JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 105, NO. A8, PAGES 18,793–18,806, AUGUST 1, 2000

Conditions for the validity of the incompressible assumption for the ballooning instability in the long-thin magnetospheric equilibrium

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Abstract. Conditions under which the compressibility can be neglected for the magnetospheric ballooning instability, which arises in the shear Alfvén branch, are clarified in the context of ideal magnetohydrodynamic plasmas and stochastic plasmas by using the normal mode analysis and the energy principle. An expansion in the small parameter, which is equal to the ratio of the $\nabla_\perp B_0$ scale length to the field line curvature radius, shows that the incompressible assumption is valid for the ballooning instability with long-thin perturbations in the low-frequency regime and in the long-thin magnetospheric equilibrium, in which the field line curvature radius is much larger than the $\nabla_\perp B_0$ scale length. When the long-thin assumption for the equilibrium is not satisfied near the equator, the calculation of the energy functional for a trial function shows that the strongly localized ballooning mode is essentially incompressible if the plasma β at the equator is much larger than $6/\Gamma$, where Γ is the ratio of specific heats. For the stochastic plasmas near the equator the strongly localized ballooning mode is essentially incompressible irrespective of the β value. These results justify the incompressible assumption made in a previous ballooning stability analysis for the long-thin magnetospheric equilibrium. It is suggested that before the substorm onset, the near-Earth plasma sheet becomes more taillike, and the long-thin assumption for the equilibrium becomes more likely to be satisfied on average, and thus the near-Earth plasma sheet becomes more favorable to the onset of the ballooning instability without the strong stabilizing influence of the compressibility.

1. Introduction

The investigation of the hydromagnetic stability of high-β plasma confined by the magnetic fields is of interest in such varied fields as the study of fusion plasma confinement and dynamical processes in space and astrophysical plasmas. The ballooning instability is a pressure-driven ideal magnetohydrodynamic (MHD) instability in a high-β plasma, and it occurs where the pressure gradient vector ∇p_0 and the field line curvature vector are in the same direction. Therefore the plasma sheet and the outer edge of the ring current are potentially subject to the ballooning instability [*Miura et al.*, 1989; *Ohtani et al.*, 1989a, b]. *Miura et al.* [1989] showed by numerical eigenmode analysis that the plasma sheet is subject to the ballooning instability, and *Ohtani et al.* [1989b] showed by numerical eigenmode analysis that the outer edge of the ring current is subject to the ballooning instability and/or interchange instability. *Viñas and Madden* [1986] investigated effects of the azimuthal shear flow on the ballooning instability and applied their results to the plasmapause. *Lakhina et al.* [1990] investigated the ballooning instability in the plasma sheet region in the presence of parallel plasma flow. *Hameiri et al.* [1991] give a general discussion of the ballooning instability in space plasmas. *Sundaram and Fairfield* [1997] studied stability of resistive MHD tearing and ballooning modes in the tail current sheet.

Paper number 2000JA900035.
0148-0227/00/2000JA900035$09.00

Theoretical and observational investigations of the ballooning instability have centered on the plasma sheet stability against the ballooning instability because of great interest in the possible relevance of the instability to substorm dynamics [*Roux et al.*, 1991a, b; *Korth et al.*, 1991; *Ullaland et al.*, 1993; *Ohtani and Tamao*, 1993; *Hurricane et al.*, 1995, 1999; *Samson et al.*, 1996; *Lee and Min*, 1996; *Hurricane*, 1997; *Liu*, 1997; *Pu et al.*, 1997, 1999; *Wu et al.*, 1998; *Bhattacharjee et al.*, 1998; *Cheng and Lui*, 1998; *Lee*, 1998, 1999; *Pritchett and Coroniti*, 1999; *Horton et al.*, 1999]. The ballooning instability has also been studied quite intensively to investigate the origin of a class of geomagnetic pulsations (hydromagnetic waves) [e.g., *Miura et al.*, 1989; *Ohtani et al.*, 1989a, b; *Chen and Hasegawa*, 1991; *Chan et al.*, 1994; *Cheng and Qian*, 1994]. *Holter et al.* [1995] found diamagnetic hydromagnetic oscillations with periods of ~45–65 s, which are observed during the most active phase of the substorm breakup. These oscillations are consistent with diamagnetic ballooning modes, which are strongly localized near the equator [*Miura et al.*, 1989].

It is well known that the ballooning instability is analogous to the gravitational Rayleigh-Taylor instability, in which the effective gravity is given by equating the gravitational drift with the combined ∇B_0 and curvature drifts. Thus the effective gravity g_{eff} is given by [e.g., *Goldston and Rutherford*, 1995]

$$g_{\mathrm{eff}} = \left(\frac{v_\perp^2}{2} + v_\parallel^2 \right) \frac{1}{R_c}, \tag{1}$$

where R_c is the radius of curvature of the field line. When averaged over a thermal distribution of particle velocities $v_\perp$

and $v_\parallel$, the g_{eff} is written as [e.g., *Goldston and Rutherford*, 1995]

$$g_{\text{eff}} = \frac{2p_0}{\rho_0 R_c}. \tag{2}$$

Substitution of g_{eff} into g in the growth rate of the gravitational Rayleigh-Taylor instability, which is equal to $(g|\nabla\rho_0/\rho_0|)^{1/2}$ [*Chen*, 1974], yields the growth rate of the interchange (flute) instability, which is the pressure-driven version of the Rayleigh-Taylor instability,

$$\gamma_{\text{MHD}} = \left(\frac{2|\nabla p_0|}{\rho_0 R_c}\right)^{1/2}. \tag{3}$$

This growth rate of the interchange (flute) instability is also an important measure of the growth rate of the ballooning instability [e.g., *Miura et al.*, 1989]. In deriving (3) we equated the density scale length $|\nabla\rho_0/\rho_0|^{-1}$ to the pressure gradient scale length $|\nabla p_0/p_0|^{-1}$. Although the interchange or flute instability extends uniformly along the entire length of the field line, the ballooning instability is localized to a finite region of unfavorable curvature. Therefore the growth rate of the ballooning instability γ should be smaller than the peak value of γ_{MHD}, which occurs somewhere along the field line (equatorial plane in the case of the plasma sheet).

Miura et al. [1989] and *Ohtani et al.* [1989a] derived coupled eigenmode equations of the ballooning instability, which show the coupling of the Alfvén mode and the slow mode in the one-fluid MHD and in the two-fluid equations, respectively. After deriving those equations, Miura et al. assumed intuitively that the parallel component of the velocity perturbation vanishes and that the ballooning mode in the magnetosphere is incompressible, because the ballooning instability arises in the shear Alfvén branch, and the slow mode is only stabilizing. By that assumption they simplified considerably the coupled eigenmode equations to the single second-order differential equation, which is amenable to the numerical analysis. Miura et al. solved the simplified eigenmode equation and investigated the stability of the plasma sheet against the ideal MHD ballooning instability using a two-dimensional (2-D) plasma sheet model of *Kan* [1973] as an equilibrium state. Miura et al. found that the plasma sheet is subject to the ballooning instability and the fundamental symmetric mode is destabilized by the instability. Later, *Lee and Wolf* [1992] employed the energy principle [*Bernstein et al.*, 1958] to test the stability of the Kan's model of the plasma sheet [*Kan*, 1973] against the ideal MHD ballooning instability. They tried several test functions of an arbitrary form to find an unstable solution. However, all of their test functions turned out to be stable, and they could not find an unstable solution. From their results they questioned the existence of the unstable ballooning mode, which was found by Miura et al. in the Kan's model of the plasma sheet. Since Lee and Wolf retained the stabilizing compressible term in their energy principle approach and sought a compressible unstable solution, the discrepancy of the above two approaches is due to the treatment of the compressibility in the stability analysis and the choice of the equilibrium state. In principle, the validity of the incompressible assumption can be verified a posteriori after obtaining the growth rate [e.g., *Goldston and Rutherford*, 1995]. However, Miura et al. did not rigorously prove that the incompressible assumption is valid for their obtained unstable mode. The purpose of the present paper is to show by the normal mode analysis and the energy principle that the incompressible assumption used for the ballooning instability [*Miura et al.*, 1989] is valid for long-thin perturbations in a low-frequency regime and in the long-thin equilibrium, where the magnetic field scale length parallel to the field line (field line curvature radius) is much larger than its scale length perpendicular to the unperturbed magnetic field ($\nabla_\perp B_0$ scale length) on average. Thus we will show that the incompressible ballooning instability is a viable MHD instability in the plasma sheet because the long-thin ordering of the equilibrium is indeed satisfied by Kan's model of the plasma sheet, except for a tiny region near the equator [see *Miura et al.*, 1989, Figure 4].

By including in the eigenmode equation an ion diamagnetic drift term, which is the most dominant nonideal MHD term, *Miura et al.* [1989] showed that the ballooning instability destabilizes the drift Alfvén mode [*Tamao*, 1984] in the magnetosphere, which is propagating westward. *Ohtani et al.* [1989a, b] further extended the eigenmode analysis of Miura et al. by employing two-fluid equations. *Pu et al.* [1997] also used two-fluid equations in their study of the ballooning instability based on the local approximation. Although the real frequency of the unstable ballooning mode due to the presence of the ion diamagnetic drift term in the eigenmode equation is important, we neglect the ion diamagnetic drift term in the present analysis and treat only the ideal MHD ballooning instability in the plasma sheet. We will also show that the eigenmode analysis adopted by Miura et al. is equivalent to the energy principle approach adopted by *Lee and Wolf* [1992] when the energy minimized condition is taken into account and the compressibility term is retained. Therefore the eigenmode equations of Miura et al. derived by the normal mode analysis are also justified from the energy principle point of view. The nonideal MHD and kinetic effects on the ballooning instability have also been investigated intensively. *Chen and Hasegawa* [1991] investigated effects of the anisotropic pressure and kinetic effects on the ballooning instability. *Chan et al.* [1994] made a numerical stability analysis including the anisotropic pressure for the self-consistent equilibrium based on the formulation by Chen and Hasegawa. *Cheng and Qian* [1994] also studied effects of the anisotropic pressure and kinetic effects including trapped particle effects and did a numerical stability analysis. *Cheng and Lui* [1998] studied stabilizing effects of trapped electrons and the finite ion Larmor radius. *Horton et al.* [1999] discussed the kinetic effects on the interchange mode and the ballooning mode in the near-Earth plasma sheet. Beyond some distance down the tail the field curvature radius becomes smaller than the ion Larmor radii of the bulk of ions (S. Machida, personal communication, 1998) and the nonadiabatic effects due to the stochastic ion dynamics [e.g., *Büchner and Zelenyi*, 1989; *Chen*, 1992] become important in the ballooning instability [*Hurricane et al.*, 1994, 1995]. The stochastic ion dynamics may be important even as close as in the near-Earth plasma sheet [*Lui et al.*, 1992]. Although the above kinetic effects on the ballooning instability may be important in the near-Earth plasma sheet, the present paper is limited to the basic ideal MHD limit and to the stochastic limit and investigates within these limits the effects of the difference of the background equilibrium configuration on the ballooning instability.

