Phase Transition in a Foreign Exchange Market—Analysis Based on an Artificial Market Approach

Kiyoshi Izumi and Kazuhiro Ueda

***Abstract*—In this study, we propose an artificial market approach, which is a new agent-based approach to foreign exchange market studies. Using this approach, emergent phenomena of markets such as the peaked and fat-tailed distribution of rate changes were explained. First, we collected the field data through interviews and questionnaires with dealers and found that the features of dealer interaction in learning were similar to the features of genetic operations in biology. Second, we constructed an artificial market model using a genetic algorithm. Our model was a multiagent system with agents having internal representations about market situations. Finally, we carried out computer simulations with our model using the actual data series of economic fundamentals and political news. We then identified three emergent phenomena of the market. As a result, we concluded that these emergent phenomena could be explained by the phase transition of forecast variety, which is due to the interaction of agent forecasts and the demand-supply balance. In addition, the results of simulation were compared with the field data. The field data supported the simulation results. This approach therefore integrates fieldwork and a multiagent model, and provides a quantitative explanation of micro–macro relations in markets.**

***Index Terms*—Artificial markets, foreign exchange markets, genetic algorithms, micro–macro problems, multiagent system.**

I. Introduction

RECENTLY, large economic changes have brought to our attention the behavioral aspects of economic phenomena. One example is that large fluctuations in exchange rates are said to be mainly caused by bandwagon expectations[1] [1]. This fact shows that an exchange market has the features of multiagent systems.

1) *Autonomous Agents:* Each dealer makes a decision based on his own trading rules and information.
2) *Interaction:* Each dealer learns the market situation by interacting with each other.
3) *Emergence:* there are emergent phenomena such as rate bubbles at the upper (market) level, which are not directly designed at the lower (agent) level.

These features are related to the micro–macro problem in economics. Agents in economic systems are interacting with each other and there are complex relations between the *micro* behavior of agents and the *macro* behavior of whole systems. In complex economic systems, agents should be *adaptive* to the change of whole systems: they must always change their own models of economic systems in order to improve their prediction.

Most conventional market models in economics, however, ignore the multiagent features by assuming a rational expectations hypothesis (REH). REH assumes that all agents are homogeneous and forbids essential differences of agents' forecasts. Recently, this assumption has been criticized and the multiagent features have been said to be important for analysis of the micro–macro relation in markets [2], [3].

Among several alternative approaches, there are *multiagent* models of markets on the agent-based approach [4]–[7]. They make market models with artificial adaptive agents and conduct computer simulations. Then the studies analyze the evolution of models and use the results of the analysis to understand the actual markets.

There are, however, two problems in the multiagent models constructed up to now. First, their agents do not have explicitly internal representations about market situations. Instead, the agents were described by condition-action rules or parameters standing for the amount of assets. So these models could not examine the relationship between the change of agents' opinions or cognition about markets and the macro behavior of markets in detail. Second, they do not use actual data series about economic fundamentals and political news. Instead, they used artificial data generated by stochastic processes such as auto-regressive processes. They can not, therefore, directly investigate the actual exchange rate dynamics.

The purpose of this study is to propose a new agent-based approach of foreign exchange market studies, an *artificial market approach*. This approach integrates fieldwork and multiagent models in order to provide a quantitative explanation of the micro and macro relation in markets.

II. Framework of the Artificial Market Approach

The artificial market approach consists of three steps (Fig. 1).

1) *Observation in the Field:* First, the field data were gathered by interviewing an actual dealer and questionnaires, as described in the Section III. Then, we investigate learning patterns and interaction patterns of the dealers. As a result of analysis, hypotheses are proposed about the dealers' behavioral pattern: decision rules, learning rules, and interaction pattern.

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[1]The word "bandwagon" here means that many people join others in doing something fashionable or likely to be successful. That is, many agents (or participants) in a market ride along with the recent trend.

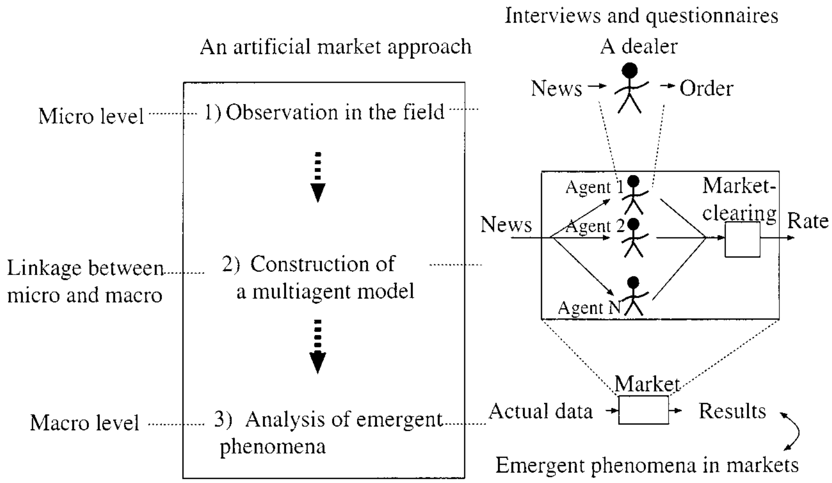

Fig. 1. Framework of artificial market approach.

2) *Construction of a Multiagent Model:* Second, a multiagent model was implemented based on our hypotheses, as described in Section IV. Artificial markets are virtual markets operating on computers. They consist of computer programs as virtual dealers, a market-clearing mechanism, and rate determination rules. The model provides the linkage between the behavioral pattern of agents at the micro level and the rate dynamics at the macro level.
3) *Analysis of Emergent Phenomena:* Finally, we conducted simulations using actual data of economic fundamentals in order to evaluate the model in Section V. Based on the simulation results, we verified whether the model could explain three emergent phenomena of markets in the real world, the peaked and fat-tailed distributions of rate changes, the negative correlation between trading amounts and rate fluctuations, and the contrary opinions phenomena. An emergent phenomena here means the rate dynamic patterns, which are not directly designed by each dealer, but appear at the market level. In addition, the results of computer simulations were compared with the field data.

This approach has two advantages over previous studies. First, agents in our model have explicit representations about market situations. Thus, the simulation results about change of agents' opinions can be compared directly with the field data gathered by the interview and questionnaires. Second, our model used actual data about economic fundamentals and news in the simulation. Thus, we can verify directly whether the model can replicate the emergent phenomena of rate dynamics in the real world. These advantages enable our model to be bound to the real world data at both micro and macro levels.

III. Observation in the Field

In this section, we observed the actual dealers' behavior by using interviews and questionnaires. Based on these field data, we propose a hypothesis of dealers' learning. This hypothesis is also used in the construction of a multiagent model as a rule of agents' interaction and learning.

We first observed temporal changes of dealers' forecast rules by interviews and extracted several features of dealers' learning in the Section III-A. Then, the features were verified using questionnaires data in the Section III-B. Finally, based on the results of the fieldwork, we pointed out several similarities between the features of dealers' learning and genetic operations in biology in the Section III-C.

A. Interviews—Features of Learning

We held interviews with two dealers who usually engaged in yen–dollar exchange transactions in Tokyo foreign exchange market. The first dealer (X) is a chief dealer in a bank. The second dealer (Y) is an interbank dealer in the same bank. They have more than two years of experience on the trading desk.

The interviewees were asked to explain the rate dynamics of the two years from January 1994 to November 1995, when the interview took place. Concretely, we asked each dealer to do the following things.

1) Divide these two years into several periods according to his recognition of market situations.
2) Talk about which factors he regarded as important in his rate forecasts for each period.
3) Rank the factors in order of weight (importance) and explain the reason for the ranking.
4) In case his forecast factors changed between periods, describe the reasons for the reconsideration.

The division of the two years and the ranking of factors are shown in Tables I and II. From the interview data of the two dealers, we found three basic features in the acquisition of prediction methods in the market.

1) *Market Consensus:* Fashions of interpretation of factors in markets. For example, the weight of the trade balance factor was not constant, although there are always the large trade surpluses of Japan throughout these two years. The dealers said that it was because they were sensitive not to the value of economic index, but to the market consensus.
2) *Communication and Imitation:* The dealers communicated with other dealers to infer a new market consensus

TABLE I
RESULTS OF THE INTERVIEWS WITH DEALER X

1994	I Jan	II Feb-Jun	III Jul-Oct	IV Nov-Dec
Actual	→	↘	→	→
Forecast	→	↘	→	→
Ranking of factors	1.Value of Mark 2.Seasonal factors*1	1.Chart trends 2.Trade 3.Politics	1.Chart trends 2.Deviation 3.Politics	1.Seasonal factors*1

1995	V Jan	VI Feb-Apr	VII May-Jul	VIII Aug-Sep	IX Oct-Dec
Actual	↗	↘	↗	↗	→
Forecast	↗	↘	↘	↗	→
Ranking of factors	1.Seasonal factors*1	1.Trade 2.Politics 3.Mexico 4.Chart trends		1.Deviation*2 2.Intervention	

Division into periods was determined by the dealers. Actual trends in reates and each dealer's forecast for each period are shown in terms of the three basic kinds of trends (downward, sideways, and upward). Factors in the forecast are listed and ranked in order of importance.
*1: Dealer said that rates did not move at the beginning and end of the year.
*2: Dealer forecasted that rates would return to the previous level after large deviation.

