学位論文（要約）

Cosmological implications from supersymmetric axion models -- origin of matter and its fluctuations

(超対称性アクシオンモデルにおける物質とその揺らぎの起源)

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THESIS

Cosmological implications from supersymmetric axion models

– origin of matter and its fluctuations

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Abstract

The matter-antimatter asymmetry and the existence of dark matter are unsolved mysteries that current cosmology and the Standard Model cannot explain. Supersymmetry (SUSY), the symmetry of bosons and fermions, adds a full new set of partner particles to the Standard Model and the partner particles are thought to be candidates of dark matter. On the other hand, axion is a particle beyond the Standard Model, which is introduced as a solution of the strong CP problem. Thus, we consider axion models in the framework of SUSY that solve these problems. In SUSY axion models, axino, a supersymmetric partner of the axion exists and is also a candidate for dark matter. In this thesis, we investigate SUSY axion models that solve the matter-antimatter asymmetry and the dark matter problem simultaneously. The Affleck-Dine (AD) mechanism is a promising baryogenesis model that explains the matter-antimatter asymmetry, and can generate a non-topological soliton, Q-ball during generating the baryon number. If one assumes that Q-balls decay into dark matter particles, baryons and dark matter have same origin, Q-ball. Therefore the Q-ball decay can naturally explain the observational fact that the energy densities of the two components are at some order. In the thesis, we assume axino dark matter and the gauge mediated SUSY breaking to produce Q-balls. The decay takes place well before the Big Bang Nucleosynthesis (BBN) and also the decay into the supersymmetric particles of the Minimal Supersymmetric Standard Model (MSSM) is kinematically prohibited until the very end of the decay. As a result, we can safely make their abundances small enough for the successful BBN.

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Chapter 1

Introduction

Cosmology makes use of every scale of physics, from the quantum scale to the scale of galaxies. By the combinations of the theories of gravity and observations, cosmology has unveiled the history and the structures of the universe, for example, the Big Bang theory and the accelerated expanding of the universe. Furthermore, with the knowledge of high energy physics, there are promising and interesting theories, which give solutions to the early history of the universe, such as inflation models, which expect exponential expansions of the early universe.

Still there exist unsolved mysteries of the universe. The observation tells us that ordinary matter only consists 5 % of the energy of the universe. Other components are called dark matter and dark energy. Dark matter, which consists almost one thirds of the energy, does interact through gravity but does not seem to interact through electro magnetic force. Thus dark matter does not emit, nor absorb photon, if it does, the interaction has to be very small. Besides the dark matter problem, another puzzling observation result that contradicts to the theory is the baryon asymmetry. It seems to be natural that if there exist same amounts of matter and anti-matter, but obviously the universe mainly consists of matter.

As the Standard Model has not given a concrete solution for dark matter, the baryon asymmetry and other cosmological problems, cosmology motivates physics beyond the Standard Model. One of the promising theory beyond the Standard Model is supersymmetry (SUSY), the symmetry of bosons and fermions, adds a full new set of partner particles to the particles of the Standard Model. It is quite natural to consider such symmetry because of symmetric nature of physics theories. In SUSY, there consequently exist the lightest supersymmetric particle (LSP). The LSP is stable with R-parity conservation and in most cases scarcely interacts with other particles. These properties are ideal to be dark matter. SUSY also provides a key concept for the baryon asymmetry. The Minimal Supersymmetric Standard Model (MSSM) provides many "flat directions", and they carry baryon number. As we do not observe any supersymmetric particle, SUSY has to be broken at some energy scale. After the SUSY breaking, the flat directions are lifted and can gain large VEV. These flat directions can produce baryon number by rotations due to the SUSY breaking effect. This idea is called the Affleck-Dine mechanism.

So far, we have mentioned the motivations for physics beyond the Standard Model from cosmological observational results, but of course there are other unsolved mysteries that the Standard Model itself has. One of them is the strong CP problem. The strong CP problem is a fine-tuning problem, a problem of the existence of the θ term in QCD. The θ term is written as $\mathcal{L}_\theta = \theta F_{\mu\nu}\tilde{F}^{\mu\nu}$ where $F^{\mu\nu}$ is a gluon field strength and θ is a parameter. $\mathcal{L}_\theta$ violates CP, but from experiments, it is suggested that the term has to be unnaturally small. A promising solution is introducing a new particle, axion, then θ becomes the phase of a dynamical field, axion. Thus when the axion settles at the bottom of its potential, the θ term becomes effectively zero. The axion and its supersymmetric partner are also candidates for dark matter.

Cosmology not only gives motivations for physics beyond the Standard Model, but also the universe and its history can play a rule as a huge observatory for high energy physics at large energy scales where terrestrial experiments cannot reach. Cosmic Microwave Background (CMB) observations are one of them, and especially have developed drastically in recent years. By observing the perturbations of the temperature when photons finally came to be able to move freely after the recombination, we can observe the perturbation of matters at early universe indirectly. These perturbations are known to be generated by many inflation models that assume particle physics models. Thus we can constrain these models using CMB data.

In this thesis, we take the axion as a key ingredient and investigate how the axion that is introduced by physics beyond the Standard Model can solve the cosmological problem, and how cosmological observations can have constraint on axion model parameters. Firstly, we investigate the dark matter problem with the supersymmetric partner of the axion, axino as LSP. The AD mechanism can generate not only baryon number, but also can generate dark matter through a consequent object of the AD mechanism. The object is called Q-ball, non-topological soliton. If Q-balls stay as dark matter or decay into dark matter, the baryon and dark matter have same origin, an AD field, a flat direction of the MSSM. This is quite attractive because the energy densities of baryon and dark matter are surprisingly at same order $\rho_{\rm DM} \sim 5\rho_b$, thus such scenarios have been investigated for different dark matter candidates. The properties of Q-balls depend on its potential. The potential is related to the kind of SUSY breaking which is necessary for the AD mechanism to occur. Thus Q-ball's properties vary according to SUSY breaking mechanisms. For example, If one takes the gravity mediated SUSY breaking, Q-balls become unstable, on the other hand, for the gauge mediated SUSY breaking, Q-balls are stable. We take the gauge mediation as the SUSY breaking factor, and assume that Q-balls decay into the axino LSP partly and mainly into nucleons, *i.e.* baryons. Thus the model can solve the dark matter and the baryon asymmetry problem simultaneously from the same origin.

In our research of the Q-ball decay, we assume that dark matter consists of nothing but axino. The axion can also become LSP and dark matter, but if an axion model produces scale invariant perturbations, there can exist constraints on the amount of the axion perturbation from the observation of the cosmological perturbation. In the appendix chapter, we investigate the cosmological perturbations generated by a simple SUSY axion model. The model produce isocurvature perturbations, and the perturbations have a specific feature, the blue spectrum at large scales while invariant at small scales. It is interesting, because such blue spectrum is, contrary to its specific feature, still allowed by the observations. Using recent CMB data, we constrain the axion model parameters.

The structure of the thesis is as following: in the second chapter, we explain the CP problem and how axion can solve the problem. In the third chapter, after we review SUSY and the MSSM, we also introduce axino, a supersymmetric partner of the axion. In the fourth chapter, we review the basis of cosmology, the Friedmann-Robertson-Walker metric and inflation. In the fifth chapter, we briefly review the AD mechanism and Q-ball. The sixth chapter is devoted to the research. We investigate a model, which assume Q-balls from the AD mechanism with the gauge mediated SUSY breaking, and the decay of Q-balls into the nucleons and the axinos. With the BBN constraints and other observational constraints, we calculate the allowed region of the axino and Q-ball parameters. In the appendix chapter, we assume a SUSY axion model, which generate isocurvature fluctuations. Performing Markov Chain Monte Carlo (MCMC) analysis with CMB data from WMAP and Planck observations, we constrain the axion model parameters.

Chapter 2

Strong CP problem and axion

In this chapter, following recent reviews [1–3]. We explain the strong CP problem in the quantum chromodynamics (QCD) and how it can be solved by introducing a light pseudoscalar particle, the axion.

2.1 Strong CP problem

In QCD, the Lagrangian is described by

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}, \tag{2.1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right]$ is the gluon field strength tensor, and the gluon fields are represented by $A_\mu = \sum_a A_\mu^a \tau^a$, τ^a is a generator of $SU(3)_C$. g represents the coupling constant of QCD. Here, for a while, we omit the kinetic and mass terms of the quarks q, the fields represented by the fundamental representation of $SU(3)_C$ gauge group. The gauge transformations of A_μ and $F_{\mu\nu}$ are described by

$$A_\mu \to UA_\mu U^{-1} + \frac{i}{g}\partial_\mu UU^{-1}, \tag{2.2}$$

$$F_{\mu\nu} \to UF_{\mu\nu}U^{-1} \tag{2.3}$$

where U denotes a space-time dependent $SU(3)$ matrix.

Let us take the temporal gauge $A_0(x) = 0$. The vacuum of this Lagrangian satisfies

$$F_{\mu\nu} = 0. \tag{2.4}$$

The classical vacuum corresponds to the zero field $A_i(x) = 0$, but the gauge fields which are generated by gauge transformations from the zero field, $A_\mu = \frac{i}{g}\partial_i UU^{-1}$ also satisfy the condition. In this case, U has to be time-independent unitary matrix $\partial_0 U = 0$. Because

$$A_0(x) \to \frac{i}{g}\partial_0 UU^{-1} = 0, \tag{2.5}$$

and this leads to

$$A_i(\vec{x}) = \frac{i}{g}\partial_i U(\vec{x})U^{-1}(\vec{x}). \tag{2.6}$$

They are called the pure gauge.

The nature of a vacua can be described by how U goes to unity as $|\vec{x}| \to \infty$, such as

$$U_n \to e^{i2\pi n}, \tag{2.7}$$

where n is a integer and called the *topological winding number*. The vacua which have different winding number are topologically different and cannot reach each other by a continuous gauge transformation. n is also described by

$$n = \frac{1}{24\pi^2}\int d^3x \mathrm{Tr}\epsilon_{ijk}\left[U^{-1}\left(\partial^i U\right)U^{-1}\left(\partial^j U\right)U^{-1}\left(\partial^k U\right)\right]. \tag{2.8}$$

In the pure gauge case, n is expressed using A_μ such as,

$$n = \frac{ig^3}{24\pi^2}\int d^3x \mathrm{Tr}\left(\epsilon_{ijk}A^i A^j A^k\right). \tag{2.9}$$

The pseudoscalar density $\frac{1}{2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu} = \mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}$, $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$, can be described by a total divergence such as,

$$\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu K^\mu, \tag{2.10}$$

where

$$K^\mu = \epsilon^{\mu\alpha\beta\gamma}\mathrm{Tr}\left(A_\alpha F_{\beta\gamma} - \frac{2i}{3}gA_\alpha A_\beta A_\gamma\right). \tag{2.11}$$

As $\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}$ is described by the total derivative of K, this does not contribute to any perturbative calculation.

Now for the pure gauge, the non-zero component is only K^0, which is written by

$$K^0 = -\frac{2gi}{3}\epsilon^{ijk}\mathrm{Tr}\left(A_i A_j A_k\right), \tag{2.12}$$

then

$$\begin{aligned}\int dx^4 \partial_0 K^0 &= \int dx^4 \mathrm{Tr}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right) \\ &= \frac{2gi}{3}\frac{24\pi^2}{g^3 i}\int dt\partial_0 n \\ &= \frac{16\pi^2}{g^2}\left(n_{t=\infty} - n_{t=-\infty}\right).\end{aligned} \tag{2.13}$$

Here $n_{t=\infty} - n_{t=-\infty}$ becomes non-zero if there exist a transition between different vacua. The solution that gives such a transition of vacua is called the *instanton*.

The true vacuum of QCD is a superposition of the vacua which have the winding number n, $|n\rangle$

$$|\theta\rangle = \sum_n e^{-i\theta n}|n\rangle. \tag{2.14}$$

This vacuum is called the θ-vacuum. Now let us see the transition from one θ-vacuum to another θ-vacuum by calculating the transition amplitude,

$$\begin{aligned}
{}_+\langle\theta^{'}|\theta\rangle_- &= \sum_{m,n} e^{im\theta^{'}-in\theta}\ {}_+\langle m|n\rangle_- \\
&= \sum_{\kappa=m+n,\,\nu=m-n} e^{\frac{i}{2}\left(\theta^{'}+\theta\right)\nu} e^{\frac{i}{2}\left(\theta^{'}-\theta\right)\kappa}\ {}_+\langle\nu|0\rangle_- \\
&= \delta\left(\theta^{'}-\theta\right)\sum_\nu e^{i\theta\nu}\ {}_+\langle\nu|0\rangle_- \\
&= \delta\left(\theta^{'}-\theta\right)\sum_\nu \int [\mathcal{D}A]_\nu \exp\left(i\int dx^4\mathcal{L}_{\text{eff}}\right),
\end{aligned} \tag{2.15}$$

where $[dA]_\nu$ means the field configuration which gives $n=\nu$. The effective Lagrangian becomes

$$\mathcal{L}_{\text{eff}} = \mathcal{L} + \frac{g^2\theta}{16\pi^2}\text{Tr}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right). \tag{2.16}$$

Considering the nature of QCD vacuum adds an extra term such as

$$\mathcal{L}_\theta = \frac{g^2\theta}{32\pi^2}F^{a\mu\nu}\tilde{F}^a_{\mu\nu}. \tag{2.17}$$

This term is antisymmetric under the parity exchange and symmetric under the charge reversal, thus it violates P and CP symmetry.

