that takes into account main sources of performance degradation that these devices experience during seismic events is considered. To deal with the variability of future excitations, a stochastic excitation model is adopted in the present formulation. First excursion probabilities are used as measures of system reliability. The associated reliability analysis is carried out by combining an advanced simulation technique with an adaptive parametric meta-model. The effectiveness of the methodology is demonstrated by means of one application problem.

Keywords: advanced simulation methods, reliability analysis, sliding bearings, stochastic excitation.

1. General

Isolation concepts have been used for the protection of a number of critical facilities such as hospitals, buildings, bridges, industrial and data center facilities, etc. One of the difficulties in the analysis of isolated systems has been the consideration of realistic models for the non-linear behavior of the isolation devices. Another challenge has been the efficient prediction of the dynamic response under future ground motions considering their potential variability as well as the efficient control of competing objectives related to the protection of the structure and the minimization of the base displacement. Among existing conventional isolation devices, friction-based isolators are suitable for a wide range of applications (Bomdonet and Filiatrault 1997); (Villaverde 2017). In this context, the reliability analysis of isolated systems plays an important role to ensure structural safety and integrity (Chen et al. 2007); (Jensen and Kusanovic 2014).

This contribution presents a framework for the reliability analysis of involved structural systems equipped with friction-based devices. Specifically, devices composed of sliding concave bearings are considered. An experimentally validated model that takes into account main sources of performance degradation that these devices experience during seismic events is implemented in this study (Benzoni and Seible 1998). To take into account the variability of future excitations, a stochastic model for the description of ground motions is implemented (Boore 2003); (Atkinson and Silva 2000). First excursion probabilities are used as measures of the system reliability. Reliability is quantified as the probability that response quantities of interest will not exceed acceptable performance bounds within a particular reference period. Such probabilities are estimated by an adaptive Markov Chain Monte Carlo procedure (Au and Beck 2001). From a numerical and practical point of view, the reliability analysis of stochastic dynamical systems requires a large number of finite element analyses. These analyses correspond to finite element re-analyses over the uncertain parameter space that characterizes the structural parameters and the excitation. Consequently, the computational demands involved in the reliability analysis may be large or even excessive. To deal with this difficulty, a model reduction technique based on substructure coupling for dynamic analysis is considered in the present implementation (Craig and Kurdila 2006). The technique is combined with an adaptive finite element model parametrization scheme for approximating the different quantities involved in the definition of the reduced-order models in terms of the uncertain structural parameters (Angelikopoulos et al. 2015); (Jensen and Papadimitriou 2019).

2. Isolator Modeling

A sliding bearing, which is shown schematically in Figure 1, is composed of an upper steel plate with a housing cap for the slider, a bottom plate with a concave semi-spherical stainless steel surface, and a steel slider. The low friction material that interfaces the stainless concave surface is usually made of an un-lubricated polymer composite liner. A simplified scheme that shows a typical force distribution on the slider of the sliding concave isolator is depicted in Figure 2.

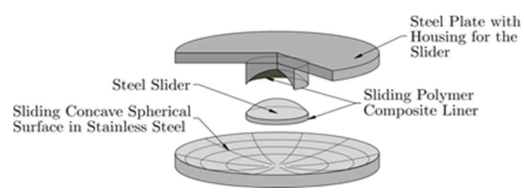

Figure 1. Sliding bearing

In Figure 2, A is the radius of the sliding surface and a is the radius of the slider. According to the simplified Coulomb model, the force-displacement relationship is expressed as

$$\mathrm{f(t)} = \frac{W}{R} u_{is}(t) + \mathrm{sgn}\big(\dot{u}_{is}(t)\big)\mu W \tag{1}$$

where W represents the applied vertical load, $u_{is}(t)$ the horizontal relative displacement between the slider and the concave base, $\dot{u}_{is}(t)$ the corresponding velocity, R the radius of the concave surface, μ the friction coefficient of the sliding system due to the composite material at the bottom of the slider and the stainless steel overlay of the concave base, and $f(t)$ the horizontal force. It is seen that the horizontal force $f(t)$ is partially resisted by the force μW due to the frictional characteristics of the contact between the slider and steel concave surface. The remaining force is resisted by the linear component $(W/R)u_{is}(t)$ associated with the restoring stiffness W/R.