TABLE II
RESULTS OF THE INTERVIEW WITH DEALER Y

1994	I Jan-May	II Jun	III Jul-Dec
Actual	↘	↘	→
Forecast	↘	→	→
Ranking of factors	1. Trade 1. Order*1 3. Chart trends	1. Rate level	1.Order*1 2. Chart trends

1995	VI Jan-Feb	V Mar-Apr	VI May-Jul	VII Aug-Dec
Actual	↘	↘	→	↗
Forecast	↘	↘	→	↗
Ranking of factors	1. Politics 2. Value of Mark 2. Announcement	1. Politics 1. Order*1 1. Intervention	1. Chart trends 2.Order*1	1. Intervention 2. Politics

Forecast factors are ranked in order of importance.
*1: Directions and the amount of orders that the dealer received from other dealers or customers.

in order to get information on which factors were regarded important and then replaced a (part of) their prediction method with that of other dealers which better explained recent rate dynamics, when switching prediction method.

3) *Learning Promoted by Forecast Errors:* When the forecast of the interviewee was quite different from the actual rate, he recognized the need to change his weights. For example, in the period VII of the dealer X, when the rate reached the level of 92 yen, he suddenly recognized that the trend had changed. He then discarded his old opinions about factors and adopted new opinions.

From the above features, we propose the following hypothesis at the micro level in markets. When the forecasts based on a dealer's own opinion markedly differs from the actual rates, each dealer replaces (part of) their opinions about factors with other dealers' successful opinions.

B. Questionnaires—Verification of Features

If the hypothesis in the Section III-A is true, the frequency of successful weights in a market must be larger after the trend changed. Thus, the following proposition holds: *the market average of each factor's importance must shift to the value of successful weights.*

In order to verify this proposition, we undertook a questionnaire to 12 dealers in March 1997. The questionnaires are undertaken just after the market trends changed from the upward trend to the downward trend for dollars in March 1997. All 12 answerers are dealers who usually deal with exchange transactions in a bank.

Each dealer i was asked the following three questions about 22 factors that affected exchange rates.

1) Write the importance of each factor k in the previous trend (the upward trend of the dollar) in 11 discrete values from 0 to 10: $w_i^k(t)$.
2) Write the importance of each factor k in the recent trend (the downward trend of the dollar) in 11 discrete values from 0 to 10: $w_i^k(t+1)$.
3) Write forecasts made before the trend changed: $\tilde{R}_i$.

The 22 factors are: 1) economic activities; 2) price indexes; 3) short-term interest rates; 4) money supply; 5) trade balance; 6) employment prospects; 7) personal consumption; 8) intervention; 9) mark–dollar rates; 10) commodity markets; 11) stock prices; 12) bonds prices; 13) short-term chart trends (under one week); 14) long-term chart trends (over one month); 15) exchange rate policy of the Bank of Japan; 16) exchange rate policy of the Federal Reserve Bank; 17) trading by export and import firms; 18) trading by insurance firms; 19) trading by securities firms; 20) trading by other banks; 21) trading by foreign investors; and 22) the other factors.

The proposition implies that the market averages of each factor's importance change toward the averages that are weighted with forecast accuracy of the factor's weights. As mentioned in Section III-A, the interview data suggest that the factors' importance which can forecast more accurately have lager frequency after dealers change their opinions. Hence, if the proposition is true, the market averages of each factors' importance must change to the averages which are weighted with their forecast accuracy.

We calculated the market averages $\bar{W}^k$ of the importance about the each factor k both in the previous (t) and recent $(t+1)$ trend

$$\bar{W}^k(s) = \frac{1}{n}\sum_{i=1}^{n} w_i^k(s) \tag{1}$$

where n stands for the number of dealers, 12, and $s = \{t, t+1\}$.

Then, weighted averages of the importance about the each factor k in the previous trend t were calculated. The weight of dealer i's importance is defined using the dealer i's forecast error

$$e_i = |\tilde{R}_i - R| \tag{2}$$

TABLE III
CORRELATION BETWEEN DIFFERENCES

Number of samples	Correlation	Significance level
22	0.284	P < 0.1

Number of samples are the number of factors.

where $\tilde{R}_i$ is the forecast rate of the dealer i and R is the actual rate value. The weight of dealer i's importance f_i is inversely proportion to the forecast error

$$f_i = \frac{E - e_i + 1}{\sum_{j=1}^{n}(E - e_j + 1)} \tag{3}$$

where E is the maximum value of forecast error among 12 dealers. The weight f_i is defined using a difference between the maximum value of forecast error and each dealer's forecast error. Thus, the dealer with a small forecast error has a larger weight value of importance and vice versa, i.e., the weight f_i reflects the forecast accuracy of dealer i. The numerator is added (one) so that all dealers' importance can have nonzero contribution to the weighted average. The denominator is necessary because the sum of weights f_i must be one.

The weighted average of each factor k is calculated as the following:

$$\bar{W}^k_{\mathrm{weighted}}(t) = \frac{1}{n}\sum_{i=1}^{n} f_i w_i^k(t). \tag{4}$$

If the proportion is true, the market average after the trend change $\bar{W}^k(t+1)$ must be nearer the weighted average $\bar{W}^k_{\mathrm{weighted}}(t)$ from the market average before the trend change $\bar{W}^k(t)$. Thus, the two differences, the differences between $\bar{W}^k(t+1)$ and $\bar{W}^k(t)$, and the differences between $\bar{W}^k_{\mathrm{weighted}}(t)$ and $\bar{W}^k(t)$ must have positive correlation

$$\bar{W}^k(t+1) - \bar{W}^k(t) \propto \bar{W}^k_{\mathrm{weighted}}(t) - \bar{W}^k(t). \tag{5}$$

Using the questionnaire data, we tested the above correlation and verified the proposition. As a result, there were positive correlations between the two differences (Table III). Namely, successful opinions that can forecast more accurately are considered to spread in the market.

In summary, the hypothesis implies that the learning pattern of actual dealers is similar to the adaptation in an ecosystem. In our multiagent model, the adaptation of agents in the market will be described with a genetic algorithm (GA), which is based on ideas of population genetics.

C. Similarities to Genetic Operations

We introduce heterogeneity of market participants' prediction methods into our foreign exchange market model, unlike conventional econometric models. In many conventional models, market participants are assumed to compute the rational expectation solution. Under the heterogeneity, however, all market participants must know all the other market participants' prediction methods in order to compute the rational expectation solution. This is impossible in the real world. Instead of this unrealistic assumption, we assume that each market participant can know the prediction methods of other participants that he or she communicated. Then he or she may adopt other market participants' prediction methods. In order to describe such a learning process of market participants in our model, we use an analogy from biology in view of the field data.

When a prediction method of a dealer is regarded as an individual in a biological framework, several similarities between the features of interaction of dealers' forecast and genetic operations in biology can be found.

First, the imitation behavior is similar to selection operation in biology. Individuals in biological populations propagate according to their fitness, meaning that fit individuals will thrive and unfit individuals will become extinct. This is called "selection" in biology. Similarly, successful prediction methods spread over the market as market consensus, but unsuccessful prediction methods disappear.

Second, the forecast accuracy, the difference between the forecast and the actual rate, can be considered to correspond to "fitness" in a biological framework.

Finally, the communication behavior of dealers corresponds to "crossover." In biological reproduction, part of each individual's chromosomes may be exchanged for part of another individual's chromosome.

Given the similarities between the features of interaction of dealers' forecast and genetic operations, we used a GA to describe agents' learning in our artificial market model, where a GA is a computer algorithm that models the genetic operations based on population biology.

IV. CONSTRUCTION OF A MULTIAGENT MODEL

This section describes the construction of an artificial market: a multiagent model of a foreign exchange-rate market. Then the simulation method using the model and overview of the simulation results are explained. The name of the model is A GEnetic-algorithmic Double Auction Simulation in TOkyo Foreign exchange market[2] (AGEDASI TOF).[3]

AGEDASI TOF is an artificial market with 100 agents, as illustrated in Fig. 2. Each agent is a virtual dealer, which has dollar and yen assets and changes positions in the currencies for the purpose of making profits. Every week consists of the five steps.

1) *Perception:* Each dealer perceives forecast factors from weekly data on the Tokyo foreign exchange market.
2) *Prediction:* Dealers predict the future rate.
3) *Strategy-Making:* Dealers determine the trading strategy every week.
4) *Rate Determination:* The equilibrium rate is determined from the supply and demand in the market
5) *Adaptation:* Each agent improves his or her prediction method by learning from the other agents.

A. Step 1—Perception

Each agent first interprets raw data and perceives news about factors affecting the yen–dollar rate. We assume that all agents interpret raw data in the same way.

[2]The source code of AGEDASI TOF is available online at http://www.carc.aist.go.jp.

[3]AGEDASI TOF is the name of a Japanese dish, fried tofu. It is very delicious.

