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Unsupervised Deep Neural Network for Near-real-time Damage Assessment of Structures Subject to Earthquake Excitations

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Abstract: This paper introduces a Deep Neural Network (DNN) framework for near-real-time damage assessment using the structural response data from an earthquake event. The proposed network is constructed by Convolutional Autoencoder (Conv-AE), which is a powerful self-supervised DNN that can effectively recognize patterns from two-dimensional data such as image by using two-dimensional convolutional layers. The network is trained using the correlation matrix of acceleration time-series data obtained from sensors of the target structure at a healthy state. A structural damage index (SDI) is proposed at a local scale to locate the structural damage, which increases as the structure deviates from the healthy state due to the occurrence of structural damage. To demonstrate the proposed method, structural analysis is performed for a 20-story shear building at its healthy state under 2,000 artificial seismic ground motions, which are pre-assigned to the train and test datasets in the ratio of 9:1. After training the proposed networks, SDI is computed for each of the simulated damage conditions using real ground motions. Furthermore, damage assessment criteria are proposed based on the pre-obtained SDI for various damage conditions. Through further development, the proposed damage assessment framework is expected to help reduce the time required for disaster response by providing near-real-time damage assessment using the pre-established DNN and damage assessment criteria.

Keywords: earthquakes, structural response, deep neural network, convolutional autoencoder, damage assessment, structural damage index.

1. Introduction

Structural health monitoring (SHM) plays an important role in maintenance efforts to assure the safety and integrity of in-service civil engineering systems. A failure to identify critical structural damage may reduce the service life of the structural system or cause functional failures. Therefore, it is essential to implement a proper SHM process that can evaluate the integrity of the engineering system and detect possible damage to structural systems. SHM-based damage identification processes usually consist of four steps: (1) detecting the existence of the damage, (2) locating the damage, (3) identifying the types of damage, and (4) quantifying the severity of the damage (Entezami and Shariatmadar 2018). In general, SHM process are categorized into long-term and short-term SHM (Dawson 1976). A long-term SHM utilizes periodically updated information about the ability to perform the intended function, which is affected by aging and degradation resulting from its operational environments. On the other hand, a short-term SHM aims at rapid screening of change in the structural condition and providing information about the integrity of the structure in near-real-time.

With rapid developments in sensing technologies and data science, the vibration-based damage identification methods have been gaining attention especially among the applications of machine learning to SHM. These methods are built upon the fact that a change in structural characteristics, such as mass and stiffness, changes the vibration characteristics such as mode shape and natural frequency. During extreme events such as earthquakes, structural characteristics may change due to the local damage or failure of the structural system. Therefore, it is important to detect the structural damage immediately to protect human lives, and maintain or recover the serviceability of infrastructures. In particular, vibration-based pattern recognition methods are considered suitable for near-real-time damage detection. This is because these methods use the pre-trained statistical model, such as a deep neural network (DNN; Pathirage et al. 2018), using features from the vibration signals. This requires low computational time to recognize the changes in vibration characteristics.

Autoencoder (AE) is one of the well-known DNN methods used for anomaly detection (An and Cho 2015). The aim of AE is to learn patterns for a set of data by reconstructing input data identically by passing through the bottleneck, i.e. dimensionality reduction. In the same way, AE pre-trained by the data from the healthy state can reconstruct data at the healthy state only while the reconstruction error increases in the damage condition. This means that the reconstruction error can be used as a measure of structural damage for the purpose of damage identification.

This paper proposes a DNN-based framework for near-real-time damage identification using the vibration characteristic of raw acceleration data from an earthquake event. The correlation matrices at zero lag for various time windows are selected as the vibration characteristics describing localization as well as severity of damage. The proposed network hinges on convolutional AE (Conv-AE) composed of convolutional layers for the pattern recognition of highly nonlinear two-dimensional data. The proposed framework consists of the four steps: (1) Correlation matrices from artificial ground motions only at a healthy state are prepared as input data; (2) The network is trained to reconstruct the training input data and verified by the test data; (3) Computational simulations with real ground motions are performed for various damage conditions to verify the pre-trained network; and (4) The proposed index of structural damage is calculated for every sensor location in near-real-time. A numerical example of the linear structure under seismic ground motions is investigated to demonstrate the proposed framework.