Since the ballooning instability is a pressure-driven ideal MHD instability growing in a fast MHD timescale, it is tempting to consider that this instability occurring in the geomagnetic tail plays a role in the tail dynamics, possibly in the onset of the substorm expansion phase. The importance of the near-

Earth region or the geosynchronous region in the substorm onset has been emphasized by intensive observations and the localized nature of the substorm onset in the dawn to dusk direction is well known. The importance of the near-Earth region in the substorm onset has also been supported by evidence that the auroral arc that brightens first during a substorm maps to the inner edge of the plasma sheet. Whereas the ion tearing instability in the 2-D model of the thick quasi-neutral sheet is stabilized by the electron compressibility effect due to a normal magnetic field component [*Lembege and Pellat*, 1982; *Pellat et al.*, 1991], which is stronger nearer to Earth, and a small k_y is favorable for the tearing instability, where y is the dawn-to-dusk direction, a large k_y is favorable for the ballooning instability, and the growth rate of the ballooning instability in the plasma sheet is larger nearer to Earth [see *Miura et al.*, 1989, Figure 6]. Therefore the localized nature of the ballooning instability in the y direction (large k_y) and its preference for the near-Earth region in the plasma sheet are favorable for the substorm onset. *Roux et al.* [1991a, b] suggested, on the basis of in situ observations of an isolated dispersionless substorm by a geostationary satellite and ground-based observations, that the near-Earth plasma sheet is subject to the ballooning instability, and they attributed the partial cancellation of the tail current, the resulting particle injection, and the development of a westward traveling surge to the development of the ballooning instability. *Wu et al.* [1998] did a 3-D linear MHD stability analysis of the 2-D static equilibrium in the plasma sheet and showed the presence of an unstable ballooning mode. The importance of understanding the nonlinear process of the ballooning instability in the substorm onset has been emphasized [e.g., *Samson et al.*, 1996; *Hurricane et al.*, 1999]. *Voronkov et al.* [1997] investigated by MHD simulations the instability of a shear flow embedded in a pressure gradient region (near-Earth region) in the presence of the gravity. They showed that the unstable mode grows faster than the Kelvin-Helmholtz instability growth rate, owing to the existence of the gravitational Rayleigh-Taylor instability, which mimics the ballooning instability. Recently, a study of the nonlinear development of the ballooning instability in the near-Earth plasma sheet was carried out by *Pritchett and Coroniti* [1999], who used a 3-D full particle simulation; their simulation shows that the near-Earth plasma sheet does indeed become subject to the ballooning instability, when the plasma β exceeds a critical β calculated by using the incompressible assumption [*Miura et al.*, 1989]. Their simulation demonstrates clearly that the westward propagating drift Alfvén wave can be destabilized by the ballooning instability in the parameter range predicted by *Miura et al.* [1989]. Thus their simulation supports the validity of the incompressible assumption and the analysis of Miura et al.

In the following, the minimization procedure in the energy principle is briefly reviewed in section 2. The relationship between the parallel velocity perturbation and the compressibility in the 2-D equilibrium is clarified in section 3. The relation of the ballooning eigenmode equations derived by *Miura et al.* [1989] for the long-thin perturbations to the energy principle is discussed in section 4. The validity of the incompressible assumption for the ballooning instability with the long-thin perturbations in the low-frequency regime and in the long-thin magnetospheric equilibrium is shown in section 5. When the long-thin equilibrium is not valid near the equator, a separate discussion of the validity of the incompressible assumption is presented by calculating the energy functional for a trial function in section 6. The physical picture of why the incompressible assumption is valid in the long-thin, taillike equilibrium is given in section 7. Discussions and summary are given in section 8, and, in particular, it is shown that the long-thin equilibrium corresponds to the taillike equilibrium. The conditions of the validity of the incompressible assumption are also clarified for stochastic taillike plasmas. The explicit formula of the growth rate of the ballooning instability in the plasma sheet is also given, and the relevance of the present analysis to the substorm onset is discussed.

2. Review of the Minimization Procedure in the Energy Principle

Since the energy principle is derived from the normal mode analysis, the normal mode analysis and the energy principle are equivalent [*Bernstein et al.*, 1958; *Freidberg*, 1987]. The eigenmode equation of the ballooning instability can be derived from the energy principle by obtaining the Euler equation of the variational principle $\delta\omega^2 = 0$ [*Bernstein et al.*, 1958], where ω is the angular frequency and the perturbed quantities are proportional to $\exp(-i\omega t)$.

Since the powerful technique of the energy principle lies in the minimization of the potential energy δW_F with respect to the displacement, let us briefly review the minimization procedure in the energy principle, which is employed in section 4. The potential energy δW_F for the ballooning instability can be written as [*Freidberg*, 1987]

$$\delta W_F = \frac{1}{2}\int_P d\mathbf{r}\left\{\frac{|\delta\mathbf{B}_\perp|^2}{\mu_0} + \frac{\mathbf{B}^2}{\mu_0}|\nabla\cdot\boldsymbol{\xi}_\perp + 2\boldsymbol{\xi}_\perp\cdot\boldsymbol{\kappa}_c|^2 + \Gamma p|\nabla\cdot\boldsymbol{\xi}|^2 - 2(\boldsymbol{\xi}_\perp\cdot\nabla p)(\boldsymbol{\xi}_\perp^*\cdot\boldsymbol{\kappa}_c)\right\}, \tag{4}$$

where P denotes the unperturbed plasma volume, $\boldsymbol{\xi}$ is the displacement vector, Γ is the ratio of specific heats, $\boldsymbol{\kappa}_c = (\mathbf{e}\cdot\nabla)\mathbf{e}$, with $\mathbf{e}$ being the unit vector in the direction of the unperturbed field $\mathbf{B}_0$, and the asterisk denotes the complex conjugate. The first term in the integrand represents the energy required to bend magnetic field lines. It is the dominant potential energy contribution to the shear Alfvén wave. The second term corresponds to the energy necessary to compress the magnetic field and describes the major potential energy contribution to the compressional Alfvén wave. The third term represents the energy required to compress the plasma in a finite β plasma (nonzero p). It is the main source of potential energy for the sound wave. Each of the contributions just described is stabilizing. The last term can be positive or negative and thus can drive the ballooning instability. The powerful technique of the energy principle lies in minimizing the respective energy term with respect to the components of the displacement vector $\xi_\parallel$ and ξ_y, where $\xi_\parallel$ is the component of the displacement vector parallel to the unperturbed magnetic field and y is directed from dusk to dawn. Since $\xi_\parallel$ appears only in the $\Gamma p|\nabla\cdot\boldsymbol{\xi}|^2$ term, the general minimizing condition for δW_F with respect to $\xi_\parallel$ can be written as [*Freidberg*, 1987]

$$\mathbf{e}\cdot\nabla(\nabla\cdot\delta\mathbf{u}) = 0, \tag{5}$$

where $\mathbf{e} = \mathbf{B}_0/B_0$ and $\delta u_\parallel = 0$ on the ionospheric boundary was used. Now let us minimize δW_F with respect to ξ_y by assuming infinite k_y (ballooning limit). Since ξ_y affects δW_F

through $\nabla \cdot \boldsymbol{\xi}_\perp$ term in the second and third terms in the integrand of (4), the minimization condition gives in the limit of infinite k_y, [*Lee and Wolf*, 1992],

$$B_0 \delta B_\parallel + \mu_0 \delta p = 0. \tag{6}$$

Lee and Wolf [1992] derived the potential energy integral δW_F for the magnetospheric configuration by using these two constraints (equations (5) and (6)).

3. Parallel Velocity Perturbation and Compressibility in the 2-D Equilibrium

We remark here that in the work of *Miura et al.* [1989] the coupled set of eigenmode equations describing the ballooning instability were simplified considerably by assuming $\delta u_\parallel = 0$. This assumption is based on an intuitive consideration that since the ballooning instability is essentially an instability of the shear Alfvén mode, which has no $\delta u_\parallel$, $\delta u_\parallel$ was neglected in their analysis for simplicity. This assumption may also be consistent with a kinetic consideration that the macroscopic parallel flow velocities for electrons and ions must vanish, because the parallel velocity moments calculated by using perturbed distribution functions for electrons and ions vanish owing to the cancelation of contributions due to the upgoing particles and the downgoing particles when those particles are not passing particles.

We derive explicitly in this section the relationship between the parallel velocity perturbation $\delta u_\parallel$ and the compressibility factor in the 2-D equilibrium. The electromagnetic perturbations are expressed by perturbed quantities $\delta\Phi$, $\delta A_\parallel$, and $\delta B_\parallel$, which are the scalar potential, the parallel component of the vector potential, and the parallel component of the magnetic field, respectively. By making use of these perturbations, perturbed electric and magnetic fields are written as [*Miura et al.*, 1989]

$$\delta \mathbf{E} = -[(\mathbf{e} \cdot \nabla)\delta\Phi - i\omega \delta A_\parallel]\mathbf{e} - i\mathbf{k}_\perp \delta\Phi + i\omega \delta \mathbf{A}_\perp, \tag{7}$$

$$\delta \mathbf{B} = \nabla \times (\delta A_\parallel \mathbf{e} + \delta \mathbf{A}_\perp), \tag{8}$$

$$\delta B_\parallel = i(\mathbf{e} \times \mathbf{k}_\perp) \cdot \delta \mathbf{A}_\perp. \tag{9}$$

By linearizing the equation of motion, we obtain

$$-i\omega\rho_0 \delta \mathbf{u} = \mathbf{j}_0 \times \delta \mathbf{B} + \delta \mathbf{j} \times \mathbf{B}_0 - \nabla \delta p. \tag{10}$$

Taking a dot product of this with $\mathbf{e}$ yields

$$\delta u_\parallel = \frac{i}{\omega \rho_0}(\mathbf{e} \cdot \mathbf{j}_{0\perp} \times \delta \mathbf{B} - \mathbf{e} \cdot \nabla \delta p), \tag{11}$$

where

$$\mathbf{j}_0 = \mathbf{j}_{0\perp} = B_0^{-1} \mathbf{e} \times \nabla_\perp p_0. \tag{12}$$

For simplicity, let us assume as in the work of *Miura et al.* [1989] that

$$\mathbf{k}_\perp = k_\perp \hat{\mathbf{y}}, \tag{13}$$

where $\hat{\mathbf{y}}$ is the unit vector in the y direction. If we assume that

$$\delta \mathbf{A}_\perp = \delta A_n \mathbf{n}, \tag{14}$$

where $\mathbf{n} = \hat{\mathbf{y}} \times \mathbf{e}$ is the unit vector in the normal direction, we obtain

$$\mathbf{e} \cdot (\mathbf{j}_{0\perp} \times \delta \mathbf{B}) = -i(\mathbf{k}_\perp \cdot \mathbf{j}_{0\perp})\delta A_\parallel. \tag{15}$$

Since the parallel electric field must vanish in the ideal MHD, we obtain from (7) that

$$i\omega \delta A_\parallel = (\mathbf{e} \cdot \nabla)\delta\Phi. \tag{16}$$

Using (15), (16), and the fact that $\mathbf{k}_\perp \cdot \mathbf{j}_{0\perp}$ is constant along the field line in the 2-D equilibrium, we obtain from (11),

$$\delta u_\parallel = -\frac{i}{\omega^2 \rho_0}(\mathbf{e} \cdot \nabla)[(\mathbf{k}_\perp \cdot \mathbf{j}_{0\perp})\delta\Phi + \omega \delta p]. \tag{17}$$