This θ term is changed by the chiral transformations, considering the anomaly. Let us see this by considering the QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}\text{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) - \frac{1}{4}\text{Tr}\left(F^{\text{em}}_{\mu\nu}F^{\text{em}\mu\nu}\right) + \bar{q}\left(i\not{D} - m\right)q. \tag{2.18}$$

The path integral measure dq is not invariant under the chiral transformation such as

$$\begin{aligned}
q^{'}(x) &= \exp\left(i\alpha(x)\gamma_5/2\right)q(x) \\
\bar{q}^{'}(x) &= \bar{q}(x)\exp\left(i\alpha(x)\gamma_5/2\right),
\end{aligned} \tag{2.19}$$

then, the measure is changed by

$$\mathcal{D}q^{'}\mathcal{D}\bar{q}^{'} = \mathcal{D}q\mathcal{D}\bar{q}\exp\left[i\int d^4x\frac{\alpha(x)}{2}\frac{g^2}{8\pi^2}\text{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)\right], \tag{2.20}$$

due to the anomaly. The generating functional is then rewritten by

$$\int \prod_f \mathcal{D}q_f \mathcal{D}\bar{q}_f \mathcal{D}A_\mu \exp\left(i\left[\int d^4x - \frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) - \frac{1}{2}\mathrm{Tr}\left(F^{\mathrm{em}}_{\mu\nu}F^{\mathrm{em}\mu\nu}\right)\right.\right.$$
$$\left.\left. +\bar{q}_f\left(i\not{D} - e^{i\alpha\gamma_5/2}m_f e^{i\alpha\gamma_5/2}\right)q_f + \frac{g^2(\theta+\alpha)}{16\pi^2}\mathrm{Tr}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)\right]\right). \quad (2.21)$$

Here f stands for kinds of fermions. If the mass of fermions is 0, it is possible to remove the extra term which violates CP by an axial rotation with $\alpha = -\theta$. In reality, however quarks are all massive, and hence, we cannot eliminate θ-term by chiral transformations. Because of the mass term, one has to diagonalize the mass matrix such as

$$\mathcal{L}_{\mathrm{mass}} = -\bar{q}_{fR} m_{ff'} q_{fL} + h.c.. \quad (2.22)$$

In order to go to a physical basis, it is necessary to perform chiral transformation with $\alpha = \arg\det M$. Thus θ in the extra term actually becomes,

$$\bar{\theta} = \theta - \arg\det M. \quad (2.23)$$

The extra term violates CP and generates a neutron electric dipole moment d_n. It can be calculated, for example in [2], $d_n \simeq 4.5\times 10^{-15}\bar{\theta}e\,\mathrm{cm}$. There exists strong experimental bound $|d_n| < 2.9\times 10^{-26}e\,\mathrm{cm}$ [4] and this bound implies $|\bar{\theta}| < 0.7\times 10^{-11}$. Because it is natural to suppose a parameter with no dimension to be $O(1)$, such a small $\bar{\theta}$ remains problematic. This is called the *strong CP problem*.

2.2 Peccei-Quinn mechanism and Axion

The promising solution to the strong CP problem is to add a chiral symmetry. This is quite natural because the chiral symmetry can rotate the θ [2-1] term away. This chiral symmetry is global chiral $U(1)$ symmetry, which is called $U(1)_{\mathrm{PQ}}$ symmetry [5] [6]. The dynamical CP-conserving field which is the phase of $U(1)_{\mathrm{PQ}}$ is called *axion*. The axion is the Nambu-Goldstone boson of the $U(1)_{\mathrm{PQ}}$ symmetry [7], [8]. The PQ symmetry has to be spontaneously broken, which is realized when the axion settles down to the minimum of its potential. Under a $U(1)_{\mathrm{PQ}}$ transformation, the axion field $a(x)$ translates to

$$a(x) \to a(x) + \alpha f_a, \quad (2.24)$$

where f_a represents the breaking scale of the PQ symmetry. The Lagrangian that includes the new field axion has extra terms of axion and the interactions of axion and other field,

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \theta\frac{g^2}{16\pi^2}\mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - \frac{1}{2}\partial_\mu a\partial^\mu a + \mathcal{L}_{\mathrm{int}}\left[\frac{\partial^\mu a}{f_a}, \Phi\right] + \xi\frac{a}{f_a}\frac{g^2}{16\pi^2}\mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}). \quad (2.25)$$

[2-1] Here we set $\bar{\theta} \to \theta$ as there is no fear of confusing.

The last term above represents an effective potential for the axion field, and its minimum occurs at $\langle a \rangle = -\theta \frac{f_a}{\xi}$, that is

$$\left\langle \frac{\partial V_{\text{eff}}}{\partial a} \right\rangle = -\frac{\xi}{f_a} \frac{g^2}{16\pi^2} \left\langle \text{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) \right\rangle \Big|_{\langle a \rangle = -\theta \frac{f_a}{\xi}} = 0. \tag{2.26}$$

At this minimum the θ term is canceled out, so the axion field is a dynamical solution to the strong CP problem. Additionally including the effects of the QCD anomaly serves to generate a potential for the axion field which is periodic in the effective vacuum angle $\theta + \langle a \rangle \frac{\xi}{f_a}$:

$$V_{\text{eff}} \simeq \cos\left(\theta + \xi \frac{\langle a \rangle}{f_a}\right). \tag{2.27}$$

The PQ solution can be obtained by minimizing this potential with respect to $\langle a \rangle$,

$$\langle a \rangle = -\frac{f_a}{\xi}\theta. \tag{2.28}$$

Thus the Lagrangian wirtten in terms of $a_{\text{phys}} = a - \langle a \rangle$ no longer violates CP invariance. In order to make the Standard Model invariant under a $U(1)_{\text{PQ}}$ transformation, one must introduce two Higgs doublets H_u and H_d which couple to up-type and down-type quarks respectively to absorb independent chiral transformations. This is called the Peccei-Quinn-Weinberg-Wilczeck (PQWW) model. The axion is the phase of the Higgs doublets, which are written by

$$H_u = \frac{v_u}{\sqrt{2}} e^{i\frac{ax}{v_F}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \; H_d = \frac{v_d}{\sqrt{2}} e^{i\frac{a}{xv_F}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{2.29}$$

where $x \equiv \frac{v_d}{v_u}$ and $v_F^2 = v_u^2 + v_d^2 \sim 250\,\text{GeV}$ (electroweak scale). Under a $U(1)_{\text{PQ}}$ transformation, the axion, the quarks and the Higgs doublets translate to

$$a \to a + \alpha v_F, \tag{2.30}$$

$$u_R \to e^{-i\alpha x}, \; d_R \to e^{-i\frac{\alpha}{x}}. \tag{2.31}$$

From these rules of translation, one can see the Yukawa interacting term

$$\mathcal{L}_{\text{Yukawa}} = -\Gamma_{ij}^u \bar{q}_{iL} H_u u_{jR} - \Gamma_{ij}^d \bar{q}_{iL} H_d d_{jR} + h.c. \tag{2.32}$$

is invariant under $U(1)_{\text{PQ}}$ transformations. Now we perform an axial rotation

$$u_i \to e^{-i\frac{ax}{2v_F}\gamma_5} u_i, \; d_i \to e^{-i\frac{a}{2xv_F}\gamma_5} d_i, \tag{2.33}$$

in order to cancel out the dependence of the axion in the Yukawa interaction term. By the chiral anomaly, we get the interaction term between the axion and the gluons such as

$$\mathcal{L}_{agg} = -\frac{g^2}{16\pi^2} N_g \left(x + \frac{1}{x} \right) \frac{a}{v_F} \mathrm{Tr}(F^{\mu\nu} \tilde{F}_{\mu\nu}). \tag{2.34}$$

Comparing this to eq.(2.25), one finds

$$f_a = v_F, \tag{2.35}$$

and

$$\xi = N_g \left(x + \frac{1}{x} \right), \tag{2.36}$$

where N_g is the number of generations of the quarks.

2.3 Invisible Axion Models

The PQWW model we explained in the previous section was actually ruled out by experiment. The branching ratio of the decay of K^+ is

$$\mathrm{BR}(K^+ \to \pi^+ + a) \simeq 3 \times 10^{-5} \left(x + \frac{1}{x} \right)^2 \tag{2.37}$$

[9], [1]. On the other hand, the bound obtained at the experiment is $\mathrm{BR}(K^+ \to \pi^+ + \mathrm{nothing}) < 3.8 \times 10^{-8}$ [10]. Thus the branching ratio obtained by the PQWW model is far above the experimental bound. In this model, the axion is in the phases of Higgs doublets, so the PQ breaking scale f_a is bound to the electroweak scale. The branching ratio of K^0 decay is inverse proportional to f_a, thus we need to have larger f_a. Actually the interactions between the axion and the quarks or the photons are also inverse proportional to the PQ breaking scale, so if one can set $f_a \gg v_F$, axion becomes invisible in experiments. In this section, we are going to briefly explain so-called invisible axion models.

First, we introduce the Kim-Shifman-Vainstein-Zakharov (KSVZ) model [11], [12]. In this model, a scholar field σ and a super heavy quark Q are added to the model. σ is embedded with the axion degree of freedom and has VEV: $\langle \sigma \rangle = f_a \gg v_F$. Q has the mass $M_Q \simeq f_a$. Only these two fields carry PQ charges. Under a $U(1)_{\mathrm{PQ}}$ transformation, these field translate to

$$\sigma \to e^{2i\alpha}, \tag{2.38}$$

$$Q_L \to e^{i\alpha} Q_L, \ Q_R \to e^{i\alpha} Q_R. \tag{2.39}$$

In a similar way as the PQWW model, the Yukawa interaction terms are invariant under this transformation.

$$\mathcal{L}_{\text{Yukawa}} = -\Gamma \bar{Q}_L \sigma Q_R + h.c. \tag{2.40}$$

After $U(1)_{PQ}$ breaking, σ field can be written by axion such as,

$$\sigma = \frac{f_a}{\sqrt{2}} e^{i\frac{a}{f_a}}. \tag{2.41}$$

By construction, axion in the KSVZ model does not couple with leptons, so by performing a axial rotation $Q_i \to e^{-i\frac{a}{2f_a}\gamma_5} Q_i$, the strong and electromagnetic anomaly terms become

$$\mathcal{L}_{\text{anomaly}} = -\frac{a}{f_a}\left(\frac{g^2}{16\pi^2}\text{Tr}(F^{\mu\nu}F_{\mu\nu}) + 3e_Q^2\frac{\alpha}{2\pi}\text{Tr}\left(F^{\text{em}\mu\nu}F^{\text{em}}_{\mu\nu}\right)\right), \tag{2.42}$$

where e_Q is the electromagnetic charge of Q. Note that in the KSVZ model, the axion mass is written by [1]

$$m_a \simeq 6.3\,\text{eV}\left(\frac{10^6\,\text{GeV}}{f_a}\right). \tag{2.43}$$

Another invisible axion model is so called the Dine-Fischler-Srendnicki-Zhitnitskii (DFSZ) model [13], [14]. In this model, two Higgs doublets H_u, H_d and a scalar field ϕ are added to the original PQ model, where the axion is embedded in ϕ. The Standard Model quarks and ϕ, the two Higgs doublets carry PQ charges. There are couplings between the Standard Model quarks and the Higgs doublets, and couplings between ϕ and the Higgs doublets. After the spontaneous breaking of $SU(2)_W \times U(1)_Y \times U(1)_{PQ}$, there appear the couplings of the Standard Model quarks and the axion. In the similar way as the previous models, by performing chiral rotations, one gets the anomaly term such as,

$$\mathcal{L}_{\text{anomaly}} = -\frac{a}{F_a}\frac{g^2}{16\pi^2}\text{Tr}(F^{\mu\nu}F_{\mu\nu}), \tag{2.44}$$

where $F_a \simeq \frac{f_a}{2N_g}$ ($\langle\phi\rangle \gg v_F$), and N_g is the number of the generations of the Standard Model quarks. Then the axion mass in the DFSZ model is written by [1],

$$m_a \simeq 12\,\text{eV} N_g\left(\frac{10^6\,\text{GeV}}{f_a}\right). \tag{2.45}$$

There are astrophysical and cosmological bounds on the PQ breaking scale F_a (f_a in the KSVZ model) in axion models, such as

$$10^9\,\text{GeV} \lesssim F_a \lesssim 10^{11}\,\text{GeV}. \tag{2.46}$$

Astrophysics gives the lower bound. Because the axions are very light and very weakly coupled, the axion interaction temperature is much lower than the typical temperature of the interaction inside stars. Thus, axions are emitted from stars. In order not to effect the evolutions of stars, the axion mass should be small enough, that is, F_a should be large enough. This lower bound comes from the observation of supernova 1987A [15], [16]. The axion mass has to be small enough not to shorten the neutrino burst duration. On the other hand, the upper bound comes from cosmology. This is only valid only If one considers the axion as dark matter (especially, cold dark matter). The energy density of the axion cannot exceed the observational amount of cold dark matter, thus this gives the upper bound of F_a [3]. Note that the upper bound exists only if the PQ symmetry breaking takes place after the inflation. Otherwise F_a can still take $F_a > 10^{11}$ GeV. In the chapter 6, as we consider the supersymmetric partner of the axion as dark matter, this upper bound does not exist.