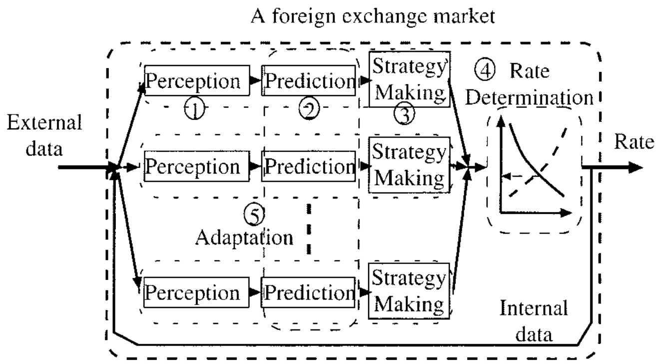

Fig. 2. Framework of the model.

$x^k(t)$ is defined as data that are made by interpreting raw data k between the end of week $t-1$ and the beginning of week t or news data about data k in this week. In the present study, it is assumed that all agents interpret raw data in the same way. Thus, the results of interpretation, the data $x^k(t)$'s, are the same for all agents.

The data $x^k(t)$ were made by coding from a newspaper article about weekly change of 17 raw data in Table IV. Those values range discretely from -3 to $+3$. Plus values indicate that the data change causes dollar appreciation according to the traditional economic theories.[4] Minus values indicate dollar depreciation. The absolute value of coding data was determined by the expression in the article. For example, a comment "Gross Domestic Product of United States *increased largely*" is interpreted as news for dollar appreciation by the traditional economic theories and this comment is coded as "Economic activities: $+3$." In contrast, a comment "Gross Domestic Product of United States *decreased largely*" is interpreted as news for dollar depreciation by the traditional economic theories and this comment is coded as "Economic activities: -3." In the same way, a comment "The Governor of the Bank of Japan announced admission of *a little more dollar appreciation*" is coded as "Announcement: $+1$" and a comment "The Governor of the Bank of Japan announced admission of *a little more dollar depreciation*" is coded as "Announcement: -1."

External data are defined as the data of economic fundamentals or political news (1–14 in Table IV) because they are data of the events in the real world. Internal data are defined as data of short-term or long-term trends of the chart (15–17 in Table IV) because they are calculated using the rate that the model made in the simulation.

B. Step 2—Prediction

After perception, using the above data, each agent predicts the future change of the rate.

Each agent has her own weights of the 17 data. $w_i^k(t)$ is defined as a weight of each datum k in each agent i's prediction of the future rate at week t or agent i's importance about news k. The value of $w_i^k(t)$ ranges among nine discrete values $\{\pm 3, \pm 1, \pm 0.5, \pm 0.1, 0\}$.

With her own importance, each agent predicts change of logarithms of the rates.

$S(t)$ A logarithm of the exchange rate at week t.

$\Delta S(t)$ Change of logarithms of the rates $S(t) - S(t-1)$.

It is assumed that each agent i predicts $\Delta S(t)$ based on the summation of products of the data $x^k(t)$ and her importance $w_i^k(t)$. For simplicity, this summation was truncated to an integer by a truncation function, trunc(). Each agent makes the prediction value $\mathbf{E}_i[\Delta S(t)]$ as the following:

$$\mathbf{E}_i[\Delta S(t)] \equiv \alpha \cdot \operatorname{trunc}\left(\sum_{i=1}^{n} w_i^k(t) x^k(t)\right) \tag{6}$$

where n stands for the number of the data, 17, and α is a scaling coefficient. The scaling coefficient α is 0.02. This value was determined using the ratio of the news data's average to the rate fluctuation's average.

The variance of agent i's forecast is calculated from the difference between the stronger yen factors and the weaker yen factors, as the following[5]:

$$\mathbf{Var}_i[\Delta S(t)] \equiv \frac{1}{\sqrt{|(wx_+)^2 - (wx_-)^2|}} \tag{7}$$

where wx_+ denotes the summation of $w_i^k(t)x^k(t) > 0$ and wx_- the summation of $w_i^k(t)x^k(t) < 0$. The wx_+ means the summation of effects of stronger yen (weaker dollar) factors and the wx_- means the summation of effects of weaker yen (stronger dollar) factors. The variance is inversely proportional to the coherence of forecast factors. Hence, the larger the variance, the lower the confidence of the forecast and vice versa.

For example, suppose news data for this week are (interest: $+2$, trade: -1, stock: -2, trend: $+2$) and the agent i's importance are (interest: $+0.1$, trade: -1.0, stock: $+0.1$, trend: $+3.0$). The mean of her forecast is the weighted average of the news data for this week, calculated as the following:

$$\begin{aligned}\mathbf{E}_i[\Delta S(t)] =& \operatorname{trunc}\{(+2)\times(+0.1)+(-1)\times(-1.0)\\ &+(-2)\times(+0.1)+(+2)\times(+3.0)\}\times 0.02\\ =& +7.0\times 0.02 = +0.14\end{aligned}$$

i.e., if the logarithm of the yen–dollar rate for the last week was $\log(125\text{yen}) = 4.82$, the agent forecasts that the rate will rise to $4.82 + 0.14 = 4.96 = \log(144\text{yen})$. The variance of her forecasts is calculated as the equation at the bottom of the next page.

C. Step 3—Strategy Making

Each agent has the dollar asset and the yen asset. Each agent decides, on the basis of her own prediction, her trading strategy (in order to buy or sell dollars). She maximizes her utility function of expected return of the next week. The strategy making process of the proposed model is common to the conventional portfolio balance model in econometrics.

Let us define the following variables about an agent i.

[4] We used purchasing power policy theory, the price-monetary model, and the portfolio balance model [8]–[10].

[5] If the denominator of (7), wx_+ and wx_- are equal, and the prediction value in the (6) is zero. In this case, the optimal position in the (11), calculated from the (10), is zero, whatever the variance is.

TABLE IV
INPUT DATA

	Data ($x_i^k(t)$)	Raw Data
1	Economic activities	Gross Domestic Product(US +, JP −), Industrial Production Index(US +, JP −), NAPM index(US +, JP −), Diffusion Indexes(US +, JP −).
2	Price	Consumer Price Index(US −, JP +), Producer Price Index(US −, JP +).
3	Interest rates	Official rate(US +, JP −), Fed Funds Rate(US +), Prime rate(US +).
4	Money supply	Money supply M1(US −, JP +), M2(US −, JP +), M3(US −, JP +).
5	Trade balance	The trade balance(US +, JP −), Balance of payments(US +, JP −).
6	Employment	Unemployment Rate(US −, JP +), Nonfarm Payrolls(US +).
7	Personal consumption	Retail sales(US +, JP −), Personal Income(US +, JP −).
8	Intervention	Buying dollar intervention(+), Selling dollar intervention(−)
9	Announcement	Announcement about stronger dollar(+), Announcement about stronger yen(−).
10	Mark	The dollar-mark(+), yen-mark rate(−).
11	Oil	Oil price(+).
12	Politics	Domestic political problems(US −, JP +), International conflict(+).
13	Stock	Nikkei225(JP −), Dow Jones(US +).
14	Bond	Treasury Bill(US +), Treasury Bond(US +), Government bond(JP −).
15	Short-term Trend 1	Change in the last week(+).
16	Short-term Trend 2	Change of short-term trend 1(+).
17	Long-term Trend	Change through five weeks(+).

US means indexes of the United States. JP means indexes of Japan.
(+) means that increase of data leads to a stronger dollar. (−) means that increase of data leads to a stronger yen.

$q_i(t)$ Amount of the dollar asset of the agent i at week t in terms of dollars.

$Q_i(t)$ Amount of whole assets (the dollar and yen assets) of the agent i at this week t in terms of yen.

$\tilde{S}_i(t) \equiv S(t-1)+\Delta S(t)$ Agent i's forecast of logarithm of yen–dollar exchange rate at the week t.

The expected return of agent i in terms of yen $\tilde{P}_i(t)$ is calculated as the following:

$$\begin{aligned}\tilde{P}_i(t) =& \frac{\{\exp(\tilde{S}_i(t)) - \exp(S(t-1))\}}{\exp(S(t-1))} q_i(t)\\ =& \{\exp(\Delta S(t)) - 1\} q_i(t)\\ \approx& \Delta S(t) q_i(t). \end{aligned} \tag{8}$$

In AGEDASI TOF, utilities of all agents $U(\tilde{P}_i(t))$ are assumed to be the same

$$U(\tilde{P}_i(t)) \equiv -\exp(-a\tilde{P}_i(t))$$

where $a > 0$ denotes risk aversion in economics. When $\tilde{P}_i(t)$ has the normal distribution $N(E[\tilde{P}_i(t)], \mathrm{Var}[\tilde{P}_i(t)])$, the logarithm of the expected utility is the following[6]:

$$\ln(\mathbf{E}[U(\tilde{P}_i(t))]) = \mathbf{E}[\tilde{P}_i(t)] - \frac{1}{2} a \mathbf{Var}[\tilde{P}_i(t)]. \tag{9}$$

Substituting the (8) into the (9), the logarithm of the expected utility is calculated as the following:

$$\begin{aligned}\ln(\mathbf{E}[U(\tilde{P}_i(t))]) =& \mathbf{E}_i[\Delta S(t)] q_i(t)\\ &- \frac{1}{2} a \mathbf{Var}_i[\Delta S(t)](q_i(t))^2. \end{aligned} \tag{10}$$

The first term of the (10) is the expected return and the second term is the risk (variance) of the position. Therefore, this equation implies that each agent tries to increase returns and reduce risk.