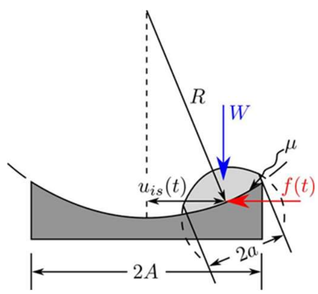

Figure 2. Force distribution on the slider of the sliding concave isolator.

Regarding the frictional force, the simplified Coulomb model indicates that the force is directly proportional to the applied vertical load and not dependent upon the apparent area of contact and sliding velocity. However, experimental results indicate three mayor effects that are responsible for the departure of the frictional performance of the device from the theoretical Coulomb's model (Lomiento et al. 2013). They include the load, cycling and velocity effects. The load effect is related to the reduction of the friction coefficient as the vertical load increases, the cycling effect corresponds to the continued reduction of the friction coefficient with the repetition of cycles, and finally the velocity effect takes into account the increase of the friction coefficient with the velocity of motion.

A model able to represent the variation of the frictional characteristics of the sliding system along the traveled path is formulated as follows. The friction coefficient is expressed as the product of three components (Lomiento et al. 2013).

$$\mu\big(\mathrm{W}, \mathrm{c(t)}, \dot{u}_{is}(t)\big) = f_W(\mathrm{W}) f_c\big(\mathrm{c(t)}\big) f_{\dot{u}_{is}}\big(\dot{u}_{is}(t)\big) \quad (2)$$

where $f_W(\cdot), f_c(\cdot)$ and $f_{\dot{u}_{is}}(\cdot)$ are functions representing the dependency of the coefficient of friction on the applied vertical load, the cycling effect, and the velocity, respectively. The load effect term is expressed as $f_W(W) = \mu_0 \exp\big(-W/W_{ref}\big)$, where μ_0 represents the theoretical slow-motion coefficient of friction under no vertical load (initial friction coefficient), and W_{ref} is a load reference value and represents the degradation rate of the slow-motion coefficient of friction. The parameters μ_0 and W_{ref} are calibrated by using experimental data. The friction degradation function, related to the cycling effect, is defined as $f_c\big(c(t)\big) = \exp(-c(t)/\ c_{ref})^{\beta}$, where c_{ref} represents the degradation rate of the friction coefficient with the cycling variable $c(t)$, and β controls the shape of the function. These parameters are obtained by least square regression of experimental results. The variable $c(t)$ is associated with the cumulative heat flux on the sliding surfaces during the time interval (t_0, t), where t_0 is the initial time of analysis. Assuming a heat flux uniformly distributed over the whole concave sliding surface, the variable $c(t)$ is written as

$$\mathrm{c(t)} = \frac{2}{a\pi^2 \mathrm{A}^2} \int_{\mathrm{t_0}}^{\mathrm{t}} \mathrm{W}\, \dot{u}_{is}(\mathrm{t})^2 \mathrm{dt} \quad (3)$$

where a is the radius of the slider, and A the radius of the sliding surface as previously pointed out. Finally, the increment of the coefficient of friction with increasing sliding velocity is described by the function $f_{u_{is}}\big(\dot{u}_{is}(t)\big) = \eta + (1-\eta)\exp\big(-|\dot{u}_{is}(t)|/\dot{u}_{is}{}^{ref}\big)$, where $\dot{u}_{is}{}^{ref}$ is a reference velocity that characterizes the variation rate, and $\eta \geq 1$ is the ratio between the fast-motion and the slow-motion coefficient of friction. The parameters $\dot{u}_{is}{}^{ref}$ and η are calibrated according to the best fit with experimental results.