By making use of (12), equations

$$\delta \mathbf{u}_\perp = -B_0^{-1} \mathbf{e} \times \delta \mathbf{E}_\perp, \tag{18}$$

$$\delta \mathbf{E}_y = -i\mathbf{k}_\perp \delta\Phi, \tag{19}$$

and the perturbed form of the adiabatic gas law [*Miura et al.*, 1989, equation (A11)],

$$i\omega \delta p = \delta \mathbf{u}_\perp \cdot \nabla_\perp p_0 + \Gamma p_0 \nabla \cdot \delta \mathbf{u}, \tag{20}$$

we obtain

$$\omega^2 \rho_0 \delta u_\parallel = -\Gamma p_0 (\mathbf{e} \cdot \nabla)(\nabla \cdot \delta \mathbf{u}), \tag{21}$$

where we used that p_0 is constant along the field line. Equation (21) is also given in the work of *Freidberg and Marder* [1973] without derivation for the 2-D MHD equilibrium. It is obvious from this equation that one of the minimizing conditions (equation (5)) in the energy principle is consistent with $\delta u_\parallel = 0$. In other words, the assumption of $\delta u_\parallel = 0$, which was adopted by *Miura et al.* [1989], leads to that $\nabla \cdot \delta \mathbf{u}$ is constant along the field line. Miura et al. assumed that $\delta u_\parallel = 0$ and thus that $\nabla \cdot \delta \mathbf{u}$ is constant along the filed line; they further assumed that this constant is zero, i.e., that the plasma is incompressible, because $\nabla \cdot \delta \mathbf{u}$ is nearly equal to zero near the ionosphere.

Freidberg [1987] argues that when the operator $\mathbf{e} \cdot \nabla$ is nonsingular, the general minimizing condition (5) becomes

$$\nabla \cdot \delta \mathbf{u} = 0. \tag{22}$$

This means that the most unstable perturbation is incompressible. Since the unstable symmetric mode obtained by *Miura et al.* [1989, Figure 7] is monotonically decreasing toward the ionosphere, the nonsingular nature of the field-aligned differential operator $\mathbf{e} \cdot \nabla$ seems to be satisfied a posteriori for their unstable solution. This seems to support a priori assumption of Miura et al. that $\nabla \cdot \delta \mathbf{u} = 0$ in the present problem of the ballooning instability in the magnetosphere. In other words, since the compressible factor $\nabla \cdot \delta \mathbf{u}$ of the symmetric unstable mode becomes evanescently small near the ionosphere, $\nabla \cdot \delta \mathbf{u}$ must be nearly equal to zero everywhere from the requirement of (5). Although this consideration of the nonsingular nature of the operator $\mathbf{e} \cdot \nabla$ in (5) seems to indicate that the incompressible assumption is valid in the work of Miura et al., physical conditions for the validity of the incompressible assumption for the ballooning instability in the long-thin equilibrium are clarified below, on the basis of the eigenmode analysis in section 4, the expansion scheme adopted by Miura et al. in section 5, and the calculation of the energy functional in section 6.

4. Relation of the Ballooning Eigenmode Equations Derived by *Miura et al.* [1989] for the Long-Thin Perturbations to the Energy Principle

Miura et al. [1989] derived straightforwardly the eigenmode equation of the ballooning instability from the MHD equations by assuming long-thin and low-frequency perturbations. Here and in the following, the long-thin perturbations mean that the wavelength parallel to the magnetic field is much larger than the wavelength perpendicular to it, and the long-thin equilibrium means that the scale length of the equilibrium field configuration is long in the direction parallel to the unperturbed magnetic field, but its scale length perpendicular to the unperturbed magnetic filed is small. The long-thin ordering of the plasma equilibrium has been commonly used for mirror machines [e.g., *Weitzner*, 1980; *Tang and Catto*, 1981; *Lee and Catto*, 1981; *D'Ippolito et al.*, 1982]. In order to adopt the long-thin orderings for the perturbations and the equilibrium, we introduce two smallness parameters ε_1 and ε_2 defined by

$$\varepsilon_1 \sim \frac{k_\parallel}{k_\perp} \sim \frac{1}{k_\perp l_\perp} \qquad \varepsilon_2 \sim \frac{l_\perp}{l_\parallel}, \tag{23}$$

respectively, where $k_\parallel \sim |(\mathbf{e}\cdot\nabla)\ln\delta f|$, $k_\perp \sim |\nabla_\perp \ln \delta f|$, $l_\parallel^{-1} \sim |(\mathbf{e}\cdot\nabla)\mathbf{e}|$, and $l_\perp^{-1} \sim |\nabla_\perp \ln B_0|$. Here f is the MHD variable. Notice that $\varepsilon_1 \sim (k_\parallel l_\parallel)^{-1}$ is derived from (23) if $\varepsilon_1 \sim \varepsilon_2$. Miura et al. further adopted a low-frequency assumption, i.e., $|\omega^2| \sim k_\parallel^2 V_A^2$. This assumption along with the long-thin ordering (equation (23)) for the perturbation ($\varepsilon_1 \ll 1$) leads to $|\omega|^2/k_\perp^2 \ll V_A^2$, which could eliminate the fast magnetosonic mode in their derivation of eigenmode equations. Notice that in *Kan*'s [1973] two-dimensional equilibrium model of the tail plasma sheet, which was used in the numerical eigenmode analysis of Miura et al., the long-thin ordering for the equilibrium ($\varepsilon_2 \ll 1$) that $l_\perp/l_\parallel \ll 1$ or $|\boldsymbol{\kappa}_c| \ll |\boldsymbol{\kappa}_b| = |\nabla_\perp \ln B_0|$ is indeed satisfied except for a tiny region around the equatorial plane [see *Miura et al.*, 1989, Figure 4], which is only 0.4% of the total field line length extending from the equator to the ionosphere. The derivation of the ballooning eigenmode equation by Miura et al. is transparent and straightforward, on the basis of the neglect of $O(\varepsilon_1^2)$ and higher-order terms. Since their derivation of the one-dimensional eigenmode equation of the ballooning instability is unique in the sense that it is different from the conventional derivation of the ballooning eigenmode equation by obtaining the Euler equation from minimization of ω^2 in the energy principle [*Bernstein et al.*, 1958], it is necessary to show explicitly how the eigenmode equation derived by them, based on the expansion in ε_1, is equivalent to the energy principle. Therefore we show in this section explicitly that the coupled set of eigenmode equations derived by Miura et al. for nonzero $\delta u_\parallel$ are equivalent to the energy principle [*Bernstein et al.*, 1958], which was applied to the ballooning instability in the magnetosphere [*Lee and Wolf*, 1992]. For this purpose we need to make the quadratic form of ω from their basic equations. Since we treat the ideal MHD, we neglect the ion diamagnetic drift term $\omega_{\perp t}$ of Miura et al. The self-adjoint property of the force operator is essential for showing that ω^2 is pure real [*Bernstein et al.*, 1958].

By neglecting terms of $O[(k_\parallel/k_\perp)^2] = O(\varepsilon_1^2)$ the diamagnetic relation

$$B_0\delta B_\parallel + \mu_0\delta p \cong 0 \tag{24}$$

is obtained [*Miura et al.*, 1989, equation (A19)], which is the same as (6) derived by *Lee and Wolf* [1992] from the minimization condition with respect to ξ_y for infinite k_y. Notice here, however, that in the derivation of the diamagnetic equation (24) by *Miura et al.* [1989], the assumption of the infinite $k_\perp = k_y$ as assumed by Lee and Wolf is not necessary.

We start with a basic equation describing the ideal MHD ballooning instability [*Miura et al.*, 1989, equation (A20)],

$$V_A^2 B_0(\mathbf{e}\cdot\nabla)[B_0^{-1}k_\perp^2(\mathbf{e}\cdot\nabla)\delta\Phi] + \omega^2 k_\perp^2\delta\Phi = 2B_0\rho_0^{-1}\omega\delta p(\mathbf{k}_\perp\times\mathbf{e})\cdot\boldsymbol{\kappa}_c. \tag{25}$$

Equation (25) must be accompanied by the adiabatic law (equation (20)) to express δp in the right-hand side of (25) by $\delta\Phi$. In deriving (25) the diamagnetic condition (24) was already used. If there is no right-hand side, (25) is the equation of the shear Alfvén wave, which is guided along the field line. In order to close (25), we need to express the compressible factor $\nabla\cdot\delta\mathbf{u}$ in (20) by $\delta\Phi$. *Miura et al.* [1989] simplified the coupled equations by assuming that $\delta u_\parallel = 0$ everywhere along the field line, which led to $\nabla\cdot\delta\mathbf{u} = $ const along the field line (see equation (21)). They further assumed that the constant is zero, i.e., $\nabla\cdot\delta\mathbf{u} = 0$ along the field line. Thus they eliminated the slow mode, which has nonzero $\delta u_\parallel$ and nonzero $\nabla\cdot\delta\mathbf{u}$. Then substitution of (18), (19), and (20) with $\nabla\cdot\delta\mathbf{u} = 0$ into the right-hand side of (25) gives the one-dimensional eigenmode equation used in their numerical eigenmode analysis.

If we do not assume that $\nabla\cdot\delta\mathbf{u} = 0$, then the minimizing condition (5) with respect to $\xi_\parallel$ must be used for the explicit calculation of $\nabla\cdot\delta\mathbf{u}$. The integration of (5) with the aid of the boundary condition $\delta u_\parallel = 0$ at the ionosphere gives [*Freidberg*, 1987; *Lee and Wolf*, 1992]

$$\nabla\cdot\delta\mathbf{u} = \text{const} = \frac{\int_{-s_b}^{s_b} B_0^{-1}\nabla\cdot\delta\mathbf{u}_\perp\, ds}{\int_{-s_b}^{s_b} B_0^{-1}\, ds}, \tag{26}$$

where s is the coordinate along the field line, which satisfies $s = -s_b$ at the southern ionosphere, $s = 0$ at the equator, and $s = s_b$ at the northern ionosphere. From ideal MHD equations we obtain [*Miura et al.*, 1989, equation (A12)]

$$\nabla\cdot\delta\mathbf{u}_\perp = i\omega B_0^{-1}\delta B_\parallel - \delta\mathbf{u}_\perp\cdot(\boldsymbol{\kappa}_c + \boldsymbol{\kappa}_b), \tag{27}$$

where $\boldsymbol{\kappa}_b = \nabla_\perp \ln B_0$. The force balance of the static equilibrium gives [*Miura et al.*, 1989, equation (A6)]

$$\nabla_\perp p_0 = \mu_0^{-1}B_0^2(\boldsymbol{\kappa}_c - \boldsymbol{\kappa}_b). \tag{28}$$

From (20), (24), (27), and (28) one obtains

$$\nabla\cdot\delta\mathbf{u}_\perp = -2\delta\mathbf{u}_\perp\cdot\boldsymbol{\kappa}_c - \Gamma\mu_0 p_0 B_0^{-2}\nabla\cdot\delta\mathbf{u}. \tag{29}$$