Chapter 3

Supersymmetry and MSSM

In this section, we review supersymmetry. Supersymmetry (SUSY) is able to solve problems in the Standard Model such as the hierarchy problem and also enables the gauge coupling unification for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ at the Grand Unified Theory scale. Besides, the new symmetry gives a promising candidate for dark matter. After a review of SUSY, we take a look at the Minimal Supersymmetric Standard Model which is the minimal supersymmetric extension of the standard model. There exist many reviews and introductions about SUSY and the MSSM. In this thesis, we roughly follow [17] and [18]. In the last section, we introduce *axino*, a supersymmetric partner of the axion.

3.1 Supersymmetry

In the Standard Model, the Higgs boson gives mass to the W and the Z bosons via the spontaneously symmetry breaking of the electroweak symmetry $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$. The masses of the quarks and the charged leptons are also generated via the couplings to the Higgs boson. The mass of the Higgs particle is effected by radiative corrections due to loop integrations from the interaction. The one loop radiative correction to the Higgs mass m_H is expressed by the renormalized mass m_H and the bare mass $m_{H,0}$,

$$\begin{aligned} \delta m_H^2 &= m_H^2 - m_{H,0}^2 \\ &= -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{\rm UV}^2, \end{aligned} \tag{3.1}$$

where $\Lambda_{\rm UV}$ is a cut off scale, at which scale new physics enters. Now as we know the Higgs mass is about 125-6 GeV [19] [20], if we suppose the cut off scale as Planck scale $\sim 10^{19}\,$GeV and $\lambda_f \sim O(1)$, we need a tremendous fine tuning cancellation such as

$$\begin{aligned} m_H^2 &= m_{H,0}^2 + \delta m_H^2 \\ O(100\,{\rm GeV})^2 &= O(10^{19}\,{\rm GeV})^2 - O(10^{19}\,{\rm GeV})^2. \end{aligned} \tag{3.2}$$

This is called the hierarchy problem in the Standard Model. SUSY solves this problem by introducing a symmetry between bosons and fermions. The new symmetry introduces bosonic fields for each fermion in the Standard Model and vice versa, so that the quadratic divergent radiative corrections to the Higgs boson mass are canceled with each other. From the interaction between the Higgs boson and scalar fields that is written by $\mathcal{L}_{\rm int} = -\lambda_s |H|^2 |\phi|^2$, the radiative corrections to the Higgs mass from one loop integration becomes

$$\delta m_H^2 = \frac{\lambda_s^2}{16\pi^2}\Lambda_{\rm UV}^2. \tag{3.3}$$

Here the difference of the sign between the radiative corrections from fermions and boson comes from the spin statistics. Thus with two new bosons to each Dirac fermion and supposing $\lambda_s = |\lambda_f|$, the radiative corrections from bosons and fermions cancel out without any fine tuning. The corrections by higher loop integrals also cancel thanks to the new symmetry.

The symmetry which relates bosons and fermions requires a supersymmetric Lagrangian to be invariant under supersymmetric transformations, which turn bosonic states into fermionic states and vice versa. Such transformations are described by spinor operators Q_α,

$$Q_\alpha \left|B\right\rangle = \left|F\right\rangle , \;\; \bar{Q}_{\dot\alpha} \left|F\right\rangle = \left|B\right\rangle . \tag{3.4}$$

Note that here we consider $Q_\alpha^{\mathcal{N}=1}$, which corresponds to a simple SUSY. α is a spinor index and $\bar{Q}_{\dot\alpha} = Q_\alpha^\dagger$. These spinor operators satisfy the following algebras,

$$\{Q_\alpha\,, Q_\beta\} = \{\bar{Q}_{\dot\alpha}\,, \bar{Q}_{\dot\beta}\} = 0, \tag{3.5}$$

$$\{Q_\alpha\,, \bar{Q}_{\dot\beta}\} = 2\left(\sigma^\mu\right)_{\alpha\dot\beta} P_\mu, \tag{3.6}$$

and

$$\left[P^\mu\,, Q_\alpha\right] = \left[P^\mu\,, \bar{Q}^{\dot\alpha}\right] = 0. \tag{3.7}$$

where P_μ is the four-momentum generator of space-time translations. Here $\sigma^\mu = (\sigma^0, \sigma^i)$, and σ^i are Pauli matrices. They are defined as

$$\begin{aligned} \sigma^0 &= \bar\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \;\; \sigma^1 = -\bar\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \sigma^2 &= -\bar\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \;\; \sigma^3 = -\bar\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned} \tag{3.8}$$

In SUSY, a one particle state $|\Omega\rangle$ and the states which are generated by acting SUSY generators Q_α and $\bar{Q}_{\dot\beta}$ make up a *supermultiplet*. The members of one supermultiplet are *superpartners* each other. One can find that each supermultiplet contains same numbers of bosons and fermions by defining the fermion number operator $(-)^F = (-1)^{2s}$ where s is spin. This operator satisfies

$$(-)^F |B\rangle = |B\rangle \, , \, (-)^F |F\rangle = -|F\rangle. \tag{3.9}$$

Then, taking trace of $(-)^F\{Q_\alpha \, , \bar{Q}_{\dot{\beta}}\}$ gives

$$\begin{aligned} \mathrm{Tr}\{(-)^F\{Q_\alpha \, , \bar{Q}_{\dot{\beta}}\}\} &= 2\,(\sigma^\mu)_{\alpha\dot{\beta}}\, P_\mu \mathrm{Tr}\{(-)^F\} \\ &= \mathrm{Tr}\{-Q_\alpha(-)^F \bar{Q}_{\dot{\beta}} + (-)^F \bar{Q}_{\dot{\beta}} Q_\alpha\} = 0, \end{aligned} \tag{3.10}$$

where we used eq.(3.6) and the anticommutation relation $\{(-)^F, \, Q_\alpha\} = 0$. Thus

$$\begin{aligned} \mathrm{Tr}\{(-)^F\} &= \sum_{\text{boson}} \langle B| \, (-)^F \, |B\rangle + \sum_{\text{fermion}} \langle F| \, (-)^F \, |F\rangle \\ &= n_B - n_F = 0 \end{aligned} \tag{3.11}$$

The particles in the same supermultiplet share the equal mass from the commutation relation of $-P^2$ and $Q_\alpha, \bar{Q}_{\dot{\beta}}$. In the similar way, from the fact that $Q_\alpha, \bar{Q}_{\dot{\beta}}$ also anticommute with the generators of the gauge transformations, the superpartners reside in the same representations of the gauge group.

Mainly supermultiplets fall into two categories. The first one is called a *chiral* supermultiplet (ϕ, ψ). A chiral supermultiplet consists of a single Weyl fermion ψ ($n_F = 4$), two real scalars ϕ ($n_B = 2$) and one complex scalar field F ($n_B = 2$). F does not have its kinetic term and called an *auxiliary* field. The auxiliary field is introduced in order to make the supersymmetric algebras to close even quantum mechanically (off-shell) and does not propagate. The other kind of supermultiplets is called a *gauge* supermultiplet $\left(A^a_\mu, \lambda^a\right)$ where a is a index for the adjoint representation of the gauge group. A gauge supermultiplet contains a spin-1 vector boson A^a_μ and its superpartner, a spin-1/2 Weyl fermion λ^a. Before the spontaneous breaking of the gauge symmetry, the vector boson is massless, thus the fermion is also massless. A massless spin-1 boson has three degrees of freedom ($n_B = 3$) (one degree of freedom is absent due to the inhomogenous gauge transformation) and a Weyl fermion has four degrees of freedom ($n_F = 4$). There exists an auxiliary field D^a with one bosonic degree of freedom ($n_B = 1$). Note that so far we counted the degrees of freedom off-shell, on-shell, a spin-1/2 Weyl fermion has two real degrees of freedom ($n_F = 2$) according to two helicity states and a spin-2 boson also has only two degrees of freedom ($n_B = 2$). On-shell, the auxiliary fields F and D^a are eliminated. There is another supermultiplet if one includes the spin-2 graviton (again with two helicity states $n_B = 2$ on-shell). The superpartner of the graviton is called *gravitino* with spin-3/2, which is massless ($n_F = 2$ on-shell) if supersymmetry is unbroken.

3.2 Lagrangian in SUSY

Supersymmetric Lagrangians are invariant under the supersymmetric transformation (up to total derivative). Let us first take a look at the Lagrangian that consists of free chiral supermultiplets $\Phi_i = (\phi_i, \psi_i, F_i)$.

$$\mathcal{L}_{\text{chiral free}} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i. \tag{3.12}$$

The index i represents all gauge and flavor degrees of freedom. This Lagrangian is invariant under the supersymmetry transformation,

$$\delta\phi_i = \epsilon\psi_i, \tag{3.13}$$

$$\delta(\psi_i)_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi_i + \epsilon_\alpha F_i, \tag{3.14}$$

$$\delta F_i = -i\epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi_i. \tag{3.15}$$

where ϵ^α is a infinitesimal anti-commuting spinor.

The general interaction part of the Lagrangian which is renormalizable and invariant under supersymmetric transformation is written by,

$$\mathcal{L}_{\text{chiral int}} = \left(-\frac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i \right) + c.c. \tag{3.16}$$

Here W^{ij} and W^i are defined by

$$W^{ij} = \frac{\delta^2}{\delta\phi_i \delta\phi_j} W, \tag{3.17}$$

and

$$W^i = \frac{\delta W}{\delta \phi_i}. \tag{3.18}$$

Here W is called the *superpotential*,

$$W = \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k, \tag{3.19}$$

where M^{ij} is a symmetric matrix and y^{ijk} is a symmetric Yukawa coupling for the interaction of one scalar field and two fermions. Thus, W^{ij} and W^i are explicitly

$$W^{ij} = M^{ij} + y^{ijk} \phi_k, \tag{3.20}$$

$$W^i = M^{ij} \phi_j + \frac{1}{2} y^{ijk} \phi_j \phi_k. \tag{3.21}$$

The classical equations of motion for F_i and F^j give the expression of the auxiliary fields by the superpotential, such as

$$F_i = -W_i^*, \;\; F^{*i} = -W^i. \tag{3.22}$$

By inserting eq.(3.22) into eqs.(3.12) and (3.16), we obtain the supersymmetric Lagrangian for chiral supermultiplets,

$$\begin{aligned}\mathcal{L}_{\text{chiral}} &= \mathcal{L}_{\text{chiral free}} + \mathcal{L}_{\text{chiral int}} \\ &= -\partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i - V(\phi, \phi^*) \\ & \quad -\frac{1}{2} M^{ij} \psi_i \psi_j - \frac{1}{2} M_{ij}^* \psi^{\dagger i} \psi^{\dagger j} - \frac{1}{2} y^{ijk} \phi_i \psi_j \psi_k - \frac{1}{2} y_{ijk}^* \phi^{*i} \psi^{\dagger j} \psi^{\dagger k}. \end{aligned} \tag{3.23}$$

Here the scalar potential $V(\phi, \phi^*)$ is

$$\begin{aligned} V(\phi, \phi^*) &= -W^i W_i^* \\ &= M_{ik}^* M^{kj} \phi^{*i} \phi_j + \frac{1}{2} M^{il} y_{jkl}^* \phi_i \phi^{*j} \phi^{*k} + \frac{1}{2} M_{il}^* y^{jkl} \phi^{*i} \phi_j \phi_k \\ & \quad + \frac{1}{4} y^{ijl} y_{knl}^* \phi_i \phi_j \phi^{*k} \phi^{*n}. \end{aligned} \tag{3.24}$$

From the total Lagrangian, one finds the scalar fields and the fermions share the same squared mass matrix $M_{ij}^* M^{jk}$. This corresponds to the fact that superpartners have same masses in each supermultiplet.