Under bi-directional excitations, the resultant horizontal force can be decomposed into a restoring force and a friction force, as in the one-dimensional case. The restoring force is associated with the pendulum behavior of the isolator due to the curvature of the sliding surface, while the friction force is activated at the interface between the sliding surface and the slider. The restoring force is directed towards the geometric center of the sliding concave surface as shown in Figure 3.

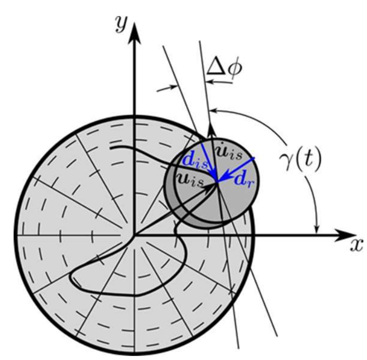

Figure 3. Direction of the restoring and frictional forces. d_r: direction of restoring force. d_{is}: direction of friction force.

Moreover, the friction force vector direction d_{is} shows an angular shift $\Delta\phi$ with respect to the velocity vector $\big(\dot{u}_{is}(t)\big)$. Such angular shift has been estimated to be around 0.24 radians for the types of isolators investigated (Lomiento et al. 2012). For illustration purposes, the direction of the frictional force and velocity vector for a given instant are plotted in Figure 3. Additional information about the bi-directional case can be found in (Lomiento et al. 2013).

3. Reliability Analysis

The performance of isolated structural systems is quantified by means of a set of response functions $r_i(t,\boldsymbol{\theta},\boldsymbol{z}), i=1,\ldots,n_r, t\in[0,T]$, where $\boldsymbol{\theta}\in\Omega_{\boldsymbol{\theta}}\subset\boldsymbol{R}^{n_{\theta}}$is a vector of uncertain structural parameters, $\boldsymbol{z}\in\Omega_{\boldsymbol{z}}\subset R^{n_z}$ is a vector of uncertain variables involved in the characterization of the stochastic excitation model, and n_r, $n_{\boldsymbol{\theta}}$, and $n_{\boldsymbol{z}}$ are the number of response functions of interest, the number of uncertain structural parameters, and the number of uncertain variables defining the stochastic excitation, respectively. In terms of the excitation, the ground acceleration is modeled as a non-stationary stochastic process. In particular, a stochastic model based on a point-source model is considered in the present implementation (Boore 2003); (Atkinson and Silva 2000). The probability that performance conditions are satisfied within a particular reference period T is used as reliability measure. In this context, a failure event F can be defined in terms of a performance function $h(\boldsymbol{\theta},\boldsymbol{z})$ as $F(\boldsymbol{\theta},\boldsymbol{z})=h(\boldsymbol{\theta},\boldsymbol{z})\leq 0$, where

$$h(\boldsymbol{\theta},\boldsymbol{z}) \;=1-\max_{i=1,\ldots n_r}\left(\max_{t\in[0,T]}\frac{|r_i(t,\boldsymbol{\theta},\boldsymbol{z})|}{r_i^*}\right) \qquad (4)$$

and $r_i^*, i=1,\ldots,n_r$ are the corresponding acceptable response levels. The probability of failure P_F can be expressed in terms of the probability integral

$$P_F=\int_{h(\boldsymbol{\theta},\boldsymbol{z})\leq 0} q(\boldsymbol{\theta})p(\boldsymbol{z})d\boldsymbol{\theta}d\boldsymbol{z} \qquad (5)$$

where $q(\boldsymbol{\theta})$ and $p(\boldsymbol{z})$ are multidimensional probability density functions that characterize the structural parameters and the uncertain variables involved in the excitation model, respectively. For systems under stochastic excitation, the reliability estimation for a given value of the model parameters constitutes a high dimensional problem (Koutsourelakis et al. 2004). This problem is solved by applying an advanced simulation technique. In particular, Subset simulation is implemented in this work (Au and Beck 2001).