2. Theoretical Backgrounds

2.1 Autoencoder

A traditional AE consists of encoder and decoder with a single hidden layer (Vincent et al. 2010). One can construct a *deep* AE by introducing multiple hidden layers. A conceptual illustration of AE is shown in the Fig. 1.

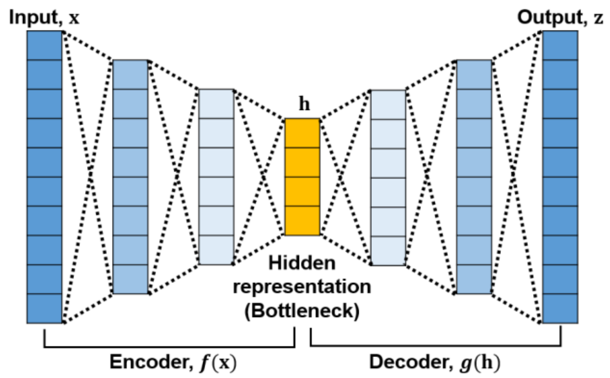

Figure 1. Conceptual illustration of AE.

Encoder: The mapping function $\boldsymbol{f}(\mathbf{x})$, which transforms a d-dimensional input vector $\mathbf{x} \in R^d$ into an r-dimensional hidden representation $\mathbf{h} \in R^r$, is called an encoder. Here, the dimension of the hidden layer is smaller than that of the input layer, i.e. $d > r$. $\boldsymbol{f}(\mathbf{x})$ is usually written with the following nonlinear transformation:

$$\mathbf{h} = \boldsymbol{f}(\mathbf{x}) = \boldsymbol{\sigma}(\mathbf{W}\mathbf{x} + \mathbf{b}) \tag{1}$$

where $\mathbf{W} \in R^{r \times d}$ denotes the mapping weight matrix of the encoder, $\mathbf{b} \in R^r$ is the bias vector and $\boldsymbol{\sigma}$ is the activation function, which is usually a nonlinear function such as sigmoid, tangent hyperbolic, Rectified Linear Unit (ReLU) and Exponential Linear Unit (ELU) function. In this paper, the hidden layers employ the ELU function

$$\mathrm{ELU}_\alpha(x) = \begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases} \tag{2}$$

Decoder: The mapping function $\boldsymbol{g}(\mathbf{h})$, which transforms the hidden representation $\mathbf{h}$ back into a reconstructed vector $\mathbf{z} \in R^d$, is called a decoder. Note that the dimension of $\mathbf{x}$ and $\mathbf{z}$ are same. $\boldsymbol{g}(\mathbf{h})$ is usually written with the following nonlinear transformation:

$$\mathbf{z} = \boldsymbol{g}(\mathbf{h}) = \boldsymbol{\sigma}(\widehat{\mathbf{W}}\mathbf{h} + \hat{\mathbf{b}}) \tag{3}$$

where $\widehat{\mathbf{W}} \in R^{d \times r}$ denotes the mapping weight matrix of the decoder, $\hat{\mathbf{b}} \in R^d$ is the bias vector and $\boldsymbol{\sigma}$ is the activation function described above.

To optimize the parameters $\boldsymbol{\theta} = [\mathbf{W}, \mathbf{b}, \widehat{\mathbf{W}}, \hat{\mathbf{b}}]$ using the training data, the mean squared error (MSE) is often used as the loss function

$$L(\mathbf{x}) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{2}\left\|\boldsymbol{x}^{(i)} - \boldsymbol{g}\left(\boldsymbol{f}\left(\boldsymbol{x}^{(i)}\right)\right)\right\|^2 \tag{4}$$

where m is the number of samples, $\boldsymbol{x}^{(i)}$ is the ith input, $\boldsymbol{f}(\cdot)$ and $\boldsymbol{g}(\cdot)$ are the mapping functions of the encoder and decoder, respectively. Generally, $L(\mathbf{x})$ in Eq. (4) is difficult to optimize because of its non-linearity, thus a gradient descent based optimizer such as Adam proposed by Kingma and Ba (2014) are commonly used. As shown in Eq. (4), AE is trained to reconstruct the input.

2.2 Convolutional Autoencoder

Conv-AE combines the standard AE with the convolutional layer to employ the convolutional operation instead of the matrix multiplication used in the general fully connected layer (Goodfellow et al. 2016). In other words, the mapping matrix of $\boldsymbol{f}(\mathbf{x})$, $\mathbf{W}$ consists of multiple small filters called the kernel instead of one large weight matrix. The convolutional layer is generally used when the input is two-dimensional data with multiple features, such as image and video, to capture local characteristics of data.

