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Reliability Evaluation for Complex System Based on Bayesian Theory and Multi-source Information Fusion

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Abstract: For large complex systems, it is of great importance to study effective approaches for reliability evaluation. In this paper, a system reliability evaluation method based on Bayesian theory and multi-source information is proposed. Firstly, model the reliability of system network, including the establishment of system reliability block diagram and the determination of the system and components' life distributions. Then, the posterior distributions and posterior moments of component reliability are determined. With the posterior moments, failure information of components can be converted to the prior moment of the system reliability according to the system structure, and the system prior distribution can be obtained. Finally, combined with the system-level field data, the posterior distribution of system reliability can be obtained on the basis of Bayesian theory, and be utilized to evaluate the reliability of system. To fill out the shortage of field data, the proposed method in this paper can make full use of various quantitative and qualitative prior information with high accuracy.

Keywords: system structure, reliability evaluation, Bayesian theory, multi-source information.

1. Introduction

Reliability assessment of satellite platforms is an important issue in satellite engineering. Due to the limitation of test cost and time, the sample size of the satellite is usually very small, and the problem of non-failure often occurs. This makes it pretty risky to use field information directly for reliability assessment.

For a satellite communication system, the availability, reliability, MTTF, cost effectiveness and sensitivity of a satellite communication system could be analyzed by using mathematical modeling (Nagiya and Ram 2013). Commonly used stochastic processes, including the compound Poisson, gamma, and inverse Gaussian processes, were adopted as the stochastic time scale under dynamic operating conditions of a system (Hong et al. 2019). Meanwhile, Markov process (Kumar and Kumar 2020) and semi-Markov process (Li et al. 2018) can also be applied to analyze dynamic systems.

Many Bayesian melding approaches to combine multilevel information (Peng et al. 2013, Xu et al. 2019, Yang et al. 2020) have been proposed for reliability assessment of complex systems. There are also some methods aiming at system reliability improvement by using an N-modular redundancy framework (Baek et al. 2019) and memetic algorithm (Ramezani et al. 2017). In addition, reliability models with cost consideration are often established to optimize satellite operation and maintenance strategy (Kim et al. 2012).

Fault trees and Bayesian networks (BN) have strong modeling capabilities for complex systems. Based on the fault tree method, the estimate of system failure probability can be obtained by using Bayes theory and Markov chain Monte Carlo (MCMC) algorithm (Hamada et al. 2004). However, the fault tree-based analysis method cannot express the correlation between the system units, and the BN method can be used to describe the correlation between the components (Wilson and Huzurbazar 2007, Wang et al. 2019, Zheng et al. 2019). The polymorphism of each unit of a system (Li et al. 2014) and a specific and complex missile system (Wilson et al. 2007) can also be modeled by BN method.

The field data of satellite platform has the characteristics of small samples and no failure, which makes the classic reliability assessment methods into a dilemma. However, satellite units usually have a lot of prior information, so this paper uses Bayesian method to effectively integrate the prior information and field information, so as to effectively solve the problem of insufficient samples on site. In the case of independent component failure, the system reliability evaluation requires functional decomposition and structural analysis of the satellite platform, and then the reliability model can be established with different information. The method proposed in this paper is based on the two most typical reliability structures of satellite platforms, including series construction and parallel construction.

The reminder of this paper is organized as follows. Section 2 details the proposed reliability model. Next, we verify the proposed models, and demonstrate the detailed analytical procedure with a specific case study in Section 3. Finally, Section 4 concludes this paper.

2. Reliability Model

The satellite platform is a typical complex system with multi-layer structure similar to a pyramid. The process of evaluating its reliability fusing multi-source information is shown in Figure 1.

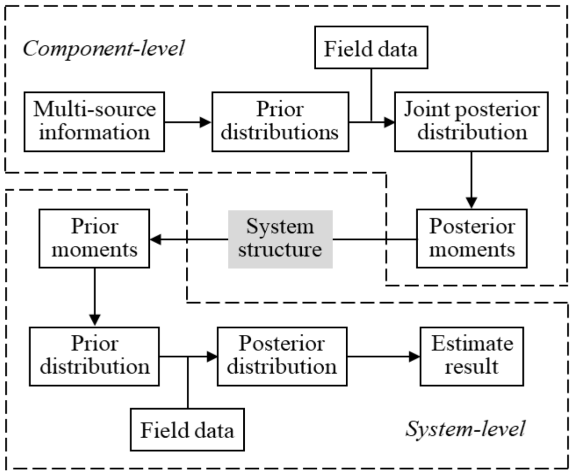

Figure 1. Procedure of system reliability evaluation

2.1 Prior distributions and information fusion of the components

It is assumed that according to different information sources such as expert information, historical life information, similar product information, performance monitoring information, etc., a point estimate of reliability can be given to determine the corresponding prior distribution as incomplete information. The incomplete information of the parameter R_τ described in the form of point estimate can be expressed as

$$\int_0^1 R_\tau \pi(R_\tau) dR_\tau = R_0. \tag{1}$$

Assuming that the parameters of the prior distribution of R_τ are a and b, so the prior distribution is recorded as $\pi(R_\tau \mid a,b)$. Then from the principle of maximum entropy method, a set of parameters a and b should be found to maximize the entropy of $\pi(R_\tau \mid a,b)$ on the premise of satisfying Eq. (1). The constrained minimization problem can be expressed as

$$\begin{cases} \max H = -\int_0^1 \pi(R_\tau)\ln[\pi(R_\tau)]dR_\tau \\ s.t.\ \int_0^1 R_\tau \pi(R_\tau) dR_\tau = R_0 \end{cases}. \tag{2}$$

The parameters a and b can be solved by optimization algorithm, so that the incomplete information can be converted into the prior distributions.