4. Numerical Implementation

The reliability analysis of complex structural systems is computationally very demanding due to the large number of dynamic analyses required during the corresponding simulation process. In fact, the reliability estimation requires the evaluation of the system response at a large number of samples in the uncertain parameter space (of the order of hundreds or thousands). Consequently, the computational cost may become excessive when the computational time for performing a dynamic analysis is significant. To cope with this difficulty, a parametric model reduction technique is considered in the present implementation. In particular, a method based on substructure coupling is implemented here (Craig and Kurdila 2006). Details of the technique can be found in (Jensen and Papadimitriou 2019) and (Jensen et al. 2020). Once a parametric reduced-order model of the structure has been defined, the equation of motion of the structural system in terms of a reduced set of generalized coordinates can be established. Such set of generalized coordinates are defined in terms of the so-called dominant fixed-interface modal coordinates of all substructures and of the vector of physical coordinates at the independent interfaces. According to this methodology, the evaluation of the reduced-order matrices at a given design is direct, since it depends only on the value of the different substructure matrices at a number of support points. Such points are selected based on an adaptive scheme (Jensen et al. 2020). The equation of motion in terms of the reduced-order model together with the equation for the isolation system can be integrated efficiently by an appropriate step-by-step integration scheme.

5. Application Problem

5.1 Model Description

The structural system, which is shown in Figure 4, consists of a seven floor three-dimensional reinforced concrete building model under stochastic ground acceleration. Material properties of the reinforced concrete structure have been assumed as follows: Young's modulus $E=2.3\times10^{10}N/m^2$; Poisson ratio $\nu=\;0.2$; and mass density $\rho=2{,}500kg/m^3$. The height of each floor is $3.3\ m$ leading to a total height of $23.1\ m$ for the structure. The floors are modeled with shell elements with a thickness of $0.2\ m$ and beam elements of rectangular cross section of dimension $0.5\ m\ \times 0.7\ m$. Each floor is supported by 56 columns of square cross section of dimension $0.5\ m\ \times 0.5\ m$. The finite element model has approximately $38{,}000$ degrees of freedom. A 5% of critical damping is added to the model. The structural system is equipped with 56 sliding bearings in its isolation system. The sliding bearings are characterized by the experimentally validated model introduced in Section 2, with model parameters given in (Jensen et al. 2020). The fundamental period of the base-isolated system is about $2.97\ s$, while the period of the fixed system (without the isolators) is close to $0.97\ s$. Thus, the increase of the fundamental period is significant due to the isolation system. The building is excited horizontally by a ground acceleration applied at -45^o with respect to the axis x, as shown in the figure. The ground excitation is modeled as indicated in previous sections. It is assumed that the bending stiffness of the column elements of the different floors is uncertain. For demonstration purposes, such uncertainty is modeled in terms of three global parameters, namely, θ_1, θ_2 and θ_3. The parameter θ_1 is associated with the bending stiffness of the column elements of the first floor, θ_2 is related to the bending stiffness of the column elements of floors 2 to 4, and θ_3 is associated with the bending stiffness of the column elements of floors 5 to 7. These parameters are modeled as independent and identically distributed log-normal random variables with mean values equal to 1.0, and coefficient of variations of 15%. The global parameters scale the nominal values of the bending stiffness of the column elements. A parametric reduced order model is constructed for performing a reliability analysis. The dimension of the reduced-order model represents a reduction of more than 90% with respect to the full finite element model. The reader is referred to (Jensen et al. 2020) for more information about the reduced-order model.

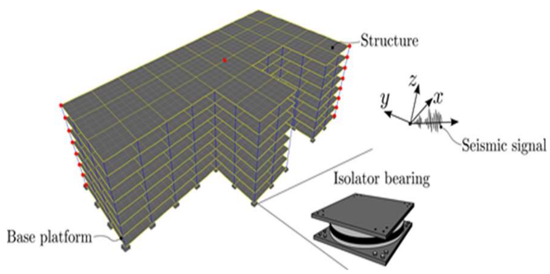

Figure 4. Isometric view of finite element model with base isolation system.