Each agent is assumed to divide her whole assets between the dollar asset and the yen asset with the optimal ratio which maximizes the (10). The optimal position of her dollar asset $q_i^*(t)$ is the following:

$$q_i^*(t) = \frac{1}{a} \frac{\mathbf{E}_i[\Delta S(t)]}{\mathbf{Var}_i[\Delta S(t)]}. \tag{11}$$

In order to coincide her holding position with the optimal position, each agent orders the same quantity as the difference between the optimal position $q_i^*(t)$ and the previous holding position $q_i(t-1)$

$$\text{order quantity} : \Delta q_i^*(t) \equiv q_i^*(t) - q_i(t-1). \tag{12}$$

[6]This calculation result is obtained by Taylor extension.

$$\begin{aligned}\mathbf{Var}_i[\Delta S(t)] =& \frac{1}{\sqrt{\{2\times(+0.1)+(-1)\times(-1.0)+2\times 3.0\}^2 - \{(-2)\times 0.1\}^2}}\\ =& 0.161\end{aligned}$$

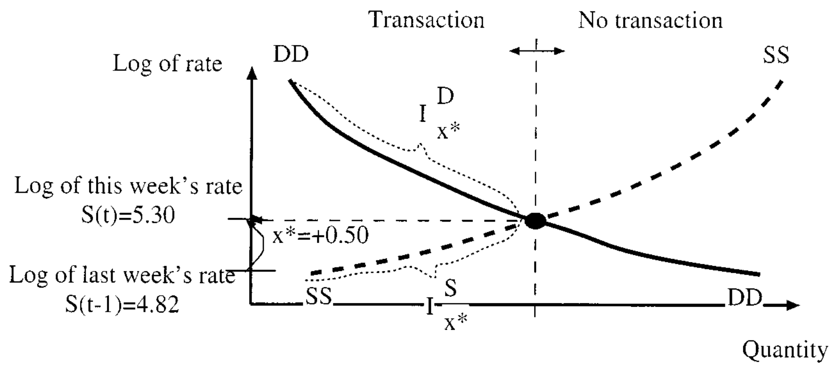

Fig. 3. Determination of rate.

If $\Delta q_t^{j*} > 0$, then the agent orders to buy dollars, i.e., submits a bid. This week's rate $S(t)$ is not determined until the rate-determination step and each agent forecasts that $S(t)$ will be her predicted value $S(t-1) + \mathbf{E}_i[\Delta S(t)]$ as a result of trading in this step. If the trading rate is lower than her predicted rate, each buyer is willing to buy the optimal order quantity $\Delta q_i^*(t)$ of dollars. If the trading rate is higher than her predicted rate, she forecasts that this trading will make a loss, thus she does not take any action. To sum up, each buyer has the following strategy. When $\Delta q_t^{j*} > 0$

$$\begin{cases} \text{buy dollar } \Delta q_i^*(t), & \text{if rate} \le S(t-1) + \mathbf{E}_i[\Delta S(t)] \\ \text{no action}, & \text{if rate} > S(t-1) + \mathbf{E}_i[\Delta S(t)] \end{cases}. \tag{13}$$

If $\Delta q_t^{j*} < 0$, then the agent orders to sell dollars, i.e., submits an ask. In the same way as buyers, if the trading rate is higher than her predicted rate, each seller are willing to sell the optimal order quantity $\Delta q_i^*(t)$ of dollars. If the trading rate is lower than her predicted rate, she forecasts that this trading will make a loss, thus she does not take any action. To sum up, each seller has the following strategy. When $\Delta q_t^{j*} < 0$

$$\begin{cases} \text{no action}, & \text{if rate} < S(t-1) + \mathbf{E}_i[\Delta S(t)] \\ \text{sell dollar } \Delta q_i^*(t), & \text{if rate} \ge S(t-1) + \mathbf{E}_i[\Delta S(t)] \end{cases}. \tag{14}$$

For example, the optimal amount of the dollar assets for the above-mentioned agent i is given by

$$q_i^* = \frac{+0.14}{0.161} = +0.87.$$

If the agent i's previous position of the dollar asset $q_i(t-1)$ is -0.74, she will want to buy dollars in order to optimize the dollar asset as the following:

$$\text{order quantity } \Delta q_i^*(t) = +0.87 - (-0.74) = +1.61.$$

She places an order to buy when the current rate is lower than her expected rate $4.82 + 0.14 = 4.96$

$$\text{trading strategy} = \begin{cases} \text{buy dollar } 1.61, & \text{if rate} \le 4.96 \\ \text{no action}, & \text{if rate} > 4.96 \end{cases}.$$

D. Step 4—Rate Determination

After the submission of orders, the demand (respectively, supply) curve is made by the aggregation of orders of all agents who want to buy (respectively, sell). The demand and supply then determine the equilibrium rate, where the quantity of demand and that of supply are equal (Fig. 3). The rate in this week $S(t)$ is the equilibrium rate.

The demand curve $\mathbf{DD}_t(x)$ is made by aggregation of the whole bids $(\Delta q_i^*(t) > 0)$ of agents having higher order rates than x

$$\mathbf{DD}_t(x) = \sum_{i \in I_x^D} \Delta q_i^*(t)$$
$$\left(I_x^D \equiv \{i: \Delta q_i^*(t) > 0 \text{ and } \mathbf{E}_i[\Delta S(t)] \ge x\}\right). \tag{15}$$

The supply curve $\mathbf{SS}_t(x)$ is made by aggregation of the whole asks $(\Delta q_i^*(t) < 0)$ of agents having lower order rates than x

$$\mathbf{SS}_t(x) = -\sum_{i \in I_x^S} \Delta q_i^*(t),$$
$$\left(I_x^S \equiv \{i: \Delta q_i^*(t) < 0 \text{ and } \mathbf{E}_i[\Delta S(t)] \le x\}\right). \tag{16}$$

The exchange rate of the artificial market is decided to the equilibrium rate, where quantity of demand and that of supply are equal

$$S(t) = S(t-1) + x^*$$
$$(\mathbf{DD}_t(x^*) = \mathbf{SS}_t(x^*)). \tag{17}$$

Buyers (Sellers) with higher (lower) order rates can execute their exchanges and coincide their holding position q_i^t with the optimal position q_i^*. However, the other agents cannot execute their exchanges and $q_i(t)$ remains the previous holding position $q_i(t-1)$

$$q_i(t) = \begin{cases} q_i^*, & \text{if } i \in I_{x^*}^S \text{ or } I_{x^*}^D \\ q_i(t-1), & \text{otherwise} \end{cases}. \tag{18}$$

E. Step 5—Adaptation

After the rate determination, each agent improves her prediction method (combinations of the weights $w_i^k(t)$) by referring to other agents' prediction methods. Our model uses GAs to describe the learning interaction between agents.

As shown by its name, the fundamental ideas of GA come from population genetics. In GA, the frequencies of the chromosomes in a population and the values of the chromosomes

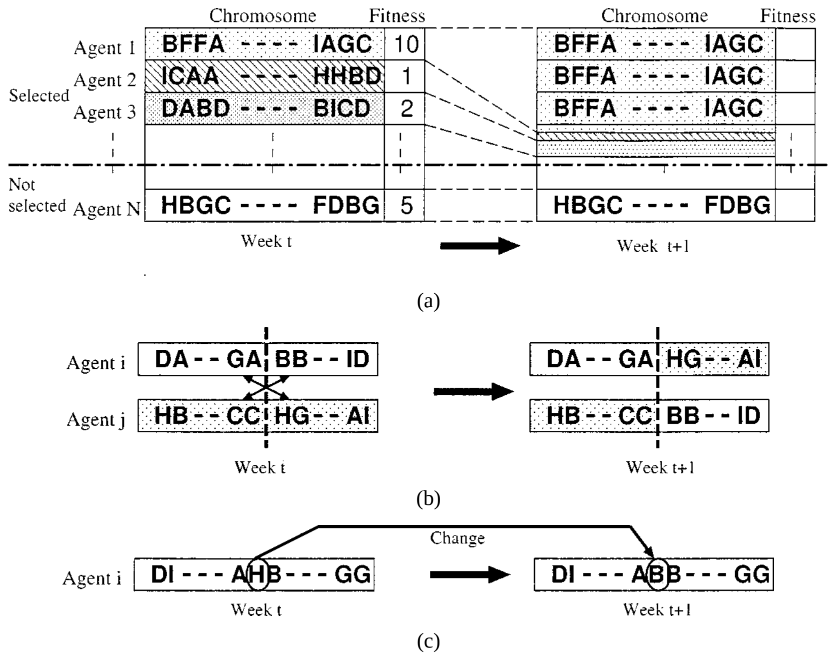

Fig. 4. Genetic algorithm. (a) Selection. (b) Crossover. (c) Mutation.

are changed with three operations: selection, crossover, and mutation. With selection, each chromosome in the population can reproduce its copies at a possibility proportionate to its fitness. Then, a frequency of a chromosome with high fitness value increases and a frequency of a chromosome with low fitness value decreases in the next generation. The crossover operator generates new chromosomes by recombining the pair of the existing chromosomes. The mutation operator generates new ones by randomly changing the value of a position within chromosomes.