In convolutional operation, the dimension of input is $\mathbf{x} \in R^{n \times l \times l}$ where n is the number of features and l is the dimension of one feature map, and the convolutional layer has m kernels, then the dimension of the output layer is $\mathbf{h} \in R^{m \times p \times p}$, where $p \leq l$. In the de-convolutional operation, so-called the inverse convolutional operation, in contrast, the dimension of the output is larger than that of the input, i.e. $p > l$. The procedure to obtain the hidden representation from the input through the convolutional operation is called convolutional encoder, $\boldsymbol{f}(\mathbf{x})$. In reverse, the procedure to obtain the reconstructed input from hidden representation through the de-convolutional operation is called convolutional decoder, $\boldsymbol{g}(\boldsymbol{f}(\mathbf{x}))$.

3. DNN-based Damage Assessment Framework

3.1 Damage identification using correlation matrix

As mentioned above, the vibration characteristics reflect changes in the structure. To capture the change in vibration characteristics, the correlation matrix at zero lag is used as the input data of the Conv-AE network in this paper. The correlation matrix of signals at lag τ, $\mathbf{R}(\tau)$, is a matrix containing the cross-correlations of all pairs of signals at lag τ as elements. The cross-correlation between two discrete signals, $\mathbf{x}$ and $\mathbf{y}$, is defined as follows:

$$R_{\mathbf{xy}}(\tau) = \begin{cases} \sum_{t=0}^{T-\tau-1} x_{t+\tau} y_t^* & \tau \geq 0 \\ R_{\mathbf{xy}}(-\tau) & \tau < 0 \end{cases} \tag{5}$$

where $R_{\mathbf{xy}}(\tau)$ is the cross-correlation between $\mathbf{x}$ and $\mathbf{y}$ at lag τ, T is the length of $\mathbf{x}$ or $\mathbf{y}$, and y_t^* is the conjugate pair of y_t. If the signal data is obtained from n sensors, the correlation matrix as lag τ is $n \times n$ matrix, i.e. $\mathbf{R}(\tau) \in R^{n \times n}$. The correlation matrix of structural responses at the healthy condition differs from that of the damage condition. Furthermore, the element of correlation matrix corresponding to the sensor location close to the damage varies greatly compared to other elements.

In the proposed framework, the Conv-AE is pre-trained with the dataset of $\mathbf{R}(0)$ calculated with the structural responses of the healthy structure for various excitations such as White Gaussian Noise (WGN), and the seismic ground motion. To characterize the system status at different time scales, various lengths of time window is applied to obtain multiple $\mathbf{R}(0)$ and each is uses as a feature map

(Zhang et al. 2019). The network is trained to learn the hidden representation of vibration characteristics of a healthy structure through reconstructing input data. The test dataset is stored in the database for analysis after training.

After pre-training, the real-time simulation is performed to verify the pre-trained network by calculating the reconstruction error of the correlation matrix every time step. The reconstruction error is low at the healthy state since the network is pre-trained to reconstruct the data from a healthy structure. During the simulation, if the damage occurs at a certain time step, the reconstruction error increases. By capturing the change in reconstruction error, the damage can be identified in near-real-time. However, the reconstruction error cannot be used as a measure of structural damage directly for accurate damage identification since the MSE is one scalar value that cannot locate damage. For this reason, the reconstruction error should be calculated in element-wise, i.e. the reconstruction error matrix or MSE matrix. To quantify the damage, this paper proposes a new structural damage index (SDI) for damage identification in the next section.

3.2 Proposed structural damage index

An example of the MSE matrix of between the correlation matrix at a healthy and damage condition is shown in Fig. 2. Note that some values in the MSE matrix stand out, which indicates that the damage occurred at the location between certain sensors. In addition, the correlation between sensor signals adjacent to damage changes the most. Therefore, damage identification and localization can be employed by monitoring the change in the MSE matrix. In this paper, it is assumed that adjacent elements in the correlation matrix indicate the correlation between signals from adjacent sensors.