5.2 Results

The system reliability is defined in terms of the relative displacement between the different floors of the building. The corresponding failure event is defined as

$$F(\boldsymbol{\theta},\boldsymbol{z}) = \max_{t\in[0,T],i=1,\ldots,24}\left(\frac{|\delta_i(t,\boldsymbol{\theta},\boldsymbol{z})|}{\delta^*}\right) \geq 1, \qquad (6)$$

where $\delta_i(t,\boldsymbol{\theta},\boldsymbol{z})$ denotes the maximum relative displacement between the different floors at control point $i = 1,\ldots,24$, and δ^* is the corresponding acceptable response level. The control points are located over the height of the structure at different corners and at the center of each floor. Some of these points, represented as dots, are shown in Figure 4. Figure 5 shows the failure probability in terms of the threshold. For comparison purposes, the cases of fixed- and isolated-base are considered. An average of five independent runs of subset simulation is used in the figure. It is observed that the effect of the isolation system is quite dramatic in terms of the system reliability. In other words, the beneficial effect of the sliding bearings in protecting the structure is significant.

The effect of considering the simplified Coulomb model for the sliding bearings is examined in Figure 6. In this case, the reliability of the system is considered in terms of the absolute displacement at the top of the building. This is done in order to take into account the displacement experienced by the isolation system. Figure 6 shows the corresponding failure probability with respect to the threshold by using the experimentally validated model and the simplified friction model. It is seen that the simplified model overestimates the system reliability. This is due to the fact that the simplified model underestimates the displacement of the sliding bearings. Thus, the importance of considering the experimentally validated model for the performance of the friction-based devices is apparent.

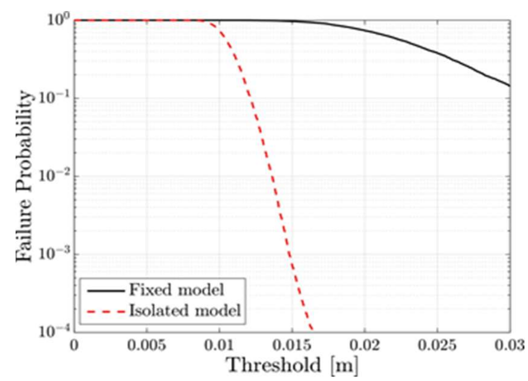

Figure 5. Probability of failure in terms of the threshold.

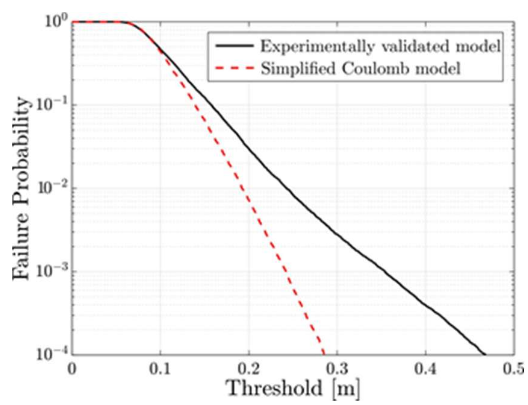

Figure 6. Probability of failure in terms of the threshold. Failure event associated with the absolute displacement at the top of the building.

5.3 Computational Demand

Validation calculations show that the computational demands involved in the reliability analysis are decreased by using the proposed parametric reduced-order model without compromising the accuracy of the reliability estimates. Actually, a speedup value of more than 6 is obtained in this case for the on-line computations. In this context, the speedup is the ratio of the execution time for performing the reliability analysis by using the full finite element model and the reduced-order model. The gain in computational savings for this structural model is significant considering the complexity of the model equipped with friction-based devices. Finally, it is noted that once a parametric reduced-order model has been defined, several scenarios in terms of reliability analyses can be performed in an efficient manner. Therefore, even higher speedup values can be obtained for the reliability analysis as a whole.