When our model uses a GA in the adaptation step, a gene represents a symbol which is made by transformation of a weight $w_i^k(t)$. Each weight $w_i^k(t)$ is transformed as the equation at the bottom of the page. A chromosome of each agent i represents a string of all weights about the 17 data in the Table IV, i.e., her prediction method

$$\text{chromosome } \mathbf{w}_i(t) = \left(w_i^1(t), w_i^2(t), \ldots, w_i^{17}(t)\right).$$

For example, a set of weights $\mathbf{w}_i(t) = (+0.1, -3, 0, +1, \ldots, +0.5)$ becomes a chromosome $\mathbf{DIEB} \cdots \mathbf{C}$. A population of chromosomes represents a set of $\mathbf{w}_i(t)$ in the foreign exchange market. Each chromosome can be regarded as an agent's belief system about the exchange rate, i.e., it represents which data are regarded as the important causes of the rate change. It must be noted that the belief systems can differ among agents.

In our model, the fitness value of each chromosome $F(\mathbf{w}_i(t))$ is calculated using the difference between the predicted rate change $\mathbf{E}_i[\Delta S(t)]$ and the actual rate change $\Delta S(t) = x^*$

$$\begin{aligned} \text{fitness of } \mathbf{w}_i(t) : F(\mathbf{w}_i(t)) &= -|\mathbf{E}_i[\Delta S(t)] - x^*| \\ &= -\left|\alpha \cdot \operatorname{trunc}\left(\sum_{i=1}^{n} w_i^k(t) x^k(t)\right) - \Delta S(t)\right|. \end{aligned} \quad (19)$$

Hence, the more precisely a chromosome predicts the rate, the higher its fitness value.

In order to improve her prediction, each agent changes her own belief system with three operators in Goldberg's simple GA [11]: selection, crossover, and mutation (Fig. 4).

Selection: The selection operator replaces some chromosomes with others that have higher fitness values [see Fig. 4(a)]. This percentage of selection is called a *generation gap* G. The chromosomes that are randomly selected with probability G are replaced the other chromosomes that are randomly selected with probability proportional to their fitness value $F(\mathbf{w}_j(t))$

$$\mathbf{W}_i(t+1) = \mathbf{W}_j(t), \quad \begin{pmatrix} i: \text{selected with probability } G, \\ j: \text{selected with probability proportional to } F(\mathbf{w}_j(t)) \end{pmatrix}. \quad (20)$$

The selection operator is regarded as the imitation of other agent's belief system that can predict the rate change more precisely. Therefore, belief systems predicting less precisely

$$w_i^k(t) = \begin{pmatrix} +3 & +1 & +0.5 & +0.1 & 0 & -0.1 & -0.5 & -1 & -3 \\ \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow \\ \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \mathbf{E} & \mathbf{F} & \mathbf{G} & \mathbf{H} & \mathbf{I} \end{pmatrix}$$

TABLE V
DATA SETS OF SIMULATION

	Training period	Forecast period
Data set 1	1986 - 1987	1988 - 1993
Data set 2	1992 - 1993	1994 - 1995
Kinds of input data	1) External data 2) Internal data 3) Actual rate data	1) External data

TABLE VI
OVERVIEW OF RESULTS

	Data set 1 ('88-'93)	Data set 2 ('94-'95)
Strong dollar	0	17
Strong dollar bubble	**42**	1
Sideway	30	48
Strong yen bubble	11	**25**
Strong yen	17	9

Groups that replicated the actual rate path are shown in bold numbers.

disappear from the market. Namely, it is regarded as the propagation of successful prediction methods.

Crossover: A pair of agents sometimes exchanges parts of their weights for news data [see Fig. 4(b)]. We use the usual single-point crossover, uniform crossover, and the crossover operation that occurs at a certain rate (*crossover rate, pcross*)

$$\begin{aligned}\mathbf{W}_i(t+1) &= \left(w_i^1(t), \ldots, w_i^k(t), w_j^{k+1}(t)+1, \ldots, w_j^{17}(t)\right)\\ \mathbf{W}_j(t+1) &= \left(w_j^1(t), \ldots, w_j^k(t), w_i^{k+1}(t)+1, \ldots, w_i^{17}(t)\right)\\ &\quad \begin{pmatrix} i,j: \text{selected with probability } pcross, \\ k: \text{randomly selected from } \{1,\ldots,17\}. \end{pmatrix}.\end{aligned} \tag{21}$$

The crossover operator works like the agent's communication with other agents and it creates new combination of weights.

Mutation: Each agent has a small probability of changing the weight used for news [see Fig. 4(c)]. First, an agent i is selected with uniform probability (*mutation rate, pmut*). Next, one point k is selected with uniform probability from $\{1,\ldots,17\}$. Finally, a new weight value $w_i^k(t+1)$ for next week $t+1$ at the point k is randomly selected with uniform probability from $\{\pm 3, \pm 1, \pm 0.5, \pm 0.1, 0\}$

$$\begin{aligned} w_i^k(t+1) = &\text{randomly selected from}\\ &\times \{\pm 3, \pm 1, \pm 0.5, \pm 0.1, 0\},\\ &\begin{pmatrix} i: \text{ selected with probability } pmut, \\ k: \text{randomly selected from } \{1,\ldots,17\}. \end{pmatrix}.\end{aligned} \tag{22}$$

This mutation operator works like the independent change of each agent's prediction method.

After the adaptation step, this week ends and the model proceeds to the next week's perception step.

F. Simulation Methods

Using our model, we carried out extrapolation simulations about two periods, 1988–1993 and 1994–1995 (Table V). These two periods were selected because there were exchange rate bubbles in these periods, the dollar appreciation bubble in 1990 and the yen appreciation bubble in 1995.

About each data set, we generated 100 simulation paths by repeating the following procedure a hundred times.

1) *Initialization:* In the initial population, a hundred agents have weights that were generated randomly

$$w_i^k(0): \text{randomly generated from } \{\pm 3, \pm 1, \pm 0.5, \pm 0.1, 0\}.$$

Each agent's position of both the dollar asset and yen asset are zero

$$q_i(0) = 0, \quad Q_i(0) = 0.$$

2) *Training Period:* We trained the agents in our model using the 17 real-world data items (the external and internal data) in Table IV and the actual rate data in this period. During the training period, we skipped the rate determination step. In the adaptation step, the fitness function for the GA was the cumulative absolute value of the difference between the forecast mean $\mathbf{E}_i[\Delta S(t)]$ of each agent and the actual rate change $\Delta S(t)$ instead of using (19)

$$\text{Fitness of } \mathbf{w}_i(t): F(\mathbf{w}_i(t)) = -\sum_{\tau=0}^{t} \left|\mathbf{E}_i[\Delta S(\tau)] - \Delta S(\tau)\right|.$$

Each weekly data item in the training period was used repeatedly two hundred times.

In this study, we used the following GA parameter set $\{pcross = 0.3, pmut = 0.003, G = 0.8\}$. We carried out out-of-sample forecast tests under various parameter sets in our preceding study [12]. As a result, this parameter set had the smallest forecast errors.

3) *Forecast Period:* We conducted the extrapolation simulations for the out-of-sample period. In the forecast period, our model forecasted the rates in the rate determination step using only the external data (1–14 in Table IV). The internal data (15–17 in Table IV) in the perception step and the fitness value [see (19)] in the adaptation step were calculated on the basis of the rates that were generated by our model in the rate determination step.

G. Overview of Simulation Results

About each data set, the 100 simulation paths were classified into five groups in Table VI. If a simulation path does not exceed the starting value of yen–dollar rate ±15%, it belongs to the *sideway* group. If a simulation path rises (dropped) over 15% and finally returns to the range, the starting value ±15%, it is classified into the *strong dollar bubble* group (*strong yen bubble* group). If a simulation path rises (dropped) over 15% and does not return, it is classified into *strong dollar* group (*strong yen* group).

We analyzed the simulation results and found the following points.