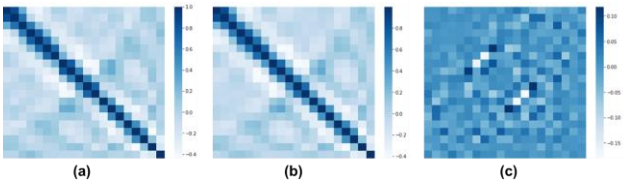

Figure 2. Heat maps of correlation matrix at (a) healthy and (b) damage condition, and (c) MSE matrix.

Based on these facts, the proposed SDI is defined as follows:

$$\mathrm{SDI}_i(t) = \ln\left[1 + \frac{\mathrm{MSE}_i(t)}{\mathrm{MSE}_i^{\mathrm{DB}}(t)}\right] \tag{6}$$

where $\mathrm{MSE}_i(t)$ denotes the maximum element among values adjacent to i-th diagonal element in the MSE matrices at time t, and $\mathrm{MSE}_i^{\mathrm{DB}}(t)$ denotes the value at the same location as $\mathrm{MSE}_i(t)$ in the most similar MSE matrices in the database. The effect of different form of correlation matrix is eliminated by dividing $\mathrm{MSE}_i(t)$ by $\mathrm{MSE}_i^{\mathrm{DB}}(t)$. In addition, a logarithm with "+1" is applied to make SDI positive. Note that the more severe damage, the greater the proposed SDI. This is because $\mathrm{MSE}_i(t)$ increases when damage occurs, whereas $\mathrm{MSE}_i^{\mathrm{DB}}(t)$ does not increase since MSE matrices in the database are obtained from responses at a healthy condition only. Fig. 3 summarizes the proposed framework. As mentioned, the framework is composed of four steps: (1) Responses of the healthy structure under artificial ground motions are pre-processed in the form of correlation matrices as input data. (2) The network is trained to learn hidden representations by reconstructing input data. (3) During the real-time simulation, the MSE matrix is obtained and (4) the proposed SDI for every sensor location is calculated in near-real-time.

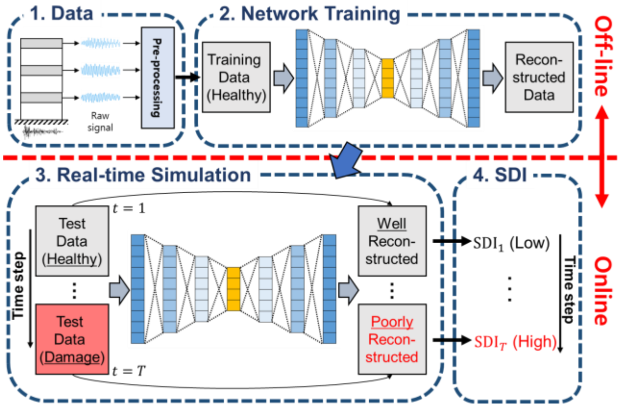

Figure 3. Process of proposed damage assessment framework.

4. Numerical Example

4.1 Structural properties

As a target structure, a linear 20-degree of freedoms (DoF) shear building is used. This is a simplified version of the LA 20-story structure presented in Spencer et al. (1998). The mass and stiffness of each story can be found in Mousavi and Ghorbani-Tanha (2012). The first 5 natural frequencies of the target structure are 0.26, 0.72, 1.18, 1.62 and 2.06 Hz, and the corresponding mode shapes are shown in Fig. 3 where m_i, c_i and k_i denote the i-th mass, damping coefficient and stiffness respectively. As proposed by Spencer et al. (1998), inherent modal damping ratios are given as follows:

$$\xi_i = \min\left(\frac{\omega_j}{50\omega_1}, 0.1\right) \tag{7}$$

where ξ_i is the modal damping ratio of i-th mode and ω_j is the natural frequency of the j-th mode.

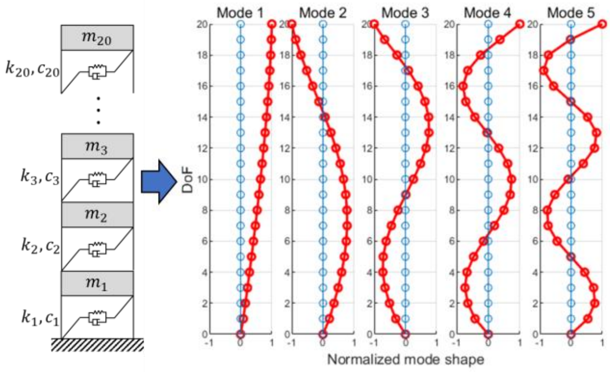

Figure 4. Target structure and its mode shapes.