Using $\delta\Phi$, the n component of $\delta\mathbf{u}_\perp$, which appears in the first term in the right-hand side of (29), can be written as

$$\delta u_n = iB_0^{-1}(\mathbf{e}\times\mathbf{k}_\perp)\cdot\mathbf{n}\delta\Phi. \tag{30}$$

Integration of (29) from the southern ionosphere to the northern ionosphere with the aid of (26) and (30) yields

$$\nabla\cdot\delta\mathbf{u} = -2i\frac{\int_{-s_b}^{s_b} B_0^{-2}(\mathbf{e}\times\mathbf{k}_\perp)\cdot\boldsymbol{\kappa}_c\delta\Phi\, ds}{\int_{-s_b}^{s_b} B_0^{-1}\, ds + \Gamma\mu_0 p_0\int_{-s_b}^{s_b} B_0^{-3}\, ds}. \tag{31}$$

Since the fast mode was eliminated, this compression factor must be due to the slow mode compression or rarefaction. From (20), (25), (30), and (31) we obtain the integrodifferential ballooning eigenmode equation for finite compressibility,

$$V_A^2 B_0(\mathbf{e}\cdot\nabla)[B_0^{-1}k_\perp^2(\mathbf{e}\cdot\nabla)\delta\Phi] + \omega^2 k_\perp^2 \delta\Phi = 2B_0 p_0^{-1}\left[B_0^{-1}\delta\Phi(\mathbf{e}\times\mathbf{k}_\perp)\cdot\nabla_\perp p_0 - 2\Gamma p_0 \cdot \frac{\int_{-s_b}^{s_b} B_0^{-2}(\mathbf{e}\times\mathbf{k}_\perp)\cdot\boldsymbol{\kappa}_c\delta\Phi\, ds}{\int_{-s_b}^{s_b} B_0^{-1}\, ds + \Gamma\mu_0 p_0 \int_{-s_b}^{s_b} B_0^{-3}\, ds}\right](\mathbf{k}_\perp\times\mathbf{e})\cdot\boldsymbol{\kappa}_c. \quad (32)$$

Equation (32) was obtained for long-thin perturbations ($\varepsilon_1 \ll 1$) by using only the minimization condition (5) of δW_F with respect to $\delta u_\parallel$ and the diamagnetic condition (24). Notice that in the energy principle of *Lee and Wolf* [1992] the diamagnetic condition (6) was obtained from the minimization condition of δW_F with respect to ξ_y and by assuming infinite k_y. However, in the analysis of *Miura et al.* [1989] the fast mode is eliminated automatically because of the low-frequency assumption of $|\omega^2| \sim k_\parallel^2 V_A^2$ and the long-thin perturbations, and the diamagnetic condition is obtained from the neglect of $O(\varepsilon_1^2)$ terms. Therefore, in derivation of the diamagnetic condition (24) by Miura et al., the assumption of infinite k_y is not necessary.

By multiplying (32) by $V_A^{-2}B_0^{-1}k_\perp^{-2}\delta\Phi^*$ (complex conjugate of $\delta\Phi$) and then by integrating from the southern ionosphere to the northern ionosphere, we obtain the following quadratic form:

$$\omega^2 \int_{-s_b}^{s_b} V_A^{-2}B_0^{-1}|\delta\Phi|^2\, ds = \int_{-s_b}^{s_b} B_0^{-1}k_\perp^2|(\mathbf{e}\cdot\nabla)\delta\Phi|^2\, ds - 2\mu_0 \cdot \int_{-s_b}^{s_b} B_0^{-3}[(\mathbf{e}\times\mathbf{k}_\perp)\cdot\nabla_\perp p_0][(\mathbf{e}\times\mathbf{k}_\perp)\cdot\boldsymbol{\kappa}_c]|\delta\Phi|^2\, ds + 4\Gamma\mu_0 p_0 \frac{\left|\int_{-s_b}^{s_b} B_0^{-2}(\mathbf{e}\times\mathbf{k}_\perp)\cdot\boldsymbol{\kappa}_c\delta\Phi\, ds\right|^2}{\int_{-s_b}^{s_b} B_0^{-1}\, ds + \Gamma\mu_0 p_0\int_{-s_b}^{s_b} B_0^{-3}\, ds}, \quad (33)$$

where we used that $\delta\Phi$ vanishes at the ionosphere. The right-hand side of (33) agrees with the potential energy δW_F derived by *Lee and Wolf* [1992]. When δW_F or ω^2 is negative, the configuration is unstable. The first term in the right-hand side of (33) is positive definite and represents stabilization by the line bending. This term vanishes for the interchange (flute) mode. The last term in the right-hand side is also positive definite and represents stabilization by the slow mode compression or rarefaction. The second term in the right-hand side is negative if $\nabla_\perp p_0 \cdot \boldsymbol{\kappa}_c > 0$ and represents the driving term of the ballooning instability. Therefore we could prove that the set of eigenmode equations derived by *Miura et al.* [1989] for the low-frequency and long-thin perturbations is equivalent to the energy principle in this specific problem of the ballooning instability in the magnetosphere [*Lee and Wolf*, 1992]. It is easily shown that if we minimize ω^2 calculated from (33) and obtain the Euler equation, the Euler equation becomes equal to the eigenmode equation (equation (32)).

It is obvious that the stabilizing term by the slow mode compression or rarefaction (the last term in the right-hand side of (33)) vanishes for the field-aligned antisymmetric mode. However, for field-aligned symmetric modes ($\delta\Phi$ is symmetric with respect to the equator) the last term in the right-hand side of (33) gives a stabilizing term. In the numerical calculation of *Miura et al.* [1989] the last term in the right-hand side of (32) is neglected, on the basis of the assumption of the incompressibility. Let us assume, for the moment, that the long-thin assumption for the equilibrium is valid along the entire field line. Then the order of magnitude calculation of each term in (32) using (28), $|\omega^2| \sim k_\parallel^2 V_A^2$, and the long-thin orderings for the perturbations and the equilibrium assuming $\varepsilon \sim \varepsilon_1 \sim \varepsilon_2 \ll 1$ shows that both terms in the left-hand side of (32) are $O(1)$. The first and second terms in the right-hand side of (32) are $O(\varepsilon)$ and $O(\varepsilon^2)$, respectively. Therefore the neglect of the compressible term of $O(\varepsilon^2)$ in (32) in Miura et al.'s analysis for the long-thin equilibrium is justified. The discrepancy between numerical results of Miura et al. and *Lee and Wolf* [1992] concerning the stability can therefore be attributed to the absence of the stabilizing compressible term in the analysis of Miura et al., which is justified for their long-thin equilibrium. The reason why Lee and Wolf could not find any unstable trial function seems to be due to the fact that they used shorter field lines [*Lee*, 1999] than the field lines of Miura et al.

In summary, the one-dimensional ballooning eigenmode equation obtained straightforwardly by the low-frequency assumption ($|\omega^2| \sim k_\parallel^2 V_A^2$) and the long-thin assumption for perturbations ($\varepsilon_1 \ll 1$) is equivalent to the energy principle. The neglect of $\nabla \cdot \delta\mathbf{u}$ term in the numerical analysis of *Miura et al.* [1989] is justified by the fact that the compressible stabilizing term (the last term in the right-hand side of (32)) is $O(\varepsilon^2)$ for the low-frequency perturbation ($|\omega^2| \sim k_\parallel^2 V_A^2$) and the long-thin orderings for the perturbations and the equilibrium, which mean that $\varepsilon \sim \varepsilon_1 \sim \varepsilon_2 \ll 1$.

5. Validity of the Incompressible Assumption for the Long-Thin Perturbations and Equilibrium

In this section we show directly by a normal mode analysis without using the minimization condition (5) and the diamagnetic condition (24) that $\nabla \cdot \delta\mathbf{u}$ can be neglected in the basic equations for the low-frequency assumption and long-thin orderings for the perturbations and the equilibrium ($\varepsilon \sim \varepsilon_1 \sim \varepsilon_2 \ll 1$). The validity of the incompressible assumption of *Miura et al.* [1989] could have been verified a posteriori using the growth rate obtained by using these assumptions. Such an assumption of the incompressibility is quite commonly used in the stability analysis of the instabilities, which are essentially the instability of the shear Alfvén branch, because the shear Alfvén mode is incompressible. *Goldston and Rutherford* [1995] showed a posteriori that the incompressible assumption is valid under the normal conditions for the pressure-driven Rayleigh-Taylor instability (interchange or flute instability) and the resistive-tearing instability in the poloidal plane with a strong toroidal field, both of which arise in the linearly polarized shear Alfvén branch. Following a procedure of Goldston and Rutherford, we will now verify the validity of the incompressible assumption for the ballooning instability for the long-thin perturbations in the long-thin magnetospheric equilibrium ($\varepsilon \sim \varepsilon_1 \sim \varepsilon_2 \ll 1$), which also arises in the shear Alfvén branch.

Taking a dot product of (10) with $\mathbf{k}_\perp$ yields

$$-i\omega\rho_0\mathbf{k}_\perp \cdot \delta\mathbf{u} = \delta\mathbf{B} \cdot (\mathbf{k}_\perp \times \mathbf{j}_0) + \mathbf{B}_0 \cdot (\mathbf{k}_\perp \times \delta\mathbf{j}) - ik_\perp^2\delta p. \quad (34)$$

For a 2-D equilibrium configuration used by *Miura et al.* [1989], $\mathbf{k}_\perp \times \mathbf{j}_0 = 0$ holds. Therefore the first term in the right-hand side of (34) vanishes. Then substitution of

$$\delta\mathbf{j} = \mu_0^{-1}\nabla \times \delta\mathbf{B} \quad (35)$$

into (34) yields after some algebra

$$-i\omega\rho_0\mathbf{k}_\perp \cdot \delta\mathbf{u} = \mu_0^{-1}B_0(\mathbf{e} \cdot \nabla)(\mathbf{k}_\perp \cdot \delta\mathbf{B}) - ik_\perp^2(\delta p + \mu_0^{-1}B_0\delta B_\parallel). \quad (36)$$

Using $\nabla \cdot \delta\mathbf{B} = 0$ in (36) shows that the first term in the right-hand side of (36) is $O(\varepsilon^2)$ smaller than the last term in the right-hand side. Therefore, by neglecting terms of $O(\varepsilon^2)$ in the right-hand side, we obtain from (36)

$$-i\omega\rho_0\mathbf{k}_\perp \cdot \delta\mathbf{u} = -ik_\perp^2(\delta p + \mu_0^{-1}B_0\delta B_\parallel). \quad (37)$$

Notice that in deriving (37) we only neglected the first term in the right-hand side of (36), which is $O(\varepsilon^2)$. We did not neglect other terms, so (37) may include other $O(\varepsilon)$ or higher-order terms. The purpose of the following calculation is to express the right-hand side of (37) by using only $\nabla \cdot \delta\mathbf{u}$ and $\delta\mathbf{u}_\perp$ in order to compare the compressible factor $\nabla \cdot \delta\mathbf{u}$ with the term $i\mathbf{k}_\perp \cdot \delta\mathbf{u}$, which is a dominant constituent term of $\nabla \cdot \delta\mathbf{u}$. For this purpose we use the adiabatic gas law (equation (20)) and the induction equation. Therefore (37) does not contradict (24), because the left-hand side of (37) may be $O(\varepsilon)$ or higher order. Although the left-hand side of (37) indeed turns out to be $O(\varepsilon)$ smaller than the right-hand side, our purpose in the following is to show that $\nabla \cdot \delta\mathbf{u}$ is negligible and not to show that (24) is valid. Therefore we do not neglect $O(\varepsilon)$ or higher-order terms until we can express the right-hand side of (37) by using only $\nabla \cdot \delta\mathbf{u}$ and $\delta\mathbf{u}_\perp$.