First, our model was capable of generating the characteristics of actual time series. The first reason for this is that the percentages of groups that contained the movement of the actual path

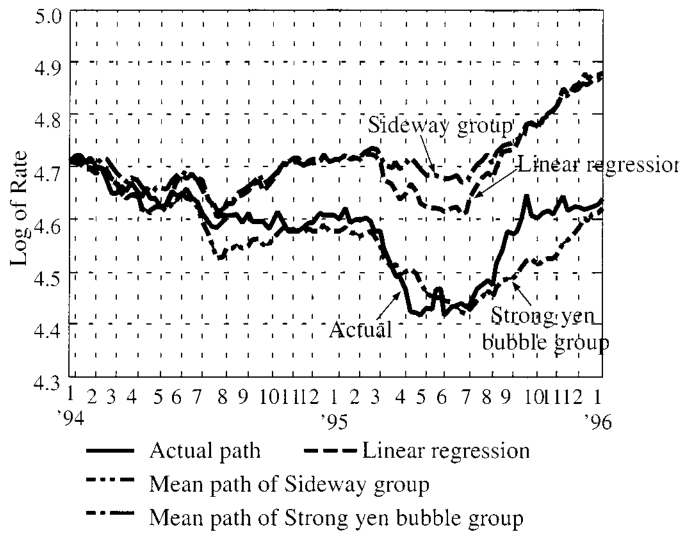

Fig. 5. Rate movement. Atual rate path, path extracted by linear regression, mean path of the sideway group simulation, and mean path of the strong yen bubble group simulation in the data set 2 are shown.

were the highest or the second highest among the five groups (Table VI). The second reason is that the percentages of bubble groups having the same direction as the actual path were much higher than those of bubble groups having the opposite direction, i.e., our model could replicate the direction of actual rate movement. The third reason is that the bubbles of the simulation paths occurred in the same year (1990 in the data set 1 and 1995 in the data set 2) as the bubbles in the actual rate path, i.e., our model could also replicate the timing of the bubbles.

Second, we found that the rate bubbles were caused by the endogenous reasons, the trend following behavior of agents and the agents' sensitivity to external news. In order to analyze the effect of exogenous factors on the rate bubbles, we carried out linear regression only using the same external data (no. 1–14 in Table IV) as our model. As shown by Fig. 5, the path extracted by linear regression moves in a way similar to the mean path of the sideway group, i.e., the linear regression paths did not have the rate bubbles; the rate bubbles seems to be caused by other reasons than exogenous factors. Then, we investigated the conditions that cause the bubble by comparing the market averages of the weights of the 17 data in the bubble group paths with those in the other group paths about the data set 2.[7] First, we chose the four external data that have the largest absolute values of the market averages and we compared the time variances of these data in the bubbles group with those in the other groups (Table VIIa). The result is that the variances of the bubble group are significantly larger than those of the other groups. Namely, one of the conditions of the bubble is that the interpretations of the external data in the market change flexibly from one period to another period. We also compared the time average of the weights of the internal data in the bubble group with those in the other groups (Table VIIb). The result is that the averages of the bubble group are positive, whereas those in the other groups are negative and that the differences are significant, i.e., the agents

[7]We found similar results about data set 1 [12].

TABLE VII
COMPARISONS OF FACTORS' WEIGHTS. (a) FUNDAMENTAL FACTORS: COMPARISON OF TIME VARIANCE. (b) TREND FACTORS: COMPARISON OF TIME AVERAGE

	Price	Interest	Intervention	Announcement
Bubble	1.279	1.210	0.759	0.923
The others	1.152	1.077	0.413	0.336

(a)

	Short-term Trend 1	Long-term Trend
Bubble	0.105	0.113
The others	−0.102	−0.229

(b)

All differences are significant at the 99.9% level.

forecasting that the recent chart trend will continue (the bandwagon expectations) is also a condition of the bubble.

V. Analysis of Emergent Phenomena

In this section, we analyzed simulation results in order to examine the emergent phenomena of markets. First, we outlined the simulation results and defined the idea, the phase transition of forecast variety. Second, we identified the mechanism of phase transition based on the analysis of computer simulation. Finally, we described our investigation of three emergent phenomena by the phase transition of forecast variety.

In the following sections, we analyzed five simulation paths that are selected randomly from the strong yen bubble group simulation using the data set 2. These five simulation paths occupy 20% of the strong yen bubble group because there are 25 simulation paths. We illustrate the results of the above analysis considering one typical path. However, the pattern of these results are common among the selected five paths and we found similar results about the simulation using the data set 1.

A. *Phase Transition of Forecasts Variety*

First, we outline the simulation results and found the phase transition of forecast variety.

Each simulated path in the strong yen bubble group is divided into two periods: a highly fluctuated period and a low fluctuated period. The simulated rate moved flat from March 1994 to December 1994, while the rate drop quickly and then rose dramatically from January 1995 to December 1995. Because these two periods have quite different characteristics, we called these qualitatively distinct periods *phases*: the low fluctuated period is defined as a *flat phase*, while the highly fluctuated period is defined as a *bubble phase*.

Let us begin comparison of the characteristics of these phases. Fig. 6 shows the percentage of agents who forecast a rise of the dollar and that of agents who forecast a drop of the dollar in the form of four-week averages. In the flat phase, agents' forecasts distribute symmetrically around the last week's rate. In other words, the variety of forecasts is rich because there are forecasts in both sides. In the bubble phase, agents' forecasts lean to one side, i.e., the variety of forecasts is poor because most agents have the same forecasts.

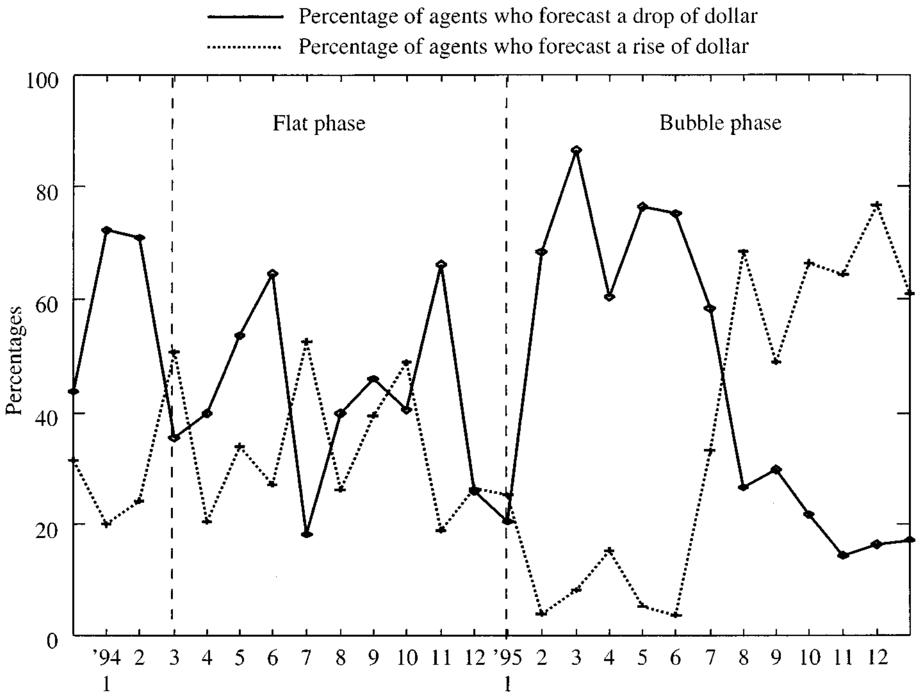

Fig. 6. Percentages of agents' forecasts.

TABLE VIII
RESULTS OF FACTOR ANALYSIS

	Econometrics category		News category		Trend category	
	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
1	Economic activities -0.5256	Trade 0.4885	Announcement 0.497	Intervention -0.4132	Short-term Trend 1 0.5628	Long-term Trend 0.4938
2	Price 0.5009	Interest -0.3578	Employment 0.3719	Politics 0.3751	Stock -0.4688	Stock 0.2283

For each factor, the two data with the largest loading are shown.

In the flat phase, the amounts of supply and demand are balanced, so the trading amounts are larger at the equilibrium. Supply and demand tend to meet around the last week's rate because there are sufficient amounts of supply and demand around the last week's rate. Hence, the rate fluctuation is smaller in the flat phase.

By contrast, in the bubble phase, the amounts of supply and demand are one-sided, so the trading amounts are smaller at the equilibrium. Supply and demand tend to meet apart from the last week's rate because there are not sufficient amounts of opposite orders around the last week's rate. Hence, the rate fluctuation is larger in the bubble phase.

We define such drastic changes of the variety of agents' forecasts between the two phases as *phase transition of forecasts variety*. In Section V-B, we examined the mechanism of phase transition.

B. Mechanism of Phase Transition

In order to identify the mechanism of phase transition, we first classified the 17 data items in Table IV using factor analysis. Next, using this categorization, we examined temporal change of agents' opinions.

1) Classification of Data Weights: The 17 news data in Step 1 were, as a result of a factor analysis of dynamic patterns of their weights,[8] classified into six factors, which are shown in Table VIII. The matrix that is analyzed by factor analysis is a list of 12 weights[9] of 100 agents every ten weeks during the forecast period. Because this matrix includes the weight value in different weeks, it can represent the temporal change of weights.

The weights of economic activities and price data have the largest loading value of the first factor. We call the first factor the price monetary factor because these two data are used in the price monetary approach to econometrics. The second factor is related to trade and interest rate data, which are included in the portfolio balance approach in econometrics, so we call it the portfolio balance factor. The third factor is related to announcement and employment data, so we call it the announcement factor. The fourth factor is related to intervention and politics data, so we call it the politics factor. The fifth factor is related to short-term trends and stock data, so we call it the short-term factor. And the sixth factor is related to long-term trend data, so we call it the long-term factor.

We combined these six factors into three categories: the price monetary and portfolio balance factors are classified into an

[8]The proportion of explanation by these six factors is 67.0%.