Next, we include gauge supermultiplets $\left(A_\mu^a, \lambda^a, D^a\right)$ in the supersymmetric Lagrangian. The gauge transformation are

$$A_\mu^a \to A_\mu^a + \partial_\mu \Lambda^a + g f^{abc} A_\mu^b \Lambda^c, \tag{3.25}$$

$$\lambda^a \to \lambda^a + g f^{abc} \lambda^b \Lambda^c, \tag{3.26}$$

where Λ^a is a parameter for the infinitesimal gauge transformation, g is the gauge coupling, f^{abc} are the totally antisymmetric structure constants which satisfy $\left[T^a, T^b\right] = i f^{abc} T^c$ (T^a representationsofthegaugegroup). For Abelian gauge groups, $f^{abc} = 0$.

The free part of the Lagrangian for gauge supermultiplets is written by,

$$\mathcal{L}_{\text{gauge free}} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + i\lambda^{\dagger a} \bar{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2} D^a D^a, \tag{3.27}$$

where $F_{\mu\nu}^a$ is the field strength which is given by

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c. \tag{3.28}$$

The covariant derivative of λ^a is

$$\nabla_\mu \lambda^a = \partial_\mu \lambda^a + g f^{abc} A_\mu^b \lambda^c. \tag{3.29}$$

This supersymmetric Lagrangian for the gauge supermultiplets is invariant under supersymmetric transformation such as

$$\delta A^a_\mu = -\frac{1}{\sqrt{2}}\left(\epsilon^\dagger\bar{\sigma}_\mu\lambda^a + \lambda^{\dagger a} + \lambda^{\dagger a}\bar{\sigma}_\mu\epsilon\right), \tag{3.30}$$

$$\delta\lambda^a_\alpha = \frac{i}{2\sqrt{2}}\left(\sigma^\mu\bar{\sigma}^\nu\epsilon\right)_\alpha F^a_{\mu\nu} + \frac{1}{2}\epsilon_\alpha D^a, \tag{3.31}$$

$$\delta D^a \frac{i}{\sqrt{2}}\left(-\epsilon^\dagger\bar{\sigma}^\mu\nabla_\mu\lambda^a + \nabla_\mu\lambda^{\dagger a}\bar{\sigma}^\mu\epsilon\right). \tag{3.32}$$

Next we are going to introduce a general supersymmetric Lagrangian with chiral supermultiplets and gauge supermultiplets including the interaction between both suplermultiplets. Fields in a same supermultiplets share in the same gauge representation, thus they transform under the gauge transformation such as

$$\phi_i \to \phi_i + ig\Lambda^a(T^a\phi)_i, \tag{3.33}$$

$$\psi_{i\alpha} \to \psi_{i\alpha} + ig\Lambda^a(T^a\psi_\alpha)_i, \tag{3.34}$$

$$F_i \to F_i + ig\Lambda^a(T^aF)_i. \tag{3.35}$$

The derivatives of the fields are replaced by the covariant derivatives in order to keep the Lagrangian invariant under the gauge transformations. They are written by,

$$\nabla_\mu\phi_i = \partial_\mu\phi_i - igA^a_\mu\left(T^a\phi\right)_i, \tag{3.36}$$

$$\nabla_\mu\psi_i = \partial_\mu\psi - igA^a_\mu\left(T^a\psi\right)_i. \tag{3.37}$$

Besides, we need the condition

$$W^i\left(T^a\phi\right)_i = 0, \tag{3.38}$$

in order to keep the Lagrangian (especially the scalar potential part $V(\phi,\phi^*)$) invariant under the gauge transformations.

The general interaction part of the total Lagrangian is written by

$$\mathcal{L}_{\text{int}} = -\sqrt{2}g\left(\phi^*T^a\psi\right)\lambda^a - \sqrt{2}g\lambda^{\dagger a}\left(\psi^\dagger T^a\phi\right) + g\left(\phi^*T^a\phi\right)D^a. \tag{3.39}$$

With this interaction part, the total Lagrangian $\mathcal{L}_{\text{chiral}} + \mathcal{L}_{\text{gauge free}} + \mathcal{L}_{\text{int}}$ is invariant under the supersymmetric transformations of gauge supermultiplets eqs.(3.30), (3.31) and

(3.32). The supersymmetric transformations of chiral supermultiplets which keep the total Lagrangian invariant are modified to

$$\delta\phi_i = \epsilon\psi_i, \tag{3.40}$$

$$\delta\psi_{i\alpha} = -i\left(\sigma^\mu\epsilon^\dagger\right)_\alpha \nabla_\mu\phi_i + \epsilon_\alpha F_i, \tag{3.41}$$

$$\delta F_i = -i\epsilon^\dagger\bar{\sigma}^\mu\nabla_\mu\psi_i + \sqrt{2}g\left(T^a\phi\right)_i \epsilon^\dagger\lambda^{\dagger a}. \tag{3.42}$$

The equation of motion for the D^a field leads to

$$D^a = -g\left(\phi^* T^a \phi\right). \tag{3.43}$$

Eqs.(3.22) and (3.43) give the expression for the scalar potential

$$\begin{aligned} V(\phi,\phi^*) &= F^{*i}F_i + \frac{1}{2}D^aD^a \\ &= W^iW_i^* + \frac{1}{2}g^2\sum_a\left(\phi^*T^a\phi\right)^2. \end{aligned} \tag{3.44}$$

The first term is called *F-term*, which is determined by the interaction between the scalars and the fermions and the mass terms, on the other hand, the second term is *D-term* determined by the gauge interactions.

So far, we did not use the language of superfields and superspace. A supermultiplet can be represented by a *superfield* Φ or V, which contains all components of the supermultiplet $\Phi \ni (\phi,\psi,F)$ or $V \ni (A_\mu,\lambda_\alpha,D)$. In this thesis, we do not explain superfields and superspace in detail, but in many SUSY reviews, they explain the concept (for example in [17], [18]). Using the superfields, the superpotential is written by

$$W = \frac{1}{2}M^{ij}\Phi_i\Phi_j + \frac{1}{6}y^{ijk}\Phi_i\Phi_j\Phi_k. \tag{3.45}$$

Actually, one can add a term proportional to Φ_i to the superpotential only if Φ_i is a gauge singlet. As we will concentrate on the minimal extension to the Standard Model where there is no such a chiral supermultiplet, we will omit this term in the following.

3.3 Minimal Supersymmetric Standard Model

So far we briefly reviewed SUSY and introduced the supersymmetric Lagrangian that is invariant under the spupersymmetric transformation, the transformation into boson from fermion and vice versa. Now we are going to review a supersymmetric extension of the Standard Model, the Minimal Supersymmetric Standard Model (MSSM). The Standard

Model is spontaneously broken $SU_C(3)\otimes SU_L(2)\otimes U_Y(1)$ gauge theory. The super partners in a supermultiplets inhabit the same gauge representation. Now let us take a look at the contents of the MSSM.

In the MSSM, there exist chiral supermultiplets with quarks in the Standard Model and their superpartners, the *squarks*. Table. 3.1 shows chiral supermultiplets for the quarks/squarks where i represents three generations for the quarks in the Standard Model. The chiral supermultiplet Q_i is a doublet under $SU(2)_L$. Fig.3.2 shows chiral supermultiplets for the leptons and their superpartners the *sleptons* and Table.3.3 shows for the Higgs and their superpartners, the *higgsinos*. Note that there are two Higgs $SU(2)_L$ doublets in the MSSM while there is only one $SU(2)_L$ Higgs doublet in the Standard Model. Besides the cancellation of the anomalies, another reason is that the superpotentials in SUSY have to be holomorphic. We need two Higgs supermultiplets to give masses to the up and down-type quarks and the charged leptons.

supermultiplets	spin 0 (squark)	spin 1/2 (quark)	$SU_C(3)\otimes SU_L(2)\otimes U_Y(1)$
Q_i	$\begin{pmatrix} \tilde{u}_{Li} \\ \tilde{d}_{Li} \end{pmatrix}$	$\begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$	$\left(3, 2, \frac{1}{6}\right)$
$\bar{U}_i$	$\tilde{u}^*_{Ri}$	$u^\dagger_{Ri}$	$\left(3^*, 1, -\frac{2}{3}\right)$
$\bar{D}_i$	$\tilde{d}^*_{Ri}$	$d^\dagger_{Ri}$	$\left(3^*, 1, \frac{1}{3}\right)$

Table 3.1: Chiral supermultiplets for quarks/squarks

supermultiplets	spin 0 (slepton)	spin 1/2 (lepton)	$SU_C(3)\otimes SU_L(2)\otimes U_Y(1)$
L_i	$\begin{pmatrix} \tilde{\nu}_{Li} \\ \tilde{e}_{Li} \end{pmatrix}$	$\begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}$	$\left(1, 2, -\frac{1}{2}\right)$
$\bar{E}_i$	$\tilde{e}^*_{Ri}$	$e^\dagger_{Ri}$	$(1, 1, 1)$

Table 3.2: Chiral supermultiplets for leptons/sleptons

supermultiplets	spin 0 (Higgs)	spin 1/2 (higgsino)	$SU_C(3)\otimes SU_L(2)\otimes U_Y(1)$
H_u	$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$\begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$	$\left(1, 2, \frac{1}{2}\right)$
H_d	$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$\begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$	$\left(1, 2, -\frac{1}{2}\right)$

Table 3.3: Chiral supermultiplets for Higgs/higgsinos

The gauge bosons of the Standard Models make up gauge supermultiplets (Table. 3.4). Corresponding to $SU_C(3)\otimes SU_L(2)\otimes U_Y(1)$, there are three gauge supermultiplets. The gauge supermultiplet for the $SU_C(3)$ gauge interaction consists of the gluon g and the *gluino* $\tilde{g}$. For the electroweak gauge interaction $SU_L(2) \otimes U_Y(1)$, there exist gauge bosons and fermions (*gauginos*), the W bosons $W^{\pm}, W^0$ and the *winos* $\tilde{W}^{\pm}, \tilde{W}^0$, the B boson B^0 and the *bino* $\tilde{B}^0$. These electroweak bosons and their superpartners mix into the Z boson Z^0 and the photon γ after spontaneous symmetry breaking of the electroweak symmetry. The corresponding gauge fermions are the *zino* $\tilde{Z}^0$ and the *photino* $\tilde{\gamma}$.

supermultiplets	spin 1/2 gaugino	spin 1 gauge boson	$SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$
G_1	$\tilde{g}$	g	$(8, 1, 0)$
G_2	$\tilde{W}^{\pm}, \tilde{W}^0$	$W^{\pm}, W^0$	$(1, 3, 0)$
G_3	$\tilde{B}^0$	B^0	$(1, 1, 0)$

Table 3.4: gauge supermultiplets

Now we have the supermultiplets in the MSSM that are shown in Tables. (3.1)-(3.4). The general superpotential of the MSSM is written by

$$\begin{aligned} W_{\text{MSSM}} &= \epsilon^{\alpha\beta}\left(y_u^{ij}\bar{U}_{ia}Q_{j\alpha a}H_{u\beta} - y_d^{ij}\bar{D}_{ia}Q_{j\alpha a}H_{d\beta} - y_e^{ij}\bar{E}_i L_{j\alpha}H_{d\beta} + \mu H_{u\alpha}H_{d\beta}\right) \\ &+ \epsilon^{\alpha\beta}\left(\alpha_1^{ijk}L_{i\alpha}L_{j\beta}\bar{E}_k + \alpha_2^{ijk}L_{i\alpha}Q_{j\beta a}\bar{D}_k^a + \alpha_3^i L_{i\alpha}H_{u\beta}\right) \\ &+ \epsilon^{abc}\beta^{ijk}\bar{U}_{ai}\bar{D}_{bj}\bar{D}_{ck}. \end{aligned} \tag{3.46}$$

where $\epsilon^{\alpha\beta}$ is the antisymmetric symbol $\epsilon^{12} = -\epsilon^{21} = -\epsilon_{12} = \epsilon_{21} = 1$. ϵ^{abc} is also totally antisymmetric tensor $\epsilon^{123} = -\epsilon_{123} = 1$ where $a = 1, 2, 3$ is the color index of 3 and its adjoint representations of $SU_C(3)$. The first line of eq.(3.46) contains terms with Yukawa couplings that give masses to quarks and leptons and a mass term for two Higgs bosons. The terms in the second line violate the total baryon or lepton number conservation. These terms do not exist in the Standard Model that automatically preserves the total baryon or lepton numbers. The existence of the baryon and lepton number violating terms leads to proton decay $p \to e^+ + \pi^0$. The decay time can be estimated to be within seconds. This is quite contrary to the experiments which suggest that the lifetime of proton is much longer than the age of the universe. Thus we need to constrain the MSSM to preserve the total baryon and lepton number by imposing extra symmetry, *R-parity*. The R-parity is defined as

$$P_R = (-1)^{3(B-L)+2s}, \tag{3.47}$$

where B and L are the baryon/lepton number respectively and s is the spin of the particle. All of the Standard Model particles including the Higgs boson have $P_R = 1$ and their superpartners, the *sparticles* have $P_R = -1$. The R-symmetry means that there only exist interactions involving even-number sparticles. If the R-symmetry exist, the lightest sparticle, the lightest supersymmetric particle, LSP is stable. Thus, if LSP is electrically neutral, which means that it scarcely interacts with other particles, LSP is a promising candidate for dark matter (weakly interacting massive particle, WIMP). With the R-symmetry, the superpotential of the MSSM is written by

$$\begin{aligned} & W_{\mathrm{MSSM\,R}} \\ = \; & \epsilon^{\alpha\beta}\left(y_u^{ij}\bar{U}_{ia}Q_{j\alpha a}H_{u\beta} - y_d^{ij}\bar{D}_{ia}Q_{j\alpha a}H_{d\beta} - y_e^{ij}\bar{E}_i L_{j\alpha}H_{d\beta} + \mu H_{u\alpha}H_{d\beta}\right). \end{aligned} \tag{3.48}$$