The perturbed form of the induction equation is

$$-i\omega\delta\mathbf{B} = -\mathbf{B}_0\nabla \cdot \delta\mathbf{u} + (\mathbf{B}_0 \cdot \nabla)\delta\mathbf{u} - (\delta\mathbf{u} \cdot \nabla)\mathbf{B}_0, \quad (38)$$

and the parallel component of this is

$$-i\omega\delta B_\parallel = -B_0\nabla \cdot \delta\mathbf{u} + \mathbf{e} \cdot [(\mathbf{B}_0 \cdot \nabla)\delta\mathbf{u}] - \mathbf{e} \cdot [(\delta\mathbf{u} \cdot \nabla)\mathbf{B}_0]. \quad (39)$$

Substitution of (20) and (39) into (37) yields after some algebra

$$-\omega^2\rho_0 k_\perp \cdot \delta\mathbf{u} = ik_\perp^2\left\{(\delta\mathbf{u}_\perp \cdot \nabla_\perp)\left(p_0 + \frac{B_0^2}{2\mu_0}\right) + (\Gamma p_0 + \mu_0^{-1}B_0^2)\nabla \cdot \delta\mathbf{u} - \mu_0^{-1}B_0^2\mathbf{e} \cdot [(\mathbf{e} \cdot \nabla)\delta\mathbf{u}] + \frac{1}{2\mu_0}\delta u_\parallel(\mathbf{e} \cdot \nabla)B_0^2\right\}. \quad (40)$$

For the interchange or flute instability there is no field-aligned variation of the perturbation. Then, for $(\mathbf{e} \cdot \nabla)B_0 \sim 0$, (40) becomes similar to (19.24) of *Goldston and Rutherford* [1995] derived for the interchange (flute) instability. Using the vector identity

$$\mathbf{e} \cdot [(\mathbf{e} \cdot \nabla)\delta\mathbf{u}_\perp] = -\boldsymbol{\kappa}_c \cdot \delta\mathbf{u}_\perp \quad (41)$$

and

$$(\delta\mathbf{u}_\perp \cdot \nabla)\left(p_0 + \frac{B_0^2}{2\mu_0}\right) = \mu_0^{-1}B_0^2\delta\mathbf{u}_\perp \cdot \boldsymbol{\kappa}_c, \quad (42)$$

(40) can be further reduced to

$$-\omega^2\rho_0\mathbf{k}_\perp \cdot \delta\mathbf{u} = ik_\perp^2\left\{2(\delta\mathbf{u}_\perp \cdot \nabla_\perp)\left(p_0 + \frac{B_0^2}{2\mu_0}\right) + (\Gamma p_0 + \mu_0^{-1}B_0^2)\nabla \cdot \delta\mathbf{u} - \mu_0^{-1}B_0^2(\mathbf{e} \cdot \nabla)\delta u_\parallel + \frac{1}{2\mu_0}\delta u_\parallel(\mathbf{e} \cdot \nabla)B_0^2\right\}. \quad (43)$$

By substituting (21) into (43), the right-hand side of (43) can be expressed only by using $\nabla \cdot \delta\mathbf{u}$ and $\delta\mathbf{u}_\perp$. In the left-hand side of (43), $\mathbf{k}_\perp \cdot \delta\mathbf{u} = k_y\delta u_y$ holds. From

$$\delta\mathbf{A} = \delta A_n\mathbf{n} + \delta A_\parallel\mathbf{e} \quad (44)$$

and the Coulomb gauge

$$\nabla \cdot \delta\mathbf{A} = 0, \quad (45)$$

we obtain

$$\delta A_n = -\frac{B_0}{|\boldsymbol{\kappa}_c|}(\mathbf{e} \cdot \nabla)\left(\frac{\delta A_\parallel}{B_0}\right), \quad (46)$$

where we used

$$\nabla \cdot \mathbf{n} = -\boldsymbol{\kappa}_c \cdot \mathbf{n}. \quad (47)$$

From (16) and (46) we obtain

$$|\delta A_n| \sim \left|\frac{k_\parallel^2}{\omega|\boldsymbol{\kappa}_c|}\delta\Phi\right|. \quad (48)$$

Making use of (42) and (48) and noting that

$$|\delta u_y| \sim \left|\frac{\omega}{B_0}\delta A_n\right|, \quad (49)$$

we obtain that the ratio of the first term in the right-hand side of (43) to the left-hand side of (43) is $O(1)$. Therefore the left-hand side of (43) is comparable to the first term in the right-hand side of (43). Here it is important to notice from (30), (48), and (49) that δu_y and δu_n are of the same order. It is also easily shown that the magnitude of the third term in the right-hand side of (43), which is ε^{-1} larger than the fourth term in the right-hand side of (43), is smaller than the magnitude of the second term in the right-hand side of (43). This means that the magnitude of the second term in the right-hand side of (43) must be at most comparable to or smaller than the magnitude of the left-hand side of (43). Therefore, from (43) and $|\omega^2| \sim k_\parallel^2V_A^2$, we obtain

$$\left|\frac{\nabla \cdot \delta\mathbf{u}}{i\mathbf{k}_\perp \cdot \delta\mathbf{u}}\right| \leq \left|\frac{\mu_0^{-1}B_0^2}{\Gamma p_0 + \mu_0^{-1}B_0^2}\varepsilon^2\right| < \varepsilon^2. \quad (50)$$

Therefore the compressible factor $\nabla \cdot \delta\mathbf{u}$ is much smaller than one of its constituent parts, i.e., $i\mathbf{k}_\perp \cdot \delta\mathbf{u}$. This indicates that the incompressible assumption that $\nabla \cdot \delta\mathbf{u} \sim 0$ is valid for the ballooning instability with long-thin perturbations in the low-frequency ($|\omega^2| \sim k_\parallel^2 V_A^2$) regime and in the long-thin equilibrium. This is in strong contrast with situations for other instabilities, which do not arise in the shear Alfvén branch. For example, for the Kelvin-Helmholtz instability in the 2-D transverse configuration [*Miura and Pritchett*, 1982; *Miura*, 1997], in which the magnetic field is transverse to the plane including the sheared flow, the compressibility gives a strong stabilizing influence. Therefore the compressibility cannot be neglected for calculation of the growth rate except for the case with the fast mode Mach number much smaller than unity, and the growth rate is reduced substantially by the compressibility, because the Kelvin-Helmholtz instability in such a configuration arises in the fast magnetosonic branch.

By using (28) the ratio of the second term in the right-hand side of (20) to the first term in the right-hand side of (20) (convective change of the pressure) is expressed by using (50) as

$$\left|\frac{\Gamma p_0 \nabla \cdot \delta \mathbf{u}}{\delta \mathbf{u}_\perp \cdot \nabla_\perp p_0}\right| \sim \left|\frac{\Gamma p_0}{\Gamma p_0 + \mu_0^{-1} B_0^2}\varepsilon\right| < \varepsilon. \tag{51}$$

Therefore it is indeed justified to neglect the second term in the right-hand side of (20) compared with the first term in the right-hand side of (20). This again shows that the incompressible assumption is valid for the long-thin perturbations and equilibrium. By comparison of the first term in the right-hand side of (37) with the left-hand side of (37) it is easily shown by using (20) and (51) that the left-hand side of (37) is indeed $O(\varepsilon)$ smaller than the first term of the right-hand side of (37). Thus (37) and (24) are consistent.

When the compressibility is neglected, the eigenmode equation (32) can be written as

$$B_0(V_A/k_\perp)^2(\mathbf{e} \cdot \nabla)[(k_\perp^2/B_0)(\mathbf{e} \cdot \nabla)\delta\Phi] + (\omega^2 + \gamma_{\mathrm{MHD}}^2)\delta\Phi = 0, \tag{52}$$

where γ_{MHD} is given by (3). Using the WKB approximation for the field-aligned differentiation, the dispersion equation of the ideal MHD ballooning instability with incompressible assumption, which is also equation (14) of *Miura et al.* [1989] without the ion diamagnetic drift term, can be obtained from (52) as

$$\gamma^2 = \gamma_{\mathrm{MHD}}^2 - k_\parallel^2 V_A^2, \tag{53}$$

where γ is the growth rate ($\omega = \omega_r + i\gamma = i\gamma$). The necessary condition for the instability is therefore

$$\gamma_{\mathrm{MHD}}^2 > k_\parallel^2 V_A^2, \tag{54}$$

which can be rewritten as

$$\beta > \beta_{\mathrm{crt}} = k_\parallel^2 L_p R_c = \frac{L_p R_c}{L_\parallel^2}, \tag{55}$$

where $L_\parallel = k_\parallel^{-1}$ and $L_p = |\nabla p_0/p_0|^{-1}$. Since $|\omega^2| = \gamma^2 \sim k_\parallel^2 V_A^2$ from the low-frequency assumption, we obtain from (53)

$$\gamma^2 \sim k_\parallel^2 V_A^2 \sim \tfrac{1}{2}\gamma_{\mathrm{MHD}}^2. \tag{56}$$

Therefore we obtain

$$\gamma \sim \frac{1}{\sqrt{2}}\gamma_{\mathrm{MHD}}. \tag{57}$$

In the numerical analysis of Miura et al. the growth rate of the ideal MHD ballooning instability is given by $\gamma \sim 0.85\ \gamma_{\mathrm{MHD}}$ (see their Figure 6), where γ_{MHD} is defined at the equator. Therefore, (57) with γ_{MHD} calculated at the equator gives a fairly good approximate formula for their numerically obtained growth rate.