[9]Five data in Table IV are discarded because they are always zero during the forecast period or both their market average and variance are so small that they have little influence on the rate change.

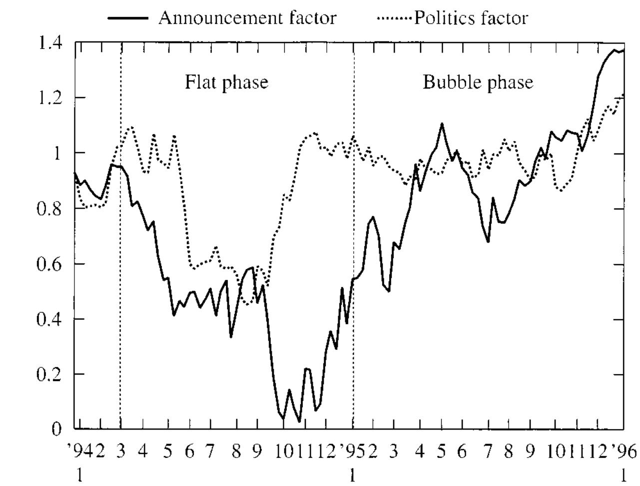

Fig. 7. Temporal change of news category.

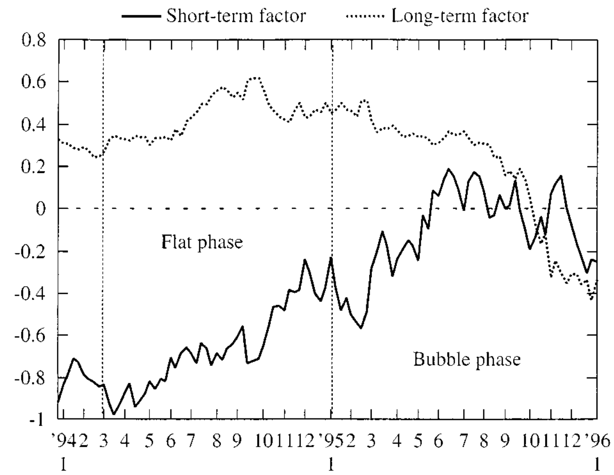

Fig. 8. Temporal change of trend categories.

econometrics category, the announcement and politics factors are classified into a news category, and the short-term and long-term factors are classified into a trend category.

In order to verify the result of this classification, we compared the categories acquired from the simulation with those acquired from the questionnaire data in the Section III-B. As a result, we extracted eight factors from the matrix with factor analysis.[10] The factors can be clearly classified into the three categories: econometrics, news, and trend categories. Thus, the actual dealers also classify data in the same way as the simulation results.

2) Dynamics of Categories: About each category, the following matters are examined: differences of its value between the flat phase and the bubble phase, temporal changes of agent groups, and distribution patterns in the market.

The weights of the econometric categories are relatively stable during the flat phase and bubble phase. However, their influence on rates is not so large because its absolute value is small. Only the portfolio balance factor has large absolute values of its market averages. Especially during the first half of the bubble phase, they are roughly twice as before. It is because the correlation coefficient between the trade data and rate changes is much larger during this period than before. This fact implies that the agents regarded the trade data as more important just before the bubble started because the trade data could explain the rate change better than other data.

Fig. 7 illustrates market averages of all agents' scores of the news category. These factors' weights rapidly increased just before the rate bubble started, i.e., they were not so paid attention in the flat phase. However, from the end of the flat phase to the bubble phase, they are recognized as important factors. The very strong market consensus is established since the end of the flat phase. Over 90% of agents have plus weights of these data in the bubble phase. The correlation coefficient between component data of the News category and rate changes is much larger than the other data from June 1994 to April 1995. The large correlation made market opinions about these data converge.

Fig. 8 illustrates market averages of all agents' scores of the short-term factor and long-term factor. These factors show distinctive dynamic patterns. About the short-term factor, the market average continuously rose to the plus until May 1995. After it fluctuated at the plus, it returned to the minus in December 1995. By contrast, concerning the long-term factor, its market average moves steadily until June 1995. Since July 1995, it drops to the lowest level. There is a *positive feedback* by both the short-term and long-term trend in the bubble phase. The positive feedback means that the plus weights of trend data make the continuing trends. However in the end of the bubble phase, this positive feedback weakened because the weight of the long-term data changed to the minus. Because of the large correlation before the bubble started, the weights of the trend category got larger and the positive feedback started. However, after the rate passed the lowest point in May 1995, the correlation coefficients became much smaller because the lack of opposite order lead the forecasts made by the trend data to the failure. Then, the positive feedback weakened.

In order to verify these results, we compared the dynamics of weights in the computer simulation with temporal changes of the rank of factors in the interview data (Tables I and II).[11] Both in the computer simulation and the interview data of the dealer X, the weight of the trade balance factor was large in the first half of the bubble phase. Both the dealer X and Y regarded news factors as important during the bubble. These interview data justify the simulation results that market opinions about the news category converged in the bubble phase. Both of the two dealers emphasized the importance of market sentiment during the bubble. The market sentiment can be considered as representation of market trend. Hence, their stress on the market sentiment supports the simulation results that the trend factors magnified rate fluctuation.

3) Mechanism: Let us summarize the main points that have been made in the above sections concerning the phase transition of rate dynamics.

1) In the flat phase, the weights of the news and trend categories are different among agents. In other words, there are variant opinions about these two categories. Hence, the variety of forecasts is rich. It leads to large

[10] Our preceding paper shows the results of comparison with the field data in detail [13].

[11] Our preceding paper shows the results of comparison with the interview data in detail [14].

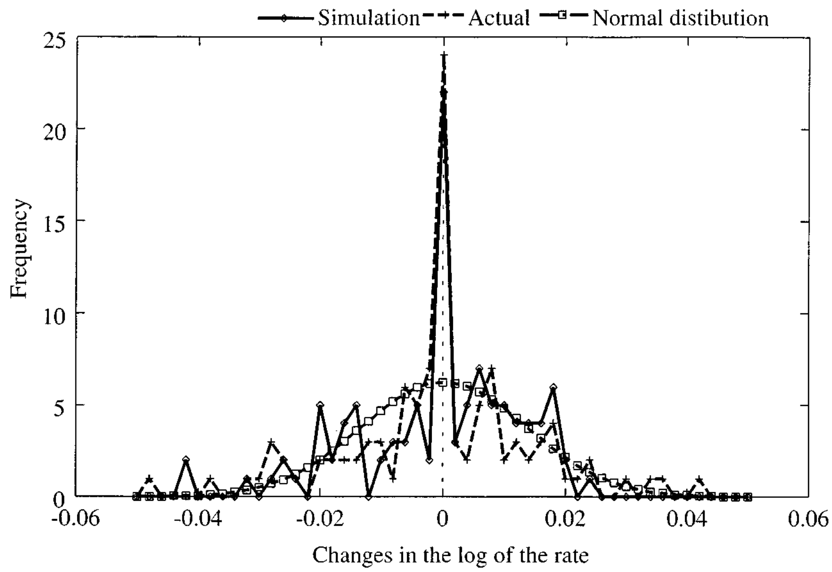

Fig. 9. Distribution of rate change. Gaussian distribution has the same mean and variance as the distribution of actual rate changes.

trading amounts and small rate fluctuation. Opinions about the econometrics category are stable and common in the market, but their influence is not so large in these periods.

2) In the latter half of the flat phase, from summer 1994, the trade, announcement, and politics data appeared frequently. Then, many agents focused on these data because their correlation to the rate change is large.
3) Opinions about these data converged in the market. Moreover, agents believed that the short-term and long-term trend will have continued. This belief made the trend further. Because of such positive feedback, the bubble phase started. In the bubble phase, the variety of forecasts is poor. It leads to small trading amounts and large rate fluctuation.
4) In May 1995, almost all forecasts in the market converged. Because there is no opposite order in the market, the downward trend vanished. Then the trend reversed and the bubble collapsed.
5) After the rate passed the lowest point in May 1995, the correlation coefficients between the trend data and the rate change became much smaller. Then, the weight of the long-term data became negative and the positive feedback was weakened. Finally, the bubble phase ended.

C. Three Emergent Phenomena

Based on the idea, phase transition of forecast variety, we analyzed the following emergent phenomena in markets by the artificial market approach: peaked and fat-tailed distribution of rate changes, negative correlation between trading amounts and rate fluctuation, and contrary opinions phenomena.

1) Peaked and Fat-Tailed Distribution: The distribution of rate changes is different from Gaussian distribution, i.e., exchange rate changes have peaked fat-tailed (i.e., leptokurtosis) distributions [15]. Moreover, many statistical studies also reveal that there is indeed evidence of autocorrelation of rate variance.

TABLE IX
KURTOSIS

Actual	Simulation	Normal distribution
0.564	0.477	0.000

Kurtosis of normal distribution is a theoretical value.