We have discussed the supersymmetric extension of the Standard Model, but as the superpartners of the Standard Model particles have not discovered yet, SUSY has to be broken. If SUSY would be unbroken, the sparticles should have shared same masses as their superpartners in the Standard Model, which means that they should have discovered by experiments. The effect of the SUSY breaking has to be constrained in order not to bring back quadratic divergence in the Higgs mass which is cancelled by introducing SUSY. Such SUSY breaking is called ”soft” SUSY breaking. The SUSY breaking occurs spontaneously in a hidden sector different from the visible sector where the MSSM particle inhabit, then this SUSY breaking communicates via the messenger sector to the visible sector. The property of the SUSY breaking depends on the mediating interaction. The gravity mediated SUSY breaking is mediated by gravitational interactions. The typical mass of the particles is assumed to be

$$m_{\mathrm{sparticle}} \sim \frac{M_{\mathrm{SUSY}}^2}{M_P}. \tag{3.49}$$

If one assumes $m_{\mathrm{sparticle}} \sim 1\,\mathrm{TeV}$, which leads to the SUSY breaking scale $M_{\mathrm{SUSY}} \sim 10^{12}\,\mathrm{GeV}$ and the gravitino obtains mass of $m_{\mathrm{sparticle}} \sim 1\,\mathrm{TeV}$. While the gauge mediated SUSY breaking is via gauge interactions between the messenger sector and the visible sector. The sparticle mass is roughly estimated as

$$m_{\mathrm{sparticle}} \sim \frac{g^2}{16\pi^2}\frac{M_{\mathrm{SUSY}}^2}{M_S}, \tag{3.50}$$

where g is the gauge coupling constant and M_S is the mass scale of the messenger sector. Thus if one takes $m_{\mathrm{sparticle}} \sim 1\,\mathrm{TeV}$ and $M_S \sim M_{\mathrm{SUSY}}$, M_{SUSY} becomes comparably small $\sim O(100\,\mathrm{TeV})$. The gravitino mass becomes also small, approximately $\sim \frac{M_{\mathrm{SUSY}}^2}{M_P} \lesssim O(\mathrm{GeV})$ in the case of the gauge mediation, while in the gravity mediation, it is close to the other sparticle masses.

3.4 Axino

In this section, we briefly explain the *axino*, the supersymmetric partner of the axion. As SUSY is a symmetry between bosons and fermions, if there exist supersymmetry, each elemental particle has its superpartner whose spin differs by 1/2 and each superparter shares the same mass and internal quantum numbers except spin. Furthermore, with the convention of the R-parity, LSP becomes a strong candidate of the dark matter. The axino can be LSP, which makes the axino a candidate of the dark matter. Reviews about the axino as the dark matter are [21], [22], for example.

When an axion model is supersymmetrized, a superpartner of the axion a, the axino $\tilde{a}$ and a real scalar field, the *saxion* s are introduced. The axion superfield which includes the axion, the saxion and the axino $A \ni (a, s, \psi_a)$ is written by

$$A = \frac{s + ia}{\sqrt{2}} + \epsilon_{\alpha\beta}\sqrt{2}\theta^\alpha\psi_a^\beta + \epsilon_{\alpha\beta}\theta^\alpha\theta^\beta F_A, \tag{3.51}$$

θ is a complex anticommuting two-component spinor. F_A is an axial field which does not carry the physical degree of freedom. The axino is written as 4-spinors using ψ_a,

$$\tilde{a} = \begin{pmatrix} \psi_a \\ \bar{\psi}_a \end{pmatrix}. \tag{3.52}$$

The superpotential responsible for the spontaneous PQ symmetry breaking is written as [23]

$$W_{\mathrm{PQ}} = f_Z Z(S_1 S_2 - V_a^2), \tag{3.53}$$

where Z is a gauge singlet superfield, S_1 and S_2 are PQ scalars, and f_Z is a Yukawa coupling. V_a is a dimensional parameter corresponding to the PQ-breaking scale. While Z does not have PQ charge, S_1 and S_2 translates into

$$S_1 \to e^{i\alpha Q_\sigma} S_1,\ S_2 \to e^{-i\alpha Q_\sigma} S_2, \tag{3.54}$$

under $U(1)_{\mathrm{PQ}}$ transformation. At the vacuum, $S_1 \simeq S_2 \simeq V_a$, the PQ symmetry is spontaneously broken. The axion superfield is obtained as $(S_1 - S_2)/\sqrt{2}$.

Now let us see the supersymmetrization of the two invisible axion models the KSVZ and the DFSZ model. In the KSVZ model, new heavy quark fields that carry PQ charges are introduced. Using the new quark fields, the superpotential is written by [24]

$$W_{\mathrm{KSVZ}} = W_{\mathrm{PQ}} + f_Q Q_L \bar{Q}_R S_1. \tag{3.55}$$

The second term leads to the anomaly coupling $aF\tilde{F}$ at low energy after the heavy quarks are integrated out.

On the other hand, in the DFSZ model, two Higgs doublets, H_d and H_u, and the MSSM (s)quarks, carry PQ charges and couples to the axion multiplet as [25]

$$W_{\mathrm{DFSZ}} = W_{\mathrm{PQ}} + \frac{f_s}{M_P} S_1^2 H_u H_d. \tag{3.56}$$

The anomalous coupling is obtained after electroweak symmetry breaking through the coupling of the axion to the Higgs doublets which further couples to the light (s)quarks.

In the chapter 6, we will consider these two supersymmetric axion models.

Chapter 4

The basis of modern cosmology

This chapter explains basis of cosmology, Friedmann-Robertson-Walker universe, Einstein equation, and inflation. Throughout this thesis without any explicit statement, natural units are adopted, where the speed of light, the reduced Planck constant and the Boltzmann constant are unity, $c = \hbar = k_B = 1$ [26]. In appendix A, we also include a review of the perturbation theory of the universe [27, 28].

4.1 The background universe and Einstein field equations

In order to investigate the nature of space and time, we define the space-time metric. If there exist curvatures in the space time, it is impossible to define the inertial space, thus we can only define a local metric using the metric tensor $g_{\mu\nu}$, such as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{4.1}$$

The cosmological principle is the assumption that considers no special point and direction in the universe. This principle describes the homogeneous and isotropic universe. Adapting the cosmological principle, the metric of the homogeneous and isotropic universe at the uniform time is described by the metric

$$ds^2 = a^2(\tau) \left[-d\tau^2 + \frac{dr^2}{1 - Kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right], \tag{4.2}$$

where $a(\tau)$ is the scale factor normalized to 1 at present (*i.e.* $a(\tau_0) = 1$), K is a constant, which describes the curvature of the universe and τ is called conformal time and defined by $d\tau = \frac{dt}{a(\tau)}$. In the following we use $\frac{d}{d\tau} = {}'$, $\frac{d}{dt} = \dot{}$ This metric is called Friedmann-Robertson-Walker (FRW) metric. $K = -1, 0, 1$ respectively corresponds to closed, flat and open universes.

The covariant derivative is the derivative of general metrics. Vectors and tensors change their components by parallel transports in curved surfaces. The changes are expressed by using the Christoffel symbols $\Gamma^{\mu}{}_{\nu\lambda}$,

$$dA^{\mu} = -\Gamma^{\mu}{}_{\nu\lambda} A^{\nu} dx^{\lambda}, \tag{4.3}$$

$$dA_{\mu} = \Gamma^{\nu}{}_{\mu\lambda} A_{\nu} dx^{\lambda}, \tag{4.4}$$

$$\Gamma^{\mu}{}_{\lambda\nu} = \frac{1}{2} g^{\mu\rho} \left(\partial_{\nu} g_{\rho\lambda} + \partial_{\lambda} g_{\rho\nu} - \partial_{\rho} g_{\lambda\nu} \right). \tag{4.5}$$

Then the covariant derivative is written by

$$\nabla_{\nu} A^{\mu} = \partial_{\nu} A^{\mu} + \Gamma^{\mu}{}_{\nu\lambda} A^{\lambda}, \tag{4.6}$$

$$\nabla_{\nu} A_{\mu} = \partial_{\nu} A_{\mu} - \Gamma^{\lambda}{}_{\nu\mu} A_{\lambda}. \tag{4.7}$$

Note that the covariant derivative is not commutative.

$$\nabla_{\alpha} \nabla_{\beta} A^{\mu\nu}{}_{\lambda} - \nabla_{\beta} \nabla_{\alpha} A^{\mu\nu}{}_{\lambda} = R^{\mu}{}_{\nu\alpha\beta} A^{\nu} \neq 0 \tag{4.8}$$

$$R^{\mu}{}_{\nu\alpha\beta} = \Gamma^{\mu}{}_{\beta\nu,\alpha} - \Gamma^{\mu}{}_{\nu\alpha,\beta} + \Gamma^{\mu}{}_{\alpha\lambda} \Gamma^{\lambda}{}_{\beta\nu} - \Gamma^{\mu}{}_{\beta\lambda} \Gamma^{\lambda}{}_{\nu\alpha} \tag{4.9}$$

From now on, we express $\nabla_{\mu} =; \mu, \partial_{\mu} =, \mu$. $R^{\mu}{}_{\nu\alpha\beta}$ is the Riemann curvature tensor, the Ricci curvature and the Riemann curvature scholar are defined as

$$\begin{aligned} R_{\mu\nu} &= R^{\lambda}{}_{\mu\lambda\nu} \\ &= \Gamma^{\lambda}{}_{\mu\nu,\lambda} - \Gamma^{\lambda}{}_{\nu\lambda,\mu} + \Gamma^{\lambda}{}_{\rho\lambda} \Gamma^{\rho}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\rho\nu} \Gamma^{\rho}{}_{\nu\lambda}, \end{aligned} \tag{4.10}$$

$$R = g^{\mu\nu} R_{\mu\nu}. \tag{4.11}$$

Let us calculate $R_{\mu\nu}, R$.

$$\begin{aligned} &R_{00} = -3\ddot{a}a. \\ &R_{ij} = \delta_{ij} \left(a\ddot{a} + 2\dot{a}^2 + 2K \right). \\ &R_{0i} = R_{i0} = 0. \end{aligned} \tag{4.12}$$

$$R = g^{\mu\nu} R_{\mu\nu} = 6 \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{K}{a^2} \right]. \tag{4.13}$$

Next we introduce the energy-momentum tensor $T_{\mu\nu}$. The energy-momentum tensor of perfect fluid can be expressed using the energy density ρ, the pressure P and the comoving velocity of an observer u^{μ} ($g^{\mu\nu} u_{\mu} u_{\nu} = -1$), such as

$$T^{\mu}_{\nu} = (\rho + P) u^{\mu} u_{\nu} + P \delta^{\mu}{}_{\nu}, \tag{4.14}$$

The comoving velocity is written by

$$u^{\mu} = \frac{1}{a} \delta^{\mu}{}_{0}. \tag{4.15}$$

Imposing the energy-momentum conservation, the covariant derivatives of the energy-momentum tensors

$$T^{\mu}_{\nu;\mu} = \frac{\partial T^{\mu}_{\nu}}{\partial x^{\mu}} + \Gamma^{\mu}{}_{\alpha\mu} T^{\alpha}_{\nu} - \Gamma^{\alpha}{}_{\mu\nu} T^{\mu}_{\alpha} \tag{4.16}$$

become 0 for the 0 th and i-th components.