4.2 Data generation and pre-processing

To obtain responses from the seismic ground motion of various characteristics, structural analysis is performed under the artificial ground motion with the magnitude from 6 to 9, which is generated from the method proposed in Rezaeian (2010). In addition, WGN, which simulates live loads, is applied to every DoF before and after an earthquake event. 2,000 artificial ground motions are generated and a total time length of one simulation is 655.36s. As the structural response, horizontal accelerations of every DoF are recorded. To capture the effect of the signal length, various lengths of the time window, with the length of 1.28s, 10.24s, and 81.92s, are applied to responses at each time step. Each time window characterizes the status of the target structure at different time scales. The interval of two signal segments is set as 1.28s, i.e. the length of the short-time window. The correlation matrices are obtained from signal segments at each time step and the dataset of the correlation matrix is divided into the training and test dataset with a ratio of 9:1.

Note that the form of the correlation matrix is different for the vibration situation. The situation can be divided into three categories: (1) the vibration during the WGN force, (2) the earthquake, and (3) after the earthquake or free vibration. The typical form of the correlation matrix according to each situation is shown in Fig. 5. As mentioned in Section 3.2, however, the effect of different forms of the matrix is eliminated at the stage of calculating the proposed SDI.

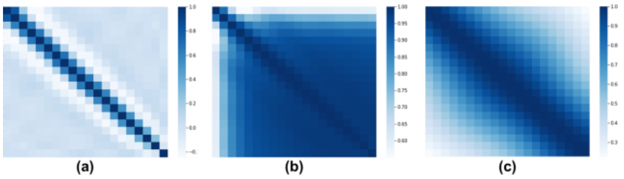

Figure 5. Typical heat maps of correlation matrix during (a) WGN, (b) earthquake, and (c) free vibration.

4.3 Network training

The architecture of Conv-AE used in this example, inspired by Na et al. (2018), is illustrated in Fig. 6. As mentioned, Conv-AE consists of two main parts: (1) the convolutional encoder, and (2) decoder. The convolutional encoder is composed of six convolutional layers with 64, 64, 128, 128, 256, and 256 filters respectively. Stride 2 is used at the 4th and 6th layers for dimension reduction. After the convolutional calculation, the output of the last convolutional layer is reshaped into the one-dimensional vector by Flatten layer, which is followed by one Dense layer with 256 nodes representing the hidden representation. ELU activation function is used in all layers. Since the proposed Conv-AE has deep network architecture, the batch normalization is used after every convolutional layer to prevent the gradient from vanishing and exploding problem (Ioffe and Szegedy 2015). The decoder has the inverse architecture of the encoder and uses the same hyper-parameters as the encoder. The shape of the output layer is the same as the input layer. The value of the MSE loss function is calculated as the last step of the forward-propagation and the value is back-propagated through the network to optimize parameters $\boldsymbol{\theta}$.

Conv-AE was constructed using the Python deep learning library Keras with the Tensorflow backend and trained on a server with 2x Intel(R) Xeon(R) Gold 6126 2.60GHz and 2 NVIDIA TITAN RTX graphics cards. The number of epochs and batch size is set to 200 and 32 respectively. Rectified Adam (RAdam) optimizer proposed by Liu et al. (2019) with the learning rate of 0.003 is used for optimizing the loss function. The training process took about 6 hours and the final training and test loss value were 1.64×10^{-4} and 1.96×10^{-4}, respectively. The convergence of the loss function was fast and stable without showing overfitting or exploding of the test loss.