6. Calculation of the Energy Functional for a Trial Function

If the long-thin assumption for the equilibrium is valid along the whole field line, the discussion in sections 4 and 5 is enough to show that the incompressible assumption is valid in the ballooning instability in the magnetosphere. However, in Kan's model of the plasma sheet shown in Figure 4 of *Miura et al.* [1989], the long-thin assumption for the equilibrium is valid on average, but it is not satisfied in the tiny region around the equator. Therefore it seems more likely that the long-thin equilibrium in the magnetosphere includes necessarily a region near the equator, where the long-thin assumption for the equilibrium is not valid. In such a case, the previous discussion in sections 4 and 5 is not valid near the equator, and we need a separate discussion to see whether the incompressible assumption is valid near the equator. Therefore, in this section we calculate and compare each energy term in the right-hand side of (33) to clarify when the compressible stabilizing term can be neglected. We consider here that $|\boldsymbol{\kappa}_c| \gg |\boldsymbol{\kappa}_b|$ is valid at $|s| \le |s_\delta|$ near the equator and that $|\boldsymbol{\kappa}_c|$ is strongly peaked at the equator as is valid for the field line A in Kan's model of the plasma sheet shown in Figure 4 of Miura et al. Furthermore, we assume that the contribution to the integral in the numerator of the second term of the right-hand side of (32) from the long-thin part, where $|\boldsymbol{\kappa}_c| \ll |\boldsymbol{\kappa}_b|$ is satisfied, is negligible. Then the integral in the numerator of the second term of the right-hand side of (32) can be written as

$$\int_{-s_b}^{s_b} B_0^{-2}(\mathbf{e} \times \mathbf{k}_\perp) \cdot \boldsymbol{\kappa}_c \delta\Phi\, ds \cong \int_{-s_\delta}^{s_\delta} B_0^{-2}(\mathbf{e} \times \mathbf{k}_\perp) \cdot \boldsymbol{\kappa}_c \delta\Phi\, ds. \tag{58}$$

From (28) we also obtain

$$\nabla_\perp p_0 \cong \mu_0^{-1} B_0^2 \boldsymbol{\kappa}_c \tag{59}$$

at $|s| \le |s_\delta|$. Since $\mathbf{k}_\perp \cdot \mathbf{j}_{0\perp} = -B_0^{-1}(\mathbf{e} \times \mathbf{k}_\perp) \cdot \nabla_\perp p_0$ is constant along the field line, (58) can be rewritten by using (58) and (59) as

$$\int_{-s_b}^{s_b} B_0^{-2}(\mathbf{e} \times \mathbf{k}_\perp) \cdot \boldsymbol{\kappa}_c \delta\Phi\, ds \cong \mu_0^{-1} B_0^{-1}(\mathbf{e} \times \mathbf{k}_\perp) \cdot \nabla_\perp p_0 \int_{-s_b}^{s_b} B_0^{-3}\delta\Phi\, ds, \tag{60}$$

where we replaced in the integral in the right-hand side of (60) the interval $(-s_\delta, s_\delta)$ for the integration with $(-s_b, s_b)$ by assuming that the contribution to the integral from the integration from $s = -s_b$ to $s = -s_\delta$ is negligible. Then (32) can be rewritten as

$$V_A^2 B_0(\mathbf{e} \cdot \nabla)[B_0^{-1} k_\perp^2 (\mathbf{e} \cdot \nabla)\delta\Phi] + \omega^2 k_\perp^2 \delta\Phi = -2\rho_0^{-1} k_\perp^2 (\boldsymbol{\kappa}_c \cdot \nabla_\perp p_0)\left[\delta\Phi - \frac{2\langle\delta\Phi\rangle}{1 + \frac{2}{\Gamma}\langle\beta^{-1}\rangle}\right], \tag{61}$$

where we defined a weighted flux average of $g(s)$ with the weighting function $B_0^{-3}(s)$ by

$$\langle g(s)\rangle = \frac{\int_{-s_b}^{s_b} B_0^{-3}(s) g(s)\, ds}{\int_{-s_b}^{s_b} B_0^{-3}(s)\, ds}. \quad (62)$$

By dividing both sides of (61) by $V_A^2 B_0 k_\perp^2$, multiplying them by $\delta\Phi^*$, and integrating from $s = -s_b$ to s_b, we obtain after some algebra

$$\delta W'_F = \delta W'_B + \delta W'_C, \quad (63)$$

where

$$\delta W'_F = \omega^2 \mu_0 \rho_0 \langle |\delta\Phi|^2\rangle, \quad (64)$$

$$\delta W'_B = \langle B_0^2 |(\mathbf{e}\cdot\nabla)\delta\Phi|^2\rangle, \quad (65)$$

$$\delta W'_C = -2\mu_0^2 (B_0^{-1}|\nabla_\perp p_0|)^2 \cdot \left(\langle |\delta\Phi|^2\rangle - \frac{2}{1 + \frac{2}{\Gamma}\langle \beta^{-1}\rangle} \left|\langle \delta\Phi\rangle\right|^2 \right), \quad (66)$$

where we assumed that $\delta\Phi\ (\pm s_b) = 0$ and where we used a simplification similar to (58)–(60) in deriving (66). Equation (66) is also valid for the interchange (flute) mode ($\delta\Phi(s)$ = const), if we use a different boundary condition $(\mathbf{e}\cdot\nabla)\delta\Phi(\pm s_b) = 0$, which is satisfied for zero ionospheric conductivities. Notice that (63) is a simplified form of the quadratic form of (33). The functional $\delta W'_F$ is proportional to the energy functional δW_F in (4), $\delta W'_B$ is proportional to the energy required to bend the magnetic field lines, and $\delta W'_C$ is proportional to the part of the energy functional including the curvature term. More specifically, the first term in the bracket of the right-hand side of (66) is the ballooning driving term, and the second term is the compressional stabilizing term.

In order to calculate the weighted average quantities in (64)–(66) we need a specific model of $B_0(s)$ and a specific trial function $\delta\Phi(s)$. In the long-thin equilibrium the weighting function $B_0^{-3}(s)$ is strongly peaked at the equator [see *Miura et al.*, 1989, Figure 3]. Therefore we assume as a specific model of $B_0^{-3}(s)$,

$$B_0^{-3}(s) = B_0^{-3}(0)\exp(-|s|/s_B), \quad (67)$$

where we assume that $s_B \ll s_b$. Then we obtain simply

$$\langle \beta^{-1}(s)\rangle = 3\beta^{-1}(0), \quad (68)$$

where $\beta(0)$ is the plasma β at the equator. For the specific field model (equation (67)) we obtain the stability condition of the interchange (flute) mode ($(\mathbf{e}\cdot\nabla)\delta\Phi(s) = 0$) by setting $\delta\Phi(s) = \delta\Phi(0)$ in (66) and substituting (68) into (66). From $\delta W'_C > 0$ we find that the interchange mode is completely stabilized by the compressibility when $\beta(0) > 6/\Gamma$, where Γ = 5/3 for the adiabatic 3-D plasma. This is consistent with the findings of *Horton et al.* [1999] that the interchange mode is stabilized by the compressibility when $\beta(0) > 1.5$–3.0. The difference of the critical $\beta(0)$ between the present calculation and that of Horton et al. is due to the difference of the specific field models. We also find from (66) that for $\beta(0) \ll 6/\Gamma$ the interchange (flute) mode becomes essentially incompressible.

Since the ballooning mode is strongly localized near the equator [see, e.g., *Miura et al.*, 1989, Figure 7], we can use as a trial function of the energy principle

$$\delta\Phi(s) = \delta\Phi(0)\exp(-|s|/s_\Phi), \quad (69)$$

where we assume that $S_\Phi \ll S_b$. Then $\delta W'_C$ can be calculated as

$$\delta W'_C = -2\mu_0^2 (B_0^{-1}|\nabla_\perp p_0|)^2 |\delta\Phi(0)|^2 \cdot \left[\frac{s_\Phi}{s_\Phi + 2s_B} - \frac{2}{1 + \frac{6}{\Gamma}\beta^{-1}(0)} \left(\frac{s_\Phi}{s_\Phi + s_B}\right)^2 \right]. \quad (70)$$

Equation (70) indicates that when $\beta(0) \gg 6/\Gamma$, the compressible stabilizing term can be neglected in $\delta W'_C$ for $s_\Phi \ll s_B/4$. This means that when the ballooning eigenmode function is more strongly peaked than $B_0^{-3}(s)$ at the equator, where $\beta(0) \gg 6/\Gamma$ is satisfied, the ballooning mode becomes essentially incompressible. In such a limit we obtain

$$\delta W'_F = \delta W'_B + \delta W'_C = -B_0^2(0)|\delta\Phi(0)|^2 \frac{(2L_{\text{peq}}^2 - \beta^2(0)s_\Phi^2)}{4 s_\Phi s_B L_{\text{peq}}^2}, \quad (71)$$

where L_{peq} is the pressure scale length at the equator, which is defined by

$$B_0^{-1}(s)|\nabla_\perp p_0| = \text{const} = B_0^{-1}(0)\frac{p_0}{L_{\text{peq}}}. \quad (72)$$

From (71) the condition for the ballooning instability in the same incompressible limit is

$$\beta(0) > \frac{L_{\text{peq}} R_{\text{ceq}}}{s_\Phi^2}, \quad (73)$$

where R_{ceq} is the curvature radius at the equator, which is calculated by using (59) at the equator. This condition for the ballooning instability is similar to the instability condition (55) obtained by the WKB approximation. Since the unstable ballooning eigenmode found by *Miura et al.* [1989, Figure 7] using the incompressible assumption satisfies $\beta(0) \gg 6/\Gamma$ and is strongly localized near the equator, the incompressible assumption in their calculation can be justified a posteriori. In the numerical eigenmode analysis of the ballooning instability in the outer edge of the ring current by *Ohtani et al.* [1989b], however, the long-thin equilibrium is not valid along the entire length of the dipole-like field line [see *Ohtani et al.*, 1989b, Figure 2], and $\beta(0) \gg 6/\Gamma$ is not satisfied at the equator. Therefore the coupled eigenmode equations including both Alfvén mode and slow mode were solved in their calculation.

7. Physical Picture of the Validity of the Incompressible Assumption

The detailed discussion in sections 4–6 suggests that the incompressible assumption is more valid in the long-thin equilibrium including the equatorial region, where the long-thin ordering for the equilibrium may not be valid. In order to understand physically why the incompressible assumption is more valid in the long-thin equilibrium, we must compare the relative importance of each term in the bracket of the right-hand side of (32). Notice that the second term in the bracket of

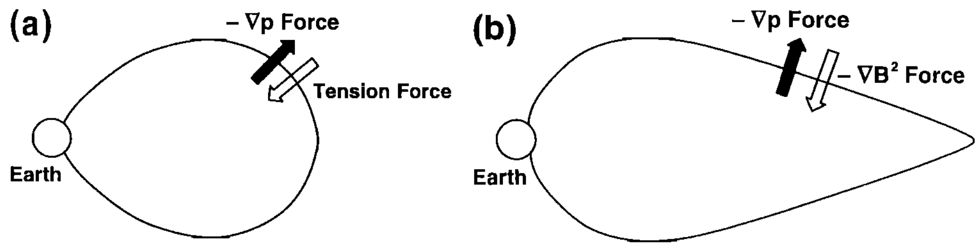

Figure 1. Schematic view of field lines of dipole-like field and taillike field. (a) In the dipole-like field, $|\boldsymbol{\kappa}_b| \ll |\boldsymbol{\kappa}_c|$ holds, and the pressure gradient force (solid arrow) is nearly balanced with the tension force (open arrow) owing to the field line curvature. (b) In the taillike field the long-thin assumption for the equilibrium $|\boldsymbol{\kappa}_b| \gg |\boldsymbol{\kappa}_c|$ holds on average, and the pressure gradient force (solid arrow) is nearly balanced with the magnetic pressure gradient force (open arrow).