$$-\frac{\partial \rho}{\partial t} - \Gamma^{\mu}{}_{0\mu}\rho - \Gamma^{\alpha}{}_{0\mu} T^{\mu}_{\alpha} = 0 \tag{4.17}$$

$$\frac{\partial \rho}{\partial t} + \frac{\dot{a}}{a} 3\left[\rho + P\right] = 0. \tag{4.18}$$

The actual universe consists of multi components and if one labels the energy density and pressure of the i-th components as ρ_i and P_i then, the total energy density and total pressure are given by

$$\rho = \sum_i \rho_i, \tag{4.19}$$

$$P = \sum_i P_i. \tag{4.20}$$

By defining the critical density such as

$$\rho_c \equiv \frac{3H_0^2}{8\pi G}, \tag{4.21}$$

($H = \frac{\dot{a}}{a}$ is called the Hubble parameter, and the Hubble constant at present $H_0 = 100h\,\mathrm{km/sec/Mpc}$), then the density parameter for a single component i is defining by

$$\Omega_i = \frac{\rho_i(t_0)}{\rho_c}, \tag{4.22}$$

where $\rho_i(t_0)$ represents the energy density of today. Eq.(4.18) express the continuous property of the energy density of the expanding universe. Einstein's theory of gravity tells us the interaction between the space-time and matter of the universe. The Einstein field equations are given by

$$G^{\mu}{}_{\nu} \equiv R^{\mu}{}_{\nu} - \frac{1}{2} g^{\mu}{}_{\nu} R = 8\pi G T^{\mu}{}_{\nu} - \Lambda g^{\mu}{}_{\nu}. \tag{4.23}$$

The left hand side $G^{\mu}{}_{\nu}$, is called the Einstein tensor and expresses the geometry of the universe and the right hand side describes cosmological constituents. Here λ is the cosmological constant, which is the one of the unsolved mystery of the cosmology. Let us calculate the Einstein tensors in the FRW metric, which are written by

$$\begin{aligned} G^{0}{}_{0} &= -3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2}\right], \\ G^{i}{}_{0} &= G^{0}{}_{i} = 0, \\ G^{i}{}_{j} &= -\left[2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2}\right]\delta^{i}{}_{j}. \end{aligned} \tag{4.24}$$

Now assuming the components of the universe consist of perfect fluid , we can obtain the two independent components of the Einstein field equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3}, \tag{4.25}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{4.26}$$

Eq.(4.25) is especially called the Friedemann equation. From Eq.(4.25) and (4.26), we can derive the continuous equation, Eq.(4.18), this is due to the fact that $T^{\mu}{}_{\nu;\mu} = 0$ is included in $G^{\mu}{}_{\nu;\mu} = 0$. Actually, the three equations, the Friedmann equation Eq.(4.25), the continuous equation Eq.(4.18), and the equation of state $p = p(\rho)$ are the basic equations of the universe which decide the evolution and dynamics of the universe.

So far we have worked with the homogenous and isotropic universe. The universe without any inhomogeneities and anisotropies are referred as background or unperturbed universe. The observed universe is not exactly homogenous and isotropic. Actually the observed universe can be assumed as a unperturbed universe at 0-th order. In the next section, we introduce the perturbed universe and how the perturbations of the universe evolve.

4.2 Inflation

The standard cosmological models that assume the homogeneous and isotropic universe have discovered the expanding universe, the Big Bang Nucleosynthesis (BBN), and the thermal history of the universe, so on. But there still exist unsolved problems such as the horizon problem, why our universe's temperature is so homogeneous over the regions which do not have causality each other, and the flatness problem which involves the fine tuning of the curvature. The inflation models [29, 30] assume that at the early time, the universe experienced the exponential expansion,

$$a(t) \propto e^{Ht}. \tag{4.27}$$

With this exponential expansion, the regions that have almost same temperatures can have been inside the horizon in which every two points have causality, and this solves the horizon problem. On the other hand, the curvature problem is also solved by the large scale factor of the early universe. In addition to solving these problem, inflation models also provide perturbations of the universe and even though these primordial perturbations have quantum origin, the perturbations freeze out when the scales of the perturbations become larger than the horizon scale and generate the structure of the universe afterwards. In this section we explain one of the most simple inflation models, the single slow-roll inflation model.

The stage of the history of the universe when the exponential expansion takes place is called the de-Sitter stage, and at this stage from Eq.(4.26), one finds that in order to obtain $\ddot{a} > 0$, we need $P < -\frac{\rho}{3}$. Here we ignore the K term due to the fact that a^{-2} decreases rapidly. In order to gain such pressure, we need some substance that is not matter nor radiation. Now let us consider a scalar field, inflaton φ. The lagrangian of the inflaton is given by

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi), \tag{4.28}$$

where $V(\varphi)$ is the potential of the inflaton. The action of the inflaton is written by

$$S = \int d^4x \sqrt{-g} \mathcal{L}, \tag{4.29}$$

where $g = det g_{\mu\nu}$, by using the variational method, we obtain the Klein-Gordon equation.

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi \right) = \frac{\partial V(\varphi)}{\partial \varphi}. \tag{4.30}$$

For the FRW metric without perturbations Eq.(4.2), the field equation of the inflaton becomes

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{\nabla^2 \varphi}{a^2} + \frac{dV}{d\varphi} = 0. \tag{4.31}$$

The motion of the inflaton is comparable to the motion of a particle, which moves with fraction in the potential. Then the energy-momentum tensor is given

$$T_{\mu\nu} = -2\frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} + g_{\mu\nu}\mathcal{L} = \partial_\mu \varphi \partial_\nu \varphi + g_{\mu\nu} \left[-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - V(\varphi) \right]. \tag{4.32}$$

Now we express the inflaton by a classical field, which has the infinite wavelength and a quantum perturbative part

$$\varphi(t, \mathbf{x}) = \varphi_0(t) + \delta\varphi(t, \mathbf{x}). \tag{4.33}$$

For a while, we just deal with the classical field $\varphi(t) = \varphi_0(t)$. Then the field equation of the inflaton becomes

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0, \tag{4.34}$$

and the energy-momentum tensor is given by

$$T_{00} = \frac{\dot{\varphi}^2}{2} + V(\varphi), \tag{4.35}$$

$$T_{i0} = T_{0j} = 0, \tag{4.36}$$

$$T_{ij} = \left[\frac{\dot{\varphi}^2}{2} + V(\varphi) \right] \delta_{ij}. \tag{4.37}$$

$\rho_\varphi = -T_{00}, P_\varphi = T_{ii}$ are respectively

$$\rho_\varphi = \frac{\dot{\varphi}^2}{2} + V(\varphi), \tag{4.38}$$

$$P_\varphi = \frac{\dot{\varphi}^2}{2} - V(\varphi). \tag{4.39}$$

In order to have inflation, one needs the slow-roll condition

$$|\dot{\varphi}|^2 \ll V(\varphi), \tag{4.40}$$

$$|\ddot{\varphi}| \ll \frac{dV(\varphi)}{d\varphi}. \tag{4.41}$$

From (4.40),

$$P_\varphi \simeq -\rho_\varphi. \tag{4.42}$$

Now from the Friedmann equation, only considering the energy density of the inflaton,

$$H^2 \simeq \frac{8\pi G_N}{3}\rho_\varphi \simeq \frac{8\pi G_N}{3}V(\varphi), \tag{4.43}$$

and using $P_\varphi \simeq -\rho_\varphi$ and the continuous equation Eq.(4.18), one finds $\dot{\rho_\varphi} = 0$, $\rho_\varphi = \text{const}$. Thus the Hubble parameter becomes constant.

$$H = \text{const} = H_I. \tag{4.44}$$

Now we define the time when the inflation begins as t_i and the scale factor at t_i as a_i, then during the inflation, a is

$$a(t) = a_i e^{H_I(t-t_i)}. \tag{4.45}$$

Thus by imposing the slow roll conditions, we obtained the exponential expansion of the universe. Besides with the slow roll conditions, from Eq.(4.41), approximately

$$\dot{\varphi} \simeq -\frac{1}{3H}\frac{dV}{d\varphi}. \tag{4.46}$$

Using this, the slow roll conditions can be rewritten in terms of the flatness of the potential.

$$\frac{\frac{dV}{d\varphi}}{V} \ll H^2, \tag{4.47}$$

$$\frac{d^2V}{d\varphi^2} \ll H^2. \tag{4.48}$$

These condition also can be expressed by the slow roll parameter ϵ, θ,

$$\epsilon = \frac{M_{pl}^2}{2}\left(\frac{\frac{dV}{d\varphi}}{V}\right)^2, \tag{4.49}$$

$$\theta = M_{pl}^2 \left(\frac{\frac{d^2V}{d\varphi^2}}{V} \right), \tag{4.50}$$

where we used the Planck mass $M_{pl} \equiv \sqrt{8\pi G_N} \simeq 2.4 \times 10^{18}\,\mathrm{GeV}$. By using Eqs.(4.43), (4.46), ϵ is also expressed as $\epsilon = -\frac{\dot{H}}{H}$.

The e-fold number N shows the amount of exponential growth. N is defined by

$$\frac{a(t_f)}{a(t_i)} = \exp \int_{t_i}^{t_f} H dt \equiv e^N. \tag{4.51}$$

$\frac{a(t_f)}{a(t_i)}$ is the proportion of the scale factors at the beginning and the end of the inflation. If the Hubble parameter is constant during the inflation,

$$N = \int_{t_i}^{t_f} H dt \sim H(t_f)\,(t_f - t_i)\,, \tag{4.52}$$

and from Eqs(4.46), (4.43),

$$N = \int_{t_i}^{t_f} H dt = \int H \frac{d\varphi}{\dot{\varphi}} \simeq M_{pl}^2 \int \frac{V}{\frac{dV}{d\varphi}} d\varphi = M_{pl}^2 \int_{\varphi_i}^{\varphi_f} \left(-\frac{d\ln V}{d\varphi} \right)^{-1} d\varphi. \tag{4.53}$$

Thus one can find the e-fold number becomes larger when the potential becomes more flat. In order to resolve the horizon problem, roughly speaking, we need $N \gtrsim 60$.

Chapter 5

Q-ball and Affleck Dine Mechanism

In this chapter, we briefly explain a model which solves the baryon asymmetry problem, Affleck-Dine (AD) baryogenesis, then introduce a non-topological soliton, Q-ball, that is produced via fragmentations made after AD baryogenesis.

5.1 Baryon Asymmetry

The baryon makes up only 5% of the total energy density of the universe. The dark energy makes up roughly 2/3 while the dark matter explains about 1/3 of the total energy density. Although it seems to be natural to suppose that there exist same amounts of matter and anti-matter by nature, we know almost all stars and gas are consists of matter and as it is also difficult to consider a large region which consists of mainly anti-matter, we can conclude that there exists the asymmetry of matter and anti-matter. The baryon to photon ratio is defined as

$$\eta \equiv \frac{n_B}{n_\gamma}, \ n_B = n_b - n_{\bar{b}}, \tag{5.1}$$

where n_b and $n_{\bar{b}}$ are the number densities of baryon and anti-baryon, while n_γ is the number density of photon. As $n_b \gg n_{\bar{b}}$, we can consider $n_B \sim n_b$. Actually the theory and observations of Big Ban Nucleosynthesis (BBN) constrain the amount of η [31]

$$5.1 \times 10^{-10} \leq \eta \leq 6.5 \times 10^{-10}. \tag{5.2}$$

There are many models which explain how to produce baryon asymmetry, and they are called as *baryogenesis* models. In this chapter, we are going to introduce one of the most promising model which is called the Affleck-Dine baryogenesis.

5.2 Affleck-Dine Baryogenesis

In the Affleck-Dine (AD) mechanism [32], they considered a complex scalar field $\phi = |\phi| e^{i\theta}$, which has a global $U(1)$ symmetry, then the Lagrangian can be written by

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - V(|\phi|). \tag{5.3}$$

The Noether current which is related to $U(1)$ symmetry gives the conserved charge Q is given by,

$$Q = i\left(\dot{\phi}^* \phi - \phi^* \dot{\phi}\right) = \dot{\theta}|\phi|^2. \tag{5.4}$$

This Q corresponds to the baryon number and one can see we need $\dot{\theta} \neq 0$ and $\phi \neq 0$ in order to generate the baryon number. Note that as under $U(1)$ symmetry, n_B is conserved, in order to generate baryon number, we need to break $U(1)$ symmetry.

In the MSSM model, there exist many directions along which the effective scalar potential vanishes. These directions are called *flat directions*. As they consist of the squarks, the sleptons and the Higgs, they carry the baryon and lepton numbers. In AD model, a flat direction is taken as the AD field Φ [33].