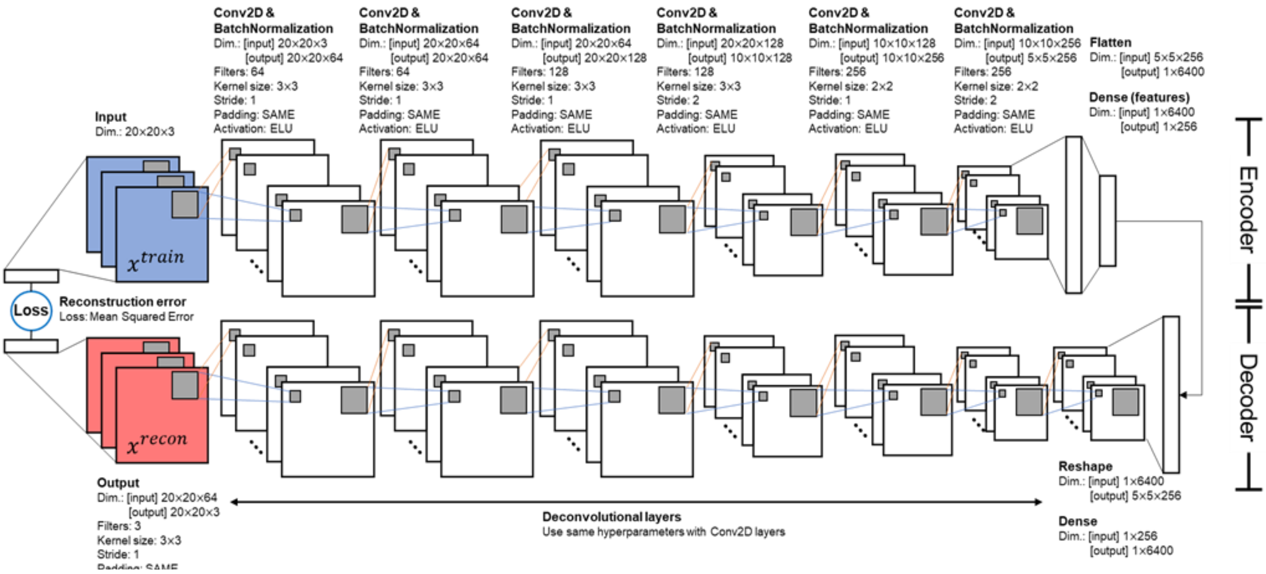

Figure 6. Conv-AE architecture in the proposed framework.

4.4 Near-real-time damage assessment

To verify the performance of the pre-trained network, a real-time simulation is performed with a real ground motion. The N-S component of El Centro earthquake in 1940 is selected as the ground motion for simulation. In addition, WGN forces that simulate the live load are applied to every DoF before and after an earthquake event. The correlation matrix is obtained every time step, and $\mathrm{SDI}_i(t)$ for every sensor is calculated simultaneously through the pre-trained Conv-AE.

In this experiment, two assumptions are introduced: (1) The stiffness degradation simulates the damage to the structure, and (2) the damage occurs when the earthquake reaches the peak ground acceleration (PGA). To verify the performance for damage localization, two damage cases are employed: (1) the stiffness degradation in k_8 and k_{13}, and (2) k_3 and k_{18}. In each case, 15%, 30%, and 50% of stiffness degradation are considered to test the quantification of the damage severity. Each scenario indicates the minor, moderate, and severe damage, respectively.

4.4.1 Case 1: degradation in k_8 and k_{13}

The results of near-real-time damage assessment for the case of the degradation in k_8 and k_{13} are shown in Fig. 7. The thick lines indicate the SDIs for the damaged DoFs. It is seen that the SDI increases as soon as the damage occurs. Note that the SDIs for the damaged DoF are distributed in a specific range for all cases of vibrations according to the damage severity. The range is illustrated by the black dashed lines in Fig. 7. The range for the minor, moderate, and severe damage is 2.0~4.5, 3.5~6.5, and 5.0~7.5, respectively. For the undamaged case, the SDI is distributed below 2.0. The fact that SDI of the damaged DoFs are distributed in the constant range at all times confirms that the proposed SDI can eliminate the effect of the vibration case or the matrix form. In addition, it is observed that some of SDIs for the undamaged DoF also increase after the earthquake and these DoFs are close to the damage location. This is because the closer to the damage location, the more vibration characteristics change.

4.4.2 Case 2: degradation in k_3 and k_{18}

The result for the case of the degradation in k_3 and k_{18} is shown in Fig. 8. As the result of Case 1, the SDIs for the damaged DoF are also distributed in the aforementioned range for all cases. During the earthquake, however, all SDIs are distributed in a higher range than a result of Case 1. This is because the first mode is dominant during the earthquake and the element adjacent to the ground, k_3, is damaged. The damage to the element close to the ground affects the vibration characteristic of other elements above it significantly. Nevertheless, the SDIs for the damaged DoF are distributed higher than those for the undamaged in all vibration cases in the constant range, which gives the same result as Case 1.