6. Conclusions

A framework for performing reliability analyses of involved structural models equipped with friction-based devices under stochastic excitation has been presented. The experimentally validated model for the performance of sliding concave isolators takes into account main sources of performance degradation experienced by these devices during seismic events. With respect to the numerical implementation, the adaptive parametric model reduction technique has been validated in an involved model. Results show that an important reduction in computational efforts can be achieved without compromising the accuracy of the reliability estimates. Numerical results indicate that the simplified theoretical Coulomb's model tends to underestimate the displacements experienced by the sliding bearings. Moreover, the simplified model leads to failure probability estimates smaller than the ones obtained by the experimentally validated model. As a consequence, the simplified model overestimates the reliability of the system. Based on the application problem and additional validation calculations, it is concluded that the beneficial effect of the sliding bearings in protecting structural system is significant. Actually, the effect of the devices is quite dramatic in terms of the system reliability.

7. Acknowledgements

This research is partially supported by Universidad Tecnica Federico Santa Maria under its program PAC (Programa Asistente Cientifico 2017). This support is gratefully acknowledged by the authors.

References

Angelikopoulos, P., Papadimitriou, C. and Koumoutsakos P. 2015. X-TMCMC: adaptive kriging for Bayesian inverse modeling. *Computer Methods in Applied Mechanics and Engineering*, 289:409-428.

Atkinson, G.M. and Silva, W. 2000. Stochastic modeling of California ground motions. *Bulletin of the Seismological Society of America*, 90(2):255-274.

Au, S.K. and Beck, J.L. 2001. Estimation of small probabilities in high dimensions by subset simulation. *Probabilistic Engineering Mechanics,* 16(4):263-277.

Benzoni, G. and Seible, F. 1998. Design of The Caltrans Seismic Response Modification Device (SRMD) Test Facility. Proc. of the USA-Italy Workshop on Protective Systems, Report No. MCEER-98-0015, *Multidiciplinary Center of Earthquake Engineering Research*, Buffalo, New York City.

Bomdonet, G. and Filiatrault, A. 1997. Frictional Response of PTFE Sliding bearings at higher frequencies. *Journal of Bridge Engineering*, 2(4):139-148.

Boore, D.M. 2003. Simulation of ground motion using the stochastic method. *Pure and Applied Geophysics*, 160(3-4):635-676.

Craig Jr., R.R. and Kurdila, A.J. 2006. Fundamentals of Structural dynamics. John Wiley & Sons, Second Edition, Hoboken, New Jersey.

Chen, J.B., Liu, W.Q., Peng Y.P. and Li, J. 2007. Stochastic seismic response and reliability analysis of base-isolated structures. *Earthquake Engineering*, 11(6):903-924.

Jensen, H.A, and Kusanovic, D.S. 2014. On the effect of near-field excitations on the reliability-based performance and design of base-isolated structures. *Probabilistic Engineering Mechanics*, 36:28-44.

Jensen, H.A. and Papadimitriou, C. 2019. Sub-structure Coupling for Dynamic Analysis. Application to Complex Simulation-Based Problems Involving Uncertainty. Series: *Lecture Notes in Applied and Computational Mechanics*, number 89, Springer.

Jensen, H.A., Mayorga, F. and Valdebenito M. 2020. On the reliability of structures equipped with a class of friction-based devices under stochastic excitation. *Computer Methods in Applied Mechanics and Engineering*. 364, 112965.

Koutsourelakis, P.S., Pradlwarter, H.J. and Schueller, G.I. 2004. Reliability of structures in high dimensions, part I: Algorithms and applications. *Probabilistic Engineering Mechanics*, 19(4):409-417.

Lomiento, G., Bonessio, N. and Benzoni, G. 2012. Effects of loading characteristics on the performance of sliding isolation devices. *15th World Conference on Earthquake Engineering*, Lisbon, Portugal.

Lomiento, G., Bonessio, N. and Benzoni, G. 2013. Friction model for sliding bearings under seismic excitation. *Journal of Earthquake Engineering*, 17:1162-1191.

Villaverde, R. 2017. Base isolation with sliding hydromagnetic bearings: Concept and feasibility study. *Structure and Infrastructure Engineering*, 13(6):709-721.