the right-hand side of (32) including the flux integral comes from the compressional term $\Gamma p_0 \nabla \cdot \delta\mathbf{u}$ in the right-hand side of (20), whereas the first term comes from the convective term $\delta\mathbf{u}_\perp \cdot \nabla_\perp p_0$ in the right-hand side of (20). We multiply the operator $\mathbf{e} \cdot \nabla$ on both sides of (21) and use (6), (20), (28), and $\nabla \cdot \mathbf{e} = -\mathbf{e} \cdot \nabla(\ln B_0)$ to obtain

$$\omega^2 \nabla \cdot \delta\mathbf{u} + \frac{V_A^2 c_s^2}{V_A^2 + c_s^2} (\mathbf{e} \cdot \nabla)^2 \nabla \cdot \delta\mathbf{u}$$

$$= -\frac{V_A^2}{V_A^2 + c_s^2} \omega^2 [2\delta\mathbf{u}_\perp \cdot \boldsymbol{\kappa}_c + \delta u_\parallel (\mathbf{e} \cdot \nabla) \ln B_0], \qquad (74)$$

where $c_s^2 = \Gamma p_0/\rho_0$. Equation (74) means that when $\boldsymbol{\kappa}_c \neq 0$, the Alfvénic perturbation δu_n becomes a source of $\nabla \cdot \delta\mathbf{u}$ perturbation and $\nabla \cdot \delta\mathbf{u}$ propagates along the field line with the slow mode speed. By taking into account only curvature effect, which means that $|\boldsymbol{\kappa}_c| \gg |\boldsymbol{\kappa}_b|$, *Southwood and Saunders* [1985] showed that the coupling of the Alfvén mode and the compressible slow mode is important in such a case. Therefore (74) is consistent with their findings. Since the stability analysis studies the steady state, when the perturbation becomes steady, the compressional effect given at the equator is averaged along the field line and $\nabla \cdot \delta\mathbf{u}$ is homogeneously distributed and becomes constant along the field line. This is what the minimization condition (5) means. Notice that the minimization condition (5) is derived from the minimization of $|\nabla \cdot \boldsymbol{\xi}|^2$ term in (4), and this is consistent with the above physical picture. Although the ballooning driving term, which is the first term in the bracket of the right-hand side of (32) is a local quantity depending on s and is strongly peaked at the equator, the compressional stabilizing term, which is the second term in the bracket of the right-hand side of (32), is a constant global quantity, which arises from the spreading of the compressional effect over the entire field line. When the magnetic field lines are more stretched and the long-thin orderings for the perturbations and the equilibrium are more satisfied, on average, the local instability driving term is more strongly peaked near the equator, but the compressional stabilizing term is averaged along the entire field line, however strongly the field lines become stretched and taillike. Therefore the local ballooning driving term becomes more important than the global compressional stabilizing term near the equator. Thus the incompressible assumption is more likely to be satisfied in the long-thin equilibrium.

8. Discussion and Summary

The present analysis shows that the incompressible assumption of the perturbation in the ballooning instability is valid for low-frequency perturbations and long-thin assumptions for the perturbations and the equilibrium. Even if the long-thin equilibrium is not valid near the equator, the results in section 6 show that the incompressible assumption is still valid for the ballooning mode, which is strongly localized near the equator, where $\beta(0) \gg 6/\Gamma$. Since the understanding of the long-thin equilibrium is the key to understanding the validity of the incompressible assumption, we clarify here what kind of equilibrium configuration the long-thin equilibrium corresponds to. The smallness parameter ε_2 for the long-thin equilibrium is $\varepsilon_2 = |\boldsymbol{\kappa}_c|/|\boldsymbol{\kappa}_b|$. Therefore the long-thin equilibrium corresponds to the case for $\varepsilon_2 \ll 1$ or $|\boldsymbol{\kappa}_c| \ll |\boldsymbol{\kappa}_b|$. In such a case we obtain from (28)

$$\nabla_\perp p_0 \cong -\mu_0^{-1} B_0^2 \boldsymbol{\kappa}_b = -\nabla_\perp \left(\frac{B_0^2}{2\mu_0} \right). \qquad (75)$$

This means that the pressure gradient force is balanced with the magnetic pressure gradient force. Such an equilibrium is possible only when the equilibrium is taillike and the field lines are stretched substantially, so that the field line curvature is small except for a tiny region around the equator. Figure 1b shows schematically this case, where the long-thin assumption for the equilibrium is valid, on average, and the configuration is taillike. The extreme limit ($\varepsilon_2 = 0$) of this case is the one-dimensional neutral sheet (Harris sheet), in which the pressure gradient force is outward away from the neutral plane, and this force is balanced with the magnetic pressure gradient force directing toward the neutral plane. In the highly stretched 2-D plasma sheet, such a long-thin assumption for the equilibrium is likely to be satisfied except near the equator. It is indeed shown in Figure 4 of *Miura et al.* [1989] that Kan's model of the plasma sheet satisfies the long-thin assumption except for a tiny region around the equator. In the opposite limit $\varepsilon_2 \gg 1$ or $|\boldsymbol{\kappa}_c| \gg |\boldsymbol{\kappa}_b|$ (short-thick equilibrium) we obtain from (28)

$$\nabla_\perp p_0 \cong \mu_0^{-1} B_0^2 \boldsymbol{\kappa}_c. \qquad (76)$$

This means that the pressure gradient force is balanced with the magnetic tension force due to the field-line curvature. Therefore this case corresponds to the case where the field line is substantially curved or dipole-like. Figure 1a shows schematically this case, where the field line is substantially curved and the tension force is nearly balanced with the pressure gradient force. We expect that the outer edge of the ring current is an example of the short-thick equilibrium, because the field line is not strongly stretched from the dipole as in the plasma sheet. *Ohtani et al.* [1989b] obtained by numerical iteration a model equilibrium representing the outer edge of the ring current. It is shown in Figure 2 of Ohtani et al. that the short-thick assumption ($|\boldsymbol{\kappa}_c| > |\boldsymbol{\kappa}_b|$) is valid in their model.

Although there is no quantitative definition of the taillike configuration and the dipole-like configuration, which is important in understanding the dynamical change of the magnetospheric configuration in substorms, the above consideration suggests that a good criterion of the equilibrium configuration determining whether it is taillike or dipole-like is the smallness parameter ε_2. That is, when $\varepsilon_2 \ll 1$, the configuration is considered to be taillike, and when $\varepsilon_2 \gg 1$, the configuration is considered to be dipole-like. Here we should note that since the dipole field is a zero-β field ($p_0 = 0$), the dipole field satisfies $\boldsymbol{\kappa}_c = \boldsymbol{\kappa}_b$ as is obvious from (28). This means that in the dipole field the tension force by the field line curvature is balanced with the magnetic pressure gradient force. It follows from this consideration that the incompressible assumption for the ballooning instability is more likely to be satisfied when the equilibrium configuration becomes more taillike. Since the near-Earth plasma sheet is stretched and is taillike during the growth phase [*Kaufmann*, 1987], the long-thin ordering for the equilibrium is more likely to be satisfied before the onset of the expansion phase. Therefore we conjecture that the incompressible assumption for the ballooning instability becomes more likely to be valid before the expansion phase onset in the near-Earth plasma sheet.

It is interesting to point out here that a different long-thin ordering was used by *Schindler* [1972] to obtain a 2-D static equilibrium of the distant plasma sheet. The smallness parameter ε_2 adopted in his calculation is equal to L_z/L_x instead of $l_\perp/l_\parallel = |\boldsymbol{\kappa}_c|/|\boldsymbol{\kappa}_b|$, where L_x and L_z are scale lengths along the x and z directions, respectively; the x axis points along the tail, and the z axis is along the dipole axis. By using this smallness parameter as the ordering parameter, *Lakhina et al.* [1990] investigated the stability of the distant tail plasma sheet and the plasma sheet boundary layer with a sheared parallel flow and found that the distant tail plasma sheet with the parallel flow is subject to the ballooning instability.

From (3) and (57) the growth rate of the incompressible ballooning instability in the long-thin equilibrium can be written as

$$\gamma = \left| \frac{p_0}{\rho_0} \frac{1}{L_{\text{peq}} R_{\text{ceq}}} \right|^{1/2}, \tag{77}$$

where p_0 and ρ_0 are the unperturbed pressure and density at the equator, respectively, and L_{peq} and R_{ceq} are the perpendicular pressure scale length and the field line curvature at the equator, respectively. Since $p_0 = n_0 k(T_i + T_e)$ holds, (77) can be rewritten as

$$\gamma = \left| \frac{k(T_i + T_e)}{m_i} \frac{1}{L_{\text{peq}} R_{\text{ceq}}} \right|^{1/2}, \tag{78}$$

where m_i is the ion mass and T_i and T_e are ion and electron temperatures at the equator, respectively. *Korth et al.* [1991] and *Pu et al.* [1992] measured L_{peq} and R_{ceq} in the near-Earth plasma sheet prior to the substorm onset. According to their results shown in Table 2 of Korth et al., the average values of L_{peq} and R_{ceq} prior to the onset are 4000 and 3500 km, respectively. If we further assume that kT_i ($\gg kT_e$) [*Korth et al.*, 1991; *Pu et al.*, 1999] is typically 10 keV [*Korth et al.*, 1991; *Pu et al.*, 1992], (78) gives the e-folding time (γ^{-1}) or the growth time of the ballooning instability in the near-Earth plasma sheet equal to 3.8 s. This e-folding time is fast enough to account for the rapid onset of the substorm expansion phase. Notice that although the present incompressible ballooning instability arises in the shear Alfvén branch, this e-folding time is much smaller than the bounce time of the Alfvén wave between the ionospheres of both hemispheres. This is because the Alfvén wave is trapped in a region around the equator (effective potential well) and is not bouncing back and forth between the ionospheres. If we use more modest parameters such as $kT_i = 1$ keV and L_{peq} and R_{ceq} are both 10,000 km at the equator, (78) gives the e-folding time of 32 s. This is still fast enough to explain the rapid substorm onset. According to *Hurricane et al.* [1999] and *Cowley and Artun* [1997], for systems in which the equilibrium is evolving slowly through marginal stability, any linearly unstable evolution must be slow compared to the Alfvén time, i.e., $\gamma\tau_A < 1$, where $\tau_A = L/V_A$ is the Alfvén time defined by the characteristic scale length L. Since the Alfvén wave is trapped near the equator and not bouncing back and forth between ionospheres, the characteristic length L in the ballooning instability should be R_{ceq} at the equator. Therefore $\tau_A = R_{\text{ceq}}/V_A$. Since (28) holds and $\boldsymbol{\kappa}_b$ and $\boldsymbol{\kappa}_c$ are in the same direction at the equator, we obtain

$$\frac{1}{L_{\text{peq}}} = \left| \frac{\nabla_\perp p_0}{p_0} \right| = \left| \frac{\mu_0^{-1} B_0^2}{p_0} (\boldsymbol{\kappa}_c - \boldsymbol{\kappa}_b) \right| < \left| \frac{\mu_0^{-1} B_0^2}{p_0} \boldsymbol{\kappa}_c \right| \tag{79}$$

at the equator. Therefore, from (77) and (79) we obtain

$$\gamma\tau_A = \left| \frac{p_0}{\rho_0} \frac{1}{L_{\text{peq}} R_{\text{ceq}}} \right|^{1/2} \frac{R_{\text{ceq}}}{V_A} < 1. \tag{80}$$

Thus the present growth rate (equation (77)) obtained for the ballooning instability satisfies the above requirement of the unstable evolution $\gamma\tau_A < 1$. It follows that the incompressible ballooning instability for the long-thin equilibrium seems to be a candidate mechanism for the rapid near-Earth substorm onset. *Korth et al.* [1991] found in their statistical study of the substorm onset that the instability conditions (equation (55)) obtained by assuming the incompressibility were satisfied prior to most of their substorm onsets. *Pu et al.* [1992] also showed by studying substorm onsets that the scale length of the ion pressure gradient is typically of the order of $10\rho_{\text{Li}}$, which is the average Larmor radius of energetic ions, and all plasma and field parameters are favorable to the onset of the ballooning instability. *Ullaland et al.* [1993] also found in a single substorm influenced by a storm sudden commencement (ssc) that the instability condition (55) is nearly satisfied prior to the substorm onset. These observational results seem to support the present view that the incompressible ballooning instability in the long-thin equilibrium is a relevant instability related to the near-Earth substorm onset.