Even though flat directions are exactly flat if SUSY is unbroken, they are lifted by SUSY breaking terms and higher order terms in the super potential. The higher order terms are non-renormalizable and the super potential due to non-renormalizable terms is written as

$$W = \lambda \frac{\Phi^n}{n M^{n-3}}, \tag{5.5}$$

where M is the large energy scale where new physics comes in, here we take $M = M_P$. The scalar potential due to the non-renormalizable parts are given by

$$V_{NR} = \lambda^2 \frac{|\Phi|^{2n-2}}{M^{2n-6}}, \tag{5.6}$$

$$V_A = a m_{3/2} \frac{\Phi^n}{M^{n-3}} + \text{h.c.}. \tag{5.7}$$

Here a is order $O(1)$ parameter and we assume that the cosmological constant vanishes. Note that this A term V_A is also non-renormalizable and breaks $U(1)$ symmetry, thus this term is key to the baryon number generation. The mass term is given by

$$V_{\text{mass}} = m_\Phi^2 |\Phi|^2. \tag{5.8}$$

In addition to these terms, there exists the Hubble induced mass term which is due to the inflaton potential,

$$V_H = -c_H H^2 |\Phi|^2, \tag{5.9}$$

where c_H is $O(1)$ parameter.

Even though there are other terms in the scalar potential, these terms above are dominant part. Thus the scalar potential for a flat direction is given by [33], [34]

$$V(\Phi) = (m_\Phi^2 - c_H H^2)|\Phi|^2 + a\frac{m_{3/2}}{M^{n-3}}\left(\Phi^n + \Phi^{*n}\right) + \lambda^2\frac{|\Phi|^{2n-2}}{M^{2n-6}}. \tag{5.10}$$

The AD field evolves depending on dominant terms of the potential. Now let see how it evolves during and after the inflation. In the following discussion, we take $c_H \sim \lambda \sim 1$ for simplicity. During the inflation, $H = H_I \gg m_\Phi \simeq O(1\,\mathrm{TeV})$, the Hubble induced mass term and non-renormalizable term are dominant. The scalar potential is approximately expressed as

$$V(\Phi) \sim -H^2|\Phi|^2 + \frac{|\Phi|^{2n-2}}{M^{2n-6}}. \tag{5.11}$$

The AD field takes the minimum $|\Phi| \sim (H_I M^{n-3})^{1/(n-2)}$ which is decided mainly by two dominant terms. After the inflation, the reheating era begins and the inflaton oscillates around its minimum of the potential, while $H > m_{3/2}$, $|\Phi|$ follows adiabatically the minimum $(HM^{n-3})^{1/(n-2)}$, where $H = 2/3t$. When $H \sim m_\Phi$, the AD field begins to oscillate from $|\Phi| \sim (m_\Phi M^{n-3})^{1/(n-2)}$ and rotate into the phase direction by the kicking of the A term under the scalar potential

$$V(\Phi) = m_\Phi^2|\Phi|^2 + a\frac{m_{3/2}}{M^{n-3}}\left(\Phi^n + \Phi^{*n}\right). \tag{5.12}$$

This rotation is supposed to generates the baryon number, however, the AD mechanism is made complicated by the existence of new object, Q-ball. While the AD field is rotating in its potential, it also feels spatial instabilities that lead to generate the instabilities of the baryon asymmetry. The condensed region develops to become Q-balls. In the next section, we look into the nature of this object.

5.3 Q-ball

Q-ball is a non-topological soliton, which appears if there exist scholar fields which carry a conserved charge associated with a global $U(1)$ symmetry. In the MSSM, the squarks, the sleptons and the Higgs carry the conserved baryon and lepton numbers. After the baryogenesis mechanism, these numbers can act as the conserved charge Q, thus these fields can form Q-balls.

Now let us consider a global $U(1)$ symmetry and a complex scalar field $\Phi = \frac{1}{\sqrt{2}}\phi$. Suppose the potential of ϕ, $U(\phi)$ which is symmetric under $\phi \to \phi e^{i\theta}$ and has a global minimum $U(0)$ at $\phi = 0$. The Lagrangian is written as

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi - U(\phi) \tag{5.13}$$

The charge Q associated with $U(1)$ is given by

$$Q = \int \left(\frac{\mathcal{L}}{\partial \dot{\phi}} i\phi + \frac{\mathcal{L}}{\partial \dot{\phi}^*}(-i\phi^*) \right) d^3x = \frac{1}{2i} \int d^3x \left(\phi^* \dot{\phi} - \dot{\phi}^* \phi \right). \tag{5.14}$$

The energy is

$$E = \int d^3x \left(\frac{1}{2} |\dot{\phi}|^2 + \frac{1}{2} |\nabla \phi|^2 + U(\phi) \right). \tag{5.15}$$

Now we calculate the scalar field ϕ which gives the lowest energy under the fixed charge Q. Using a Lagrange multiplier ω, define E_ω as

$$E_\omega \equiv E + \omega \left[Q - \frac{1}{2i} \int d^3x \left(\phi^* \dot{\phi} - \dot{\phi}^* \phi \right) \right]. \tag{5.16}$$

We need to find ω and ϕ which minimize E_ω. By dividing this into the time dependent and independent parts,

$$\begin{aligned} E_\omega &= \int d^3x \left[\frac{1}{2} |\dot{\phi}|^2 + \frac{\omega}{2} i \left(\phi^* \dot{\phi} - \dot{\phi}^* \phi \right) \right] + \int d^3x \left[\frac{1}{2} |\nabla \phi|^2 + U(\phi) \right] + \omega Q \\ &= \int d^3x \frac{1}{2} \left| \dot{\phi} - i\omega\phi \right|^2 + \int d^3x \left[\frac{1}{2} |\nabla \phi|^2 + U(\phi) - \frac{1}{2} \omega^2 |\phi|^2 \right] + \omega Q. \end{aligned} \tag{5.17}$$

The first term gives the time dependence of ϕ, such as

$$\phi(x, t) = e^{i\omega t} \phi(x). \tag{5.18}$$

Then the energy and charge are written by

$$E_\omega = \int d^3x \left[\frac{1}{2} |\nabla \phi(x)|^2 + U_\omega(\phi(x)) \right] + \omega Q, \tag{5.19}$$

where $U_\omega(\phi(x)) = U(\phi(x)) - \frac{1}{2}\omega^2 |\phi|^2$, and

$$Q = \omega \int d^3x |\phi(x)|^2. \tag{5.20}$$

Now we need to minimize eq.(5.19) with fixed ω and Q. The solution for this problem, actually which is the Q-ball, exists if

$$\frac{U(\phi)}{\phi^2} = \min, \ \text{for}\, \phi = \phi_0 > 0. \tag{5.21}$$

This means $U_\omega(\phi)$ has a negative global minimum at $\phi_0 > 0$ with the local minimum at $\phi = 0$ [36], [37]. In this case, the solution is spherically symmetric [37] $\phi(x) = \phi(r)\,, r = \sqrt{\vec{x}^2}$. By taking variation of eq.(5.19), one finds

$$\frac{d^2\phi(r)}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{\partial U_\omega}{\partial \phi} = 0, \tag{5.22}$$

with the condition

$$\frac{d\phi(0)}{dr} = \phi(\infty) = 0. \tag{5.23}$$

If one sees r as time, this equation becomes the dynamics of a point particle, which has a potential $-U_\omega$ and a damping part $-\frac{2}{r}\frac{d\phi}{dr}$. Then, the condition eq.(5.23) means that one puts a particle at some point $\phi > 0$ at $t = 0$ and the particle has to reach the origin at the end $t = \infty$. In order to find the solution, we need $\omega_0^2 < \omega^2 < U''(0)$ where $\omega_0 = \sqrt{2U(\phi_0)/\phi_0^2}$. This is because if $\omega^2 < \omega_0^2$, the particle cannot reach the origin due to the dumping term, on the other hand, if $\omega^2 > U''(0)$, the particle overshoots the origin. Fig. 5.3 shows the situations for each case. In the two following sections, we are going to

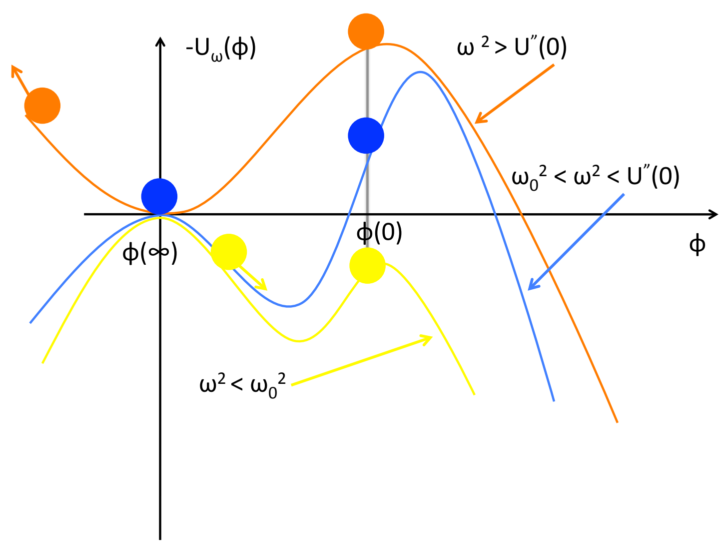

Figure 5.1: The condition for the existence of Q-balls. The blue line corresponds to $\omega_0^2 < \omega^2 < U''(0)$, the yellow one is for $\omega^2 < \omega_0^2$ (the particle stops before reaching the origin) while the orange line is for $\omega^2 > U''(0)$ (overshooting)

take a look at two extreme cases: the thin and thick-wall approximations.

5.3.1 Thin-wall approximation

In this section, we suppose that the radius of the Q-ball R is large, this corresponds the case $\omega \sim \omega_0$ [35]. Due to the large radius, we can ignore the effect of the surface of the Q-ball, then $\phi(r)$ is given by

$$\phi(r) = \begin{cases} \phi_0 & \text{for} \quad r < R \\ 0 & \text{for} \quad r > R \end{cases}. \tag{5.24}$$

Then, the energy and charges are given by

$$E = \frac{4\pi}{3} R^3 \left[\frac{1}{2} \omega^2 \phi_0^2 + U(\phi_0) \right], \tag{5.25}$$

$$Q = \frac{4\pi}{3} R^3 \omega \phi^2. \tag{5.26}$$

From these equations, we can express E in terms of Q,

$$E = \frac{1}{2} \frac{Q^2}{\phi_0^2 V} + U(\phi_0) V \geq 2 \sqrt{\frac{Q^2 U(\phi_0)}{2\phi_0^2}}, \tag{5.27}$$

where $V = \frac{4\pi}{3} R^3$ and the equality is attained if $V = \frac{Q}{\phi_0} \sqrt{\frac{1}{2U(\phi_0)}}$. Then we can obtain ϕ_0 by minimizing E.

5.3.2 Thick-wall approximation

In this section, we take a look at the case $\omega \gg \omega_0$ [38], [39]. In this case, two minima of $U_\omega(\phi)$ are very non-degenerate and the "escape point" $\bar{\phi}(0)$ (the maximal value of ϕ inside of the Q-ball) approaches the zero of $U_\omega(\phi)$ and far from the global minimum. In this limit, we can neglect the dynamics at large ϕ.