There are many uncertainties in multiple prior distributions obtained from multi-source information. In order to facilitate the inference of the posterior distribution, multiple prior distributions must be fused into a joint prior distribution to eliminate uncertainty, making the joint prior distribution more reasonable than a single prior distribution.

The fusion method based on second maximum likelihood estimation (MLE-II) is more objective, and there is not much difficulty in calculation (Wu et al. 2014). Therefore, the method based on MLE-II is considered to determine the weight of the multi-source prior distribution.

This method assumes that the field data is generated by the marginal density of the prior distribution. The larger the value of the likelihood function generated by the marginal density of the field sample is, the greater the probability that the prior distribution is the true prior distribution of the parameter is, so its weight in the joint prior distribution should also be greater.

Assuming that there are *m* sources of information, the *m* prior distributions obtained from the prior information are $\pi_i(\theta), i = 1,2,\cdots,m$, respectively. And the joint prior distribution is

$$\pi(\theta) = \sum_{i=1}^{m} \varepsilon_i \pi_i(\theta), \tag{3}$$

where ε_i denotes the weight of *i*-th prior distribution in the joint prior distribution, and

$$\sum_{i=1}^{m} \varepsilon_i = 1.$$

From the multi-source prior distributions and the joint prior distribution, their marginal distribution are

$$m(x \mid \pi_i) = \int_\Theta f(x \mid \theta)\pi_i(\theta)d\theta,\ i = 1,2,\cdots,m, \tag{4}$$

$$m(x \mid \pi) = \int_\Theta f(x \mid \theta)\pi(\theta)d\theta. \tag{5}$$

Regarding the field data $x_1, x_2, \cdots, x_n$ as being generated by the marginal distribution $m(x \mid \pi_i)$, the likelihood function of field sample *D* is

$$L(D \mid m_i) = \prod_{j=1}^{n} m(x_j \mid \pi_i). \tag{6}$$

According to the principle of MLE-II, the larger the value of $L(D \mid m_i)$, the greater the weight of the corresponding prior distribution $\pi_i(\theta)$ in the fusion prior distribution. And the fusion weight is

$$\varepsilon_i = \frac{L(D \mid m_i)}{\sum_{i=1}^{m} L(D \mid m_i)}. \tag{7}$$

2.2 Joint posterior distribution and moments of the components

After obtaining the joint prior distribution, with the field samples of the components, the posterior distributions of the unknown parameters can be obtained. When $\pi(\theta)$ denotes the prior distribution of θ and $P(D \mid \theta)$ denotes likelihood function of θ in field information, Bayes formula is

$$\pi(\theta \mid D) = \frac{\pi(\theta)P(D \mid \theta)}{\int_\Theta \pi(\theta)P(D \mid \theta)d\theta}. \tag{8}$$

From Eq. (3)(7)(8), it can be expressed as

$$\pi(\theta \mid D) = \sum_{i=1}^{m} w_i \cdot \pi_i(\theta \mid D), \tag{9}$$

where the weight of *i*-th information source is

$$w_i = \frac{L(D \mid m_i)\int_{\Theta}\pi_i(\theta)P(D \mid \theta)d\theta}{\sum_{i=1}^{m}\left[L(D \mid m_i)\int_{\Theta}\pi_i(\theta)P(D \mid \theta)d\theta\right]}$$

and $\pi_i(\theta \mid D)$ is the i-th posterior distribution of θ. That is to say, the joint posterior distribution is equal to the weighted sum of the posterior distributions corresponding to each information source.

In particular, for the exponential distribution, the prior distribution of reliability is following a negative log gamma (NLG) distribution as

$$\pi(R_\tau) = \frac{b^a}{\Gamma(a)} R_\tau^{b-1}(-\ln R_\tau)^{a-1}, \tag{10}$$

where a and b are the parameters of the NLG distribution determined from known information. Through derivation, the posterior distribution can be obtained as

$$\pi(R_\tau \mid D) = NLG(a+r, b+\frac{T}{\tau}), \tag{11}$$

where T, r and τ denote the total test time of field data, the failure number of field samples and the mission time required to assess reliability, respectively. Therefore, the component reliability and the fore M-th order moment at the mission time are

$$\hat{R}_\tau = \int_0^1 \pi(R_\tau \mid D)dR_\tau = \left(\frac{b\tau+T}{b\tau+T+\tau}\right)^{a+r}, \tag{12}$$

$$E(R_\tau^k) = \left(\frac{b\tau+T}{b\tau+T+k\tau}\right)^{a+r}, k=1,2,\cdots,M. \tag{13}$$

2.3 Prior moments of the system

Focusing on the two most common system structures, series system and parallel system, the fore M-th order moment of unit reliability $\mu_{ik}(1 \le i \le n, 1 \le k \le M)$ are utilized to determine the system reliability moment.