When the field line becomes very taillike, so that the field line curvature radius at the equator becomes smaller than the ion Larmor radius, the ion motion becomes stochastic, and the stochastic ion dynamics becomes important [e.g., *Büchner and Zelenyi*, 1989; *Chen*, 1992] and must be taken into account in the ballooning stability analysis [*Hurricane et al.*, 1995]. Existence of such stochastic plasmas has also been suggested by observations in the near-Earth plasma sheet by *Lui et al.* [1992]. We found that the ions should be stochastic near the equator for the field line A in Figure 1 of *Miura et al.* [1989], although they are likely to remain adiabatic for field lines B and C near the equator in the same figure. Therefore, for the field line A we must include the stochastic ion dynamics. According to the formulation of *Hurricane et al.* [1995, equation (23)], the ballooning eigenmode equation including the stochastic dynamics is similar to (61), but the compressibility term $2\langle\delta\Phi\rangle/(1 + 2\Gamma^{-1}\langle\beta^{-1}\rangle)$ in the right-hand side of (61) must be replaced by a weighted average of the form

$$\frac{\displaystyle\int_{-s_b}^{s_b} B_0^{-2}(s)|\boldsymbol{\kappa}_c|\delta\Phi\, ds}{\displaystyle\int_{-s_b}^{s_b} B_0^{-2}(s)|\boldsymbol{\kappa}_c|\, ds}. \tag{81}$$

This indicates that the effective compression term in the stochastic plasma can also be represented by the weighted average of $\delta\Phi$ with the weighting function $B_0^{-2}(s)|\boldsymbol{\kappa}_c|$. Since the stochastic dynamics is only important where $|\boldsymbol{\kappa}_c|$ is very large near the equator, we reasonably assume that $|\boldsymbol{\kappa}_c| \gg |\boldsymbol{\kappa}_b|$ is satisfied at $|s| < |s_\delta|$ near the equator and that the integral from $s = -s_\delta$ to $s = s_\delta$ contributes most to the above weighted integral (equation (81)). Since (59) holds at $|s| < |s_\delta|$ and $B_0^{-1}(s)|\nabla_\perp p_0|$ is constant along the field line, we obtain

$$\begin{aligned}\int_{-s_b}^{s_b} B_0^{-2}(s)|\boldsymbol{\kappa}_c|\delta\Phi(s)\, ds &\cong \int_{-s_\delta}^{s_\delta} B_0^{-2}(s)|\boldsymbol{\kappa}_c|\delta\Phi(s)\, ds \\ &\cong \mu_0 B_0^{-1}|\nabla_\perp p_0| \int_{-s_\delta}^{s_\delta} B_0^{-3}(s)\delta\Phi(s)\, ds \\ &\cong \mu_0 B_0^{-1}|\nabla_\perp p_0| \int_{-s_b}^{s_b} B_0^{-3}(s)\delta\Phi(s)\, ds,\end{aligned} \tag{82}$$

where we assumed that the contribution to the final integral from the interval from $s = -s_b$ to $s = -s_\delta$ is negligible. Therefore the eigenmode equation for the stochastic taillike plasma can be written as

$$\begin{aligned}&V_A^2 B_0(\mathbf{e}\cdot\nabla)[B_0^{-1}k_\perp^2(\mathbf{e}\cdot\nabla)\delta\Phi] + \omega^2 k_\perp^2 \delta\Phi \\ &\quad = -2\rho_0^{-1}k_\perp^2(\boldsymbol{\kappa}_c\cdot\nabla_\perp p_0)[\delta\Phi - \langle\delta\Phi\rangle],\end{aligned} \tag{83}$$

where $\langle\delta\Phi\rangle$ is the same weighted average of $\delta\Phi$ with the weighting function $B_0^{-3}(s)$ as used in the ideal MHD case in section 6. Therefore, by the similar calculation as used in section 6, $\delta W'_C$ for the stochastic plasma can be written as

$$\delta W'_C(\text{stochastic}) = -2\mu_0^2(B_0^{-1}|\nabla_\perp p_0|)^2(\langle|\delta\Phi|^2\rangle - |\langle\delta\Phi\rangle|^2). \tag{84}$$

From (84) it is obvious that there is no unstable interchange mode with $\delta\Phi(s) = \text{const}$ and that the interchange mode is only marginal in the stochastic plasma, because $\delta W'_C$(stochastic) $= 0$ for such interchange mode. For the same ballooning trial function (equation (69)), $\delta W'_C$(stochastic) becomes

$$\begin{aligned}\delta W'_C(\text{stochastic}) &= -2\mu_0^2(B_0^{-1}|\nabla_\perp p_0|)^2|\delta\Phi(0)|^2 \\ &\quad\cdot\left[\frac{s_\Phi}{s_\Phi + 2s_B} - \left(\frac{s_\Phi}{s_\Phi + s_B}\right)^2\right].\end{aligned} \tag{85}$$

Since $\delta W'_C$(stochastic) is always negative, the ballooning mode in the stochastic plasma cannot be completely stabilized by the compressibility alone. For $s_\Phi \ll s_B$ the effective compressional term in the stochastic plasma, which is the second term in the bracket of the right-hand side of (85), can be neglected, and the ballooning eigenmode equation (83) under such a condition in the stochastic plasma becomes the same as the ideal incompressible MHD eigenmode equation. This is very reasonable, because the stochastic dynamics are to change the equation of state from the adiabatic equation of state used in the ideal MHD analysis. Since the incompressible treatment does not use the equation of state and instead uses a mechanical equation $\nabla\cdot\delta\mathbf{u} = 0$ for closure of the fluid equations, the incompressible equation should be valid in the appropriate limits considered above irrespective of whether the plasma is adiabatic or stochastic. Notice that there is no condition for plasma β for the validity of the incompressible assumption in the stochastic plasma, although there is a critical β, which is set by the stabilizing tension force. By comparing (70) to (85) it is obvious that for $\beta(0) \gg 6/\Gamma$ the stochastic plasma is less stable than the ideal adiabatic MHD plasma, owing to the appearance of factor 2 in the compressional stabilizing term in (70). This is consistent with the findings of *Hurricane et al.* [1995].

The plasma β exceeds a few hundred at the equator for the field line A in Figure 1 of *Miura et al.* [1989], and therefore the field line A is not an appropriate field model in the near-Earth plasma sheet. However, according to *Lui et al.* [1992], the plasma β as large as ~70 has been observed in the near-Earth plasma sheet before the substorm onset. Therefore the plasma β much larger than 1 may not be a rare case in the near-Earth plasma sheet before the substorm onset. Therefore the present result in section 6 showing that the ideal MHD ballooning mode, which is strongly localized near the equator, is essentially incompressible for $\beta(0) \gg 6/\Gamma$ is valid in the near-Earth plasma sheet before the substorm onset, where such modest high-β is not a rare case. When the plasma in the near-Earth plasma sheet is stochastic, the incompressible assumption is valid irrespective of the β value. For such high-β the kinetic effect may also be important as has been discussed by *Cheng and Lui* [1998] and *Horton et al.* [1999].

Bhattacharjee et al. [1998] showed by an eigenmode analysis that a 2-D magnetotail, obtained by 2-D time-dependent simulations of the magnetotail in the high-Lundquist-number regime, is subject to an ideal compressible ballooning instability (symmetric mode) with high wave number along y. They also showed that the same magnetotail configuration is not subject to the ideal incompressible ballooning instability. *Lee* [1998] also showed that the analytic model equilibrium or the 2-D plasma sheet that includes the Earth's 2-D dipole field is subject to the ideal compressible ballooning instability, and therefore Lee supported the view of Bhattacharjee et al. *Hurricane* [1997] showed that a sheared MHD equilibrium with the azimuthal magnetic field component is subject to the compressible ballooning instability by the shear destabilization. As shown in section 4, the compressibility has always a stabilizing influence on the ballooning instability. Therefore it is not certain why the calculation of Bhattacharjee et al. showed that the same equilibrium is subject to the compressible ballooning instability but is not subject to the incompressible ballooning instability, because if an unstable compressible mode is found by the eigenmode analysis, that mode should become an unstable trial function in the energy principle, in which the compressible stabilizing term is neglected. The reason why Bhattacharjee et al. could not find an incompressible unstable mode may be due to the fact that they used a dynamic equilibrium solution instead of the static equilibrium. The present analysis shows that whether the incompressible assumption is valid or not depends on the background profiles of $|\boldsymbol{\kappa}_b|$ and $|\boldsymbol{\kappa}_c|$ along the field line. Although there are several numerical eigenmode analyses of the ideal MHD ballooning instability [*Miura et al.*, 1989; *Ohtani et al.*, 1989b; *Hurricane*, 1997; *Bhattacharjee et al.*, 1998; *Lee*, 1998, 1999], background profiles of $|\boldsymbol{\kappa}_b|$ and $|\boldsymbol{\kappa}_c|$ are only shown by *Miura et al.* [1989] and *Ohtani et al.* [1989b] in spite of the importance of those equilibrium quantities. It is not likely that the long-thin ordering for the equilibrium is always satisfied for the magnetospheric equilibrium. However, it is quite likely as discussed previously that during the growth phase of the substorm the field line in the near-Earth plasma sheet becomes more taillike and the plasma sheet becomes thinner [*Kaufmann*, 1987], and thus the long-thin ordering for the equilibrium becomes more likely to be satisfied, on average, in the near-Earth plasma sheet, so that the stabilizing compressible effect vanishes.

Although the present analysis is limited to the ideal MHD and stochastic plasmas without consideration of the full kinetic effects, within these limits the present results are useful in studying the linear stability of the long-thin, taillike magnetospheric equilibrium against the ballooning and interchange instabilities including the case where the long-thin assumption for the equilibrium is not valid near the equator. The present results justify the incompressible assumption made in the ballooning stability analysis of *Miura et al.* [1989] for a taillike equilibrium. Furthermore, the present results suggest that when the critical β due to the stabilizing tension force is exceeded, the ballooning instability is a viable instability in the near-Earth plasma sheet, which is strongly localized near the equator and which may become essentially incompressible before the substorm onset. It is also suggested that in the incompressible limit the growth time of the ballooning instability in the taillike equilibrium can become as short as the field line curvature radius at the equator divided by the Alfvén speed.

Acknowledgments. The author would like to thank T. Tamao for insightful comments. This work has been supported by Grant-in-Aid for Scientific Research 09640529 provided by the Ministry of Education, Science, Sports, and Culture.

Hiroshi Matsumoto thanks the referees for their assistance in evaluating this paper.

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(Received October 18, 1999; revised January 25, 2000; accepted February 28, 2000.)