First, let us consider a potential, which is written by

$$\begin{aligned} U_\omega(\phi) &= U(\phi) - \frac{1}{2} \omega^2 \phi^2 \\ &= \frac{1}{2} (m^2 - \omega^2) \phi^2 - A\phi^3 + \lambda \phi^4, \end{aligned} \tag{5.28}$$

where $m^2 - \omega^2 > 0$. Here as we mentioned, when ω is large enough, we can ignore higher terms of the potential, so we ignore the quartic term and introduce two non-dimensional parameters

$$\xi_i \equiv \sqrt{m^2 - \omega^2} x_i, \tag{5.29}$$

where $i = 1, 2, 3$, and

$$\psi \equiv \frac{A\phi}{(m^2 - \omega^2)}. \tag{5.30}$$

Then, in terms of new parameters, E_ω are written by

$$E_\omega = \frac{(m^2 - \omega^2)^{3/2}}{A^2} \int d^3\xi \left[(\nabla_\xi \psi)^2 + \frac{1}{2} \psi^2 - \psi^3 \right] + \omega Q. \tag{5.31}$$

The first term's integral $S_\psi = \int d^3\xi \left[(\nabla_\xi \psi)^2 + \frac{1}{2}\psi^2 - \psi^3\right]$ is the action of the bounce solution in the potential $\frac{1}{2}\psi^2 - \psi^3$ and this is numerically calculated in [40], [38]: $S_\psi \sim 4.85$. Therefore we are ready to minimize

$$E_\omega = S_\psi \frac{(m^2 - \omega^2)^{3/2}}{A^2} + \omega Q \tag{5.32}$$

with respect to ω. By taking a derivative by ω

$$\frac{\partial E_\omega}{\partial \omega} = (-2\omega)\frac{3}{2} S_\psi \frac{(m^2 - \omega^2)^{1/2}}{A^2} + Q = 0, \tag{5.33}$$

we find

$$\left(\frac{\omega}{m}\right)^4 - \left(\frac{\omega}{m}\right)^2 + \epsilon = 0, \tag{5.34}$$

where $\epsilon \equiv \frac{QA^2}{3S_\psi m^2}$. This equation has solutions, which satisfy $0 < \omega < m$, if

$$\epsilon = \frac{QA^2}{3S_\psi m^2} < \frac{1}{2}. \tag{5.35}$$

One finds the solution is

$$\omega = m\sqrt{\frac{1 + \sqrt{1 - 4\epsilon^2}}{2}}. \tag{5.36}$$

Then by using inserting ω into the expression of E and expand in powers of ϵ, E is written by

$$E \simeq Qm \left(1 - \frac{1}{6}\epsilon^2 - \frac{1}{8}\epsilon^4\right). \tag{5.37}$$

One can see as $E < Qm$, the soliton is stable against the decay into ϕ particle. The radius of the Q-ball R in the non-dimensional parameter $R\sqrt{m^2 - \omega^2} \sim 1$, so

$$R^{-1} \sim \sqrt{m^2 - \omega^2} \sim m\epsilon \left(1 + \frac{1}{2} + \frac{7}{8}\epsilon^4\epsilon^2\right). \tag{5.38}$$

Now let us check if the assumptions we used are correct. First, $\phi(0)$ is small enough to get $\lambda\phi^4 \ll A\phi^3$. This is correct, if

$$\lambda\phi^2 \simeq \lambda \frac{m^2 - \omega^2}{A} \simeq \lambda \frac{\epsilon^2 m^2}{A} \ll A. \tag{5.39}$$

The second assumption $0 < \omega < m$ is correct, if $\epsilon < \frac{1}{2}$. Therefore rewriting these condition in terms of Q, we get,

$$Q \ll 14.6 \frac{m}{\sqrt{\lambda} A}, \tag{5.40}$$

$$Q < 7.28 \left(\frac{m}{A}\right)^2. \tag{5.41}$$

Here we use $S_\psi = 4.85$.

5.4 Q-ball in the MSSM

So far, we explained the AD mechanism and Q-ball which is produced by the fragmentation of the baryon asymmetry via the AD baryogenesis. In this section, we discuss the properties of Q-balls from the AD mechanism with two different types of SUSY breaking: the gravity mediation and the gauge mediation. The properties of the Q-ball depend on the potential of the AD filed, so vary depending on how SUSY is broken.

5.4.1 Q-ball from the gravity mediated AD mechanism

In the gravity mediation, the potential for the AD field Φ is written by [41]

$$V_{\rm grav} = m_\Phi^2 \left[1 + K \log\left(\frac{|\Phi|^2}{M^2}\right)\right] |\Phi|^2, \tag{5.42}$$

where the K term is the 1-loop correction term and we suppose $M = M_P$. The Q-ball solution exists if $K < 0$. Because for $K < 0$, the potential $V_{\rm grav}$ grows slower than $|\Phi|^2$ at low energy scales. Thus, by rewriting $\Phi = \frac{1}{\sqrt{2}}\phi(r)e^{iwt}$, we obtain

$$U_\omega(\phi) = \frac{1}{2} m_\Phi^2 \phi^2 \left[1 + K \log\left(\frac{\phi^2}{M^2}\right)\right] - \frac{1}{2}\omega^2\phi^2. \tag{5.43}$$

Then from eq.(5.22), the equation of motion of ϕ is written by

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = -\left[\omega^2 - m_\Phi^2(1+K)\right]\phi - m_\Phi^2 \phi K \log\left(\frac{\phi^2}{M^2}\right) \tag{5.44}$$

We solve this equation of motion with the Gaussian ansatz,

$$\phi(r) = \phi_0 e^{-\frac{r^2}{R^2}}. \tag{5.45}$$

Then R and $\omega_0 \equiv \omega^2 - m_\Phi^2(1+K)$ are obtained by

$$R = \frac{\sqrt{2}}{\sqrt{|K|} m_\Phi}, \tag{5.46}$$

$$\omega_0^2 = \left[3|K| - 2|K| \log\left(\frac{\phi_0}{\sqrt{2}M}\right)\right] m_\Phi^2. \tag{5.47}$$

Then the charge and energy are given by

$$Q = \left(\frac{\pi}{2}\right)^{3/2} \omega \phi_0^2 R^3, \tag{5.48}$$

$$E = m_\Phi Q \left(1 + \frac{3}{2}|K|\right). \tag{5.49}$$

As the energy per charge is $\frac{E}{Q} = m_\Phi(1 + \frac{3}{2}|K|) > m_\Phi$, one can see the Q-ball is unstable against the decay into Φ particle.

5.4.2 Q-ball from the gauge mediated AD mechanism

In the gauge mediation, the potential term is given as [43, 44]

$$V_{\text{gauge}}(\Phi) = M_F^4 \log\left(1 + \frac{|\Phi|^2}{M_S^2}\right), \tag{5.50}$$

M_S is the mass scale for messenger particles and M_F is given $\sqrt{m_\Phi M_S}$. One can approximate this potential with a tree level mass term at low energy scales and a flat potential U_0 at high energy scales such as [45]

$$V_{\text{gauge}}(\Phi) \sim \begin{cases} U_0 \ \ (|\Phi| \gg M_S), \\ m_\Phi^2 \Phi^2 \ \ (|\Phi| \ll M_S), \end{cases} \tag{5.51}$$

Then for low energy scales, that is for large r, eq.(5.22) becomes

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = -\omega^2\phi, \tag{5.52}$$

and for high energy scales, that is for small r,

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = (m_\Phi^2 - \omega^2)\phi. \tag{5.53}$$

The boundary condition is $\phi^{'}(0) = \phi(\infty) = 0$. Thus the Q-ball solution is given by for low energy scales,

$$\phi = \phi_1 \frac{e^{-mr}}{mr}, \tag{5.54}$$

where $m = \sqrt{m_\Phi^2 - \omega^2}$ and for high energy scales

$$\phi = \phi_0 \frac{\sin(\omega r)}{\omega r}. \tag{5.55}$$

Here ω, ϕ_0 and ϕ_1 are chosen to minimize E_ω and give a continuos Q-ball solution. In the thick wall limit $\omega_0 \sim m_\Phi \ll \omega$, by defining $\xi = \omega x$ and $\psi = \frac{\phi}{\omega}$, E_ω is written by

$$E_\omega = \frac{1}{\omega}\int 4\pi\xi^2 d\xi \left(\frac{1}{2}\frac{d^2\psi}{d\xi^2} - \frac{1}{2}\psi^2 + U_0\right) + \omega Q, \tag{5.56}$$

where we ignored terms in the potential except the constant and ω^2 term. Then one can see this gives $E_\omega \sim a\omega + b\omega^{-3} + \omega Q$, where a and b are independent of ω. Minimizing E_ω leads to $\omega \propto Q^{-1/4}$, and using M as a mass parameter, $\omega \sim MQ^{-1/4}$. The radius of the Q-ball R is $R \sim \omega^{-1} \sim M^{-1}Q^{1/4}$, using this approximation, one obtains

$$Q \simeq \omega \int d^3x |\phi|^2 \simeq \omega R^3 |\phi_0|^2. \tag{5.57}$$

Thus $\phi_0 \sim MQ^{1/4}$, and the energy $E \sim \omega Q \sim MQ^{3/4}$. Except coefficients, these results coincide with the previous results in [45], [46]. At last in this section, let us see the stability of the Q-ball in the gauge mediated AD mechanism, the energy per charge is given by

$$\frac{E}{Q} \sim MQ^{-1/4}. \tag{5.58}$$

Thus if the Q is large enough, the Q-ball becomes stable against the decay. This situation is contrary to the case of the gravity mediation where the Q-ball is always unstable.

Chapter 6

Axino dark matter and baryon number asymmetry from Q-ball decay in gauge mediation

インターネット公表に関する共著者全員の同意が得られていないため、本章については、非公開

Chapter 7

Summary

In this thesis, we have seen how unsolved mysteries of cosmology motivate physics beyond the Standard Model and the theories beyond the Standard Model can help solving these problems. Here we took the axion, which is a solution to the CP problem, as a key to investigate such a combination of cosmology and physics beyond the Standard Model. Supersymmetry could give solutions to the dark matter and the baryon asymmetry problems. The lightest supersymmetric particle is a promising dark matter candidate in supersymmetry with R-symmetry. On the other hand, supersymmetry also enables the Affleck-Dine mechanism to have the AD field that after generates the baryon number. The AD field may fragment into the non-topological soliton, Q-balls. We have investigated the model that combines the supersymmetric partner of the axion, the axino, the Affleck-Dine mechanism and Q-ball. The model can explain dark matter and the baryon asymmetry simultaneously. We have found that there remain allowed parameter regions that satisfy cosmological constraint from the Big Bang Nucleosynthesis and give the necessary and sufficient amount of the dark matter and the baryon asymmetry.

We have not mentioned the possibility of the axion dark matter yet. In the Q-ball model, we assumed dark matter only consists of the axino. There also can exist the axion dark matter as well. If there exist the axion during the inflation era, the axion fluctuation produces the isocurvature fluctuation, which has the different property to the fluctuation that is simply produced by the single scalar inflation. If the isocurvature is scale invariant, the amount of isocurvature fluctuations is strongly constrained by the observational results. However, the constraints is comparably weak if the isocurvature fluctuation has a scale-dependent spectrum, more specifically, blue-tilted spectrum. In the appendix, we calculate constraints from the observational data on a SUSY axion dark matter model, which produce the blue-tilted spectrum perturbation.

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Appendix A

CMB constraint on blue-tilted isocurvature perturbations from a SUSY axion model

インターネット公表に関する共著者全員の同意が得られていないため、本章については、非公開

Appendix B

Appendix for special functions

B.1 Hankel function

The bessel equation

$$\frac{d^2u}{dz^2} + \frac{1-2\alpha}{z}\frac{du}{dz} + \left(\beta^2 + \frac{\alpha^2 - \nu^2}{z^2}\right)u = 0 \tag{B.1}$$

has a solution, which is represented by the cylinder function $Z(z)$

$$z^\alpha Z_\nu\left(\beta z\right). \tag{B.2}$$

The cylinder functions Z are general forms of the first kind Bessel functions J_{nu}, the second kind Bessel functions N_ν, the first kind Hankel functions $H_\nu^{(1)}$, the second kind Hankel functions $H_\nu^{(2)}$.

$$J_\nu(z) = \left(\frac{z}{2}\right)^2 \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{z}{2}\right)^{2n}}{n!\Gamma\left(\nu+n+1\right)}, \tag{B.3}$$

($z \neq$ negative real number)

$$N_\nu(z) = \frac{1}{\sin\nu\pi}\left[\cos\nu\pi J_\nu\left(z\right) - J_{-\nu}\left(z\right)\right], \tag{B.4}$$

$$H_\nu^{(1)} = J_\nu\left(z\right) + iN_\nu(z), \tag{B.5}$$

$$H_\nu^{(2)} = J_\nu\left(z\right) - iN_\nu(z). \tag{B.6}$$

B.2 Legendre polynomials

For the Legendre's differential equation with $n =$integerA

$$(1-z^2)\frac{d^2u}{dz^2} - 2z\frac{du}{dz} + n(n+1)u = 0, \tag{B.7}$$

the solutions are the Legendre polynomials.

$$P_n(z) = \sum_{k=0}^{[\frac{n}{2}]} \frac{(-1)^k (2n-2k)!}{2^n k!(n-k)!(n-2k)!} z^{n-2k}, \tag{B.8}$$

where $[x]$ is an integer and satisfies $x - 1 < [x] \le x$. Using Rodrigues's formula

$$P_n(z) = \frac{1}{2^n n!} \frac{d^n}{dz^n} (z^2 - 1)^n. \tag{B.9}$$

The orthogonality of the Legendre polynomials is expressed

$$\int_{-1}^{1} P_n(z) P_m(z) dz = \frac{2}{2n+1} \delta_{mn}. \tag{B.10}$$

The relation between the spherical harmonics is given by

$$P_l(\hat{\mathbf{n}} \cdot \hat{\mathbf{k}}) = \frac{4\pi}{2l+1} \sum_m Y_{lm}(\hat{\mathbf{n}}) Y^*_{lm}(\hat{\mathbf{k}}). \tag{B.11}$$

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