論 文 の 内 容 の 要 旨

論文題目

Geometric Numerical Integration Methods for Energy-Driven Evolution Equations

（エネルギー関数を持つ発展方程式に対する幾何学的数値計算法）

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This thesis is about geometric numerical integration methods for energy-driven evolution equations. Geometric numerical integration methods or structure-preserving numerical methods are rather specific-purpose methods in the sense that they exactly preserve or inherit geometric properties, such as symplecticity and energy-preservation, of differential equations. The main advantage of geometric numerical integration methods is that in many cases we can expect qualitatively better numerical solutions, especially over a long period of time, than with general-purpose methods. This thesis consists of two parts.

In the first part, we consider ordinary differential equations, especially Hamiltonian systems with emphasis on their energy-preservation property. It is a natural idea to consider numerical methods which exactly inherit the property. However, the study on energy-preserving methods has a shorter history than that on other geometric integration methods such as symplectic methods. The main reason is that no Runge-Kutta method is energy-preserving and thus we have to develop energy-preserving methods in another framework. The biggest contribution of the first part is to give an algebraic characterisation of so called continuous stage Runge-Kutta methods being energy-preserving. Moreover, from a practical point of view, we construct several efficient energy-preserving methods by using the characterisation.

In the second part, we consider partial differential equations. For partial differential equations, special care must be taken for space discretisation as well as time discretisation. It is of interest to extend several existing structure-preserving numerical methods, which have been developed only on uniform meshes, to nonuniform meshes. In a finite element context, we propose a general framework for constructing energy-preserving or dissipative integrators, and further extend this framework to discontinuous Galerkin methods. We also develop theory on energy-preserving/dissipative methods on moving meshes. Furthermore, we study the treatment of nonlocal equations, tanking the Hunter-Saxton equation as our working example.