The results from the two cases confirm that the proposed damage assessment framework based on DNN can identify and locate the damage accurately in near-real-time.

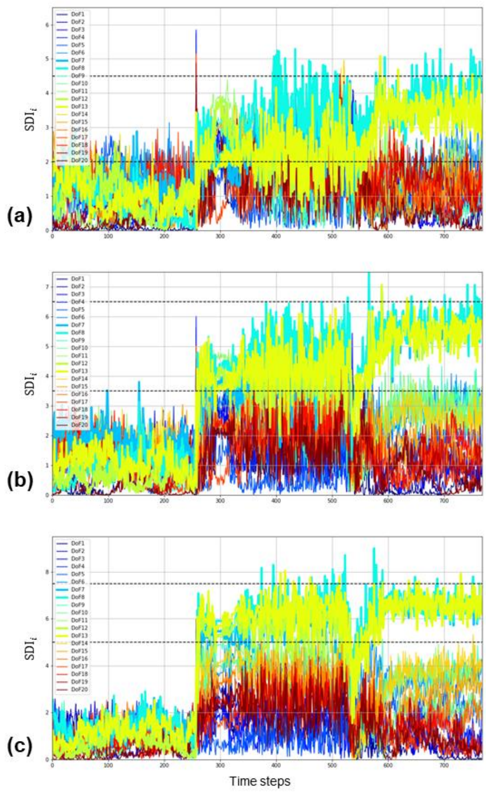

Figure 7. SDIs for (a) 15%, (b) 30%, and (c) 50% degradations in k_8 and k_{13}.

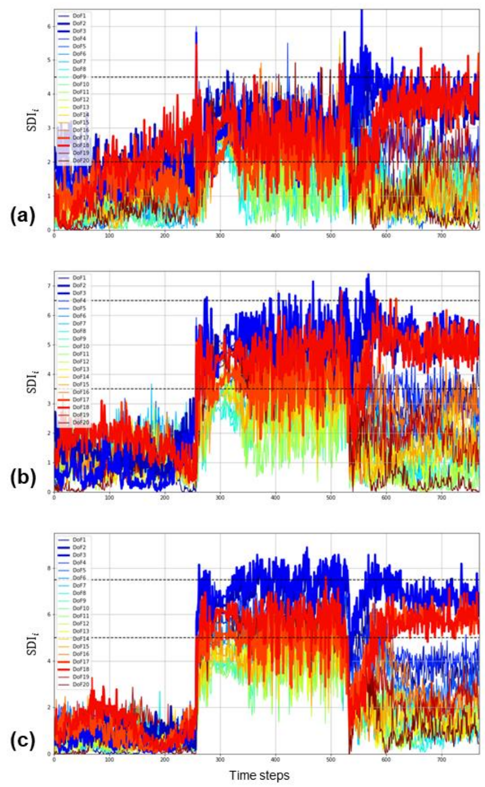

Figure 8. SDIs for (a) 15%, (b) 30%, and (c) 50% degradations in k_3 and k_{18}.

5. Conclusions

In this paper, a new DNN framework was proposed for near-real-time damage assessment of the structural system subject to the earthquake. In the proposed framework, a Conv-AE is trained only with the correlation matrix from the structural response at a healthy state to detect the damage by the reconstruction error. A new structural damage index (SDI) was proposed based on the mean square error (MSE) matrix to locate the damage accurately. The proposed SDI was configured to have consistent performance by removing the effects of the form of the correlation matrix. A numerical example of the real-time simulation was provided to verify the proposed framework. As the target structural system, the linear shear building subject to the live load, and seismic ground motion was selected. The Conv-AE was trained successfully to reconstruct the input and learn the hidden representation. The proposed SDI shows a great performance for the damage localization and quantification in near-real-time. However, the performance of the SDI varied while the structure is subject to the earthquake depending on the damage location.

Future research topics were identified as follows during this study. First, the proposed framework can be applied to a nonlinear and complex structural system. However, the patterns of changes in vibration characteristic may be different from the linear system because of its nonlinearity and complexity. This would make it difficult to identify the damage based on the correlation matrix. To address this, the proposed framework need to be modified. Second, a modified or new SDI can be developed to achieve a more stable and accurate performance of the damage identification regardless of various situations. The SDI proposed in this paper does not perform well during the earthquake excitations. To improve this, a new type of damage or health index is needed to facilitate near-real-time SHM.

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