For the series structure, each moment of the system is

$$\mu_{Sk} = E\left(R_S^k(t)\right) = E\left(\prod_{i=1}^{n} R_i(t)\right)^k = \prod_{i=1}^{n}\mu_{ik}. \tag{14}$$

For the parallel structure, each moment of the system is

$$\begin{aligned}\mu_{Pk} &= E\left(R_P^k(t)\right) = E\left(1-\prod_{i=1}^{n}\left(1-R_i(t)\right)\right)^k \\ &= \int_0^1\cdots\int_0^1\left(1-\prod_{i=1}^{n}\left(1-R_i(t)\right)\right)^k \prod_{i=1}^{n}\pi\left(R_i(t)\right)dR_1\cdots dR_n \\ &= \sum_{s=0}^{k}\left((-1)^s\binom{k}{s}\prod_{i=1}^{n}\left(\sum_{m=0}^{s}\left(\binom{s}{m}(-1)^m\mu_{im}\right)\right)\right).\end{aligned} \tag{15}$$

2.4 Posterior distribution and reliability estimation of the system

Due to the complexity of the satellite platform, the distribution of the system is difficult to obtain. In practical engineering, Beta or NLG distribution are usually used to fit the reliability of the system. Specifically, when the test of platform system is frequency-counted, the Beta distribution is used to fit its reliability distribution; when the test of platform system is time-counted, the NLG distribution is used to fit its reliability distribution. In this case, the recursive least squares (RLS) method could be used to obtain the parameters of the distribution.

On the condition that the fore M-th order moment of the satellite platform $\mu_k = E(R^k), k=1,2,\cdots,M$ have been obtained, the optimization problem becomes

$$\min_{\alpha,\beta}\sum_{k=1}^{M}\left[\frac{\mu_k(\alpha,\beta)-\mu_k}{\mu_k(\alpha,\beta)}\right]^2, \tag{16}$$

where $\mu_k(\alpha,\beta)$ denotes the k-th order moment of $\beta(R;\alpha,\beta)$ or $NLG_r(R;\alpha,\beta)$.

After obtaining the value of the parameters α and β, the prior distribution of the system reliability can be determined. Then, similar to the operation in component-level, combined with the field data of the system, the posterior distribution of the system reliability can be deduced, so as to calculate the system reliability.

3. Case Study

This section uses simulation study to verify the effectiveness of the proposed method. Assume that the lifetimes of the components C1, C2 and C3 all follow exponential distributions, and their average lifespans are $\theta_1 = 60$, $\theta_2 = 70$ and $\theta_3 = 80$. The system structure is shown in Figure 2. According to the calculation principles of series and parallel constructions, the reliability of the system at the beginning can be obtained that is 0.9665.

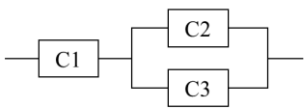

Figure 2. Reliability block diagram of the system

In the stochastic simulation, it is assumed that the system has been working for time $\tau = 2$ when the system just started working soon. The field information of each component is Type-II censoring life test data whose number of test samples is 15 and the number of failures is 10. By using three exponential distribution models, a set of field data is randomly generated as $T_1 = 644.6$, $T_2 = 680.24$ and $T_3 = 797.28$. Let the prior distributions of the three units are all information-free priors which is $NLG(0,0)$, and the posterior distributions of the three units can be obtained that are $\pi_1(R_\tau) \sim NLG(10,322.3)$, $\pi_2(R_\tau) \sim NLG(10,340.12)$ and $\pi_3(R_\tau) \sim NLG(10,398.64)$, respectively. Therefore, the fore 5th order moments of the reliability of each unit and system can be obtained as shown in Table 1.

Table 1. The fore 5th order moments of the reliability of components and system

Order	1st	2nd	3rd	4th	5th
C1	0.9695	0.9400	0.9115	0.8839	0.8573
C2	0.9710	0.9430	0.9159	0.8896	0.8642
C3	0.9752	0.9512	0.9277	0.9049	0.8828
System	0.9688	0.9386	0.9095	0.8814	0.8542

When the order of moments reaches the 5th order, the fitting effect can achieve good accuracy. Hence, in practical engineering, the order of the moments can be taken to the 5th order. It can be seen that the system reliability evaluation result at this time is 0.9688, which proves the feasibility of this method.

4. Conclusions

In this paper, a Bayesian reliability assessment method of the satellite platform is studied. The satellite platform is a typical complex system whose system-level data is usually rare. Therefore, the underlying component data of the system is converted to the system-level for evaluation. The proposed method combines the Bayesian melding and the multi-source information to make full use of the prior information to evaluate the systematic reliability. It can provide a feasible idea for the reliability assessment of the satellite platform system and be applied to engineering practice. Subsequent research may consider analyzing more complex system structures and expanding the applicability of evaluation method.

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