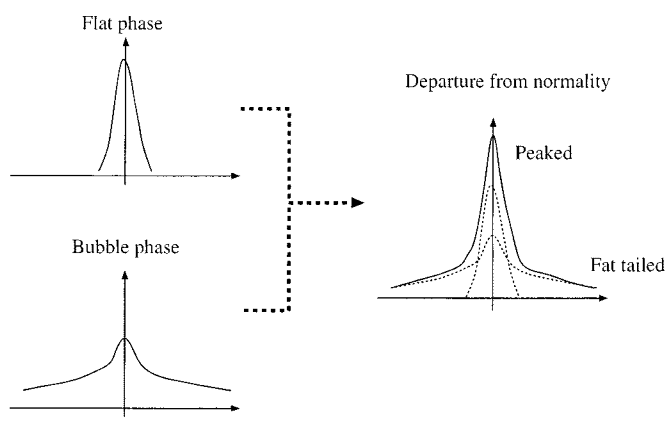

Fig. 10. Mechanism of peaked and fat-tailed distribution.

TABLE X
COMPARISON BETWEEN FLAT AND BUBBLE PHASE

	Flat phase		Bubble phase
Rate variance	1.39×10^{-4}	$<$	2.25×10^{-4}
Trading volume	0.745	$>$	0.549

Both differences between the two phases are significant at the 10% level.

The rate changes in the bubble group simulation also have peaked fat-tailed distributions like the actual rate (Fig. 9). In fact, the kurtosis of the simulated rate changes is near that of the actual rate changes (Table IX).

The mechanism of such leptokurtosis (peaked and fat-tailed distributions) of rate changes can be explained by the idea, the phase transition (Fig. 10). The rate changes in the bubble phase are larger than those in the flat phase. Namely, the distribution

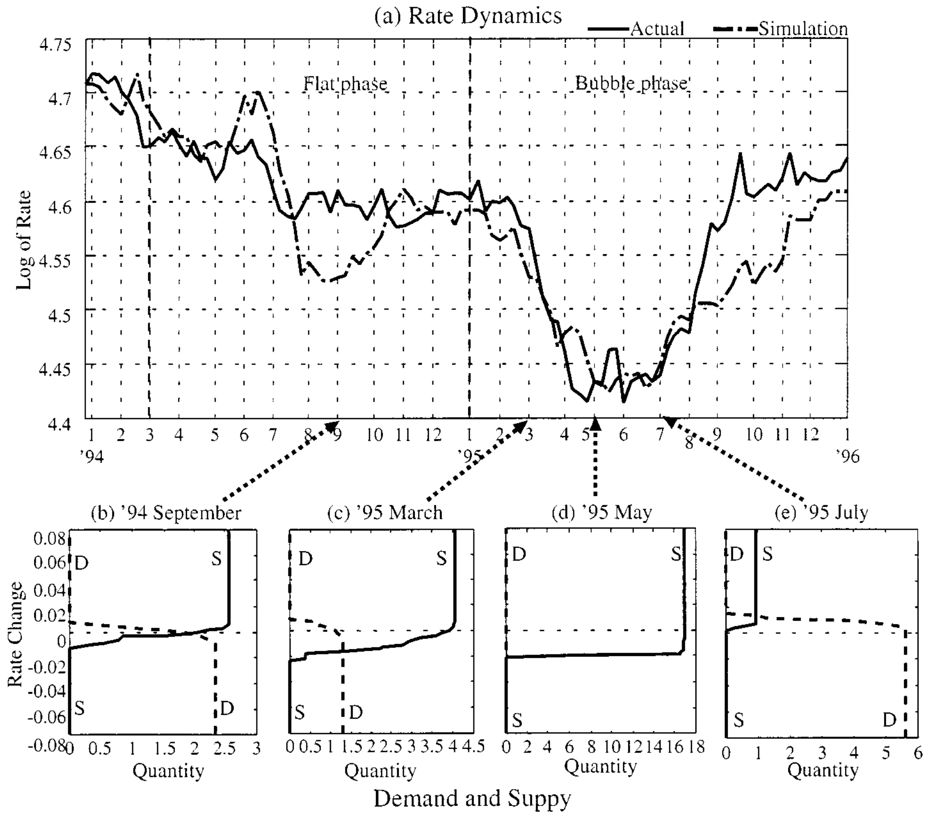

Fig. 11. Supply and demand in the typical simulation path.

of the rate changes in the bubble phase has a large variance, while that in the flat phase has a small variance. Because of the combination of these two distributions, the distribution of the rate changes during the whole periods is peaked and fat-tailed.

Moreover, there was positive autocorrelation (0.4469, $P < 0.05$) of absolute value of rate changes in the simulation as well as in the actual path from 1994 to 1995 (0.1891, $P < 0.05$). The mechanism of the autocorrelation of rate fluctuation can be explained in the same way. As Table X shows, rate variance in the bubble phase is significantly larger than that in the flat phase. Thus, in the bubble phase, large rate changes tend to be followed by large changes. In the flat phase, small changes tend to be followed by small changes. Hence, the rate variance shows the autocorrelation.

2) Volume and Fluctuation: As the empirical knowledge of dealers, it is said that there is negative correlation between trading volume and rate fluctuation [16]. Namely, when the rate fluctuates more, the volume is smaller; when the rate moves flat, the volume is larger. The interview data also show that there is a negative correlation between the transaction amount and the width of the rate fluctuations. For example, in the period V of the dealer Y (Table II), he said that the trading volume was very small when the yen–dollar rate decreased quickly: "There was sometimes no transaction when the rate moves quickly." The econometric theory of Tauchen and Pitts [17] says that the correlation between trading volume and price variance should be negative when volume is determine by the number of active traders.

For the typical simulation path mentioned above, we calculated the correlation between the absolute values of the rate fluctuation and the transaction amounts and obtained -0.280. This shows that there is significant negative correlation between the two.

This negative correlation can be explained by the phase transition. In the flat phase, as Fig. 6 indicates, about a half of the agents forecast changes in one direction and the other half forecast changes in the other direction. Then, the supply and demand are balanced near the last week's rate, as Fig. 11(b) shows. Thus, as Table X shows, the artificial market had smaller rate variance and larger trading volume in the flat phase.

By contrast, in the bubble phase, as Fig. 6 indicates, many (but not all) of the agents forecast changes in one direction. Then, the supply and demand are one-sided, as Fig. 11(c) and (e) show. Thus, as Table X shows, the artificial market had larger rate variance and smaller trading volume in the bubble phase.

In summary, the combination of relations between trading volume and rate variance in the two phases yields the negative correlation.

3) Contrary Opinions Phenomena: Many dealers and their books say, "If almost all dealers have the same opinion, the contrary opinion will win." In fact, questionnaire data sometimes show that convergence of the dealers' forecasts leads to an unexpected result of the rate move.

The typical simulation path also shows the contrary opinions phenomena. In May 1995, almost all the agents' forecasts converge to the same forecast of the same direction—the dollar depreciation. As Fig. 11(d) indicates, there was no demand of dollars in the artificial market at that point. Because no transactions occurred, the rate did not drop further. In summary, the excessive convergence of agents' forecasts caused the contrary opinions phenomena.

The interview data also supported this result. In period VII of dealer X (Table I), he missed the quick trend change until July 1995: "Until July almost all dealers did not forecast the rate would return to the level of 100 yen by this year. It was unexpected." This is a good example of the contrary opinions phenomena. These interview data justify the simulation results that the actual rate did not move in that direction because almost all dealers' forecasts converged to the same forecast of one direction. In fact, the dealer X said, "According to my experience, when 90% or 95% of all dealers have the same opinion, the rate reaches the peak."

VI. Conclusion

We analyzed the following emergent phenomena in markets by the artificial market approach: peaked and fat-tailed distribution of rate changes, negative correlation between trading amounts and rate fluctuation, and contrary opinions phenomena.

In order to analyze the emergent properties, the phase transition of agents' forecast variety in the simulated paths was examined. Each simulated path was divided into two phases: highly fluctuated periods (bubble phases) and low fluctuated periods (flat phases). In the flat phase, a large variety of forecasts lead to large trading amounts and small rate fluctuation. By contrast, in the bubble phase, a small variety of forecasts lead to small trading amounts and large rate fluctuation. Then we classified factors into the three categories: econometrics, news, and trend category. We investigated the dynamics of agents' opinions about each category. As a result, the following mechanism of the phase transition was proposed: convergence of opinions about news factors and trade factors and positive feedback by trend factors caused phase transition from the flat phase to the bubble phase.

Based on the idea "phase transition of forecast variety," we explained the three emergent phenomena. Peaked and fat-tailed distribution of rate changes was explained by the combination of flat tailed distribution in the bubble phase and peaked distribution in the flat phase. Negative correlation between trading volume and rate fluctuation was explained by the their negative relation in two phases. Contrary opinions phenomena are explained by lack of opposite orders.

Moreover the results were compared with the field data of the interviews and questionnaires in the three points: classification of factors, dynamics of weights, and mechanisms of emergent properties. As a result, the field data supported the simulation results.

This study, therefore, shows that the artificial market approach can be effective not only for qualitative analysis, but also for quantitative analysis of actual economic systems. A further direction of this study will be to apply artificial market models to the real-world problems such as decision support systems [18].

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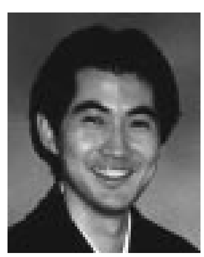

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